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Conference Report

Equation of State for Dense Matter with a QCD Phase Transition

Sanjin Benić †

Physics Department, Faculty of Science, University of Zagreb, Zagreb 10000, Croatia; sanjinb@phy.hr
† Current address: Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan.

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Abstract: We construct a dense matter equation of state (EoS) starting from a hadronic density dependent relativistic mean-field model with a DD2 parametrization including the excluded volume corrections at low densities. The high density part is given by a Nambu–Jona–Lasinio (NJL) model with multi-quark interactions. This EoS is characterized by increasing speed of sound below and above the phase transition region. The first order transition region has a large latent heat leaving a distinctive signature in the mass-radii relations in terms of twin stars.

Keywords: neutron star; equation of state; phase transition; quark matter

1. Introduction

We know from finite temperature lattice Quantum Chromodynamics (QCD) that the transition from hadrons to quarks and gluons is a crossover [1]. At finite chemical potential and especially at small temperatures and large chemical potentials, appropriate for neutron stars, the situation is completely different—first principle lattice calculations have a sign problem and so it is an open question whether the transition remains a crossover or if it becomes a first order with the corresponding critical point somewhere in the temperature-chemical potential plane.

This question could hopefully be resolved by colliding heavy ions, but one should be aware of various uncertainties related to the system size and lifetime as well as the possibility that it just may not be possible to reach sufficiently high densities where we would find deconfined quark matter. By contrast, in the case of compact stars, we have a huge system which is long lived and potentially very dense.

As on the theoretical side we still cannot get a first principle information on finite density QCD, in this work, we will assume some model equation of state (EoS). Additionally, we assume the EoS has a first order transition from hadron to quarks at finite density. Our purpose is then to provide a systematic model study of the EoS and the mass-radii (M–R) relations of compact stars which can be used to verify this scenario with simultaneous observations of masses and radii.

2. Twin Stars

Compact stars can be divided into families, such as white dwarfs, neutron stars and sometimes hybrid stars with a quark core and a nuclear mantle as a third family is considered. Purely quark stars is another example of a third family, but we do not consider such possibility as it involves more assumptions. In the case of third families, it is possible to get the so-called twin stars phenomena ([2,3]), where the neutron and the hybrid stars would have same mass but different radii. The main interest here is that, to get twin stars, we need strong first order transition and so measuring twins would provide evidence for first order transitions in QCD.

Recently, astrophysical observations pointed out the existence of several compact stars with masses at $2M_{\odot}$ [4–6], pressing a clear understanding of the interior structure in terms of its equation of state (EoS) at very high density. The purpose of our work in [7] was to revisit the twin stars scenario with the goal of understanding qualitatively and quantitatively (within a model) systematics of twin stars at $2M_\odot$. Some more recent works include Bayesian analysis [8,9], relation to the CEP [10], impact of rotation [11], twins in protoneutron stars [12] and a triplet of stars with same mass and different radii [13] (see Alford et al. [14] and Zacchi et al. [15] for classification studies).

3. Equation of state

How should a strong first order transition be engineered? We first calculate the hadron and the quark EoS separately and we use Maxwell construction—this is always first order by construction. To get strong first order, the two EoS must have different slopes on the $p - \mu_B$ plot.

For hadrons, we use the density dependent relativistic mean field model with the DD2 parameterization [16]. We can use this EoS (or any kind of hadron EoS, for that matter) until some density where the quarks from different baryons start to overlap. Increasing the number of baryons further is disfavored because of Pauli blocking effects between quarks. We mimic this effect by taking into account the finite size of hadrons via the excluded volume approach (see, e.g., [17,18] and references therein). We introduce a quantity Φ

$$\Phi = \frac{V_{\mathrm{av}}}{V} = 1 - v \sum_{i=n,p} n_i \,, \tag{1}$$

which is the ratio of the available volume V_{av} for hadrons and the total volume of the system. This can also be written in terms of the excluded volume parameter denoted by small v and the number densities, as in the second line of Equation (1). By looking at single particle energies,

$$E_i = \mu_i - V_i - \frac{v}{\Phi} \sum_{j=p,n} p_j \,, \tag{2}$$

where μ_i is the chemical potential and V_i the vector mean field, we see that v acts in a similar way as the vector mean field and so an EoS should become more stiff with excluded volume. In this work, we consider two flavor case so that the hyperon problem is avoided by a phase transition to quark matter.

For quarks, we use the Nambu–Jona–Lasinio (NJL) model with scalar and vector interactions and we add the higher order interactions in the scalar

$$\mathcal{L}_{\mathrm{scal}} = \frac{g_{20}}{\Lambda^2}(\bar{q}q)^2 + \frac{g_{40}}{\Lambda^8}(\bar{q}q)^4 \,, \tag{3}$$

and in the vector channel

$$\mathcal{L}_{\mathrm{vec}} = \frac{g_{02}}{\Lambda^2}(\bar{q}\gamma^\mu q)^2 + \frac{g_{04}}{\Lambda^8}(\bar{q}\gamma^\mu q)^4 \,. \tag{4}$$

Refer to Benić [7] and Benić et al. [19] for more details. It turns out that higher order scalar interactions do not change the chiral transition much. However, the higher order vector interactions should be more and more important as we increase the density in terms of stiffness of the EoS [7,19].

4. Impact of the Choice of the Equation of State on the Mass-Radii Relations

Now, we discuss some EoS systematics where we control the repulsion in the hadron and the quark EoS. Characteristic EoS are shown on Figure 1. If there is no repulsion (Figure 1a) in both phases, then this particular model cannot pass the $2M_\odot$ constraint. Typically, one introduces some quark repulsions as in Figure 1b. This delays the onset of quarks and at the same time reduces the latent heat. Because quark EoS becomes stiffer we can get to $2M_\odot$ and typically at first we have hybrid stars. Increasing the quark repulsions more and more it becomes hard to get the hadron and the quark EoS to cross. This is the familiar problem of vector interactions in the quark phase [20].

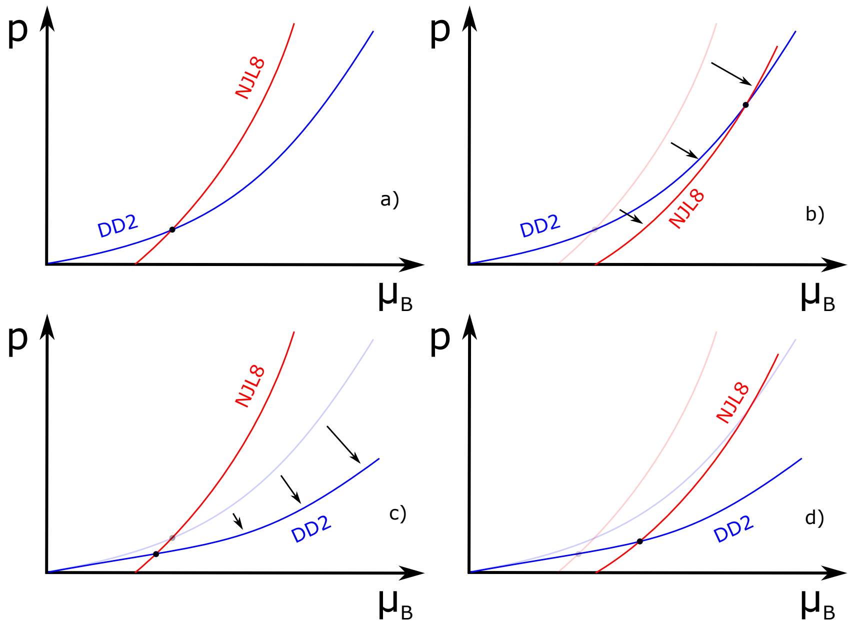

Figure 1. Generic systematics of the different possibilities of the hybrid Equation of State (EoS) within the density dependent relativistic mean field hadronic model with the DD2 parametrization and Nambu-Jona-Lasinio model with 8-quark interactions (DD2-NJL8) hybrid EoS model in terms of the absence (soft EoS) or presence (stiff EoS) of the repulsive interactions: (**a**) soft-to-soft; (**b**) soft-to-stiff; (**c**) stiff-to-soft; and (**d**) stiff-to-stiff EoS.

On the other hand, if we have some repulsions in hadron phase and no repulsion in quark phase (see Figure 1c), then we get an interesting situation that the onset of quarks is lowered and the latent heat becomes increased. Because we put more repulsions, we can pass the $2M_\odot$ limit and typically we get neutron stars. In other words, with the simultaneous combination of the increase of the repulsions in the hadron phase, large latent heat and a soft quark phase, hybrid stars quickly become unstable.

The most interesting situation is when we turn on repulsions in the hadron and the quark phase, as shown in Figure 1d. Then, we can explore also hybrid stars with very stiff quark matter EoS which was not possible previously. Because both the quark and the hadron phase are now stiff, we can easily pass the $2M_\odot$ constraint. In particular, in this window of parameters, we can get twin stars because there is also considerable latent heat.

Figure 2 summarizes the previous discussion. Without repulsions in either of the phases. we can only have the conventional hybrid stars within this particular model. Adding repulsions in either phase hybrid stars turn to neutron stars. If there are some finite quark and hadron repulsions, we can get twin stars.

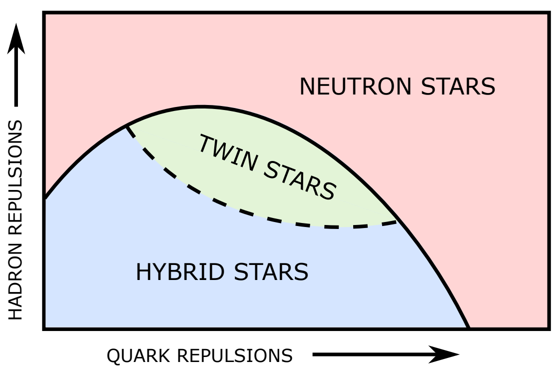

Figure 2. Summary of impact of repulsions in the hadron and in the quark phase on the M–R characteristics.

5. Results

Now, we show some selected results of model calculations performed in [7]. For a density functional approach to such class of EoS, see [21]. Further astrophysical implications are discussed in [22–24]. In the left panel of Figure 3, we show the pressure and the speed of sound as a function of the energy density. First, let us appreciate that the DD2 EoS with the excluded volume is significantly stiffer than the standard DD2 EoS. After the phase transition, we change the stiffness of quark matter by the 8-quark vector coupling. We see that its effect on the stiffness becomes more and more significant by increasing the density. It is essentially the reason why the transition can achieve considerable latent heat. In other words, changing the higher order vector coupling controls the high density part of the EoS while it does not influence the phase transition much. The latter is completely controlled by the excluded volume of the hadron EoS.

In the right panel, we show the M–R relations using this hybrid EoS model. First, because of the very stiff hadron EoS, we get stars with large radii: of the order of 14.5 km or even 15 km. Second, because the latent heat is also quite large, we can get the twin stars: the radii difference is around a 0.5 km or 1 km depending on the model details.

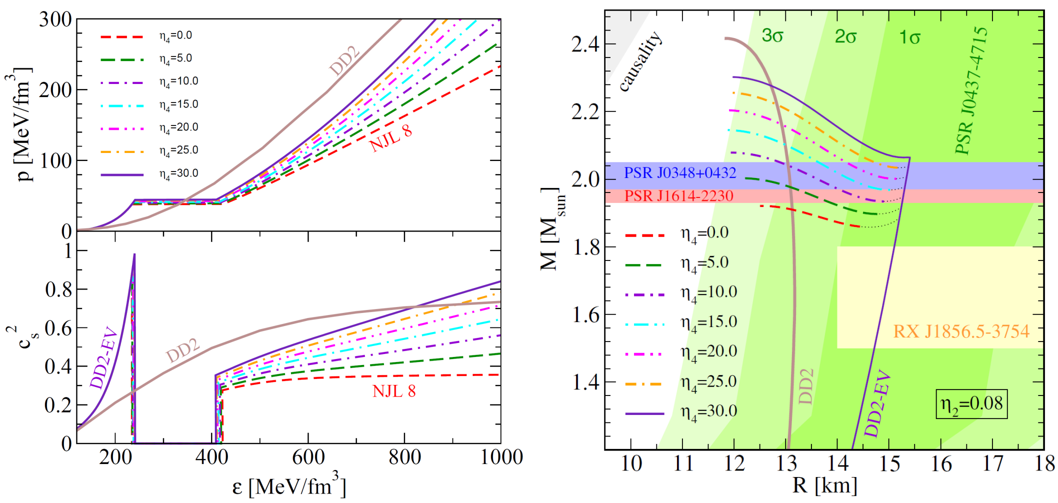

Figure 3. (**Left**) Equation of state (EoS) and speed of sound as a function of the energy density; and (**Right**) M–R relation for the corresponding EoS. Different curves correspond to a variation of the high density quark EoS in terms of a NJL 8-vector coupling parameter η_4. Figure from [7].

6. Conclusions

To conclude, we showed that, to get twin stars at $2M_\odot$, we need strong repulsions in nuclear and in quark matter. Measurements of twin stars has strong potential, as it could exclude the approaches with stiffening EoS in the transition regions [25–27]. On the other hand, the two EoS scenario (quark stars and neutron stars) cannot be excluded by radii measurements provided that the quark star has a large radii as well. In our case, twin stars at $2M_\odot$ require very stiff EoS already in the hadron phase, that is, below the hadron–quark transition. The EoS at these densities controls the radius of the star, and so in our calculation we get big, dilute stars. Measurements of very small radii may disfavor our scenario.

Finally, we make a couple of remarks on the possible caveats. Provided we measure twin stars we can say transition in beta equilibrium is strong first order but this does not mean transition in symmetric matter is also first order. Some effective models suggest chiral transition should become more strong in symmetric matter [28] but in QCD this question does not have a definite answer yet. Additionally, to get twins, we need strong first order but this is not completely correct. It is in fact sufficient for the EoS to just be very soft in a wide region of densities [29,30].

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References

1. Bhattacharya, T.; Buchoff, M.I.; Christ, N.H.; Ding, H.T.; Gupta, R.; Jung, C.; Karsch, F.; Lin, Z.; Mawhinney, R.D.; McGlynn, G.; et al. QCD Phase Transition with Chiral Quarks and Physical Quark Masses. *Phys. Rev. Lett.* **2014**, *113*, 082001.
2. Glendenning, N.K.; Kettner, C. Nonidentical neutron star twins. *Astron. Astrophys.* **2000**, *353*, L9.
3. Schertler, K.; Greiner, C.; Schaffner-Bielich, J.; Thoma, M.H. Quark phases in neutron stars and a 'third family' of compact stars as a signature for phase transitions. *Nucl. Phys. A* **2000**, *677*, 463–490.
4. Demorest, P.; Pennucci, T.; Ransom, S.; Roberts, M.; Hessels, J. Shapiro Delay Measurement of A Two Solar Mass Neutron Star. *Nature* **2010**, *467*, 1081–1083.
5. Antoniadis, J.; Freire, P.C.; Wex, N.; Tauris, T.M.; Lynch, R.S.; van Kerkwijk, M.H.; Kramer, M.; Bassa, C.; Dhillon, V.S.; Driebe, T.; et al. A Massive Pulsar in a Compact Relativistic Binary. *Science* **2013**, *340*, 1233232.
6. Fonseca, E.; Pennucci, T.T.; Ellis, J.A.; Stairs, I.H.; Nice, D.J.; Ransom, S.M.; Demorest, P.B.; Arzoumanian, Z.; Crowter, K.; Dolch, T.; et al. The NANOGrav Nine-year Data Set: Mass and Geometric Measurements of Binary Millisecond Pulsars. *Astrophys. J.* **2016**, *832*, 167.
7. Benić, S.; Blaschke, D.; Alvarez-Castillo, D.E.; Fischer, T.; Typel, S. A new quark-hadron hybrid equation of state for astrophysics—I. High-mass twin compact stars. *Astron. Astrophys.* **2015**, *577*, A40.
8. Alvarez-Castillo, D.; Ayriyan, A.; Benic, S.; Blaschke, D.; Grigorian, H.; Typel, S. New class of hybrid EoS and Bayesian M-R data analysis. *Eur. Phys. J. A* **2016**, *52*, 69.
9. Alvarez-Castillo, D.E.; Blaschke, D. Proving the CEP with compact stars? *arXiv* **2013**, arXiv:1304.7758.
10. Alvarez-Castillo, D.; Benic, S.; Blaschke, D.; Han, S.; Typel, S. Neutron star mass limit at $2M_\odot$ supports the existence of a CEP. *Eur. Phys. J. A* **2016**, *52*, 232.
11. Bejger, M.; Blaschke, D.; Haensel, P.; Zdunik, J.L.; Fortin, M. Consequences of a strong phase transition in the dense matter equation of state for the rotational evolution of neutron stars. *Astron. Astrophys.* **2017**, *600*, A39.
12. Hempel, M.; Heinimann, O.; Yudin, A.; Iosilevskiy, I.; Liebendörfer, M.; Thielemann, F.K. Hot third family of compact stars and the possibility of core-collapse supernova explosions. *Phys. Rev. D* **2016**, 94, 103001.
13. Alford, M.; Sedrakian, A. Compact stars with sequential QCD phase transitions. *Phys. Rev. Lett.* **2017**, *119*, 161104.
14. Alford, M.G.; Han, S.; Prakash, M. Generic conditions for stable hybrid stars. *Phys. Rev. D* **2013**, *88*, 083013.
15. Zacchi, A.; Tolos, L.; Schaffner-Bielich, J. Twin Stars within the SU(3) Chiral Quark-Meson Model. *Phys. Rev. D* **2017**, *95*, 103008.
16. Typel, S.; Wolter, H.H. Relativistic mean field calculations with density dependent meson nucleon coupling. *Nucl. Phys. A* **1999**, *656*, 331–364.
17. Typel, S. Variations on the excluded-volume mechanism. *Eur. Phys. J. A* **2016**, *52*, 16.
18. Typel, S.; Blaschke, D. Phenomenological Equation of State of Strongly Interacting Matter with First-Order Phase Transitions and Critical Points. *Universe* **2018**, *4*, 32.
19. Benic, S. Heavy hybrid stars from multi-quark interactions. *Eur. Phys. J. A* **2014**, *50*, 111.
20. Klähn, T.; Łastowiecki, R.; Blaschke, D. Implications of the measurement of pulsars with two solar masses for quark matter in compact stars and heavy-ion collisions: A Nambu-Jona-Lasinio model case study. *Phys. Rev. D* **2013**, *88*, 085001.
21. Kaltenborn, M.A.R.; Bastian, N.U.F.; Blaschke, D.B. Quark-nuclear hybrid star equation of state with excluded volume effects. *Phys. Rev. D* **2017**, *96*, 056024.
22. Alvarez-Castillo, D.E.; Bejger, M.; Blaschke, D.; Haensel, P.; Zdunik, L. Energy bursts from deconfinement in high-mass twin stars. *arXiv* **2015**, arXiv:1506.08645.
23. Paschalidis, V.; Yagi, K.; Alvarez-Castillo, D.; Blaschke, D.B.; Sedrakian, A. Implications from GW170817 and I-Love-Q relations for relativistic hybrid stars. *arXiv* **2017**, arXiv:1712.00451.
24. Fischer, T.; Bastian, N.U.F.; Wu, M.R.; Typel, S.; Klähn, T.; Blaschke, D.B. High-density phase transition paves the way for supernova explosions of massive blue-supergiant stars. *arXiv* **2017**, arXiv:1712.08788.

25. Masuda, K.; Hatsuda, T.; Takatsuka, T. Hadron-Quark Crossover and Massive Hybrid Stars with Strangeness. *Astrophys. J.* **2013** , *764*, 12.
26. Alvarez-Castillo, D.E.; Benic, S.; Blaschke, D.; Lastowiecki, R. Crossover transition to quark matter in heavy hybrid stars. *Acta Phys. Polon. Supp.* **2014**, *7*, 203–208.
27. Kojo, T.; Powell, P.D.; Song, Y.; Baym, G. Phenomenological QCD equation of state for massive neutron stars. *Phys. Rev. D* **2015**, *91*, 045003.
28. Ueda, H.; Nakano, T.Z.; Ohnishi, A.; Ruggieri, M.; Sumiyoshi, K. QCD phase diagram at finite baryon and isospin chemical potentials in Polyakov loop extended quark meson model with vector interaction. *Phys. Rev. D* **2013**, *88*, 074006.
29. Alvarez-Castillo, D.E.; Blaschke, D. Mixed phase effects on high-mass twin stars. *Phys. Part. Nucl.* **2015**, *46*, 846–848.
30. Ayriyan, A.; Bastian, N.U.; Blaschke, D.; Grigorian, H.; Maslov, K.; Voskresensky, D.N. How robust is a third family of compact stars against pasta phase effects? *arXiv* **2017**, arXiv:1711.03926.