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Cosmological Constant in SUGRA Models with Degenerate Vacua

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Abstract: The extrapolation of couplings up to the Planck scale within the standard model (SM) indicates that the Higgs effective potential can have two almost degenerate vacua, which were predicted by the multiple point principle (MPP). The application of the MPP to $(N = 1)$ supergravity (SUGRA) implies that the SUGRA scalar potential of the hidden sector possesses at least two exactly degenerate minima. The first minimum is associated with the physical phase in which we live. In the second supersymmetric (SUSY) Minkowski vacuum, the local SUSY may be broken dynamically, inducing a tiny vacuum energy density. In this paper, we consider the no-scale-inspired SUGRA model in which the MPP conditions are fulfilled without any extra fine-tuning at the tree-level. Assuming that at high energies, the couplings in both phases are identical, one can estimate the dark energy density in these vacua. Using the two-loop renormalization group (RG) equations, we find that the measured value of the cosmological constant can be reproduced if the SUSY breaking scale M_S in the physical phase is of the order of 100 TeV. The scenario with the Planck scale SUSY breaking is also discussed.

Keywords: supergravity; cosmological constant; Higgs boson

1. Introduction

The discovery of the Higgs boson with a mass M_H around 125 GeV constitutes a crucial step towards our understanding of the mechanism of the electroweak (EW) symmetry breaking. It allows one to determine the parameters of the Higgs potential of the standard model (SM):

$$V_{eff}(H) = m^2(\phi)H^\dagger H + \lambda(\phi)(H^\dagger H)^2\,, \tag{1}$$

where H is a Higgs doublet and ϕ is the norm of the Higgs field, i.e., $\phi^2 = H^\dagger H$. The measured Higgs mass corresponds to a relatively small value of the Higgs quartic coupling at the electroweak (EW) scale, i.e., $\lambda \simeq 0.13$. Such a value of λ is rather close to the theoretical lower bound for it that comes from the vacuum stability constraint. Indeed, when the values of the Higgs quartic coupling at the EW scale are sufficiently small, $\lambda(\phi)$ decreases with increasing ϕ and may become negative at some intermediate scale. This can lead to either instability or metastability of the physical vacuum. The extrapolation of the SM couplings up to the Planck scale, M_P, using three-loop renormalization group equations (RGEs) results in (see, for example, [1]):

$$\lambda(M_P) = -0.0143 - 0.0066\left(\frac{M_t}{\text{GeV}} - 173.34\right) + 0.0018\left(\frac{\alpha_3(M_Z) - 0.1184}{0.0007}\right) + 0.0029\left(\frac{M_H}{\text{GeV}} - 125.15\right), \quad (2)$$

where M_t is the pole mass of the top quark, M_H is the pole mass of the Higgs boson, and $\alpha_3(M_Z)$ is the value of the strong gauge coupling at the Z mass. As follows from Equation (2), $\lambda(M_P)$ tends to be rather close to zero for any phenomenologically-acceptable values of M_t and $\alpha_3(M_Z)$. Moreover, for $M_t \leq 171\,\text{GeV}$, the value $\lambda(M_P)$ tends to be larger than zero so that $\lambda(\phi)$ remains positive at any intermediate scale below M_P. In this case, the physical vacuum is stable, and the parameters of the SM can be extrapolated all the way up to M_P without any inconsistency (for the accurate calculation of the upper bound on the pole mass of the top quark, which represents the vacuum stability constraint, see [2]). It is worth noting that near the Planck scale, the value of the beta-function of $\lambda(\phi)$, $\beta_\lambda = \dfrac{d\lambda(\phi)}{d\log\phi}$, is also quite small. Therefore, for $M_t \leq 171\,\text{GeV}$, the Higgs effective potential of the SM (1) may have two vacua, which are approximately degenerate. One of them is associated with the physical vacuum, where the vacuum expectation value (VEV) of the Higgs doublet $\langle H\rangle = v/\sqrt{2}$ ($v \simeq 246\,\text{GeV}$). In the other vacuum, $\langle H\rangle$ is somewhat close to the Planck scale.

The existence of such degenerate vacua was predicted by the so-called multiple point principle (MPP) [3,4]. The MPP postulates the coexistence in Nature of many phases allowed by a given theory. It corresponds to a special (multiple) point on the phase diagram of the theory where these phases meet. At the multiple point, the vacuum energy densities of these different phases are degenerate. Provided that several phases do exist at all, the article by Dvali [5] means a proof of the multiple point principle in so far as it declares inconsistent a truly false Lorentz invariant vacuum. When applied to the SM, the MPP implies that the Higgs effective potential (1) possesses two degenerate minima taken to be at the EW and Planck scales [4]. The position of the first (physical) vacuum is determined by the mass parameter in the effective potential (1). At high scales, the ϕ^4 term in Equation (1) strongly dominates the ϕ^2 term, which can therefore be ignored in the leading approximation. Then, the derivative of $V_{eff}(H)$ near the Planck scale takes the form:

$$\left.\frac{dV_{eff}(H)}{d\phi}\right|_{\phi=M_P} \approx (4\lambda(\phi) + \beta_\lambda)\,\phi^3\,. \quad (3)$$

From Equations (1) and (3), it follows that the degeneracy of the vacua at the EW and Planck scales can be achieved only if:

$$\lambda(M_P) \simeq 0\,, \qquad \beta_\lambda(M_P) \simeq 0\,. \quad (4)$$

When the Higgs self-coupling tends to zero at the Planck scale, $\beta_\lambda(M_P)$ vanishes only for a unique value of the top quark Yukawa coupling $h_t(M_P)$. As a consequence, using the renormalization group (RG) flow and conditions (4), one can compute the pole masses of the top quark and Higgs boson [4]. In 1995, it was shown [4], using two-loop renormalization group equations, that the MPP conditions (4) can be fulfilled if:

$$M_t = 173 \pm 5\,\text{GeV}\,, \qquad M_H = 135 \pm 9\,\text{GeV}\,. \quad (5)$$

The implementation of the MPP in the two Higgs doublet extension of the SM was considered in [6–11].

The successful predictions for the Higgs and top quarks masses (5) suggest that the MPP can be used to explain the tiny dark energy density spread all over the Universe (the cosmological constant ρ_Λ), which is responsible for its accelerated expansion. In previous articles [12–24], the implementation of the MPP in models based on local supersymmetry, supergravity (SUGRA), was

studied. In SUGRA models a huge degree of fine-tuning is required to ensure that the energy density in the physical vacuum is sufficiently small. The successful application of the MPP to $(N = 1)$ supergravity implies the existence of a vacuum in which the low-energy limit of the theory is described by a pure supersymmetric model in flat Minkowski space [12–24]. According to the MPP, the physical vacuum is the one in which we live, and this second vacuum must have the same vacuum energy density. Since the vacuum energy density of supersymmetric states in flat Minkowski space vanishes, the cosmological constant problem is thereby solved to first approximation. However, in the second vacuum, the dynamical breakdown of SUSY may take place, leading to an exponentially-suppressed value of the vacuum energy density. This energy density is then transferred to the physical vacuum by the assumed degeneracy [12–24]. The results of the numerical analysis performed in the previous works indicate that in the leading one-loop approximation, the observed value of the cosmological constant can be reproduced, even if the SM gauge couplings are almost identical in both vacua.

In this paper, we review the results of the implementation of the MPP in the $(N = 1)$ SUGRA models and examine the dependence of the dark energy density on the parameters more accurately, taking into account leading two-loop corrections. The paper is organized as follows: in the next section, the cosmological constant problem within the $N = 1$ supergravity (SUGRA) models is discussed; in Section 3, the no-scale inspired SUGRA model, which results in the degenerate vacua mentioned above, is considered; in Section 4, we estimate the value of the cosmological constant in such a scenario with a low SUSY breaking scale. The corresponding scenario with Planck scale SUSY breaking and its possible implications for Higgs phenomenology are studied in Section 5. Section 6 is reserved for our conclusions and outlook.

2. The Cosmological Constant Problem, the Multiple Point Principle, and No-Scale Supergravity

Astrophysical and cosmological observations indicate that the dark energy density spread all over the Universe $\rho_\Lambda \sim 10^{-123} M_P^4 \sim 10^{-55} M_Z^4$. On the other hand, much larger contributions to ρ_Λ should come from the EW symmetry breaking ($\sim 10^{-67} M_P^4$) and QCD condensates ($\sim 10^{-79} M_P^4$). The contribution of zero-modes is expected to push the vacuum energy density even higher up to $\sim M_P^4$, i.e.,

$$\rho_\Lambda \simeq \sum_{bosons} \frac{\omega_b}{2} - \sum_{fermions} \frac{\omega_f}{2} = \int_0^\Omega \left[\sum_b \sqrt{|\vec{k}|^2 + m_b^2} - \sum_f \sqrt{|\vec{k}|^2 + m_f^2} \right] \frac{d^3\vec{k}}{2(2\pi)^3} \sim -\Omega^4 \,, \tag{6}$$

where the m_b and m_f are the masses of bosons and fermions, while $\Omega \sim M_P$. Because of the enormous cancellation between different contributions to the cosmological constant, which is needed to keep ρ_Λ around its measured value, the smallness of ρ_Λ should be regarded as a fine-tuning problem.

The smallness of the cosmological constant could be related to an almost exact symmetry. However, none of the generalizations of the SM provide any satisfactory explanation for the smallness of this dark energy density. An exact global supersymmetry (SUSY) guarantees a value of zero for the vacuum energy density. Nevertheless, the non-observation of superpartners of quarks and leptons implies that SUSY is broken. Its breakdown induces a huge and positive contribution to the dark energy density, which is much larger than M_Z^4.

It is expected that at ultra-high energies, the SM is embedded in an underlying theory that provides a framework for the unification of all interactions, including gravity, such as supergravity (for some alternative ideas, see, for example, [25–27]). The full $(N = 1)$ SUGRA Lagrangian [28,29] is specified in terms of a real gauge-invariant Kähler function $G(\phi_M, \phi_M^*)$ and analytic gauge kinetic functions $f_a(\phi_M)$. These functions depend on the chiral superfields, ϕ_M. The functions $f_a(\phi_M)$ determine the gauge coupling constants $Ref_a(\phi_M) = 1/g_a^2$, where the index a represents different gauge groups. The Kähler function is a combination of two functions:

$$G(\phi_M, \phi_M^*) = K(\phi_M, \phi_M^*) + \ln |W(\phi_M)|^2 \,, \tag{7}$$

where $K(\phi_M, \phi_M^*)$ is the Kähler potential, while $W(\phi_M)$ is the superpotential of the SUGRA model under consideration. The second derivatives of the Kähler potential define the structure of the kinetic terms for the fields in the chiral supermultiplets. Here, the standard supergravity mass units are used: $M_{Pl} = \dfrac{M_P}{\sqrt{8\pi}} = 1$.

The SUGRA scalar potential is given by [28,29]:

$$\begin{aligned} V(\phi_M, \phi_M^*) &= \textstyle\sum_{M,\bar{N}} e^G \left(G_M G^{M\bar{N}} G_{\bar{N}} - 3 \right) + \dfrac{1}{2} \textstyle\sum_a (D^a)^2, \\ D^a &= g_a \textstyle\sum_{i,j} \left(G_i T^a_{ij} \phi_j \right), \qquad G_M \equiv \dfrac{\partial G}{\partial \phi_M}, \qquad G_{\bar{M}} \equiv \dfrac{\partial G}{\partial \phi_M^*}, \\ G_{\bar{N}M} &\equiv \dfrac{\partial^2 G}{\partial \phi_N^* \partial \phi_M}, \qquad G^{M\bar{N}} = G^{-1}_{\bar{N}M}. \end{aligned} \tag{8}$$

where g_a is the gauge coupling constant, which is associated with the generator T^a of the gauge transformations. The breakdown of supersymmetry in $(N = 1)$ SUGRA models takes place in the hidden sector. This sector contains superfields (z_i) that interact with the observable ones only by means of gravity. It is assumed that at the minimum of the scalar potential (8), hidden sector fields acquire VEVs so that at least one of their auxiliary fields:

$$F^M = e^{G/2} G^{M\bar{P}} G_{\bar{P}} \tag{9}$$

gets a non-vanishing VEV, resulting in the spontaneous breakdown of local SUSY. At the same time, a massless fermion with spin 1/2, the goldstino, which is a combination of the fermionic partners of the hidden sector fields, which give rise to the breakdown of local SUSY, is swallowed up by the gravitino, which becomes massive, $m_{3/2} =< e^{G/2} >$. This phenomenon is called the super-Higgs effect [30–32].

As mentioned before, the successful implementation of the MPP in $(N = 1)$ supergravity requires us to assume the existence of a supersymmetric Minkowski vacuum [12–24]. According to the MPP, this second vacuum and the physical one must have the same energy density. Such a second vacuum is realised only when the SUGRA scalar potential has a minimum where the superpotential W for the hidden sector and its derivatives vanish, i.e.,

$$W(z_m^{(2)}) = 0, \qquad \left. \frac{\partial W(z_i)}{\partial z_m} \right|_{z_m = z_m^{(2)}} = 0 \tag{10}$$

where $z_m^{(2)}$ denote VEVs of the hidden sector fields in the corresponding minimum. Equations (10) demonstrate that in general, an extra fine-tuning is needed to get such a second supersymmetric Minkowski vacuum in SUGRA models.

The simplest Kähler potential and superpotential that obey the MPP conditions (10) are given by [12]:

$$K(z, z^*) = |z|^2, \qquad W(z) = m_0 (z + \beta)^2, \tag{11}$$

where the hidden sector of this SUGRA model includes only one singlet superfield, z. If the parameter $\beta = \beta_0 = -\sqrt{3} + 2\sqrt{2}$, the corresponding SUGRA scalar potential possesses two degenerate minima with zero energy density at the classical level. One of them is a supersymmetric Minkowski minimum associated with $z^{(2)} = -\beta$. In the other minimum of the SUGRA scalar potential, $z^{(1)} = \sqrt{3} - \sqrt{2}$, local SUSY is broken, and the gravitino gains a mass $m_{3/2} \simeq 1.49 \cdot m_0$. Therefore, this minimum can be identified with the physical vacuum. Varying the parameter β around β_0, one can get a positive or a negative contribution from the hidden sector to the total energy density of the physical vacuum. Thus, β can be always fine-tuned so that the two vacua are degenerate.

Usually, the absolute value of the vacuum energy density at the minimum of the SUGRA scalar potential (8) tends to be of the order of $m_{3/2}^2 M_{Pl}^2$. To demonstrate this, let us suppose that the function

$G(\phi_M, \phi_M^*)$ has a stationary point, where $G_M = 0$. Such a point is also an extremum of the SUGRA scalar potential. In its vicinity, local SUSY remains intact, while the energy density is $-3 < e^G >$. Then, near the global minimum of the SUGRA scalar potential (8), the vacuum energy density should be less than or equal to this value. Thus, in general, a huge fine-tuning is needed, in order to keep the cosmological constant in SUGRA models around its observed value.

This fine-tuning can be, to some extent, alleviated in the no-scale SUGRA models. It was discovered a long time ago that invariance with respect to $SU(1,1)$ symmetry transformations results in a tree-level scalar potential, which vanishes identically along some directions [28,33]. In other words, the corresponding scalar potential (8) possesses an infinite set of degenerate minima with zero vacuum energy density. The $SU(1,1)$ structure of the $N = 1$ SUGRA Lagrangian can have its roots in supergravity theories with extended supersymmetry ($N = 4$ or $N = 8$) [28].

The simplest no-scale SUGRA model involves one hidden sector superfield T and a set of chiral supermultiplets φ_σ in the observable sector. The group $SU(1,1)$ contains subgroups of imaginary translations and dilatations [33,34]. The invariance of the Lagrangian of the simplest no-scale SUGRA model under the imaginary translations, i.e.,

$$T \to T + i\beta_i\,, \qquad \varphi_\alpha \to \varphi_\alpha\,, \tag{12}$$

implies that the corresponding K*ä*hler function depends only on $T + \bar{T}$. The invariance under the dilatation transformations:

$$T \to \alpha^2 T\,, \qquad \varphi_\sigma \to \alpha\varphi_\sigma\,. \tag{13}$$

constrains the K*ä*hler potential and superpotential of the model under consideration further. If the superpotential contains trilinear terms, then the K*ä*hler function is fixed uniquely by the gauge and global symmetries of this model:

$$K(T+\bar{T}, \varphi_\sigma, \bar{\varphi}_\sigma) = -3\ln(T+\bar{T}) + \sum_\sigma C_\sigma \frac{|\varphi_\sigma|^2}{(T+\bar{T})}\,, \qquad W(\varphi_\alpha) = \sum_{\sigma,\beta,\gamma} \frac{1}{6} Y_{\sigma\beta\gamma}\varphi_\sigma\varphi_\beta\varphi_\gamma\,, \tag{14}$$

where C_σ and $Y_{\sigma\beta\gamma}$ are constants. Here, we restrict our consideration to the lowest order terms $|\varphi_\sigma|^2$ in the expansion of the K*ä*hler potential in terms of observable superfields. The contribution of higher order terms to the SUGRA scalar potential is suppressed by inverse powers of M_{Pl} and can be safely ignored.

Owing to the particular form of the K*ä*hler potential, the part of the SUGRA scalar potential associated with the hidden sector vanishes, i.e.,

$$V_{hid} = e^G \left(G_T G^{T\bar{T}} G_{\bar{T}} - 3 \right) = 0\,. \tag{15}$$

Then, the full scalar potential takes the form:

$$V = \frac{1}{3} e^{2K/3} \sum_\alpha \left| \frac{\partial W(\tilde{\varphi}_\sigma)}{\partial \tilde{\varphi}_\alpha} \right|^2 + \frac{1}{2} \sum_a (D^a)^2\,, \tag{16}$$

where the observable superfields are rescaled as $\tilde{\varphi}_\alpha = \sqrt{\frac{C_\sigma}{3}}\varphi_\alpha$. The potential (16) is positive definite. Its minimum is reached when:

$$\left\langle \frac{\partial W(\tilde{\varphi}_\sigma)}{\partial \tilde{\varphi}_\alpha} \right\rangle = < D^a > = 0\,. \tag{17}$$

As a consequence, the vacuum energy density goes to zero near the global minima of the scalar potential (16). Thus, imaginary translations (12) and dilatations (13) protect a zero value for the cosmological constant in supergravity [33]. However, the structure of the potential (16) leads to a

supersymmetric particle spectrum at low energies, and the invariance of the Kähler function with respect to symmetry transformations (12) and (13) also prevents the breaking of local SUSY [13].

3. No-Scale Inspired SUGRA Model with Degenerate Vacua

In this context, it is worth considering the no-scale-inspired SUGRA model with broken dilatation invariance [13]. Let us consider a SUGRA model that includes two hidden sector superfields (T and z) and a set of chiral supermultiplets φ_σ in the observable sector. These supermultiplets transform differently under the imaginary translations ($T \to T + i\beta$, $z \to z$, $\varphi_\sigma \to \varphi_\sigma$) and dilatations ($T \to \alpha^2 T$, $z \to \alpha\, z$, $\varphi_\sigma \to \alpha\, \varphi_\sigma$). The full superpotential of the model can be presented as a sum [13]:

$$\begin{gathered} W(z,\, \varphi_\alpha) = W_{hid} + W_{obs}\,, \\ W_{hid} = \kappa\left(z^3 + \mu_0 z^2 + \textstyle\sum_{n=4}^{\infty} c_n z^n\right), \qquad W_{obs} = \textstyle\sum_{\sigma,\beta,\gamma} \dfrac{1}{6} Y_{\sigma\beta\gamma}\varphi_\sigma\varphi_\beta\varphi_\gamma\,. \end{gathered} \tag{18}$$

The superpotential (18) contains a bilinear mass term for the superfield z and higher order terms $c_n z^n$, which explicitly break dilatation invariance. A term proportional to z is not included since it can be forbidden by a gauge symmetry of the hidden sector, if z transforms non-trivially under the corresponding gauge transformations. To avoid potentially dangerous terms, which may lead, for instance, to the so-called μ-problem [35,36], the breakdown of dilatation invariance in the superpotential of the observable sector is not allowed.

The full Kähler potential of the SUGRA model under consideration takes the form [13]:

$$\begin{aligned} K(\phi_M, \phi_M^*) \;&=\; -3\ln\left[T + \overline{T} - |z|^2 - \textstyle\sum_\alpha \zeta_\alpha |\varphi_\alpha|^2\right] \\ &+\; \textstyle\sum_{\alpha,\beta}\left(\dfrac{\eta_{\alpha\beta}}{2}\,\varphi_\alpha\,\varphi_\beta + h.c.\right) + \textstyle\sum_\beta \xi_\beta |\varphi_\beta|^2\,, \end{aligned} \tag{19}$$

where ζ_α, $\eta_{\alpha\beta}$, ξ_β are some constants. When the parameters $\eta_{\alpha\beta}$, ξ_β, and κ vanish, the dilatation invariance is restored, protecting supersymmetry and a zero value of the cosmological constant. Here, we restrict our consideration to the simplest set of terms that explicitly break dilatation invariance in the Kähler potential. Extra terms that are proportional to ξ_β normally appear in minimal SUGRA models [37–39]. The other terms $\eta_{\alpha\beta}\varphi_\alpha\varphi_\beta$ induce effective μ terms after the spontaneous breakdown of local SUSY. It is worth noticing that in the Kähler potential, only the breakdown of the dilatation invariance in the part associated with the observable sector is allowed. Any variations in the Kähler potential of the hidden sector can spoil the vanishing of the vacuum energy density at the global minima.

The Kähler potential (19) and superpotential (18) lead to the tree-level SUGRA scalar potential of the hidden sector, which can be written as:

$$V_{hid} = \frac{1}{3(T + \overline{T} - |z|^2)^2}\left|\frac{\partial W_{hid}(z)}{\partial z}\right|^2. \tag{20}$$

The scalar potential (20) is positive definite so that the vacuum energy density vanishes near its global minima. These minima are attained at the stationary points of the hidden sector superpotential. The form of the superpotential (18) guarantees that there is always a supersymmetric Minkowski minimum at $z = 0$. Indeed, near $z = 0$, the conditions (10) are fulfilled without any extra fine-tuning, and the gravitino remains massless.

In the simplest scenario when $c_n = 0$, V_{hid} has two minima, at $z = 0$ and $z = -\dfrac{2\mu_0}{3}$, corresponding to the extremum points of the hidden sector superpotential. At these points, the SUGRA scalar

potential (20) attains its absolute minimal value, namely approximately zero. In the vacuum where $z = -\frac{2\mu_0}{3}$, local SUSY is broken so that the gravitino becomes massive:

$$m_{3/2} = \left\langle \frac{W_{hid}(z)}{(T+\overline{T}-|z|^2)^{3/2}} \right\rangle = \frac{4\kappa\mu_0^3}{27\left\langle \left(T+\overline{T}-\frac{4\mu_0^2}{9}\right)^{3/2} \right\rangle}. \tag{21}$$

In the limit, when $M_{Pl} \to \infty$ but $m_{3/2}$ is kept fixed (flat limit), the effective scalar potential of the observable sector near this minimum can be presented in the following compact form [13]:

$$V_{obs} = \sum_\alpha \left| \frac{\partial W_{eff}(y_\beta)}{\partial y_\alpha} + m_\alpha y_\alpha^* \right|^2 + \frac{1}{2}\sum_a (D^a)^2, \tag{22}$$

where y_α are canonically-normalized scalar fields:

$$y_\alpha = \tilde{C}_\alpha \varphi_\alpha, \qquad \tilde{C}_\alpha = \xi_\alpha\left(1+\frac{1}{x_\alpha}\right), \qquad x_\alpha = \frac{\xi_\alpha < (T+\overline{T}-|z|^2) >}{3\zeta_\alpha}. \tag{23}$$

In Equation (22), the mass parameters, m_α, are given by:

$$m_\alpha = \frac{m_{3/2}x_\alpha}{1+x_\alpha}, \tag{24}$$

whereas the effective superpotential, which describes the interactions of observable superfields at low energies, can be written as:

$$\begin{aligned} W_{eff} &= \textstyle\sum_{\alpha,\beta} \frac{\mu_{\alpha\beta}}{2} y_\alpha y_\beta + \sum_{\alpha,\beta,\gamma} \frac{h_{\alpha\beta\gamma}}{6} y_\alpha y_\beta y_\gamma, \\ \mu_{\alpha\beta} &= m_{3/2}\eta_{\alpha\beta}(\tilde{C}_\alpha\tilde{C}_\beta)^{-1}, \qquad h_{\alpha\beta\gamma} = \frac{Y_{\alpha\beta\gamma}(\tilde{C}_\alpha\tilde{C}_\beta\tilde{C}_\gamma)^{-1}}{< (T+\overline{T}-|z|^2)^{3/2} >}. \end{aligned} \tag{25}$$

Because near the minimum, where $z = -\frac{2\mu_0}{3}$, the gravitino and all the scalar particles get non-zero masses, $m_\sigma \sim \frac{m_{3/2}\xi_\sigma}{\zeta_\sigma}$, it can be identified with the physical vacuum. Although global SUSY is softly broken in this vacuum, the effective potential (22) is still positive definite. When ξ_α, ζ_α, μ_0, and $< T >$ are all of order unity, a SUSY breaking scale $M_S \sim 1$ TeV can only be obtained for extremely small values of $\kappa \simeq 10^{-15}$.

If the high order terms $c_n z^n$ are present in Equations (18), the scalar potential of the hidden sector can have many degenerate vacua with vanishing vacuum energy density, where the gravitino may remain massless or gain a non-zero mass. As a consequence, the MPP conditions are satisfied without any extra fine-tuning at the tree-level. However the inclusion of perturbative and non-perturbative corrections in the Lagrangian of the SUGRA model under consideration is expected to spoil the degeneracy of the vacua. Of course, while these corrections depend on the structure of the underlying theory, in general, they tend to induce a huge energy density in the vacua where SUSY is broken. Furthermore, the mechanism for the stabilization of the VEV of the hidden sector field T remains unclear. As a result, the gravitino mass and the supersymmetry breaking scale are not fixed in the physical vacuum. Because of the serious shortcomings mentioned above, the SUGRA model discussed in this section should be considered as a toy example only. This example indicates that in $(N = 1)$ supergravity, there might be a mechanism that ensures the cancellation of different contributions to the total vacuum energy density in the physical vacuum. Such a mechanism can also lead to a set of degenerate vacua with broken and unbroken SUSY, resulting in the realization of the MPP.

4. Dark Energy Density in the Models with the Low SUSY Breaking Scale

Hereafter, we assume that a phenomenologically-viable SUGRA model with degenerate vacua of the type discussed in the previous section is realised in Nature. This implies that there exist at least two phases that are exactly degenerate. In the first phase, which is associated with the physical vacuum, SUSY is broken. The second phase is identified with the supersymmetric Minkowski vacuum. Because the vacuum energy density of supersymmetric states in flat Minkowski space vanishes, the cosmological constant problem in the physical vacuum is solved to first approximation by assumption. At the same time, non-perturbative effects may give rise to the breakdown of SUSY in the supersymmetric phase at low energies, leading to a small vacuum energy density that should be then transferred to our vacuum by the assumed degeneracy.

In principle, the breakdown of SUSY in the second vacuum can be caused by the strong interactions in the observable sector. Indeed, the evolution of the SM gauge couplings g_1, g_2, and g_3, which correspond to $U(1)_Y$, $SU(2)_W$, and $SU(3)_C$ gauge interactions respectively, obeys the renormalization group equations (RGEs) that can be written to first order as:

$$\frac{d\alpha_i(Q)}{dt} = \frac{b_i \alpha_i^2(Q)}{4\pi}\,, \tag{26}$$

where $t = \log Q^2$, Q is a renormalization scale, $i = 1,2,3$, and $\alpha_i(Q) = g_i^2(Q)/(4\pi)$. In the pure Minimal Supersymmetric Standard Model (MSSM) only the beta function of $\alpha_3(Q)$ exhibits asymptotically-free behaviour, i.e., $b_3 < 0$. As a result, $\alpha_3(Q)$ increases in the infrared region. When $\alpha_3(Q)$ becomes rather large at low energies, the role of non-perturbative effects is enhanced.

In order to simplify our analysis here, we assume that the values of the gauge couplings at high energies are the same in the physical and supersymmetric Minkowski vacua. Therefore, the RG flow of these couplings down to the scale M_S, where M_S is a SUSY breaking scale in the physical phase, is also identical in both vacua. Below the scale M_S, all superparticles in the physical vacuum decouple and all beta functions change. In particular, above M_S, the $SU(3)_C$ beta function $b_3 = -3$, whereas for $Q < M_S$, it coincides with the corresponding SM beta function, i.e., $\tilde{b}_3 = -7$, in the one-loop approximation. Because of this, below the scale M_S, the values of $\alpha_3(Q)$ in the physical and supersymmetric Minkowski vacua ($\alpha_3^{(1)}(Q)$ and $\alpha_3^{(2)}(Q)$) are not the same. Using the matching condition $\alpha_3^{(2)}(M_S) = \alpha_3^{(1)}(M_S)$, in the one-loop approximation, one can find the value of the strong gauge coupling in the second vacuum [12,13,15,16]:

$$\frac{1}{\alpha_3^{(2)}(M_S)} = \frac{1}{\alpha_3^{(1)}(M_Z)} - \frac{\tilde{b}_3}{4\pi} \ln \frac{M_S^2}{M_Z^2}\,. \tag{27}$$

In the second supersymmetric phase, all particles of the MSSM are massless, and the $SU(3)_C$ beta function remains the same as in the MSSM. At the scale Λ_c, where the supersymmetric QCD interaction becomes extremely strong in the second vacuum, the top quark Yukawa coupling $h_t^{(2)}(Q)$ is of the same order of magnitude as the strong gauge coupling $g_3^{(2)}(Q)$. Therefore, a large value of the top quark Yukawa coupling may give rise to the formation of a quark condensate that breaks SUSY, leading to a positive value of the vacuum energy density:

$$\rho_\Lambda \sim \Lambda_c^4\,, \tag{28}$$

where in the one-loop approximation [12,13,15,16]:

$$\Lambda_c = M_S \exp\left[\frac{2\pi}{b_3 \alpha_3^{(2)}(M_S)}\right]\,. \tag{29}$$

The induced cosmological constant (28) should be then interpreted as the physical value in our phase, by virtue of the postulated degeneracy of vacua.

Equations (27) and (29) indicate that the cosmological constant and Λ_c are determined by the values of $\alpha_3^{(1)}(M_Z)$ and the supersymmetry breaking scale, M_S, in the physical phase. Because in the one-loop approximation $\tilde{b}_3 < b_3$, the QCD gauge coupling below M_S is larger in the physical phase than in the second one while Λ_c is much lower than the QCD scale Λ_{QCD} in the SM and diminishes with increasing M_S. Using Relations (27) and (29) for $M_S \sim 1$ TeV and $\alpha_3^{(1)}(M_Z) \simeq 0.118$, one obtains $\Lambda_c \simeq 100$ eV. The measured value of ρ_Λ can be reproduced when $\Lambda_c \sim 10^{-3}$ eV. Within the MSSM, $\Lambda_c \sim 10^{-3}$ eV can be obtained in the one-loop approximation for $M_S = 10^3 - 10^4$ TeV [12,13,15,16].

In this approximation, the observed value of the cosmological constant can also be reproduced even for $M_S \sim 1$ TeV if the MSSM particle content is supplemented by an additional pair of $5 + \bar{5}$ supermultiplets, which are fundamental and antifundamental representations of the supersymmetric $SU(5)$ Grand Unified Theory (GUT) [12,13,15,16]. In the physical phase, additional bosons and fermions can gain masses around M_S because of the presence of the bilinear terms $\left[\eta(5 \cdot \bar{5}) + h.c.\right]$ in the Kähler potential of the observable sector. These states would not affect gauge coupling unification, because they form complete representations of $SU(5)$. In the second supersymmetric phase, new bosons and fermions from $5 + \bar{5}$ supermultiplets remain massless and give a substantial contribution to the β functions in this vacuum. Therefore, the one-loop beta function of the strong interaction in the supersymmetric Minkowski vacuum changes from $b_3 = -3$ to $b_3 = -2$. As a result, $\alpha_3(Q)$ and Λ_c decrease.

In this context, it is worthwhile to explore how the estimations of Λ_c change when the two-loop contributions to the corresponding beta functions are included. Here, we neglect all Yukawa couplings except that for the top quark. Then, the evolution of the gauge, top quark Yukawa, and Higgs quartic couplings in the SM is described by the following set of RGEs:

$$
\begin{aligned}
\frac{d\alpha_1}{dt} &= \frac{\alpha_1^2}{4\pi}\left[\frac{41}{10} + \frac{1}{4\pi}\left(\frac{199}{50}\alpha_1 + \frac{27}{10}\alpha_2 + \frac{44}{5}\alpha_3 - \frac{17}{10}Y_t\right)\right], \\
\frac{d\alpha_2}{dt} &= \frac{\alpha_2^2}{4\pi}\left[-\frac{19}{6} + \frac{1}{4\pi}\left(\frac{9}{10}\alpha_1 + \frac{35}{6}\alpha_2 + 12\alpha_3 - \frac{3}{2}Y_t\right)\right], \\
\frac{d\alpha_3}{dt} &= \frac{\alpha_3^2}{4\pi}\left[-7 + \frac{1}{4\pi}\left(\frac{11}{10}\alpha_1 + \frac{9}{2}\alpha_2 - 26\alpha_3 - 2Y_t\right)\right], \\
\frac{dY_t}{dt} &= \frac{Y_t}{4\pi}\left[\frac{9}{2}Y_t - \frac{17}{20}\alpha_1 - \frac{9}{4}\alpha_2 - 8\alpha_3 + \frac{1}{4\pi}\left(-12Y_t^2 - 12Y_tY_\lambda + 6Y_\lambda^2 + 36\alpha_3Y_t\right.\right. \\
&+ \left.\left.\frac{225}{16}\alpha_2Y_t + \frac{393}{80}\alpha_1Y_t - 108\alpha_3^2 - \frac{23}{4}\alpha_2^2 + \frac{1187}{600}\alpha_1^2 + 9\alpha_2\alpha_3 + \frac{19}{15}\alpha_1\alpha_3 - \frac{9}{20}\alpha_1\alpha_2\right)\right], \qquad (30) \\
\frac{dY_\lambda}{dt} &= \frac{1}{8\pi}\left[24Y_\lambda^2 + 12Y_tY_\lambda - 6Y_t^2 - 9\alpha_2Y_\lambda - \frac{9}{5}\alpha_1Y_\lambda + \frac{27}{200}\alpha_1^2 + \frac{9}{20}\alpha_1\alpha_2 + \frac{9}{8}\alpha_2^2\right. \\
&+ \frac{1}{4\pi}\left(-312Y_\lambda^3 - 144Y_tY_\lambda^2 - 3Y_t^2Y_\lambda + 30Y_t^3 + \frac{108}{5}\alpha_1Y_\lambda^2 + 108\alpha_2Y_\lambda^2 + \frac{17}{2}\alpha_1Y_tY_\lambda\right. \\
&+ \frac{45}{2}\alpha_2Y_tY_\lambda + 80\alpha_3Y_tY_\lambda - \frac{8}{5}\alpha_1Y_t^2 - 32\alpha_3Y_t^2 + \frac{1887}{200}\alpha_1^2Y_\lambda + \frac{117}{20}\alpha_1\alpha_2Y_\lambda - \frac{73}{8}\alpha_2^2Y_\lambda \\
&- \left.\left.\frac{171}{100}\alpha_1^2Y_t + \frac{63}{10}\alpha_1\alpha_2Y_t - \frac{9}{4}\alpha_2^2Y_t - \frac{3411}{2000}\alpha_1^3 + \frac{305}{16}\alpha_2^3 - \frac{1677}{400}\alpha_1^2\alpha_2 - \frac{289}{80}\alpha_1\alpha_2^2\right)\right],
\end{aligned}
$$

where $Y_t(Q) = h_t^2(Q)/(4\pi)$ and $Y_\lambda(Q) = \lambda(Q)/(4\pi)$. The running of $g_i(Q)$, $h_t(Q)$, and $\lambda(Q)$ is calculated for a given set of these couplings at the scale $Q = M_t$. The values of $g_i(M_t)$, $h_t(M_t)$, and $\lambda(M_t)$ were computed in [1]. Careful numerical analysis demonstrates that the inclusion of two-loop

corrections to the beta functions leads to minor variations of $g_i(M_S)$ and $h_t(M_S)$, which are less than 1–2%.

In the second supersymmetric phase, the RG flow of the gauge and top quark Yukawa couplings is computed using a set of two-loop RGEs:

$$\begin{aligned}
\frac{d\alpha_1}{dt} &= \frac{\alpha_1^2}{4\pi}\left[\frac{33}{5}+n+\frac{\alpha_1}{4\pi}\left(\frac{199}{25}+\frac{7}{15}n\right)+\frac{\alpha_2}{4\pi}\left(\frac{27}{5}+\frac{9}{5}n\right)+\frac{\alpha_3}{4\pi}\left(\frac{88}{5}+\frac{32}{15}n\right)-\frac{26}{5}\frac{Y_t}{4\pi}\right],\\
\frac{d\alpha_2}{dt} &= \frac{\alpha_2^2}{4\pi}\left[1+n+\frac{\alpha_1}{4\pi}\left(\frac{9}{5}+\frac{3}{5}n\right)+\frac{\alpha_2}{4\pi}\left(25+7n\right)+24\frac{\alpha_3}{4\pi}-6\frac{Y_t}{4\pi}\right],\\
\frac{d\alpha_3}{dt} &= \frac{\alpha_3^2}{4\pi}\left[-3+n+\frac{\alpha_1}{4\pi}\left(\frac{11}{5}+\frac{4}{15}n\right)+9\frac{\alpha_2}{4\pi}+\frac{\alpha_3}{4\pi}\left(14+\frac{34}{3}n\right)-4\frac{Y_t}{4\pi}\right],\\
\frac{dY_t}{dt} &= \frac{Y_t}{4\pi}\left[6Y_t-\frac{13}{15}\alpha_1-3\alpha_2-\frac{16}{3}\alpha_3+\frac{1}{4\pi}\left(-22Y_t^2+16\alpha_3Y_t+6\alpha_2Y_t+\frac{6}{5}\alpha_1Y_t\right.\right.\\
&+ \left(-\frac{16}{9}+\frac{16}{3}n\right)\alpha_3^2+\left(\frac{15}{2}+3n\right)\alpha_2^2+\left(\frac{2743}{450}+\frac{13}{15}n\right)\alpha_1^2+8\alpha_2\alpha_3\\
&+ \left.\left.\frac{136}{45}\alpha_1\alpha_3+\alpha_1\alpha_2\right)\right],
\end{aligned} \tag{31}$$

and matching conditions:

$$\alpha_i^{(2)}(M_S)=\alpha_i^{(1)}(M_S)\,, \tag{32}$$

$$Y_t^{(2)}(M_S)=Y_t^{(1)}(M_S)\,. \tag{33}$$

The parameter n appearing in Equation (31) is the number of extra pairs of $5+\bar{5}$ supermultiplets of $SU(5)$ that can survive to low energies in addition to the MSSM particle content. In the case of the MSSM, $n=0$. The matching condition (33) corresponds to $\tan\beta \gg 1$ in the physical vacuum. On the other hand, it is assumed that the values of $\tan\beta$ are much smaller than 50–60. This allows us to neglect the b-quark and τ-lepton Yukawa couplings to leading order.

Exploring the two-loop RG flow of the gauge and top quark Yukawa couplings for $Q \ll M_S$ in the supersymmetric Minkowski vacuum, one can establish the position of the Landau pole in the evolution of $Y_t(Q)$ and $\alpha_3(Q)$. The results of our numerical studies are summarised in Table 1. In the second supersymmetric phase, the two-loop corrections to the beta functions alter the running of $Y_t(Q)$ and $\alpha_3(Q)$ significantly when these parameters become of the order of unity in the infrared region. In order to illustrate this, let us first consider the limit where $n=0$ and $Y_t(Q)\simeq 0$. In this case, the Landau pole in the evolution of $\alpha_3(Q)$ disappears, and the solutions of the two-loop RGEs (31) are attracted to the infrared fixed point [20]:

$$\alpha_1(Q)\to 0\,,\qquad \alpha_2(Q)\to 0\,,\qquad \alpha_3(Q)\simeq\frac{6\pi}{7}\,, \tag{34}$$

for $Q\to 0$. However, the sufficiently large values of the top quark Yukawa coupling associated with the matching condition (33) lead to a Landau pole in the two-loop RG flow of $Y_t(Q)$ and $\alpha_3(Q)$. Nevertheless, because of the substantial cancellation between one-loop and two-loop contributions to the beta function of the strong interaction for $\alpha_3(Q)\sim 1$, the value of Λ_c computed in the two-loop approximation is considerably lower than its one-loop estimation (29) [20]. This is illustrated by the results presented in Table 1. As before, the position of the Landau pole is mainly determined by the values of $\alpha_3^{(2)}(M_S)$ and M_S. The value of Λ_c grows with increasing $\alpha_3^{(2)}(M_S)$. Assuming that the values of the dimensionless couplings in the physical and supersymmetric Minkowski vacua coincide, Λ_c decreases with increasing M_S and does not change much when the pole mass of the top quark varies. The results presented in Table 1 indicate that in the two-loop approximation, $\Lambda_c \simeq 10^{-3}$ eV

can be obtained for values of the SUSY breaking scale $M_S \simeq 50$–150 TeV, which are much lower than 10^3–10^4 TeV.

Table 1. The values of Λ_c for $\alpha_3(M_Z) = 0.115$–0.121, $M_t = 171$–176 GeV, and different SUSY breaking scales M_S in the physical vacuum. These values are calculated in the two-loop approximation using matching Conditions (32)–(33). The values of Λ_c computed in the one-loop approximation are given in the brackets.

M_S (TeV)	10^4	100	50	150
Λ_c (eV)	$1 - 2.5 \times 10^{-6}$	$0.7 - 1.8 \times 10^{-3}$	$1.9 - 4.7 \times 10^{-3}$	$3.9 - 9.6 \times 10^{-4}$
	$(2.1 - 5.3 \times 10^{-4})$	$(0.1 - 0.25)$	$(0.25 - 0.62)$	$(0.058 - 0.143)$

If, in the supersymmetric Minkowski vacuum, the low energy limit of the theory under consideration is described by the MSSM with additional pairs of $5 + \bar{5}$ supermultiplets of $SU(5)$, the Landau pole in the two-loop RG flow of $Y_t(Q)$ and $\alpha_3(Q)$ disappears entirely. Indeed, when the MSSM particle spectrum is supplemented by one extra pair of $5 + \bar{5}$ supermultiplets, the solutions of the two-loop RGEs (31) are attracted to the infrared fixed point:

$$\alpha_1(Q) \to 0\,, \qquad \alpha_2(Q) \to 0\,, \qquad \alpha_3(Q) \simeq 1.15\,, \qquad Y_t(Q) \simeq 1.01 \tag{35}$$

for $Q \to 0$. Thus, it remains unclear whether a quark condensate can form in this case.

5. Preserving the Higgs Mass Prediction in Models with Planck Scale SUSY Breaking

When local SUSY is broken near the Planck scale in the physical vacuum, the contribution induced in the visible sector to the total vacuum energy density in the second phase tends to be negligibly small. In this case, the observed value of the cosmological constant can be reproduced if in the second vacuum, the dynamical breakdown of supersymmetry takes place in the hidden sector. Here, we assume that only vector supermultiplets, which correspond to the unbroken non-Abelian gauge symmetry in the hidden sector, remain massless. These supermultiplets may give rise to the breakdown of local SUSY near the scale Λ_X, where non-Abelian interactions in the hidden sector become strong in the second phase. Indeed, at the scale Λ_X, a gaugino condensate can be formed. This condensate itself does not break global SUSY [40]. Nonetheless, in $(N = 1)$ supergravity, we can have a non-trivial dependence of the gauge kinetic function $f_X(z_m)$ on the hidden sector superfields z_m. Then, the auxiliary fields F^m of the corresponding superfields z_m can acquire non-zero VEVs:

$$F^{z_m} \propto \frac{\partial f_X(z_k)}{\partial z_m} \bar{\lambda}_a \lambda_a + ..., \tag{36}$$

which are set by $< \bar{\lambda}_a \lambda_a > \simeq \Lambda_X^3$. Because Λ_X is much lower than M_P, this results in supersymmetry breaking [41] at the scale $\tilde{M}_S$, which is many orders of magnitude lower than Λ_X, as well as a tiny non-zero vacuum energy density:

$$\rho_\Lambda \sim \tilde{M}_S^4 \sim \frac{\Lambda_X^6}{M_{Pl}^2}\,. \tag{37}$$

Because of the postulated exact degeneracy of vacua, the physical phase, in which SUSY is broken near the Planck scale, has the same energy density as the phase where the breakdown of local SUSY is induced by the gaugino condensate in the hidden sector. Then, from Equation (37), it follows that the measured cosmological constant can be reproduced if Λ_X is somewhat close to Λ_{QCD} in the physical vacuum [21–24], i.e.,

$$\Lambda_X \sim \Lambda_{QCD}/10\,. \tag{38}$$

Although there is no compelling reason to expect that the two scales Λ_X and Λ_{QCD} should be relatively close, Λ_{QCD} and M_P can be considered as the two most natural choices for the scale of

dimensional transmutation in the hidden sector in the second phase. In the case when the non-Abelian interactions, which lead to the formation of the gaugino condensate in the hidden sector, are described by $SU(3)$ SUSY gluodynamics, the corresponding value of Λ_X can be obtained if the $SU(3)$ gauge coupling $g_X(M_P) \approx 0.65$ [21–24]. This is just slightly larger than the value of the QCD gauge coupling at the Planck scale in the SM, i.e., $g_3(M_P) \approx 0.49$ [1].

In principle, there might be other vacua that have the same energy density as the first and second phases. In particular, there can exist a vacuum where local SUSY is broken near the Planck scale while the EW symmetry breaking scale is just a few orders of magnitude lower than M_P. Because in this third vacuum, the Higgs VEV is somewhat close to M_P, one has to take into account the interaction of the Higgs and hidden sector fields. Thus, the full scalar potential can be written in the following form:

$$V = V_{hid}(z_m) + V_0(H) + V_{int}(H, z_m) + \dots, \tag{39}$$

where $V_{hid}(z_m)$ is the part of the full scalar potential associated with the hidden sector, $V_0(H)$ is the part of the potential (39) that depends on the SM Higgs field only, and $V_{int}(H, z_m)$ describes the interactions of the SM Higgs doublet with the hidden sector fields. Here, it is assumed that in the observable sector, only one Higgs doublet can acquire a non-zero VEV, and all other observable fields can be ignored in the first approximation. Although in general, $V_{int}(H, z_m)$ should not be ignored, the interactions between H and hidden sector fields can be rather weak if the VEV of the Higgs field is considerably smaller than M_P (say $\langle H \rangle \leq M_P/10$) and the couplings of the SM Higgs doublet to the hidden sector fields are suppressed. In this case, the VEVs of the hidden sector fields in the physical and third vacua can be almost identical, i.e., $z_m^{(1)} \simeq z_m^{(3)}$, so that $V_{hid}(z_m^{(3)}) \ll M_P^4$. As a result, one can expect that the gauge couplings and $\lambda(M_P)$ in the first and third phases are basically the same and the value of $|m^2|$ in the Higgs effective potential (1) is much smaller than M_P^2 and $\langle H^\dagger H \rangle$ in the third vacuum. Therefore, the presence of this third vacuum, with vanishingly small energy density, again implies that $\lambda(M_P)$ and $\beta_\lambda(M_P)$ should be approximately zero in the third phase. Since the couplings in the third and physical phases are basically identical, the existence of such degenerate vacua should lead to $\lambda(M_P) \approx \beta_\lambda(M_P) \approx 0$ in the physical vacuum.

6. Summary and Concluding Remarks

The implementation of the multiple point principle (MPP) in $N = 1$ supergravity (SUGRA) implies that SUGRA scalar potential has at least two exactly degenerate minima. In the first minimum, local SUSY is broken, and it can be identified with the physical vacuum, in which we live. In the second minimum, the low energy limit of the theory under consideration is described by a pure SUSY model in flat Minkowski space. It is realized if the superpotential of the hidden sector has an extremum point where it vanishes. In general, the existence of such a second minimum requires an extra fine-tuning. In this article, we discussed the no-scale-inspired SUGRA model with broken dilatation invariance in which the MPP conditions are fulfilled without any additional fine-tuning at the tree-level.

The local SUSY in the second vacuum may be broken. In the simplest case, such a breakdown can be caused by the non-perturbative effects in the observable sector that lead to the formation of a top quark condensate near the scale Λ_c, where the $SU(3)_C$ gauge interactions become strong. This gives rise to a positive vacuum energy density $\sim \Lambda_c^4$ that should be then transferred to our phase, by virtue of the MPP. Here, we restrict our consideration to the case where all gauge and Yukawa couplings at high energies are identical in both vacua. Using the two-loop RG equations, we evaluate Λ_c, which is mainly determined by the SUSY breaking scale M_S in the physical vacuum. The value of Λ_c decreases with increasing M_S. The observed value of the cosmological constant can be reproduced when M_S varies around 100 TeV. Taking into account that in the realistic SUSY extension of the SM, the SUSY breaking parameters should be distributed around M_S, some of the sparticles in this case can be sufficiently light so that the corresponding states may be discovered at the Future Circular Collider (FCC).

If in the physical vacuum, local SUSY is broken near the Planck scale, then Λ_c is negligibly small. In this scenario, the measured value of the dark energy density, as well as small values of $\lambda(M_{Pl})$ and $\beta_\lambda(M_{Pl})$ can be reproduced if there are at least three exactly degenerate vacua. In the first (physical) vacuum, local SUSY is broken near the Planck scale, and the small value of the cosmological constant appears as a result of the fine-tuned precise cancellation of different contributions. In the second vacuum, the breakdown of local SUSY is induced by gaugino condensation in the hidden sector, which is formed at a scale Λ_X, which is slightly lower than Λ_{QCD} in the physical vacuum. In the third vacuum, local SUSY and EW symmetry are broken near the Planck scale.

Finally, it is worth noting that the estimation of the dark energy density discussed here is based on the assumption that the vacua mentioned above are degenerate to extremely high accuracy. In fact, the required accuracy must be of the order of the value of the cosmological constant in the physical vacuum. In principle, a set of approximately degenerate vacua can appear if the underlying theory allows only vacua that lead to a similar order of magnitude of space-time four-volumes of the Universe at its final stage of evolution. Vacua with very different vacuum energy densities lead to rather different expansion rates and ultimately result in very different space-time volumes for the Universe. Thus, all allowed vacua should have vacuum energy densities of the same order of magnitude, i.e., they are degenerate to the accuracy of the value of the cosmological constant in the phase where we live.

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