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Study on Anisotropic Strange Stars in $f(\mathbb{T}, \mathcal{T})$ Gravity

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Abstract: In this work, we study the existence of strange stars in the background of $f(\mathbb{T}, \mathcal{T})$ gravity in the Einstein spacetime geometry, where \mathbb{T} is the torsion tensor and \mathcal{T} is the trace of the energy-momentum tensor. The equations of motion are derived for anisotropic pressure within the spherically symmetric strange star. We explore the physical features like energy conditions, mass-radius relations, modified Tolman–Oppenheimer–Volkoff (TOV) equations, principal of causality, adiabatic index, redshift and stability analysis of our model. These features are realistic and appealing to further investigation of properties of compact objects in $f(\mathbb{T}, \mathcal{T})$ gravity as well as their observational signatures.

Keywords: general relativity; alternative gravity; compact stars; anisotropic fluid

1. Introduction

In relativistic astrophysics, compact stars, which are more compact (i.e., possess larger mass and smaller radius) than ordinary stars, have received much attention from researchers to study their ages, structures and evolutions. A neutron star is the final stage of a gravitationally collapsed star which, after exhausting all its thermo-nuclear fuel, gets stabilized by degenerate neutron pressure. A massive neutron star may again collapse into a black hole, but a lower mass neutron star may be converted into a quark star. The strange quark matter [1–3] consists of u , d , s quarks and some electrons, which may be more stable than the ordinary nuclear matter. Actually, a neutron star consists of neutrons [4] whereas a strange star consists of quarks or strange matters. In this context, it should be noted that the study of neutron stars (NS) has provided us important information about the equation of state (EOS) of neutron-rich nuclear matter [5,6]. Theoretically, NS may be converted to (strange) quark stars, which are made purely of deconfined u , d and s quark matter, i.e., strange quark matter (SQM) [7–9]. However, there may have some electron-type leptons due to charge neutrality and β -equilibrium.

In 1916, Schwarzschild et al. [10,11] first gave the interior stellar solution for isotropic fluid (having equal radial pressure (p_r) and transversal pressure (p_t)). However, much later in 1972, Ruderman [12] first observed that the interior geometry of the nuclear matter of the steller system with density of order 10^{15} gm/cc possesses anisotropic behavior (i.e., $p_r \neq p_t$). After that Herrera et al. [13] have analyzed

the local anisotropic nature for self-gravitating systems. It is interesting to note that in the experimental area, Hewish et al. [14] explained the observed compactness of many astrophysical objects (rotating neutron stars) like *Her X – 1*, *4U 1820 – 30*, *RXJ 1856 – 37* and *SAX J 1808.4 – 3658*. Thus, theoretical modeling gradually became complementary to the observational results.

The study of anisotropic stars in the background of General Relativity (GR) is important in the current research. Hossein et al. [15] studied the stable structure of stellar objects for anisotropic systems. In the context of the present Standard cosmology, leading beyond the standard model of particle theory, such analysis inevitably involves additional elements in the standard model of gravity. Consequently, Kalam et al. [16] have studied the anisotropic neutron star model by considering quintessence dark energy. To study compact stellar models, the researchers have used well known Krori–Barua (KB) metric [17,18]. Paul et al. [19] have analyzed a new exact solution for compact stars. Several astrophysical natures of neutron stars as well as quark stars have been investigated [3,20]. Bhar [21] has proposed a stable anisotropic quintessence strange star model whereas Abbas et al. [22] have studied anisotropic strange stars for quintessence dark energy model. Using the MIT bag model, Rahaman et al. [23] have observed the existence of strange stars. By considering the MIT bag model, the stability of the strange stars with anisotropic fluid has been analyzed by Abrañil and Malheiro [24]. Furthermore, for the MIT bag model, Murad [25] studied anisotropic charged strange stars.

On the other hand, the study of anisotropic stars in the background of modified gravity theory is of great interest to many researchers [26–31]. The compact stars in $f(R)$, $f(G)$, $f(\mathbb{T})$ and $f(R, G)$ gravity theories have been studied by several authors [22,32–37], where R , G and \mathbb{T} are Ricci scalar, Gauss–Bonnet scalar and the torsion scalar, respectively. Strange stars in $f(\mathbb{T})$ gravity with an MIT bag model have been studied in [38]. Compact stars, in the form of a neutron stars as well as quark stars, in the Rastall theory of gravity, have been studied by several researches [39–44], whereas strange stars with the MIT bag model in the Rastall gravity have been studied in Reference [45]. Charged anisotropic collapsing stars with heat flux in $f(R)$ gravity has been investigated by Nazar and Abbas [46].

Studies on anisotropic stars in different types of modified gravity theories are available in literature [41,47–51]. Saha and Debnath [52] have investigated anisotropic stars in $f(\mathbb{T})$ gravity with modified Chaplygin gas. The anisotropic compact stars with a quintessence field and modified Chaplygin gas in the framework of $f(\mathbb{T})$ gravity model has been studied by Saha and Debnath [53]. On the other hand, strange stars as well as gravastars in $f(R, \mathcal{T})$ gravity have been studied in [54–61], where \mathcal{T} is the trace of the energy–momentum tensor.

We note from literature survey that almost all the works under the background of $f(\mathbb{T}, \mathcal{T})$ gravity have been performed in different cosmological context [62–73]. However, there are a few astrophysical applications of $f(\mathbb{T}, \mathcal{T})$ gravity as can be found in the works of Pace and Said [74,75]. Pace and Said have studied quark star incorporating the MIT bag model [74] and neutron star models using a perturbative approach [75] in $f(\mathbb{T}, \mathcal{T})$ gravity theory whereas gravastars have been investigated by Ghosh et al. [73] in the same alternative theory of gravity.

Our motivation to study anisotropic strange stars in $f(\mathbb{T}, \mathcal{T})$ gravity lies in the above background in the presence of Einstein spacetime within the spherically symmetric geometry. Thus we investigate in the present work strange quark star with MIT bag model [74] assuming anisotropy in the matter distribution and would like to note any appreciable effects due to anisotropy in addition to GR. The organization of the paper is as follows: in Section 2, the mathematical formulation of $f(\mathbb{T}, \mathcal{T})$ gravity in Einstein spacetime is given. In Section 3, we investigate the basic stellar equations in the framework of $f(\mathbb{T}, \mathcal{T})$ gravity. In Section 4, we find out the solutions to the Einstein field equations due to anisotropic fluid source. In Section 5, we explore the physical features like energy conditions, mass-radius relations, modified Tolman–Oppenheimer–Volkoff (TOV) equations, principal of causality, adiabatic index, redshift and stability analysis of our model whereas in Section 6 we perform a comparison with the standard results of GR, in order to show the effect of our model, and why it justifies our analysis and give a relevant discussion on the results. It is to note that for a comparative study we have considered, on the arbitrary ground only, the strange star candidate *LMC X – 4* for

plotting and exhibiting physical features which has specific total mass and radius as $M = 1.29 M_{Sun}$ [76] and $R = 9.711$ km [77] respectively. In Section 7, we deliver the conclusions of the work.

2. Basic Mathematical Formalism of $f(\mathbb{T}, \mathcal{T})$ Gravity and the Einstein Field Equations

The modified theories of teleparallel gravity are those for which the scalar torsion of teleparallel action is substituted by an arbitrary function of this latter. As it is done in teleparallel, the modified versions of this theory are also described by the orthonormal tetrads and its' components are defined on the tangent space of each point of the manifold. The line element is written as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ij} \theta^i \theta^j, \tag{1}$$

with the following definitions

$$dx^\mu = e_i^\mu \theta^i; \quad \theta^i = e^i_\mu dx^\mu. \tag{2}$$

Note that $\eta_{ij} = \text{diag}(1, -1, -1, -1)$ is the Minkowskian metric and the $\{e^i_\mu\}$ are the components of the tetrad which satisfy the following identity:

$$e_i^\mu e^i_\nu = \delta^\mu_\nu, \quad e_\mu^i e^j_\mu = \delta^i_j. \tag{3}$$

In General Relativity, one use the following Levi-Civita connection which preserves the curvature whereas the torsion vanishes, such as

$$\overset{\circ}{\Gamma}{}^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} (\partial_\nu g_{\sigma\mu} + \partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu}). \tag{4}$$

In the teleparallel theory and its modified version, one keeps the scalar torsion by using Weizenbock's connection defined as:

$$\Gamma^\lambda_{\mu\nu} = e_i^\lambda \partial_\mu e^i_\nu = -e^i_\mu \partial_\nu e_i^\lambda. \tag{5}$$

From this connection, one obtains the geometric objects. The first is the torsion as defined by

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}, \tag{6}$$

from which we define the contorsion as

$$K^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu} - \overset{\circ}{\Gamma}{}^\lambda_{\mu\nu} = \frac{1}{2} (T^\lambda_{\mu\nu} + T^\lambda_{\nu\mu} - T^\lambda_{\mu\nu}), \tag{7}$$

where the expression $\overset{\circ}{\Gamma}{}^\lambda_{\mu\nu}$ designs the above defined connection. Then we can write

$$K^{\mu\nu}{}_\lambda = -\frac{1}{2} (T^{\mu\nu}{}_\lambda - T^{\nu\mu}{}_\lambda + T^\lambda{}_{\nu\mu}). \tag{8}$$

The two previous geometric objects (the torsion and the contorsion) are used to define another tensor by

$$S_\lambda{}^{\mu\nu} = \frac{1}{2} (K^{\mu\nu}{}_\lambda + \delta_\lambda^\mu T^{\alpha\nu}{}_\alpha - \delta_\lambda^\nu T^{\alpha\mu}{}_\alpha). \tag{9}$$

The torsion scalar is usually constructed from torsion and contorsion as follows:

$$\mathbb{T} = S_\sigma{}^{\mu\nu} T^\sigma{}_{\mu\nu}. \tag{10}$$

In the modified versions of teleparallel gravity, one can use a general algebraic function of scalar torsion instead of the scalar torsion only as it is done in the initial theory. So, the modified action can be given by

$$\mathbb{S} = \int d^4x e \left[\frac{\mathbb{T} + f(\mathbb{T}, \mathcal{T})}{16\pi} + \mathcal{L}_m \right], \tag{11}$$

where $T_{\mu\nu}$ is the energy–momentum tensor of the strange quark matter (SQM) distribution and \mathcal{L}_m represents the Lagrangian for the matter distribution.

We define $T_{\mu\nu}$ as follows

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}, \tag{12}$$

and also define the trace of $T_{\mu\nu}$ as $\mathcal{T} = g^{\mu\nu}T_{\mu\nu}$. As \mathcal{L}_m depends only on the metric tensor components $g_{\mu\nu}$ and not on their derivatives, so we find

$$T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2\frac{\partial\mathcal{L}_m}{\partial g^{\mu\nu}}. \tag{13}$$

Varying the action with respect to the tetrad, one obtains the equations of motion [62] as

$$\begin{aligned} & [\partial_{\xi}(ee_a^{\rho}S_{\rho}^{\sigma\xi}) - ee_a^{\lambda}S^{\rho\xi\sigma}T_{\rho\xi\lambda}](1 + f_{\mathbb{T}}) + ee_a^{\rho}(\partial_{\xi}\mathbb{T})S_{\rho}^{\sigma\xi}f_{\mathbb{T}\mathbb{T}} \\ & + \frac{1}{4}ee_a^{\sigma}(\mathbb{T}) = -\frac{1}{4}ee_a^{\sigma}f - ee_a^{\rho}(\partial_{\xi}\mathcal{T})S_{\rho}^{\sigma\xi}f_{\mathbb{T}\mathcal{T}} \\ & + f_{\mathcal{T}} \left(\frac{eT_a^{\sigma} + ee_a^{\sigma}p}{2} \right) + 4\pi eT_a^{\sigma}, \end{aligned} \tag{14}$$

with $f_{\mathcal{T}} = \partial f/\partial\mathcal{T}$, $f_{\mathbb{T}} = \partial f/\partial\mathbb{T}$, $f_{\mathbb{T}\mathcal{T}} = \partial^2 f/\partial\mathbb{T}\partial\mathcal{T}$, $f_{\mathbb{T}\mathbb{T}} = \partial^2 f/\partial\mathbb{T}^2$ and T_a^{σ} represents the stress–energy tensor for the anisotropic fluid distribution defined as

$$T_{\mu\nu} = (p_r - p_t)u_{\mu}u_{\nu} - p_tg_{\mu\nu} + (\rho + p_t)u_{\mu}u_{\nu}, \tag{15}$$

where p_r and p_t represents the radial and tangential pressures of the SQM distribution, u_{μ} is the four-velocity which satisfies the conditions $u_{\mu}u^{\mu} = 1$ and $u^{\mu}\nabla_{\nu}u_{\mu} = 0$.

By using some transformations, we can establish the following relations:

$$e_{\nu}^a e^{-1} \partial_{\xi}(ee_a^{\rho}S_{\rho}^{\sigma\xi}) - S^{\rho\xi\sigma}T_{\rho\xi\nu} = -\nabla^{\xi}S_{\nu\xi}^{\sigma} - S^{\xi\rho\sigma}K_{\rho\xi\nu}, \tag{16}$$

$$G_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathbb{T} = -\nabla^{\rho}S_{\nu\rho\mu} - S^{\sigma\rho}_{\mu}K_{\rho\sigma\nu}. \tag{17}$$

Hence from the combination of Equations (16) and (17), the field equation (14) can be written as (under the relativistic geometrized units where Newton’s gravitational constant G and the speed of light c are fixed to be the unity, i.e., $G = c = 1$):

$$\begin{aligned} \frac{1}{2}(1 + f_{\mathbb{T}})G_{\mu\nu} &= \frac{1}{4}g_{\mu\nu}\mathbb{T}(1 + f_{\mathbb{T}}) - S_{\nu\mu}^{\lambda}(f_{\mathbb{T}\mathbb{T}}\partial_{\lambda}\mathbb{T} + f_{\mathbb{T}\mathcal{T}}\partial_{\lambda}\mathcal{T}) - \frac{1}{4}g_{\mu\nu}(f + \mathbb{T}) \\ &+ \frac{1}{2}f_{\mathcal{T}}(T_{\nu\mu} + g_{\mu\nu}p_t) + 4\pi T_{\nu\mu}, \end{aligned} \tag{18}$$

$$G_{\mu\nu} = 8\pi T_{\mu\nu}^{eff}, \tag{19}$$

where

$$T_{\mu\nu}^{eff} = \frac{1}{(1 + f_{\mathbb{T}})} \left\{ \frac{g_{\mu\nu}\mathbb{T}}{16\pi} (1 + f_{\mathbb{T}}) - \frac{S_{\nu\mu}{}^\lambda}{4\pi} (f_{\mathbb{T}\mathbb{T}}\partial_\lambda\mathbb{T} + f_{\mathbb{T}\mathcal{T}}\partial_\lambda\mathcal{T}) - \frac{g_{\mu\nu}}{16\pi} (f + \mathbb{T}) + \frac{f_{\mathcal{T}}}{8\pi} (T_{\nu\mu} + g_{\mu\nu}p_t) + T_{\nu\mu} \right\}. \tag{20}$$

The covariant derivative of Equation (19) reads as

$$\begin{aligned} \nabla_\mu T_\nu{}^\mu &= \frac{1}{(4\pi + (1/2)f_{\mathcal{T}})} \left\{ (f_{\mathbb{T}\mathbb{T}}\partial_\sigma\mathbb{T} + f_{\mathbb{T}\mathcal{T}}\partial_\sigma\mathcal{T}) e^a{}_\nu [e^{-1}\partial_\lambda(ee_a{}^\alpha S_\alpha{}^{\sigma\lambda}) - e_a{}^\alpha T^\lambda{}_{\gamma\alpha} S_\lambda{}^{\gamma\sigma}] - \frac{1}{4}(1 + f_{\mathbb{T}})\partial_\nu\mathbb{T} \right. \\ &+ \nabla_\mu S_\nu{}^{\mu\lambda} (f_{\mathbb{T}\mathbb{T}}\partial_\lambda\mathbb{T} + f_{\mathbb{T}\mathcal{T}}\partial_\lambda\mathcal{T}) + S_\nu{}^{\mu\lambda} (f_{\mathbb{T}\mathbb{T}\mathbb{T}}\partial_\mu\mathbb{T}\partial_\lambda\mathbb{T} + f_{\mathbb{T}\mathbb{T}\mathcal{T}}\partial_\mu\mathcal{T}\partial_\lambda\mathbb{T} + f_{\mathbb{T}\mathbb{T}}\nabla_\mu\partial_\lambda\mathbb{T} + f_{\mathbb{T}\mathcal{T}\mathbb{T}}\partial_\mu\mathbb{T}\partial_\lambda\mathcal{T} \\ &+ f_{\mathcal{T}\mathcal{T}\mathcal{T}}\partial_\mu\mathcal{T}\partial_\lambda\mathcal{T} + f_{\mathbb{T}\mathcal{T}}\nabla_\mu\partial_\lambda\mathcal{T}) + \frac{1}{4}(f_{\mathbb{T}}\partial_\nu\mathbb{T} + f_{\mathcal{T}}\partial_\nu\mathcal{T} + \partial_\nu\mathbb{T}) - \frac{1}{2}(f_{\mathcal{T}\mathbb{T}}\partial_\mu\mathbb{T} + f_{\mathcal{T}\mathcal{T}}\partial_\mu\mathcal{T}) (T_\nu{}^\mu + \delta_\nu^\mu p_t) \\ &\left. - \frac{1}{2}f_{\mathcal{T}}\partial_\nu p_t \right\}. \end{aligned} \tag{21}$$

In the current work, we specifically focus on the existence of anisotropic strange stars in the extended teleparallel gravity and for this purpose, we use an algebraic function following the prescription by Harko et al. [62] to comply with the observational data, such that $f(\mathbb{T}, \mathcal{T}) = \omega \mathbb{T}^n \mathcal{T} - 2\Lambda$, where ω , n and Λ are arbitrary constants, specifically ω represents the matter–geometry coupling constant, n is a pure number (for simplicity assumed to be unity here) and Λ can be recognized as the Cosmological constant as has been treated in their above-mentioned work by Harko et al. [62].

For the above-mentioned functional form of $f(\mathbb{T}, \mathcal{T})$ the Equation (19) reduces to

$$G_{\mu\nu} = 8\pi T_{\mu\nu}^{eff}, \tag{22}$$

where

$$T_{\mu\nu}^{eff} = g_{\mu\nu} \left[\frac{(-\omega(\rho - p_r - 2p_t) + 2\Lambda)}{16\pi} + \frac{\omega p_t}{8\pi} \right] + T_{\nu\mu} \left(1 + \frac{\omega}{8\pi} \right), \tag{23}$$

and

$$\nabla_\mu T_\nu{}^\mu = \frac{1}{(4\pi + (1/2)\omega)} \left\{ \frac{\omega}{4} (\partial_\nu\mathcal{T}) - \frac{\omega}{2}\partial_\nu p_t \right\}. \tag{24}$$

3. Explicit Stellar Equations in $f(\mathbb{T}, \mathcal{T})$ Gravity under the Einsteinian Spacetime

To describe the interior spacetime of the spherically symmetric static stellar system, we take the metric as follows:

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{25}$$

where the metric potentials ν and λ are the functions of the radial coordinate r only.

In order to preserve the invariance under local Lorentz transformations of line element and to get the form (2), the choice of tetrad matrices is not unique. Following the References [78–82] one can get $e = \det [e^i_\mu] = e^{(v+\lambda)/2} r^2 \sin(\theta)$ and with Equations (3)–(10), the torsion scalar in terms of r becomes

$$T(r) = \frac{2e^{-\lambda}}{r^2} (e^{\lambda/2} - 1) (e^{\lambda/2} - 1 - rv'), \tag{26}$$

where the prime (') denotes the derivative with respect to the radial coordinate r .

Now the nonzero components of the Einstein tensors can be provided as follows:

$$G_0^0 = \frac{e^{-\lambda}}{r^2} (-1 + e^\lambda + \lambda' r), \tag{27}$$

$$G_1^1 = \frac{e^{-\lambda}}{r^2} (-1 + e^\lambda - v' r), \tag{28}$$

$$G_2^2 = G_3^3 = \frac{e^{-\lambda}}{4r} [2(\lambda' - v') - (2v'' + v'^2 - v'\lambda')r], \tag{29}$$

where primes stand for derivatives with respect to the radial coordinate r only.

Substituting Equation (15) into Equation (22), we find the explicit form of the Einstein field equations for the interior metric (25) as follows:

$$e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = 8\pi \left\{ \left[\frac{(-\omega(\rho - p_r - 2p_t) + 2\Lambda)}{16\pi} + \frac{\omega p_t}{8\pi} \right] + \rho \left(1 + \frac{\omega}{8\pi} \right) \right\} = 8\pi \rho^{eff}, \tag{30}$$

$$e^{-\lambda} \left(\frac{v'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = -8\pi \left\{ \left[\frac{(-\omega(\rho - p_r - 2p_t) + 2\Lambda)}{16\pi} + \frac{\omega p_t}{8\pi} \right] - p_r \left(1 + \frac{\omega}{8\pi} \right) \right\} = 8\pi p_r^{eff}, \tag{31}$$

$$\frac{e^{-\lambda}}{4r} [2(\lambda' - v') - (2v'' + v'^2 - v'\lambda')r] = -8\pi \left\{ \left[\frac{(-\omega(\rho - p_r - 2p_t) + 2\Lambda)}{16\pi} + \frac{\omega p_t}{8\pi} \right] - p_t \left(1 + \frac{\omega}{8\pi} \right) \right\} = 8\pi p_t^{eff}. \tag{32}$$

Here ρ^{eff} , p_r^{eff} and p_t^{eff} represent the effective energy density, radial pressure and tangential pressure for our system and given as

$$\rho^{eff} = \rho + \frac{\omega}{16\pi} (\rho + p_r + 4p_t + 2\Lambda), \tag{33}$$

$$p_r^{eff} = p_r + \frac{\omega}{16\pi} (\rho + p_r - 4p_t - 2\Lambda), \tag{34}$$

$$p_t^{eff} = p_t + \frac{\omega}{16\pi} (\rho - p_r + 2p_t - 2\Lambda). \tag{35}$$

In the present model we have introduced p_r in a very simple bag model and p_t by hand, treating only gravity in a serious way. However, the physical origin of p_t could be either for electrically charged strange stars [25] or for solid quark stars with rigidity [83]. Astrophysical origin as well as consequence of p_t are also available along with the latter one in the following references [12,13,18,84–97].

We assume that the SQM distribution inside the strange stars is governed by the simple phenomenological MIT Bag model equation of state (EOS) [98]. In the Bag model, by introducing ad hoc Bag functions, all the corrections of the energy and pressure functions of SQM have been maintained. We also consider that the quarks are non-interacting and massless in a simplified Bag model. The quark pressure therefore can be defined as

$$p_r = \sum_{f=u,d,s} p^f - B, \tag{36}$$

where p^f is the individual pressure of the up (u), down (d) and strange (s) quark flavors and B is the vacuum energy density (also well known as the ‘Bag’ constant) which is a constant quantity within a numerical range. In the present paper, we consider the value of the Bag constant as $B = 83 \text{ MeV}/fm^3$ [23].

Now the individual quark pressure (p^f) can be defined as $p^f = \frac{1}{3}\rho^f$, where ρ^f is the energy density of the individual quark flavor. Hence, the energy density of the de-confined quarks inside the Bag is given by

$$\rho = \sum_{f=u,d,s} \rho^f + B. \tag{37}$$

Using Equations (36) and (37), we have the EOS for SQM given as

$$p_r = \frac{1}{3}(\rho - 4B). \tag{38}$$

It is observed that ignoring critical aspects of the quantum particle physics in the framework of GR several authors [59,60,99–106] have successfully introduced this simplified form of the MIT Bag EOS to study stellar systems made of SQM.

To have non-singular monotonically decreasing matter density inside the spherically symmetric stellar system, following Mak and Harko [107], we assume a simplified form of ρ given as

$$\rho(r) = \rho_c \left[1 - \left(1 - \frac{\rho_0}{\rho_c} \right) \frac{r^2}{R^2} \right], \tag{39}$$

where ρ_c and ρ_0 are two specific constants and denote the maximum and minimum values of ρ at the center and on the surface, respectively.

Now following Moraes et al. [108] we consider p_t is related to ρ by a relation given as

$$p_t = c_1\rho + c_2, \tag{40}$$

where c_1 and c_2 are purely constants.

We define the mass function of the spherically symmetric stellar system as [55]

$$m(r) = 4\pi \int_0^r \rho_{eff}(r) r^2 dr. \tag{41}$$

At this juncture, we consider the Schwarzschild metric to represent the exterior spacetime of our system given by

$$ds^2 = \left(1 - \frac{2M}{r} \right) dt^2 - \frac{dr^2}{\left(1 - \frac{2M}{r} \right)} - r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

where M is the total mass of the stellar system.

Now, substituting Equation (41) into Equation (30) we find

$$e^{-\lambda(r)} = 1 - \frac{2m(r)}{r}, \tag{42}$$

where one can note that the metric potential e^λ is of Schwarzschild type.

In this case, the conservation equation (24) in $f(T, \mathcal{T})$ gravity takes the following form

$$-p'_r - \frac{v'}{2}(p_r + \rho) + \frac{2}{r}(p_t - p_r) = \frac{1}{(4\pi + (1/2)\omega)} \left\{ \frac{\omega\rho'}{4} - \frac{\omega p'_r}{4} - \omega p'_t \right\}. \tag{43}$$

The essential stellar structure equations required to describe static spherically symmetric sphere in $f(R, \mathcal{T})$ gravity theory are given by

$$\begin{aligned} \frac{dm}{dr} &= 4\pi r^2 \rho^{eff}, \\ \frac{dp_r}{dr} &= \frac{1}{\left[-1 + \frac{\omega}{16\pi+2\omega} \left(1 - \frac{d\rho}{dp_r} + 4\frac{dp_t}{dp}\right)\right]} \left\{ -(\rho + p_r) \left[\frac{4\pi r \rho_r^{eff} + \frac{m}{r^2}}{\left(1 - \frac{2m}{r}\right)} \right] + \frac{2}{r}(p_t - p_r) \right\} \end{aligned} \tag{44}$$

4. Solution to the Einstein Field Equations in $f(\mathbb{T}, \mathcal{T})$ Gravity for Stellar Modeling

Substituting Equation (38) in Equations (39) and (40), we have the following pressures

$$p_r = \frac{1}{3}\rho_c \left[1 - \left(1 - \frac{\rho_0}{\rho_c}\right) \frac{r^2}{R^2} \right] - \frac{4}{3}B, \tag{45}$$

$$p_t = c_1\rho_c \left[1 - \left(1 - \frac{\rho_0}{\rho_c}\right) \frac{r^2}{R^2} \right] + c_2. \tag{46}$$

Again, substituting Equations (38), (45) and (46) into Equations (33), (34) and (35), we get the following physical parameters:

$$\rho^{eff} = \frac{R^2(-2B + 3\Lambda)\omega + 2(12\pi + \omega)(r^2\rho_0 + (R-r)(R+r)\rho_c) + 6\omega c_1(R^2 + r^2\rho_0 + (R-r)(R+r)\rho_c)}{24\pi R^2}, \tag{47}$$

$$p_r^{eff} = -\frac{R^2(3\Lambda\omega + 2B(16\pi + \omega)) - 2(4\pi + \omega)(r^2\rho_0 + (R-r)(R+r)\rho_c) + 6\omega c_1(R^2 + r^2\rho_0 + (R-r)(R+r)\rho_c)}{24\pi R^2}, \tag{48}$$

$$p_t^{eff} = \frac{3(8\pi + \omega)c_1(R^2 + r^2\rho_0 + (R-r)(R+r)\rho_c) + \omega(r^2(\rho_0 - \rho_c) + R^2(2B - 3\Lambda + \rho_c))}{24\pi R^2}. \tag{49}$$

Using Equation (47), we integrate Equation (44) which yields the mass function as

$$m(r) = \frac{r^5(16\pi + \omega + 4c_1\omega)(\rho_0 - \rho_c)}{20R^2} + \frac{\rho_c r^3(16\pi + \omega + 4c_1\omega)}{12} - \frac{r^3\omega(2B - 6c_2 - 3\Lambda)}{18} + d, \tag{50}$$

where d is the integrating constant.

With the help of Equations (31), (39), (42), (45), (46) and (50), we have two unknown functions of r of the given metric (25) as

$$\lambda(r) = -\ln\left(1 - \frac{r^4(16\pi + \omega + 4c_1\omega)(\rho_0 - \rho_c)}{20R^2} - \frac{\rho_c r^2(16\pi + \omega + 4c_1\omega)}{6} + \frac{r^2\omega(2B - 6c_2 - 3\Lambda)}{9} - \frac{2d}{r}\right) \tag{51}$$

and

$$v(r) = \int \left\{ \frac{r}{1 - \frac{2m(r)}{r}} \left[-8\pi \left\{ \left[\frac{-\omega(\rho - p_r - 2p_t) + 2\Lambda}{16\pi} + \frac{\omega p_t}{8\pi} \right] - p_r \left(1 + \frac{\omega}{8\pi} \right) \right\} + \frac{1}{r^2} \right] - \frac{1}{r} \right\} dr. \tag{52}$$

5. Physical Features of Strange (Quark) Stars in $f(\mathbb{T}, \mathcal{T})$ Gravity

In this section, we study some physical features of the compact star, in order to examine the physical validity and stability of the system in the $f(\mathbb{T}, \mathcal{T})$ gravity.

5.1. Energy Conditions

According to relativistic classical field theories of gravitation, anisotropic fluid model must obey the energy conditions. There is often a linear relationship between the energy density and pressure of the matter by the name (i) Null Energy Condition (NEC), (ii) Weak Energy Condition (WEC), (iii) Dominant Energy Condition (DEC) and (iv) Strong Energy Condition (SEC) [95,109]. In the $f(\mathbb{T}, \mathcal{T})$ gravity theory these are as follows:

$$NEC : \rho_{eff} + p_{effr} \geq 0, \rho_{eff} + p_{efft} \geq 0, \tag{53}$$

$$WEC : \rho_{eff} + p_{effr} \geq 0, \rho_{eff} \geq 0, \rho_{eff} + p_{efft} \geq 0,$$

$$SEC : \rho_{eff} + p_{effr} \geq 0, \rho_{eff} + p_{efft} \geq 0, \rho_{eff} + p_{effr} + 2p_{efft} \geq 0, \tag{54}$$

$$DEC : \rho_{eff} \geq 0, \rho_{eff} - p_{effr} \geq 0, \rho_{eff} - p_{efft} \geq 0. \tag{55}$$

5.2. Mass–Radius Relation

From Equation (50), we can rewrite the mass function for our stellar system in $f(\mathbb{T}, \mathcal{T})$ gravity as follows:

$$m(r) = m_{GR} + \omega m_{MG}, \tag{56}$$

where

$$m_{GR} = -\frac{4\pi r^3 (-5R^2\rho_c + 3r^2(-\rho_0 + \rho_c))}{15R^2}, \tag{57}$$

$$\begin{aligned} m_{MG} &= \frac{r^3 (5R^2(-2B + 3\Lambda) + 30R^2c_2 + 6r^2(1 + 3c_1)\rho_0 + 2(5R^2 - 3r^2)(1 + 3c_1)\rho_c)}{90R^2} \\ &= \frac{r^3}{12} \frac{2(5R^2(-2B + 3\Lambda) + 30R^2c_2)}{15R^2} + \frac{(1 + 3c_1)}{12\pi} m_{GR}, \end{aligned} \tag{58}$$

where m_{GR} represents the mass function of the SQM distribution in the Einstein frame whereas in the expression of $m(r)$ the second term, i.e., m_{MG} , represents the mass of the matter distribution and originates due to coupling between the matter and geometry in the $f(\mathbb{T}, \mathcal{T})$ gravity.

In order to plot the Mass (M/M_\odot) versus Radius (R) curve for the strange stars due to different values of ω , we can rewrite (56), the mass function for our stellar system in $f(\mathbb{T}, \mathcal{T})$ gravity, as follows:

$$M(R) = M_{GR} + \omega M_{MG}, \tag{59}$$

where

$$M_{GR} = \frac{4}{15} \pi R^3 (3\rho_0 + 2\rho_c), \tag{60}$$

$$M_{MG} = \frac{1}{18} R^3 (-2B + 3\Lambda + 6c_2) + \frac{(1 + 3c_1)}{12\pi} M_{GR}. \tag{61}$$

It is already mentioned that the strange star candidate LMC X – 4 has specific total mass and radius as $M = 1.29 M_\odot$ [76] and $R = 9.711$ km [77] respectively. Based on this data set it can be shown that the central density of the star is $\sim 1.961 \times 10^{15}$ gm/cm³ whereas the surface density is $\sim 1.045 \times 10^{15}$ gm/cm³ [35]. It is argued that according to quantum chromodynamics at sufficiently high energy densities, hadronic nuclear matter undergoes a deconfinement transition to a new phase of quarks and gluons [110]. Based on astrophysical observations and theoretical calculations in a

model-independent way, Annala et al. [111] have inferred that the matter in the interior of maximally massive stable neutron stars with mass $M \approx 2 M_{\odot}$ exhibits characteristics of the deconfined phase, which can be interpreted as evidence for the presence of quark-matter cores.

5.3. Stability of the Stellar Model

In order to study the stability of the stellar model in a more formal manner in $f(\mathbb{T}, \mathcal{T})$ gravity, the following three issues will be discussed: (i) Modified form of the TOV equation, (ii) Herrera cracking concept and (iii) adiabatic index in the following sub-subsections.

5.3.1. Modified TOV Equation in $f(\mathbb{T}, \mathcal{T})$ Gravity Theory

To check the hydrostatic equilibrium of our proposed model, the TOV [112,113] condition is very useful which is followed by the Equation (43). There exist four different forces, such as the gravitational force (F_g), hydrostatic force (F_h), anisotropic force (F_a) and an extra force in connection to $f(\mathbb{T}, \mathcal{T})$ gravity (F_e) provided in Equation (43) and can be rewritten as

$$F_g + F_h + F_a + F_e = 0,$$

where

$$\left. \begin{aligned} F_g &= \frac{v'}{r}(\rho + p_r), \\ F_h &= -p'_r, \\ F_a &= \frac{2}{r}(p_t - p_r), \\ F_e &= -\frac{1}{4\pi + \frac{\omega}{2}} \left(\frac{\omega \rho'}{4} - \frac{\omega p'_r}{4} - \omega p'_t \right). \end{aligned} \right\} \quad (62)$$

5.3.2. Principle of Causality

Anisotropic fluid stellar model will be physically admissible if we pay special attention to making the boundaries of the radial and the transversal speeds of sound in $(0, 1)$ within the matter distribution, i.e., they are less than the speed of light c (in relativistic geometrized units, the speed of light c becomes 1). This is known as the Causality condition which is based on the concept of the Cracking condition as provided by Herrera [114] which is generally expressible as $0 \leq v_s^2 = \frac{dp}{d\rho} \leq 1$. However, instead of physical p and ρ , we have here employed p^{eff} and ρ^{eff} which are involved with the coupling factor ω . This allows us to see the contribution of the considered $f(\mathbb{T}, \mathcal{T})$ theory of gravity, in particular, the effect of the coupling parameter. Thus, the radial and transversal speeds of sound of our model are

$$v_{sr}^2 = \frac{dp_r^{eff}}{d\rho^{eff}} = -1 + \frac{2(8\pi + \omega)}{12\pi + \omega + 3\omega c_1}, \quad (63)$$

$$v_{st}^2 = \frac{dp_t^{eff}}{d\rho^{eff}} = \frac{\omega + 3(8\pi + \omega)c_1}{2(12\pi + \omega + 3\omega c_1)}. \quad (64)$$

5.4. Cracking Condition

There is an another way to check the stability of the system through the Herrera cracking condition [114,115]. Here, we check whether the square of radial speed of sound exceeds the square of transversal speed of sound or not for this anisotropic fluid star model. So, this criterion sets up

$$-1 \leq v_{st}^2 - v_{sr}^2 \leq 1 = \begin{cases} -1 \leq v_{st}^2 - v_{sr}^2 \leq 0 & \text{for potentially stable model} \\ 0 \leq v_{st}^2 - v_{sr}^2 \leq 1 & \text{for potentially unstable model} \end{cases} \quad (65)$$

Using Equation (65), we calculate the difference of two speeds and they are

$$v_{st}^2 - v_{sr}^2 = -\frac{8\pi + \omega - 3(8\pi + 3\omega)c_1}{2(12\pi + \omega + 3\omega c_1)}, \quad (66)$$

$$|v_{st}^2 - v_{sr}^2| = \left| -\frac{8\pi + \omega - 3(8\pi + 3\omega)c_1}{2(12\pi + \omega + 3\omega c_1)} \right|. \quad (67)$$

Adiabatic Index

The adiabatic index is a crucial feature to investigate regarding the stable status of an anisotropic fluid star model. It has been proposed that the radial adiabatic index should be greater than $\frac{4}{3}$, which represents the rigidity of the EOS parameter of the model [116–119]. The radial adiabatic index is defined as

$$\Gamma_r = \frac{4(8\pi + \omega)(4\pi + \omega - 3\omega c_1)(R^2(B - \rho_c) + r^2(-\rho_0 + \rho_c))}{(12\pi + \omega + 3\omega c_1)(R^2(3\Lambda\omega + 2B(16\pi + \omega)) + 6R^2\omega c_2 - 2(4\pi + \omega - 3\omega c_1)(r^2\rho_0 + (R - x)(R + r)\rho_c))}, \quad (68)$$

$$\Gamma_t = \frac{3(\omega + 3(8\pi + \omega)c_1)(R^2(8\pi + 3\omega)c_2 + (8\pi + \omega + (8\pi + 3\omega)c_1)(r^2\rho_0 + (R - r)(R + r)\rho_c))}{2(12\pi + \omega + 3\omega c_1)(R^2(2B - 3\Lambda)\omega + 3R^2(8\pi + \omega)c_2 + (\omega + 3(8\pi + \omega)c_1)(r^2\rho_0 + (R - r)(R + r)\rho_c))}. \quad (69)$$

5.5. Compactification Factor and Redshift

According to Biswas et al. [61], the compactness of the anisotropic fluid model is given by the compactification factor

$$u(r) = \frac{m(r)}{r} = \frac{r^4(16\pi + \omega + 4c_1\omega)(\rho_0 - \rho_c)}{20R^2} + \frac{\rho_c r^2(16\pi + \omega + 4c_1\omega)}{12} - \frac{r^2\omega(2B - 6c_2 - 3\Lambda)}{18} + \frac{d}{r} \quad (70)$$

and the twice of the compactification factor [?] is followed by

$$2u(r) = \frac{r^4(16\pi + \omega + 4c_1\omega)(\rho_0 - \rho_c)}{10R^2} + \frac{\rho_c r^2(16\pi + \omega + 4c_1\omega)}{6} - \frac{r^2\omega(2B - 6c_2 - 3\Lambda)}{9} + \frac{2d}{r}. \quad (71)$$

The surface redshift can be defined as

$$z_s = \frac{1}{\sqrt{1 - \frac{2m(r)}{r}}} - 1 = -1 + \frac{1}{\sqrt{1 - \frac{5R^2(r^3(-2B + 3\Lambda)\omega + 18c_3) + 2r^3(15R^2\omega c_2 + (12\pi + \omega + 3\omega c_1)(3r^2(\rho_0 - \rho_c) + 5R^2\rho_c))}{45R^2r}}}} \quad (72)$$

6. A Comparison Study Between $f(\mathbb{T}, \mathcal{T})$ gravity and Standard GR

In this section, we perform a comparative study with the standard results of GR, in order to show the effect of our model in the framework of $f(\mathbb{T}, \mathcal{T})$ gravity and why it justifies our analysis, and thereafter give a relevant discussion on the results. These aspects can be discussed as follows:

(i) We have plotted in Figure 1 the behavior of the metric potentials e^ν and e^λ with respect to the fractional radial coordinate r/R respectively both of which exhibit normal features as are expected, i.e., they should exhibit regular behavior at the center as well as on the surface of the stellar system. On the other hand, in Figure 2 the variation of the effective energy density ρ^{eff} and the effective pressures p_r^{eff} and p_t^{eff} are shown with respect to r/R respectively whereas the anisotropic feature is shown in Figure 3. We observe from all the plots of Figure 2 that a kind of monotony is evolving from the center to a minimum value and hence indicates the physically viable character of the $f(\mathbb{T}, \mathcal{T})$ gravity model under consideration. One can note that the standard solutions to the Einstein field equations for stellar modeling can be recovered for $\varpi = 0$ and $\Lambda = 0$.

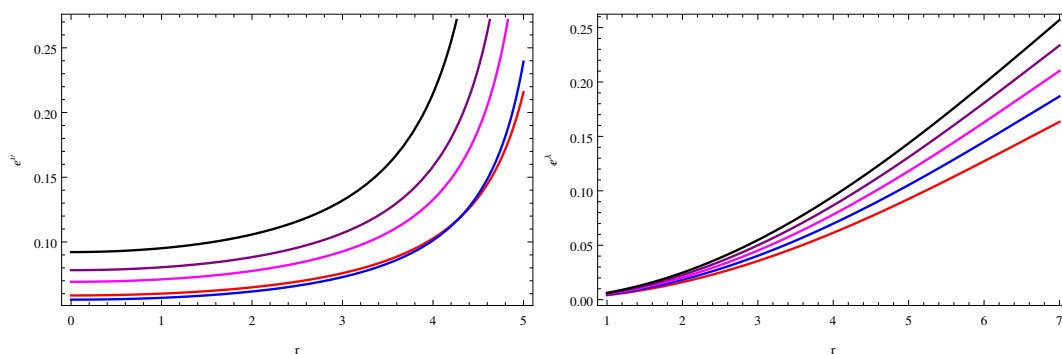


Figure 1. Plot of $e^{\nu(r)}$ and $e^{\lambda(r)}$ versus r for the strange star candidate *LMC X – 4*. The red, purple, magenta, blue and black colors represent respectively cases $\varpi = 0$, $\varpi = 0.5$, $\varpi = 1$, $\varpi = 1.5$ and $\varpi = 2$.

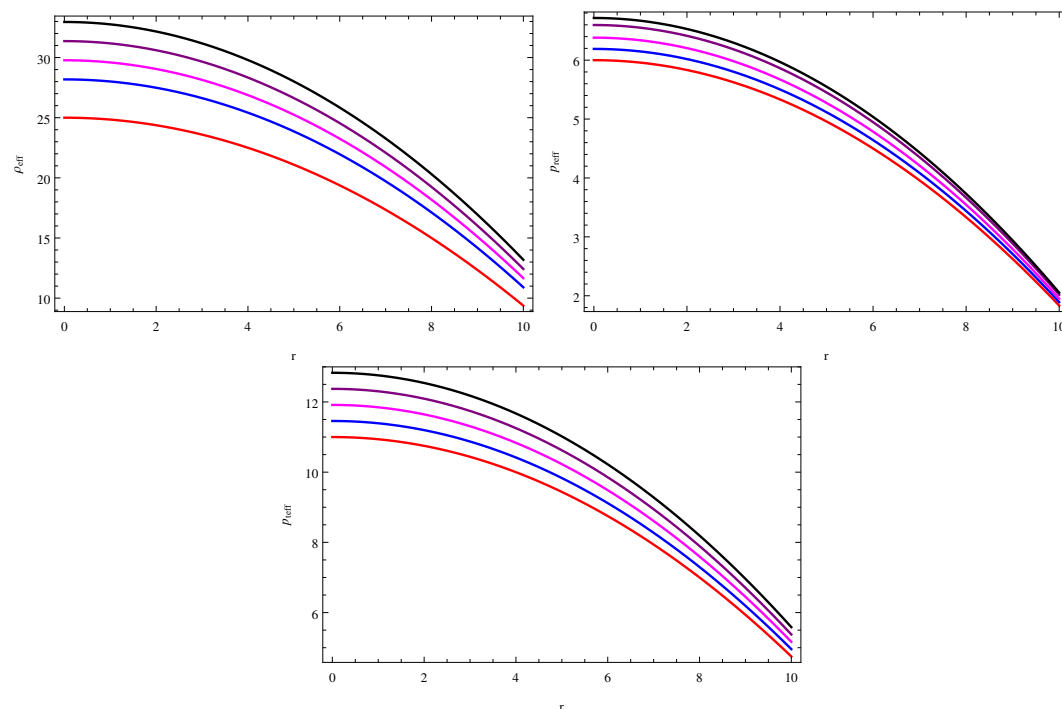


Figure 2. Plot of ρ^{eff} , p_r^{eff} and p_t^{eff} versus r for the strange star candidate *LMC X – 4*. The red, purple, magenta, blue and black colors represent, respectively, cases $\varpi = 0$, $\varpi = 0.5$, $\varpi = 1$, $\varpi = 1.5$ and $\varpi = 2$.

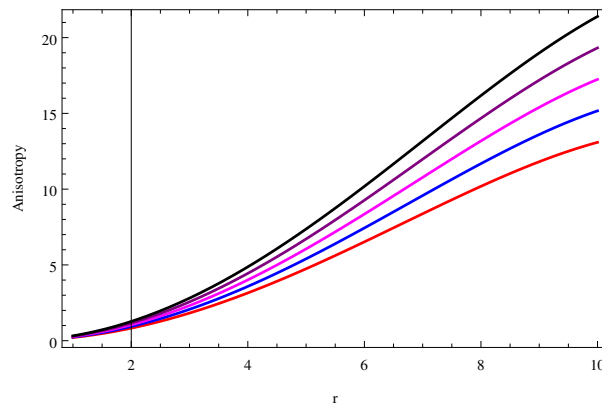


Figure 3. Plot of anisotropy versus r for the strange star candidate $LMC X - 4$. The red, purple, magenta, blue and black colors represent respectively cases $\omega = 0, \omega = 0.5, \omega = 1, \omega = 1.5$ and $\omega = 2$.

(ii) From Figure 4, we can conclude that the conditions NEC, WEC, DEC and SEC are satisfied for different values of coupling matter geometry parameter ω for our anisotropic fluid model absence of which GR features can be retrieved.

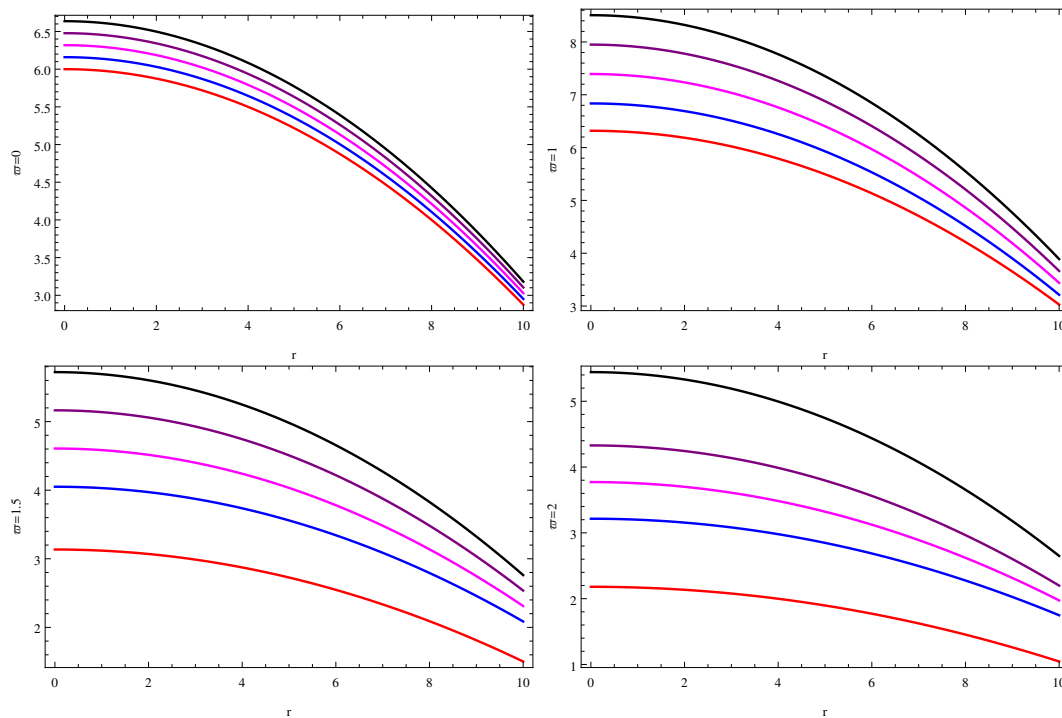


Figure 4. Plot of Null Energy Condition (NEC), Weak Energy Condition (WEC), Dominant Energy Condition (DEC) and Strong Energy Condition (SEC) versus r for the strange star candidate $LMC X - 4$ due to different chosen values of ω . The red, purple, magenta, blue and black colors represent respectively $\rho_{eff} - p_{effr}, \rho_{eff} - p_{efft}, \rho_{eff} + p_{effr}, \rho_{eff} + p_{efft}, \rho_{eff} + p_{effr} + 2p_{efft}$ with $\omega = 0, \omega = 0.5, \omega = 1, \omega = 1.5$ and $\omega = 2$ respectively.

(iii) Since $r \rightarrow 0, m(r) \rightarrow 0$ in Figure 5 so one can notice that the mass function is regular at the origin. From this mass–radius relationship, we also note a striking feature which depicts that with increasing value of radius the mass of the physical system also increases and beyond $R \sim 12$ km mass starts gradually decreasing. Thus, we observe a stable configuration of strange stars around 12 km, beyond which it suffers from instability. Moreover, we observe that in contrast to the GR case the curve allows for a massive quark star at the same radius.

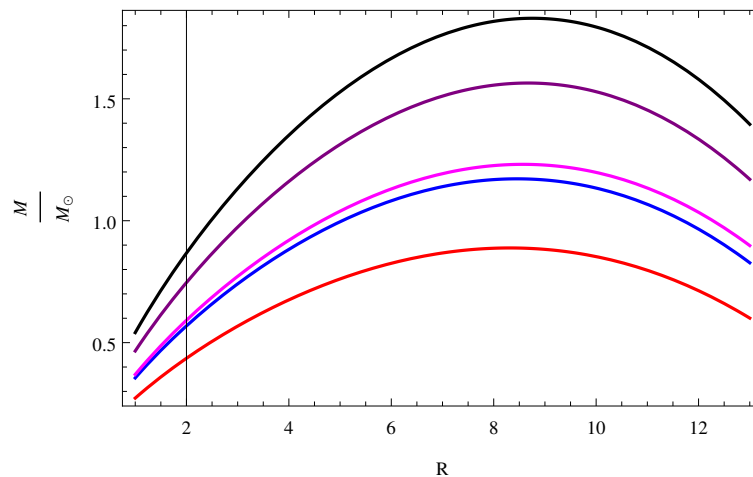


Figure 5. Plot of Mass (M/M_{\odot}) versus Radius (R in km) curve for the strange star candidate *LMC X – 4* due to different values of ω . The red, purple, magenta, blue and black colors represent respectively cases $\omega = 0$, $\omega = 0.5$, $\omega = 1$, $\omega = 1.5$ and $\omega = 2$.

(iv) Based on Equation (62), i.e., the modified TOV equation, we have drawn Figure 6 which indicates that our propounded anisotropic fluid model for different values of the coupling matter–geometry parameter ω is in hydrostatic equilibrium state under the combination of considered forces.

(v) From Equations (63) and (64), we observe that the Causality condition depends on the coupling matter–geometry parameter ω and the constant c_1 . The related Figure 7 exhibits that the Principle of Causality is preserved inside the anisotropic strange star in $f(\mathbb{T}, \mathcal{T})$ as here $0 \leq v_{sr} \leq 1$ and $0 \leq v_{st}^2 \leq 1$ if $0 \leq c_1 \leq 1$.

(vi) In a similar way, Equations (66) and (67) indicate that the Cracking condition depends on the coupling matter–geometry parameter ω and c_1 . If $0 \leq c_1 \leq \frac{1}{3}$ then we have $-1 \leq v_{st}^2 - v_{sr}^2 \leq 0$ and $0 \leq |v_{st}^2 - v_{sr}^2| \leq 1$ which is evident from Figure 8 and inconformity with the criteria imposed on the system [114,120]. It means that we have a potentially stable anisotropic star fluid model.

(vii) From Figure 9, we note that the variation of both Γ_r and Γ_t is a monotonically increasing function at all the interior points of our model with $\Gamma_r > \frac{4}{3}$ and hence provides confirmation on the stability of our model as predicted for the inside of an anisotropic, relativistic and dynamically stable stellar system [117,121,122].

(viii) Figure 10 (left panel) indicates that $u(r)$ is lying between the range $\frac{1}{4}$ and $\frac{1}{2}$. The $2u(r)$ follows the maximum allowed value $\frac{8}{9}$ for our proposed model. On the other hand, we have plotted a figure for the surface redshift with the help of Equation (72) in Figure 10 (right panel) which shows that it is a decreasing function starting from the core with higher value and on the surface of the star acquires a finite non-zero value. In this context, it is to be noted that Barraco and Hamity [123] proved that for an isotropic star and in the absence of the cosmological constant $Z_s < 2$. According to Böhmer and Harko [124] the surface redshift for an anisotropic star can reach a maximum higher value $Z_s \leq 5$, in the presence of the cosmological constant. Though, according to Ivanov [89], the maximum acceptable value of the surface redshift will be restricted up to 5.211. As our result for the surface redshift maintains the limit, i.e., $z_s \leq 5$ always, so the proposed model is quite realistic.

In a nutshell, in all the above figures, one can note that we get back the GR result for $\omega = 0$ whereas all the other non-zero values of ω correspond to $f(\mathbb{T}, \mathcal{T})$ gravity contribution, e.g., in Figure 6, the null effect due to modified gravity exhibits through the flat curve which coincides with the x -axis and hence shows the standard GR result. However, the effect of ω becomes gradually prominent as we go on increasing its value towards a higher magnitude.

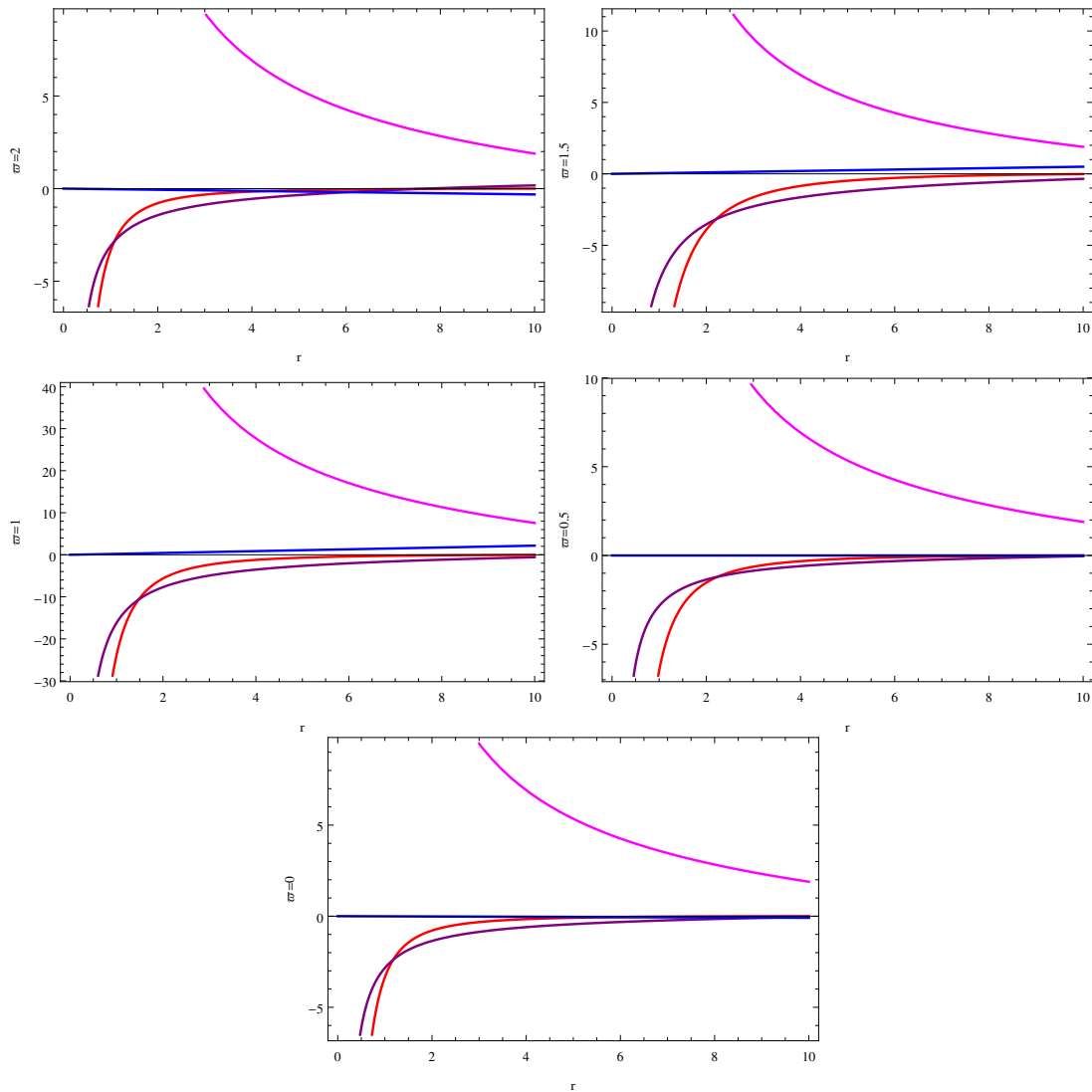


Figure 6. Plot of the different forces versus r for the strange star candidate $LMC X - 4$ due to different chosen values of ω . The red, purple, magenta, blue and black colors represent respectively F_g, F_h, F_a, F_e with $\omega = 0, \omega = 0.5, \omega = 1, \omega = 1.5$ and $\omega = 2$ respectively.

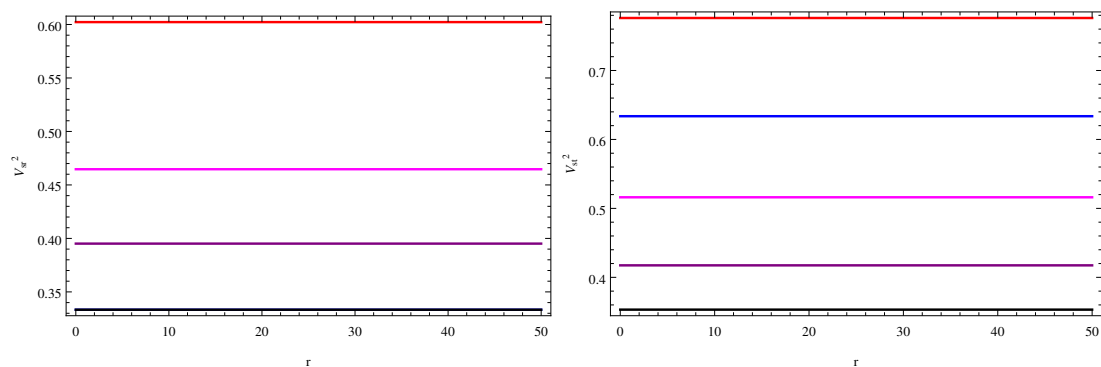


Figure 7. Plot of v_{st}^2 and v_{sr}^2 versus r for the strange star candidate $LMC X - 4$. The red, purple, magenta, blue and black colors represent respectively cases $\omega = 0, \omega = 0.5, \omega = 1, \omega = 1.5$ and $\omega = 2$.

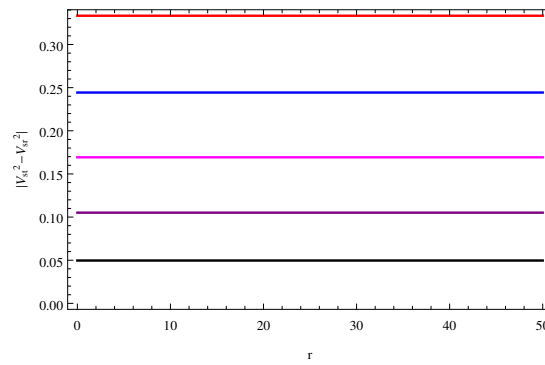


Figure 8. Plot of $|v_{st}^2 - v_{sr}^2|$ versus r for the strange star candidate *LMC X - 4*. The red, purple, magenta, blue and black colors represent respectively cases $\omega = 0, \omega = 0.5, \omega = 1, \omega = 1.5$ and $\omega = 2$.

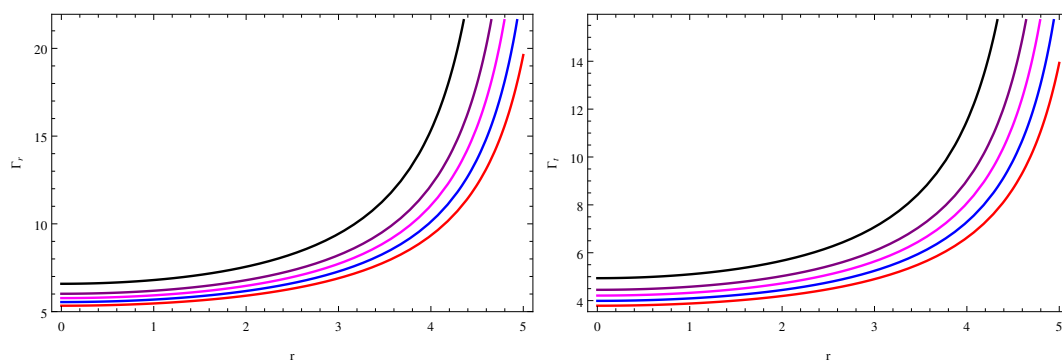


Figure 9. Plot of Γ_r and Γ_t versus r for the strange star candidate *LMC X - 4*. The red, purple, magenta, blue and black colors represent, respectively, cases $\omega = 0, \omega = 0.5, \omega = 1, \omega = 1.5$ and $\omega = 2$.

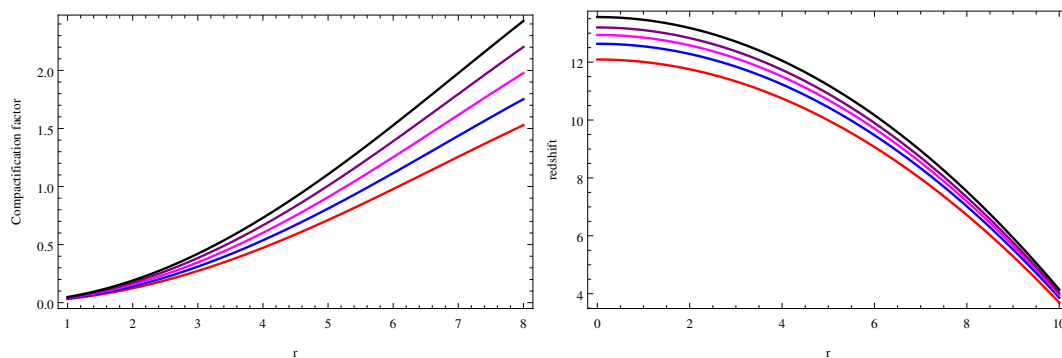


Figure 10. Plot of the compactification factor and redshift versus r for the strange star candidate *LMC X - 4*. The red, purple, magenta, blue and black colors represent respectively cases $\omega = 0, \omega = 0.5, \omega = 1, \omega = 1.5$ and $\omega = 2$.

7. Discussions and Conclusion

In this work, we present mathematical modeling of strange stars in the background of $f(\mathbb{T}, \mathcal{T})$ gravity in Einstein–Maxwell spacetime in order to obtain viable physical properties. We develop the equations of motion using anisotropic property within the spherically-symmetric strange star and thereafter explores the physical features, like the energy conditions, mass–radius relations, modified TOV equations, principle of causality, adiabatic index, redshift and stability analysis of our model.

The salient features of the presented model can be put forward as follows:

(1) For the matter-geometry coupling parameter $\omega = 0$ the usual results as available in GR can be retrieved from the results due to modified gravity $f(\mathbb{T}, \mathcal{T})$. The above statement obviously also follows from Figures 9 and 10, e.g., for $\omega = 0$ the usual form of TOV equation as in GR can easily

be retrieved because, in this situation, the force due to modified gravity vanishes, i.e., the effect of coupling also becomes ineffective. In order to verify model parameters, we have used the data for the strange star candidate *LMC X – 4* and have drawn all the Figures 9 and 10 for different values of the matter–geometry coupling parameter ω .

(2) In the present work, we have considered tetrad formalism to correspond to the metric (25), however, it is known that in spherically-symmetric solutions the diagonal tetrad is not the consistent choice. In this connection it is of note that in literature this type of approach is not at all unavailable even in the $f(\mathbb{T}, \mathcal{T})$ gravity theory [62,73–75]. The tetrad formalism has been used by Harko et al. [62] in cosmology whereas Pace and Said [74,75] as well as Ghosh et al. [73] have employed the technique in astrophysics.

(3) In this work, as a first step, we considered a very specific ansatz, viz. $f(\mathbb{T}, \mathcal{T}) = \omega T^n - 2\Lambda$, to present a simple mathematical model following the prescription by Harko et al. [62] to comply with the observational data. However, after successful execution of this specification, there are ample scopes to opt for other specified functional forms of $f(\mathbb{T}, \mathcal{T})$ gravity in the future projects following other ansatz, e.g., $f(\mathbb{T}, \mathcal{T}) = \alpha\mathcal{T} + \gamma T^2$ which has been also proposed by Harko et al. [62] or linear function in the forms (i) $f = \alpha T(r) + \beta \mathcal{T}(r) + \varphi$ as proposed by Pace and Said [74] and (ii) $f = T(r) + \mathcal{T}(r) + \omega h(\mathbb{T}, \mathcal{T}) + \mathcal{O}(\omega^2)$ as proposed by Pace and Said [75]. In the above ansatz, which has been employed in the present work, the value of n is assumed to be unity, i.e., $n = 1$, however $n \neq 1$ case also can be considered for further study.

(4) We would also like to point out here that in the present investigation, following Rahaman et al. [23], we have considered the value for Bag constant as $B = 83 \text{ MeV}/fm^3$. However, it seems that this value is quite arbitrary and needs a specific range with lower and upper values [125]. This means that further extension of the present work will provide a more realistic stellar model to validate on the physical ground.

(5) Theoretically under specific conditions some of the the up (u) and down (d) quarks get transformed into strange (s) quarks. It is argued that the cold strange matter is the true ground state of nuclear matter [1,3,126–128] and hence the u and d quarks once converted into strange matter. As a result, the entire quark matter gets converted into strange matter and in turn, the neutron star totally gets converted into a strange quark star [129]. It is pointed out through some recent simulations [130,131] that the merger process of two strange stars is possible. Consequently, the gravitational wave signal detection of these stars may enhance the probability of the strange quark matter (SQM) hypothesis.

(6) Now the curious issue may be whether the $f(\mathbb{T}, \mathcal{T})$ gravity passes the tests of binary compact star merger (BH-BH, BH-NS, NS-NS), especially clear the binary BH merging? The answer seems positive as far as the simulations based work by Bauswein et al. [130,131] on the merger process of two strange stars indicate. However, eventually, that depends on the puzzle of alternative gravity, i.e., the degeneracy in understanding both the equation of state and modified gravity (MG) [132]. On constraining recent gravitational wave observations of binary BH mergers, Motohashi and Minamitsuji [133] have classified GR solutions in MG based on their retrieval status from $MG \rightarrow GR$. According to this classification scheme, $f(\mathbb{T}, \mathcal{T})$ gravity where GR solutions are realized by the cancellation of the coupling constant, can pass the tests of binary merger (for further study following References [134,135] can be consulted).

So, based on the features as shown by Figures 9 and 10 and other aspects, we can overall conclude that our propounded model is quite realistic and appeals to further study of theoretically observable features as well as observational signatures of its predicted compact objects.

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