The Reconstruction of Non-Minimal Derivative Coupling Inflationary Potentials

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Abstract: We derive the reconstruction formulae for the inflation model with the non-minimal derivative coupling term. If reconstructing the potential from the tensor-to-scalar ratio r, we could obtain the potential without using the high friction limit. As an example, we reconstruct the potential from the parameterization $r = 8\alpha/(N+\beta)^\gamma$, which is a general form of the α-attractor. The reconstructed potential has the same asymptotic behavior as the T- and E-model if we choose $\gamma = 2$ and $\alpha \ll 1$. We also discuss the constraints from the reheating phase by assuming the parameter w_{re} of state equation during reheating is a constant. The scale of big-bang nucleosynthesis could put an upper limit on n_s if $w_{re} = 2/3$ and a low limit on n_s if $w_{re} = 1/6$.

Keywords: reconstruction; non-minimal derivative coupling inflation; reheating

1. Introduction

In the standard big-bang cosmology, inflation has successfully solved various problems, such as the flatness, horizon and monopole problems. Besides, its quantum fluctuation can produce the seed of the formation of large-scale structure [1–4]. A scalar field with a flat potential is usually chosen to investigate inflation. The most economical and fundamental candidate for the inflaton is therefore the Standard Model Higgs boson. However, the Higgs boson is disfavored by the observational data [3,5] when minimally coupled to gravity due to its large tensor-to-scalar ratio. If the kinetic term of the scalar field is non-minimal coupled to Einstein tensor, the tensor-to-scalar ratio r could be reduced to being consistent with the observational data, and the effective Higgs self-coupling λ could be the order of 1 [6,7]. This inflation model with non-minimal derivative coupling belongs to the subclass of the Horndeski theory [8], which is a general scalar–tensor theory, with field equations that are at most of the second-order derivatives of both the metric $g_{\mu\nu}$ and scalar field ϕ in four dimensions [9]. Therefore, the non-minimal derivative coupling inflation model could save the Higgs model without introducing a new degree of freedom. For more about the non-minimal derivative coupling inflation model, refer to [10–17].

The most important observables of inflation are the spectral tilt n_s and the tensor-to-scalar ratio r. To be compared with the observational data easily, they are usually expressed by the e-folding number N before the end of inflation at the horizon exit of the pivotal scale. Among them, one of the predictions that is greatly favored by the observational data may be the α-attractors, $n_s = 1 - 2/N$ and $r = 12\alpha/N^2$. Numerous inflation models make the α-attractors prediction, for example the Starobinsky model [1], the Higgs inflation with a non-minimal coupling $\xi\phi^2 R$ in the strong coupling limit $\xi \gg 1$ [18,19], the pole inflation with the kinetic term being $(\partial\phi)^2/(1-\phi^2/6\alpha)^2$ [20] and the T/E model [21,22]. It is therefore worth studying whether there are still other models that can make the prediction of

α-attractors. In this paper, we consider the non-minimal derivative coupling inflation models to investigate this α-attractors issue by reconstructing the potential. Starting from the observational data and parameterizing the observable with N, using the relationships between the observable and the potential, we can then reconstruct the potential [23,24]. By this reconstruction, the model parameters can be constrained easily and the reconstructed potential would always be consistent with the observational data [24–47].

After the inflation, it is followed by the reheating phase, which may give additional constraints on the inflation phase [46,48]. Assuming that the effective parameter w_{re} of state equation during reheating is a constant and the entropy is a conserved quantity, we can relate the e-folding number and the energy scale during reheating to those during inflation [48–54]. From these relations, the constraints on the energy scale during reheating would transfer to the constraints on the inflation model.

In this paper, we reconstruct the inflationary potentials of the non-minimal coupling inflation models and research the additional constraints from the reheating phase. The paper is organized as follows. In Section 2, we give a brief review about the inflation model with the non-minimal derivative coupling term and the reconstruction method. In Section 3, we reconstruct the potential from the parameterization of tensor-to-scalar ratio r. We discuss the constraints from the reheating in Section 4, and give the conclusion in Section 5.

2. The Relations

In this section, we develop the formulae for the reconstruction of the inflationary potential with the kinetic term non-minimal coupled to Einstein tensor. We start from the action

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{M^2} G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2V(\phi) \right], \tag{1}$$

where we choose the unit $c = M_{pl}^2 = 1/(8\pi G) = 1$ and the coupling parameter M is a constant with the dimension of mass. For the homogeneous and isotropic Universe with the Robertson–Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right) \right], \tag{2}$$

where $K = 0$ in the inflation epoch, the action (1) becomes

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[R + \left(1 + \frac{3H^2}{M^2} \right) \dot{\phi}^2 - 2V(\phi) \right]. \tag{3}$$

The kinetic term of this model is

$$(1 + \frac{3H^2}{M^2}) \frac{\dot{\phi}^2}{2} > 0, \tag{4}$$

so there are no ghosts in this model. The scale range of the parameter M is very broad. If M is extremely larger than the Hubble parameter, $M^2 \gg H^2$, the non-minimal derivative coupling term can be neglected and the model reduces to the canonical case. If M is extremely smaller than the Hubble parameter, $M^2 \ll H^2$, the non-minimal derivative coupling term dominates the inflation, and may make some new predictions different from the canonical case.

The Friedmann equation is

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{1}{3} \left[\frac{\dot{\phi}^2}{2} (1 + 9F) + V(\phi) \right], \tag{5}$$

where $F = H^2/M^2$ is the friction parameter. The equation of motion for the scalar field ϕ is

$$\frac{d}{dt} \left[a^3 \dot{\phi} (1 + 3F) \right] = -a^3 \frac{dV}{d\phi}. \tag{6}$$

For the slow-roll inflation, the slow-roll conditions are

$$\frac{1}{2}(1+9F)\dot{\phi}^2 \ll V(\phi),$$

$$|\ddot{\phi}| \ll |3H\dot{\phi}|, \tag{7}$$

$$\left|\frac{2\dot{H}}{M^2+3H^2}\right| \ll 1.$$

Under these slow-roll conditions, the background Equations (5) and (6) become

$$H^2 \approx \frac{V(\phi)}{3}, \tag{8}$$

$$3H\dot{\phi}(1+3F) \approx -V_\phi, \tag{9}$$

where $V_\phi = dV/d\phi$. With Equation (8), the friction parameter becomes

$$F \approx \frac{V(\phi)}{3M^2}. \tag{10}$$

The corresponding slow-roll parameters are

$$\epsilon_V = \frac{1}{2}\left(\frac{V_\phi}{V}\right)^2 \frac{1+9F}{(1+3F)^2}, \tag{11}$$

$$\eta_V = \frac{1}{1+3F}\frac{V_{\phi\phi}}{V}. \tag{12}$$

Using Equations (8), (9) and (11), we obtain

$$\frac{3\dot{\phi}^2(1+9F)}{2V(\phi)} \approx \epsilon_V. \tag{13}$$

The derivative of ϵ_V with respect to t is [10]

$$\begin{aligned}
\dot{\epsilon}_V = 2H\epsilon_V\Bigg[&\frac{2+21F+81F^2}{(1+9F)^2}\epsilon_V - \eta_V - \frac{1}{3}\eta_V^2\\
&-\frac{4+72F+603F^2+2538F^3+5103F^4}{3(1+3F)(1+9F)^3}\epsilon_V^2\\
&+\frac{2(2+48F+441F^2+1944F^3+3645F^4)}{3(1+3F)(1+9F)^3}\epsilon_V\eta_V\Bigg].
\end{aligned} \tag{14}$$

By using the relation $dN = -Hdt$, to the first order of slow-roll parameters, Equation (14) becomes

$$\frac{d\ln\epsilon_V}{dN} = 2\left[\eta_V - \frac{2+21F+81F^2}{(1+9F)^2}\epsilon_V\right], \tag{15}$$

where N is the e-folding number before the end of inflation at the horizon exit. The power spectrum for the scalar perturbation is [10]

$$P_\zeta \approx \frac{1+9F}{1+3F} \times \frac{H^2}{8\pi^2\epsilon_V}. \tag{16}$$

The power spectrum for the tensor perturbation is [10]

$$P_T \approx \frac{2H^2}{\pi^2}. \tag{17}$$

The scalar tilt n_s and the tensor-to-scalar ratio r are [10,55]

$$n_s - 1 = 2\eta_V - \frac{6(1+4F)}{1+9F}\epsilon_V, \tag{18}$$

$$r = \frac{16(1+3F)}{1+9F}\epsilon_V. \tag{19}$$

From Equations (15) and (18), we obtain the relation between n_s and ϵ_V,

$$n_s - 1 = \frac{d\ln\epsilon_V}{dN} - \frac{2+36F+54F^2}{(1+9F)^2}\epsilon_V. \tag{20}$$

From Equations (5) and (13), we obtain the relation between ϕ and N,

$$d\phi = \pm\sqrt{\frac{2\epsilon_V}{1+9F}}dN, \tag{21}$$

where the sign $\pm$ depends on the sign of $dV/d\phi$. Without loss of generality, in this paper, we only research the '$+$' case. Combining Equations (11) and (21), we get the relation between the potential and the slow-roll parameter,

$$\epsilon_V = \frac{1+9F}{2+6F}(\ln V)_{,N}. \tag{22}$$

By using Equations (10) and (19), Equations (16), (20) and (22) become

$$P_\zeta = \frac{2H^2}{\pi^2 r}, \tag{23}$$

$$n_s - 1 = \frac{d\ln r}{dN} - \frac{r}{8}, \tag{24}$$

$$r = 8(\ln V)_{,N}. \tag{25}$$

These relations (23)–(25) do not contain the friction parameter F, thus it is possible to reconstruct the potential from the tensor-to-scalar ratio without using the high friction limit. In the following sections, we discuss this issue.

3. The Reconstruction

In this section, we reconstruct the potential from the tensor-to-scalar ratio r. The observational data favor small r, and the α-attractor gives $r = 12\alpha/N^2$, which is small enough to be consistent with the observational data when $\alpha \ll 1$. In this section, we discuss a general parameterization of the α-attractor

$$r = \frac{8\alpha}{(N+\beta)^\gamma}, \tag{26}$$

where $\gamma > 1$, and β accounts for the contribution from the scalar field ϕ_e at the end of the inflation. From the relation (24), we obtain the spectral tilt

$$n_s - 1 = -\frac{\gamma}{N+\beta} - \frac{\alpha}{(N+\beta)^\gamma}. \tag{27}$$

With the help of relation (25), we obtain the potential

$$V = V_0 \exp\left[-\frac{\alpha}{(\gamma-1)(N+\beta)^{\gamma-1}}\right]. \tag{28}$$

Combining the slow-roll Friedmann Equation (8) and the power spectrum in Equation (23), we relate the amplitude of the power spectrum A_s to the potential,

$$A_s = \frac{2V}{3\pi^2 r}. \tag{29}$$

Substituting the reconstructed potential (28) into relation (29) and using the parameterization (26), we obtain

$$V_0 = \frac{3}{2}\pi^2 A_s r \exp\left[\frac{\alpha}{\gamma - 1}\left(\frac{r}{8\alpha}\right)^{\frac{\gamma-1}{\gamma}}\right]. \tag{30}$$

Combining Equations (28) and (22), we get the slow-roll parameter

$$\epsilon_V = \frac{1 + 3F_0 \exp\left[\alpha(1-\gamma)^{-1}(N+\beta)^{1-\gamma}\right]}{2 + 2F_0 \exp\left[\alpha(1-\gamma)^{-1}(N+\beta)^{1-\gamma}\right]}\frac{\alpha}{(N+\beta)^\gamma}, \tag{31}$$

where the amplitude of the friction parameter $F_0 = V_0/M^2$. From the condition of the end of inflation, $\epsilon_V(0) = 1$, we obtain the relation among α, β and γ

$$\frac{1 + 3F_0 \exp\left[\alpha(1-\gamma)^{-1}\beta^{1-\gamma}\right]}{2 + 2F_0 \exp\left[\alpha(1-\gamma)^{-1}\beta^{1-\gamma}\right]} \times \frac{\alpha}{\beta^\gamma} = 1. \tag{32}$$

Under the GR limit $F_0 \ll 1$, relation (32) reduces to $\alpha = 2\beta^\gamma$; under the high friction limit $F_0 \gg 1$, relation (32) reduces to $\alpha = 2\beta^\gamma/3$. From Equation (26), the tensor-to-scalar ratio r under the high friction limit is therefore smaller than that under the GR limit when β and γ is unchanged. Substituting Equation (31) into Equation (21), we get the relation between ϕ and N,

$$d\phi = \sqrt{r\left(8 + 8F_0 \exp\left[\frac{\alpha(N+\beta)^{1-\gamma}}{(1-\gamma)}\right]\right)^{-1}} dN. \tag{33}$$

Combining it with Equation (26), the relation becomes

$$d\phi = \sqrt{r\left(8 + 8F_0 \exp\left[\frac{\alpha(8\alpha/r)^{(1-\gamma)/\gamma}}{(1-\gamma)}\right]\right)^{-1}} dN. \tag{34}$$

To the first order of tensor-to-scalar ratio r, it becomes

$$d\phi = \sqrt{\frac{r}{8 + 8F_0}} dN, \tag{35}$$

and the solution is

$$\phi - \phi_0 = \begin{cases} \dfrac{2}{2-\gamma}\sqrt{\dfrac{\alpha}{1+F_0}}(N+\beta)^{\frac{2-\gamma}{2}}, & \gamma \neq 2, \\[2ex] \sqrt{\dfrac{\alpha}{1+F_0}}\ln(N+\beta), & \gamma = 2, \end{cases} \tag{36}$$

where ϕ_0 is the integration constant. Substituting Equation (36) into Equation (28), we get the reconstructed potential

$$V(\phi) = \begin{cases} V_0 \exp\left[-\lambda\left(\sqrt{1+F_0}\phi_0 - \sqrt{1+F_0}\phi\right)^{\frac{2\gamma-2}{\gamma-2}}\right], & \gamma \neq 2, \\[2ex] V_0 \exp\left[-\alpha e^{-\sqrt{1+F_0}(\phi-\phi_0)/\sqrt{\alpha}}\right], & \gamma = 2, \end{cases} \tag{37}$$

where

$$\lambda = \frac{\alpha}{\gamma - 1} \left(\frac{\gamma - 2}{2\sqrt{\alpha}} \right)^{\frac{2\gamma - 2}{\gamma - 2}}. \tag{38}$$

Therefore, we reconstruct the potential from the parameterization (26) without using the high friction limit. Furthermore, the potential (37) and parameter (38) show that the effect of the no-minimally derivative coupling term is the rescaling of the inflaton field by a factor $\sqrt{1 + F_0}$. For the α-attractors parameterization $\gamma = 2$, under the GR limit $F_0 \ll 1$, the potential reduces to [39]

$$V(\phi) = V_0 \exp\left[-\alpha e^{-(\phi - \phi_0)/\sqrt{\alpha}} \right]. \tag{39}$$

If $\alpha \ll 1$, this potential reduces to

$$V(\phi) = V_0 \left[1 - \alpha e^{-(\phi - \phi_0)/\sqrt{\alpha}} \right], \tag{40}$$

which is asymptotic behavior of the T-model and E-model.

Taking $N = 60$ and $F_0 \gg 1$, and comparing the theoretical predictions (26) and (27) with the Planck 2018 data [5], we obtain the constraints on the parameters β and γ shown in Figure 1. Taking $\gamma = 2$, $\beta = 1$ and $N = 60$, the theoretical predictions are $n_s = 0.967$ and $r = 0.0014$. With these parameters, the plot of the potential is shown in Figure 2.
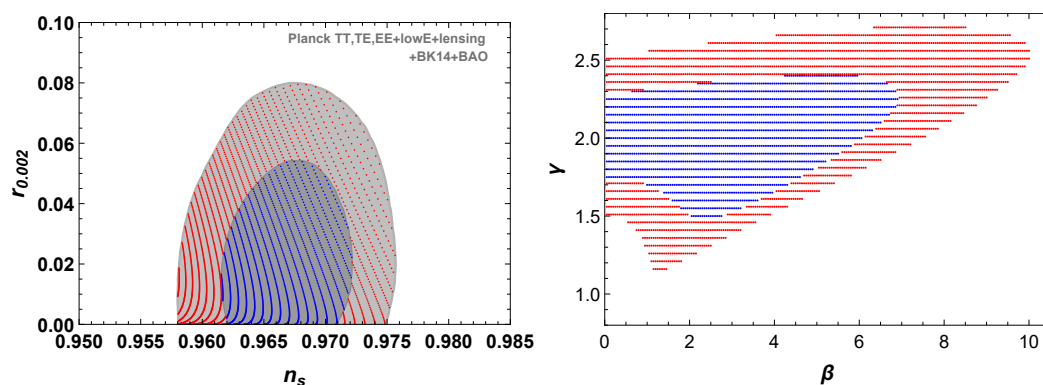

Figure 1. The constraints on n_s and $r_{0.002}$ from Planck data [5] and the theoretical predictions for the parameterization (26) in the high friction limit. The Planck constraints on n_s and r are displayed in the left panel and the constraints on β and γ for $N = 60$ are displayed in the right panel. The red and blue regions denote the 68% and 95% confidence level, respectively.
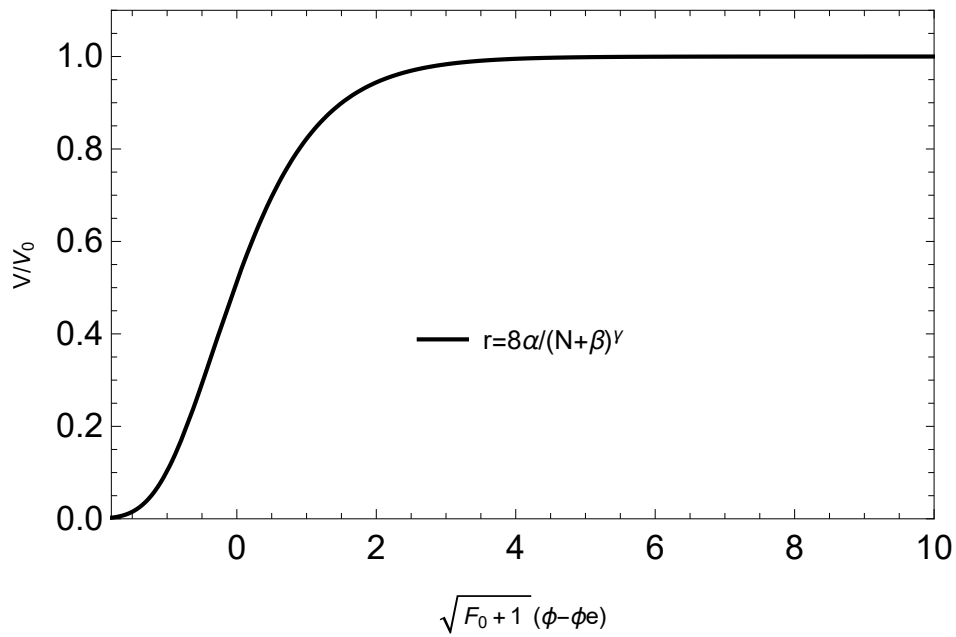
Figure 2. The reconstructed potentials are normalized with V_0 from Equation (30), and the inflaton field is normalized with $1/\sqrt{F_0 + 1}$. We choose the value of ϕ_0 that could make $\phi_e = 0$.

4. Reheating

The inflation ends when the inflaton rolls down to the minimum of the potential; around the minimum, the inflaton field will oscillate to reheat the cold universe. Because the inflation phase is followed by the reheating phase, these two phases may constrain each other, so the reheating phase may give other constraints on the inflation phase. In this section, we research the constraint from the reheating phase on the reconstructed model under the high friction limit $F \gg 1$ and the GR limit $F \ll 1$.

The relation between the pivotal scale $k_* = 0.002 \, \text{Mpc}^{-1}$ and the present Hubble parameter is

$$\frac{k_*}{a_0 H_0} = \frac{a_* H_*}{a_0 H_0} = \frac{a_*}{a_e} \frac{a_e}{a_{re}} \frac{a_{re}}{a_0} \frac{H_*}{H_0} = e^{-N - N_{re}} \frac{a_{re}}{a_0} \frac{H_*}{H_0}, \tag{41}$$

where N_{re} is the e-folding number during reheating, a_{re} is the scale factor at the end of reheating, and we assume the radiation domination phase follows the reheating phase immediately and the reheating phase follows inflation phase immediately. Because the physics of the reheating is still unknown, for simplicity, we assume a constant parameter w_{re} of state equation during reheating, and we get

$$N_{re} = \frac{1}{3(1 + w_{re})} \ln \frac{\rho_e}{\rho_{re}}, \tag{42}$$

where the relation between ρ_{re} and the temperature T_{re} is

$$\rho_{re} = \frac{\pi^2}{30} g_{re} T_{re}^4, \tag{43}$$

with g_{re} denoting the effective number of relativistic species at reheating phase. By using the condition of the entropy conservation, we get the relation between temperature T_{re} and the present cosmic microwave background temperature $T_0 = 2.725K$,

$$a_{re}^3 g_{s,re} T_{re}^3 = a_0^3 \left(2 T_0^3 + 6 \times \frac{7}{8} T_{v0}^3 \right), \tag{44}$$

where $g_{s,re}$ denotes the effective number of relativistic species for entropy, and $T_{v0} = (4/11)^{1/3}T_0$ is the present neutrino temperature. By using the above relations, we obtain [48,49]

$$N_{re} = \frac{4}{1 - 3w_{re}} \left[-N - \ln \frac{\rho_e^{1/4}}{H_*} + \frac{1}{3} \ln \frac{43}{11 g_{s,re}} + \frac{1}{4} \ln \frac{\pi^2 g_{re}}{30} - \ln \frac{k_*}{a_0 T_0} \right], \tag{45}$$

$$T_{re} = \exp \left[-\frac{3N_{re}(1 + w_{re})}{4} \right] \left[\frac{30\rho_e}{\pi^2 g_{re}} \right]^{1/4}. \tag{46}$$

The relations (45) and (46) show that N_{re} and T_{re} depend on g_{re} and $g_{s,re}$ logarithmically, thus we choose $g_{re} = g_{s,re} = 106.75$. At the end of inflation, we have $\epsilon_V \approx 1$; from Equation (13), we obtain the relation $\dot{\phi}^2 = 2V_e/(27F)$, so we have $\rho_e = 4V_e/3$. By using the observational value of the amplitude of the power spectrum [5], from Equation (16), we have

$$A_s = 3H_*^2/(8\pi^2 \epsilon_{V*}) = 2.2 \times 10^{-9}, \tag{47}$$

and Equations (45) and (46) become

$$N_{re} = \frac{4}{1 - 3w_{re}} \left(56.46 - N - \frac{\ln V_e}{4} + \frac{\ln \epsilon_{V*}}{2} \right), \tag{48}$$

$$T_{re} = \exp \left[-\frac{3N_{re}(1 + w_{re})}{4} \right] \left[\frac{4V_e}{10.675\pi^2} \right]^{1/4}. \tag{49}$$

By using Equations (28) and (31), under the high friction limit $F \gg 1$, we obtain the constraint from the reheating process on the model parameters,

$$N_{re} = \frac{4}{1 - 3w_{re}} \left[60.45 + \frac{\alpha}{4(\gamma - 1)\beta^{\gamma - 1}} + \frac{1}{4} \ln \alpha - N - \frac{\gamma}{4} \ln(N + \beta) - \frac{\alpha}{4(\gamma - 1)(N + \beta)^{\gamma - 1}} \right], \tag{50}$$

$$T_{re} = 0.01 \frac{\alpha^{1/4}}{(N + \beta)^{\gamma/4}} \exp \left[-\frac{\alpha}{4(\gamma - 1)\beta^{\gamma - 1}} + \frac{\alpha}{4(\gamma - 1)(N + \beta)^{\gamma - 1}} - \frac{3N_{re}(1 + w_{re})}{4} \right], \tag{51}$$

where $\alpha = 2\beta^\gamma/3$. Under the GR limit $F \ll 1$, the relations are

$$N_{re} = \frac{4}{1 - 3w_{re}} \left[59.90 + \frac{\alpha}{4(\gamma - 1)\beta^{\gamma - 1}} + \frac{1}{4} \ln \alpha - N - \frac{\gamma}{4} \ln(N + \beta) - \frac{\alpha}{4(\gamma - 1)(N + \beta)^{\gamma - 1}} \right], \tag{52}$$

$$T_{re} = 0.01 \frac{\alpha^{1/4}}{(N + \beta)^{\gamma/4}} \exp \left[-\frac{\alpha}{4(\gamma - 1)\beta^{\gamma - 1}} + \frac{\alpha}{4(\gamma - 1)(N + \beta)^{\gamma - 1}} - \frac{3N_{re}(1 + w_{re})}{4} \right], \tag{53}$$

where $\alpha = 2\beta^\gamma$. These two situations make almost the same constraint except the 0.5 *e*-folding difference in N_{re} and the different relations of α. Therefore, the friction parameter F has little influence on the reheating phase, and we just consider the high friction limit situation in the following.

For different kinds of β, γ, N and w_{re}, by using Equations (27), (50) and (51), we calculate the corresponding spectral tilt n_s, reheating *e*-folds N_{re} and reheating temperature T_{re}, and the results are displayed in Figure 3.
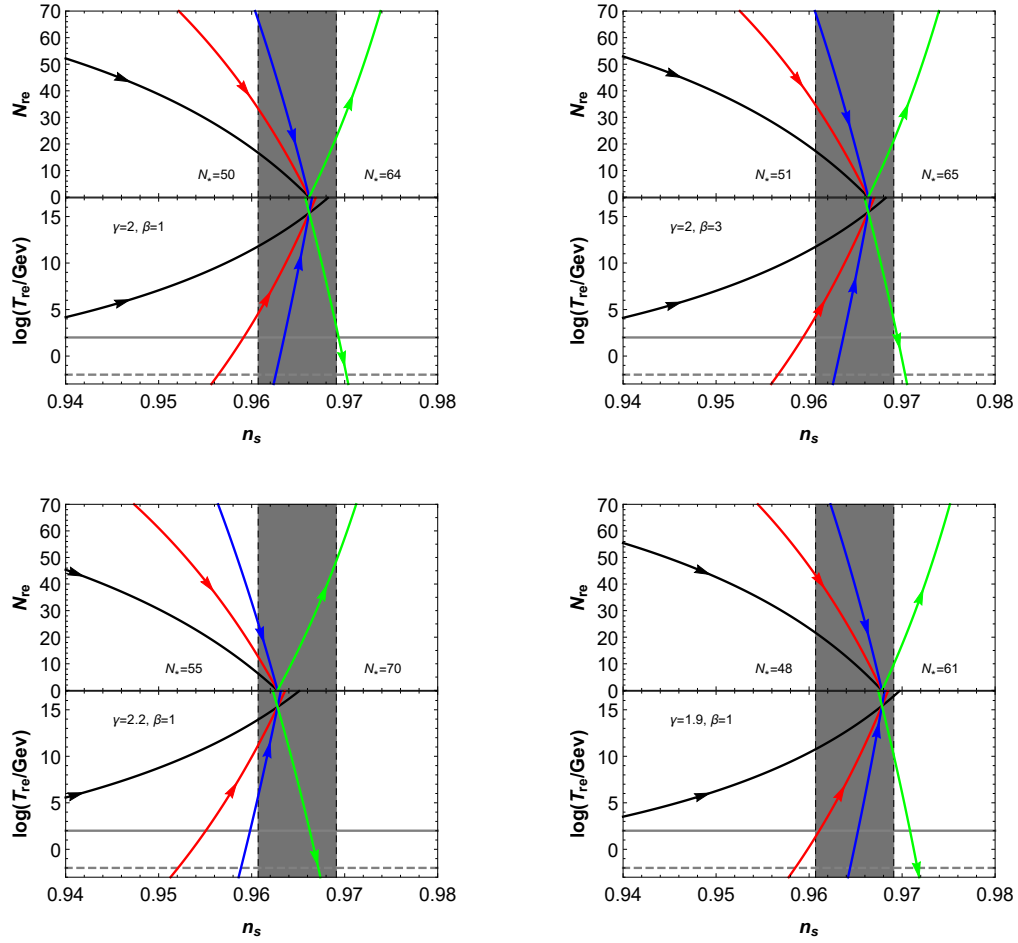
Figure 3. (**Top**) The relations between N_{re} and n_s; and (**Bottom**) the relations between T_{re} and n_s. The corresponding values of β and γ for each model are indicated in each panel. The 1σ Planck constraint $n_s = 0.9649 \pm 0.0042$ [5] is denoted by the gray band, and the 1σ Planck constraint on the e-folds N is also indicated. The black, red, blue and green lines correspond to the reheating models with $w_{re} = -1/3$, 0, 1/6 and 2/3, respectively; in each line, the arrow denotes the direction of N enlargement. The horizontal gray solid and dashed lines in the bottom panels denote the electroweak scale $T_{EW} \sim 100$ GeV and the big bang nucleosynthesis scale $T_{BBN} \sim 10$ MeV, respectively.

The figures show that different model parameters β and γ and the value of w_{re} provide different constraints on the reheating e-folds N_{re} and the reheating temperature T_{re}, while the parameter β almost does not affect the reheating process. For larger spectral tilt n_s, the allowed reheating e-folding number N_{re} with $w_{re} = -1/3$, 0 and 1/6 will become smaller, while the allowed reheating e-folding number N_{re} with $w_{re} = 2/3$ will become larger. The scale of big-bang nucleosynthesis put an upper limit on n_s if $w_{re} = 2/3$ and a low limit on n_s if $w_{re} = 1/6$.

5. Conclusions

The non-minimal derivative coupling term in the inflation model could reduce the tensor-to-scalar ratio, which can make the large tensor-to-scalar ratio models, such as the Higgs inflation, be consistent with the observations. We derive the reconstruction formulae of the inflation model with non-minimal derivative coupling. To reconstruct the potential without using the high friction limit, we consider the parameterization of the tensor to scalar ratio $r = 8\alpha/(N + \beta)^\gamma$ inspired from the α-attractor. For $\gamma = 2$, which is the α attractor, we get the same potential as obtained in [39], in the GR limit $F \ll 1$. When $\alpha \ll 1$, this potential has the same asymptotic behavior as that of T/E-model. For $\gamma \neq 2$,

the potential is the exponential form. The observational constraints on the parameters are $1.2 < \gamma < 2.7$ and $\beta < 10$. The reconstruction also show that the observational data favor the α attractor case with $\gamma \sim 2$.

The constraints on the spectral tilt n_s from the Planck data could provide constraints on the reheating process. Different model parameters provide different constraints on reheating e-folds N_{re}, reheating temperature T_{re} and reheating state equation w_{re}. For larger spectral tilt n_s, the allowed reheating e-folding number N_{re} with $w_{re} = -1/3, 0$ and $1/6$ will become smaller, while the allowed reheating e-folding number N_{re} with $w_{re} = 2/3$ will become larger. The energy scale of the reheating could also provide additional constraints on the inflation. If $\gamma = 2$, $\beta = 1$ and $w_{re} = 2/3$, the big bang nucleosynthesis scale requires $n_s < 0.967$; if $\gamma = 2$, $\beta = 1$ and $w_{re} = 1/6$, the big bang nucleosynthesis scale requires $n_s > 0.962$.

Author Contributions: Conceptualization, Z.Y.; investigation, Q.F.; data curation, Q.F.; writing—original draft preparation, Z.Y. and Y.Y.; and writing—review and editing, Z.Y. All authors have read and agreed to the published version of the manuscript.

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