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# New Scenarios of High-Energy Particle Collisions Near Wormholes

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**Abstract:** We suggest two new scenarios of high-energy particle collisions in the background of a wormhole. In scenario 1, the novelty consists of the fact that the effect does not require two particles coming from different mouths. Instead, all such scenarios of high energy collisions develop, when an experimenter sends particles towards a wormhole from the same side of the throat. For static wormholes, this approach leads to indefinitely large energy in the center of mass. For rotating wormholes, it makes possible the super-Penrose process (unbounded energies measured at infinity). In scenario 2, one of colliding particles oscillates near the wormhole throat from the very beginning. In this sense, scenario 2 is intermediate between the standard one and scenario 1 since the particle under discussion does not come from infinity at all.

**Keywords:** particle collision; wormholes; centre of mass frame; 04.70.Bw; 97.60.Lf

## 1. Introduction

During the last decade, a lot of efforts were devoted to description of high-energy collisions in the region of the strong gravitation field. This was stimulated by the observation about possibility to obtain an indefinitely large energy $E_{c.m.}$ in the centre of mass frame of two colliding particles [1] (see also earlier works [2–4]). These findings were made for the case of rotating black holes. Meanwhile, later, similar results were obtained for another strongly gravitating objects. Thus, the unbounded energies $E_{c.m.}$ were found for processes near naked singularities and wormholes. In the present article, it is the latter case which we are interested in.

One should distinguish between two kinds of energy. The first one is $E_{c.m.}$ that can be measured by an observer who is present just in the point of collision. The second one is the Killing energy $E$ measured at infinity in the asymptotically flat space-time. In the present work, we will discuss both of them. It is essential that if $E_{c.m.}$ is finite, $E$ is finite as well. This was shown in [5] for the Kerr metric and in [6] for a more general case. (There is a very special case [7] when the parameters of the metric themselves diverge, but we will not discuss it further.) Therefore, the necessary (although not sufficient) condition for obtaining unbounded $E$ consists of the consideration of processes with unbounded $E_{c.m.}$. In what follows, we will use the term super-Penrose process (SPP), if $E$ is unbounded.

For the first time, collisions with unbounded $E_{c.m.}$ near wormholes were considered in [8] for a particular type of wormholes, so-called Teo wormholes [9]. They are necessarily rotating, the corresponding space-time does not have an asymptotically flat region. In the next work, it was shown that collisions near such wormholes can also produce unbounded $E$ [10]. Later, it was noticed [11] that high-energy collisions can be realized even for static wormholes (for example, if two Schwarzschild-like wormholes are glued by means of the "cut and past" technique, see e.g., Section 15.2.1 of [12]). In the Krasnikov's scenario [11], unbounded $E_{c.m.}$ do occur but unbounded $E$ are forbidden since this would require the presence of the ergosphere where $E < 0$. Meanwhile, such a region is absent for the Schwarzschild-like metric. In our previous paper it was shown that the SPP is possible for rather general rotating wormholes [13].

It is worth stressing that there exists nontrivial dependence between the behavior of $E_{c.m.}(N)$, where $N$ is the lapse function, and the existence or nonexistence of the SPP. This relation was established in [14], where general classification was constructed. In doing so, $E_{c.m.}(N)$ itself is determined by the relative sign of radial momenta of colliding particles. It is head-on collision that leads to the existence of the SPP. In this sense, it is of interest to describe possible ways, how to realize head-on collisions. It is this point that we stress. As far as wormholes are concerned, in previous works it was assumed that two particles come from opposite mouths and meet near the throat (head-on collision). Thus, the corresponding experiment had "mixed" nature involving observers from different sides of Universe.

In the present work, we suggest two completely new, alternative scenarios. We show that an indefinitely large energies $E_{c.m.}$ and $E$ can occur even if both particles are sent from the same side of the throat. However, this requires two-step process. Also, we exploit the fact that in wormhole space-times there exist bound states (impossible for black holes) when a particle can oscillate between two turning points [15].

One reservation is in order. All scenarios connected with using wormholes for obtaining unbounded $E_{c.m.}$ share the same feature. Namely, the lapse function near the throat should be small. This leads to an indefinite growth of the curvature invariants (say, the Kretschmann scalar $K$) there. Meanwhile, one can reconcile large $E_{c.m.}$ and $K$ remaining below the Planckian scale by choosing the parameters of the system accordingly [16].

Below, we consider two types of scenarios in which high energy phenomena reveal themselves in (i) indefinite growth of $E_{c.m.}$ (with $E$ remaining modest), (ii) in the SPP. To this end, we consider separately (i) collision in static spherically symmetric wormholes and (ii) in rotating axially symmetric ones. In the first case, only $E_{c.m.}$ can be unbounded, in the second one we explain why also $E$ can be made as large as one likes.

The paper is organized as follows. In Section 2 we give basic formulas for the spherically symmetric case including the metric, equations of motion and the energy in the center of mass of two colliding particles. In Section 3 we describe a scenario in which one of two particles reflects from the potential barrier, so that an ingoing particle converts into the outgoing one. In Section 4 we show that in choosing the point of collision of the metric function being small enough, we can achieve indefinitely large $E_{c.m.}$. In Section 5 we describe another scenario in which one of colliding particles does not come from infinity but oscillates near the throat between turning points. In Section 6 we give the general metric and equations of particle motion in the case of rotating wormholes. In Section 7 we describe a general scheme of particle collisions in such a background. In Section 8 we analyze possible output of particles with ultrahigh energy. In Section 9 we discuss the role of trajectories with negative energy played in the high energy processes under consideration. In particular, we discuss how they can be used in an alternative scenario of collision. In Section 10 we summarize the results and outline some perspectives.

We use the geometric system of units in which fundamental constants $G = c = 1$.

## 2. Spherically Symmetric Case: Basic Formulas

Let us consider the spherically symmetric metric

$$ds^2 = -fdt^2 + \frac{d\rho^2}{f} + r^2(\rho)d\omega^2, d\omega^2 = d\theta^2 + \sin^2\theta d\phi^2, \tag{1}$$

where we used a so-called quasiglobal coordinate $\rho$ (see, e.g., Section 3.3.2 of [17]). Motion of free particles occurs in the plane which we choose to be the equatorial one $\theta = \frac{\pi}{2}$. Equations of motion read

$$m\dot{t} = \frac{E}{f}, \tag{2}$$

$$m\dot{\rho} = \sigma P, \tag{3}$$

$$m\dot{\phi} = \frac{L}{r^2}, \tag{4}$$

where dot denotes differentiation with respect to the proper time $\tau$, $E$ being the conserved energy, $L$ conserved angular momentum, $\sigma = \pm 1$ depending on the direction of motion,

$$P = \sqrt{E^2 - f\tilde{m}^2}, \tag{5}$$

$$\tilde{m}^2 = m^2 + \frac{L^2}{r^2}. \tag{6}$$

The forward-in-time condition $\dot{t} > 0$ is satisfied, provided $E > 0$.

If two particles 1 and 2 collide, one can define the energy in the center of mass frame according to

$$E_{c.m.}^2 = -(m_1 u_{1\mu} + m_2 u_{2\mu})(m_1 u_1^\mu + m_2 u_2^\mu) = m_1^2 + m_2^2 + 2m_1 m_2 \gamma. \tag{7}$$

Here, $u^\mu$ is the four-velocity, subscript label particles, $\gamma = -u_{1\mu} u_2^\mu$ is the Lorentz factor of relative motion. Using (2)–(4) one obtains

$$m_1 m_2 \gamma = \frac{E_1 E_2 - \sigma_1 \sigma_2 P_1 P_2}{f} - \frac{L_1 L_2}{r^2}. \tag{8}$$

In what follows we consider the manifold to be a wormhole. For simplicity, we assume that the function $r(\rho)$ has one minimum at $\rho = \rho_0$, so

$$r \geq r_0 \equiv r(\rho_0). \tag{9}$$

In this section, we restrict ourselves by pure radial motion $L = 0$ since this simplified case captures the main features of the phenomenon under discussion. Then,

$$\dot{\rho} = \sigma p, \tag{10}$$

$$p = \sqrt{\varepsilon^2 - f}, \tag{11}$$

where $\varepsilon = \frac{E}{m}$,

$$\gamma = \frac{\varepsilon_1 \varepsilon_2 - \sigma_1 \sigma_2 p_1 p_2}{f}. \tag{12}$$

## 3. Scenario of Collision 1: Two Particles Come from the Same Mouth

Let us consider the following scenario. Particle 1 has the energy $E_1 > m$ and starts its motion, say, from the right infinity. In some point it decays to two particles 2 and 3. We assume that particle 2 has the energy $E_2 < m$, whereas particle 3 has $E_3 > m$, $\sigma_3 = -1$. Then, particle 3 escapes to the left infinity. Meanwhile, particle 2 has the turning point $r_2$, where $p_2 = 0$, its position is given by

$$f(r_2) = \varepsilon_2^2. \tag{13}$$

We assume that $f$ is a monotonic function of $r$ in each half-space, so there is one value of $r_2$ but there are two turning points in terms of $\rho$ in which $r(\rho) = r_2$. It is also clear that $f$ attains its minimum $f_0$ at point $\rho_0$, $f_0 = f(r(\rho_0))$.

Particle 2 oscillates between both turning points. Let it collide in point $\rho_0$ with one more particle 4 with (for simplicity) the same mass that comes from infinity, $\varepsilon_4 > 1$, $\sigma_4 = -1$. We choose the moment of collision in such a way that particle 2 moves from the left to the right, so $\sigma_2 = +1$. From (12), we have

$$\gamma = \frac{\varepsilon_4\varepsilon_2 + p_4(\rho_0)p_2(\rho_0)}{f}. \tag{14}$$

## 4. Unbounded $\mathbf{E_{c.m.}}$

Now, we consider configurations with small $f_0 \ll \varepsilon_2 < \varepsilon_4$. Then, $p_2(\rho_0) \approx \varepsilon_2$, $p_4(\rho_0) \approx \varepsilon_4$,

$$\gamma \approx \frac{2\varepsilon_4\varepsilon_2}{f}. \tag{15}$$

When $f \to 0$, $\gamma$ grows unbounded, and so does $E_{c.m.}$

We would like to remind a reader that there are few scenarios of high energy particle collisions in which unbounded $E_{c.m.}$ is obtained in head-on collisions. The key point of such scenarios is to obtain somehow a particle that moves in the opposite direction (with respect to another particle that falls from infinity) and arrange collision in the point where the lapse function is very small. This can be realized (i) near white holes [18], (ii) in the background of a naked singularity [19], (iii) in the background of a wormhole. In case (ii) there is a two-step scenario in which a particle bounces back from an indefinitely high potential barrier and meets a new particle coming from infinity. In case (iii), there are two options. One of them (iii-a) consists of that two particles comes from opposite mouths [11]. Meanwhile, in our scenario (iii-b) all particles participating in the process, start in our universe.

Thus, in our scenario we can probe the other side of a wormhole starting the experiment on our side of it and remaining only there.

## 5. Scenario of Collision 2: Intermediate Case

In this section, we describe one more scenario. Let us remind a reader that the key ingredient for obtaining unbounded $E_{c.m.}$ is a head-on collision of two particles near the throat, under an additional condition that the metric coefficient $f$ is small enough in the corresponding point. Thus, we have two alternatives: (i) both particles come from opposite mouths [8,11], (ii) particles come from the same mouth (see above). Meanwhile, there is also one more possibility based on the property of wormholes with no analog in the black hole case. It was shown in [15] that there exist states such that a particle performs bounded motion between two turning points. Choosing an appropriate phase when particle 2 moves, say, from the left to the right, while particle 1 comes from the right infinity, for small $f_0$ we obtain the result similar to (15) with one difference: now $\varepsilon_4$ is to be replaced by $\varepsilon_1$.

To make presentation self-closed, we write down the metric in the same form as in [15]:

$$ds^2 = -dt^2(g(r) + \lambda^2) + \frac{dr^2}{g(r)} + r^2 d\omega^2. \tag{16}$$

Here, for simplicity, $g = 1 - \frac{r_+}{r}$, $r \geq r_+$, $\lambda$ is a constant, $r_+$ has the meaning of the throat radius. If $\lambda^2 < \varepsilon^2 < 1 + \lambda^2$, a trajectory oscillates between two turning points. Let collision occur in the phase when both particles move in opposite directions.

Repeating our calculations step by step, we obtain for collision of particles 1 and 2 in point $r_0$, moving in opposite directions radially, the expression

$$\gamma = \frac{\varepsilon_1 \varepsilon_2 + p_1(r_0) p_2(r_0)}{g(r_0) + \lambda^2}, \tag{17}$$

$$p(r) = \sqrt{\varepsilon^2 - (g + \lambda^2)} \tag{18}$$

instead of (14).

Choosing $r_0 = r_+$, we have

$$\gamma = \frac{2\varepsilon_1 \varepsilon_2}{\lambda^2}. \tag{19}$$

If $\lambda$ is sufficiently small, $\gamma$ can be made as big as one likes.

Such a scenario can be thought of as an intermediate case between the aforementioned scenarios in the sense that particle 2 comes neither from the left infinity nor from the right one. It was present near the throat because of initial conditions. In addition, this scenario 2 has advantage as compared to scenario 1 in that we should not arrange two-step process. It is sufficient to arrange one-step collision.

## 6. Rotating Wormholes

Now, we consider a more general metric that takes into account the effect of rotation:

$$ds^2 = -N^2 dt^2 + g_\phi (d\phi - \omega dt)^2 + \frac{d\rho^2}{A} + g_\phi d\theta^2, \tag{20}$$

where the coefficients do not depend on $t$ and $\phi$, $\omega > 0$. (To simplify formulas, we use notation $g_\phi$ for the component of the metric tensor $g_{\phi\phi}$). We suppose that the equatorial plane is a plane of symmetry and are interested in the motion within this plane only. Instead of (2)–(4), equations of motion read now

$$m\dot{t} = \frac{X}{N^2}, \tag{21}$$

$$m\frac{N}{\sqrt{A}}\dot{\rho} = P_r = \sigma P, \tag{22}$$

$$m\dot{\phi} = \frac{L}{g_\phi} + \frac{\omega X}{N^2}, \tag{23}$$

where

$$X = E - \omega L, \tag{24}$$

$$P = \sqrt{X^2 - N^2 \tilde{m}^2}, \tag{25}$$

$$\tilde{m}^2 = m^2 + \frac{L^2}{g_\phi}. \tag{26}$$

The forward-in-time condition gives us

$$X \geq 0. \tag{27}$$

We assume that our metric has a wormhole character. This means that $g_\phi$ has a minimum in some point $\rho_0$. For simplicity we assume that $N$ has also minimum in this point, $N(\rho_0) \neq 0$ and $N(\rho_0) \ll 1$.

## 7. Collisions Near Throat of Rotating Wormhole: Scenario 1

Again, we consider the two-step scenario. Our aim is to elucidate, whether or not the energy extraction from a wormhole is possible and whether or not it can be unbounded. In general, energy gain in this context is nothing else than the Penrose process [20]. Let us repeat that if it is formally (in the test particle approximation) unbounded, it is called the super-Penrose process (SPP). If the Penrose process is realized in the scenario that involves collision, it is called the collisional Penrose process (for black holes, this process is reviewed in [21]). On the first stage, particle 1 decays to particles 2 and 3. Particle 3 escapes to the left infinity while particle 2 moves to the right. Both $E_2 > 0$ and $E_3 > 0$. On the second stage, particle 4 comes from infinity and collides with particle 2 near the throat. This is head-collision like in the static case. As a result, particles 5 and 6 are created. We assume that $E_5 < 0$ and $E_6 > 0$, particle 6 escapes to the right infinity.

Here, there are two essential differences now as compared to the static case. First, we assumed that the ergoregion does exist that makes it possible to have $E < 0$. Such option was forbidden in the limit $\omega \to 0$ corresponding to the static metric. Second, we cannot put all angular momenta equal to zero. Moreover, some of them should be large.

To explain this, let us consider the conservation laws for the energy and angular momenta.

$$E_2 + E_4 = E_5 + E_6, \tag{28}$$

$$L_2 + L_4 = L_5 + L_6. \tag{29}$$

It follows from (28) and (29) that

$$X_{tot} \equiv X_2 + X_4 = X_5 + X_6. \tag{30}$$

As, by assumption, $E_5 < 0$ and $E_2 > 0$, $E_6 > E_4$. Equation (27) with $\omega > 0$ entails that $L_5 < 0$.

Furthermore, we want to have $E_6$ large positive, so $E_5$ should be large negative. Formally, $E_5 \to -\infty$, $E_6 \to +\infty$. Meanwhile, as all energies and angular momenta of particles on the 1st stage are supposed to be finite, the quantities $X_2$ and $X_4$ are finite as well. Taking into account that $X_5 > 0$ and $X_6 > 0$ for the same reason (27), we see that each of them should be finite according to (30). Therefore, we want to have configurations with $L_5 \to -\infty$, $L_6 \to +\infty$. Thus, divergences in the right hand sides of (28) and (29) should compensate each other. It is seen from (24) that $E_6 = X_6(\rho_0) + \omega(\rho_0)L_6$. Then, for finite $X_6(\rho_0), \omega(\rho_0)$ and $L_6 \to +\infty$, the energy $E_6 \to +\infty$ as well. This realizes the super Penrose process, when the energy $E$ detected by an observer at infinity is as large as one likes.

## 8. Output of Collision

In Section 7, we outlined the desired features of the process, but the question remained, whether or not it can be realized. In principle, further analysis is required that, apart from the conservation of the energy and angular momentum, also takes into account the conservation of the radial momentum. This is the most subtle and crucial point. Happily, there is no need in carrying out such analysis here since we reduced the problem to the one that has been already investigated in [13] and generalized in [14]. Namely, the following statement was proved there.

Let (i) two particles collide in the point where $N \ll 1$ but the horizon is absent (as it takes place for the wormhole metric). Then, (ii) for head-collision the energy of an escaping particle is not bounded. However, both these conditions are fulfilled now in our scenario. The aim of the first stage consisted of the possibility to prepare particle 2 that moves from the left to the right. On the 2nd stage high-energy head-on collision does occur.

It is worth stressing that both for a wormhole and a naked singularity the dependence $E_{c.m.}(N)$ for small $N$ has the same form $E_{c.m.}(N) \sim N^{-1}$ and this gives rise to unbounded $E$—see line 3 in Table 1 on page 6 in [14]. Independently of origination of head-on collision near the throat with very small $N$, once it occurred, it leads to unbounded energies at infinity $E$.

## 9. Trajectories with Negative Energies and Scenario 2

The key role in the scenario under discussion, as well as in any Penrose process, is played by the states with negative energy. Strange as it may seem, only quite recently the properties of such trajectories were elucidated and described in [22] for the Kerr metric. Later on, they were generalized in [23]. It turned out that corresponding geodesics cannot stay forever in the region external with respect to the horizon. The complete curve inevitably crosses the horizon. Correspondingly, a particle with $E < 0$ cannot oscillate between two turning points outside the horizon or move on the circular orbit. (The similar statements are valid for the Reissner-Nordström black hole [24].) In the wormhole case there is no horizon and the situation changes drastically. The particle with $E < 0$ cannot escape to either of two infinities. Therefore, it must oscillate between turning points.

Thus, in our scenario, after the 1st collision, one of particles sits on the trajectory with $E < 0$ and in the phase when it moves outward, it collides with a particle coming from infinity, creating a new particle with indefinitely large energy.

From another hand, trajectories of such a type can be used for one more scenario of collision. Omitting formulas, we describe it qualitatively. If a particle has energy $E < 0$, it cannot come from infinity or escape to infinity. Instead, it oscillates between turning points. Let collision between particle 1 coming from infinity and particle 2 oscillating inside a wormhole occur when they move in opposite directions. The reaction can be described as $1 + 2 \to 3 + 4$. If the lapse function in the point of collision (say, exactly in the throat) is small enough, we again obtain indefinitely large $E_3$, provided $L_3$ is big and negative. The essential difference between this scenario and the one described above in Section 7 consists of that there is no need in a two-step process.

## 10. Conclusions

One of the methods of obtaining the super-Penrose process consists of arranging the head-on collision in the point with a small value of the lapse function. To this end, a particle that was ingoing converts into an outgoing due to reflection from the potential barrier with subsequent collision with another particle coming from infinity. This is realized in the metric with naked singularities [5,6,19] where the potential barrier has indefinitely big height. Meanwhile, in the present work we considered wormholes, the potential barrier being finite.

In the present work, we suggested two new scenarios. In scenario 1, both particles are sent from infinity from the same side of a wormhole. It turned out that two main features are inherent to this scenario. For pure static wormholes, it warrants unbounded $E_{c.m.}$ If a wormhole is rotating, it also leads to unbounded $E$, i.e., the super-Penrose process. A separate question arises, how a remote observer who registers high-energy particles at infinity, can distinguish between a naked singularity and a wormhole.

In scenario 2, particle 1 comes from infinity while particle 2 oscillates between turning points from the very beginning. It can be considered as an intermediate scenario between a standard one (when both particles come from different mouths) and scenario 1 outlined above. In doing so, particle 2 does not come from infinity at all.

It turns out that trajectories with finite motion near the wormhole throat can play a double role. First, they can serve as initial conditions in collisions leading to unbounded $E_{c.m.}$ Second, after collisions, one of product of reaction can sit on such a trajectory. Thus, either motion along a trajectory under discussion can be specified as some initial condition or a particle can appear there as a result of a previous collision. Anyway, one cannot determine the origin of such a trajectory without additional assumptions.

All discussion was carried out in the test particle approximation. As long as the energy value does not exceed the parameters of the metric, this looks quite reasonable. Say, in the case of Kerr-like wormholes with the parameters $M$ and $a$, one can obtain the energy $1 \ll \frac{E}{m} \ll a, M$. Especially interesting is to make attempt of finding self-consistent solutions with the backreaction taken into account but this problem is beyond our task.

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