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Review

Neutrinos: Majorana or Dirac?

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Abstract: Are neutrinos with definite masses Majorana or Dirac particles? This is one of the most fundamental problems of modern neutrino physics. The solution to this problem could be crucial for understanding the origin of small neutrino masses. We review here basic arguments in favor of the Majorana nature of massive neutrinos. The phenomenological theory of $0\nu\beta\beta$-decay is briefly discussed and recent experimental data and sensitivity of future experiments are presented.

Keywords: neutrino mass; neutrino mixing; neutrino oscillations; majorana neutrino; neutrinoless double β-decay

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1. Introduction

The origin of small neutrino masses, discovered in neutrino oscillation experiments, is a major problem of modern neutrino physics. From neutrino oscillation data five neutrino oscillation parameters (two neutrino mass-squared differences and three mixing angles) were inferred with an accuracy (5–10)%. The aim of current and future experiments is to improve the accuracy of the measurement of these parameters and to answer the following basic questions:

- What is the character of the neutrino mass spectrum (Normal or Inverted Ordering?);
- What is the value of the CP phase δ?;
- How many neutrinos with definite masses ν_i exist in nature? Is the number of ν_i equal to the number of flavor neutrinos ν_l $(l = e, \mu, \tau)$ or larger (in this case exist additional sterile neutrinos)?;
- What is the nature of neutrinos with definite masses? Are they Majorana or Dirac particles?

The solution of these problems will be extremely important for the understanding of the origin of neutrino masses. We will discuss here the problem of the neutrino's nature, which, apparently, is the most fundamental one.

Neutrinos with definite masses are Dirac particles if the total lepton number L is conserved. In this case neutrino ν_i and antineutrino $\bar{\nu}_i$ have the same mass (CPT) and different lepton numbers ($L(\nu_i) = -L(\bar{\nu}_i) = 1$). Neutrinos with definite masses are Majorana particles if there no conserved lepton number (i.e there is no conserved quantum number that allows to distinguish between neutrino or antineutrino).

There is a general belief that neutrinos are Majorana particles. We will start with a general argument in favor of Majorana neutrinos. The famous two-component theory of a massless neutrino was proposed in 1957 by Landau [1], Lee, and Yang [2] and Salam [3] and was confirmed in the classical Goldhaber at al. experiment on the measurement of the neutrino helicity [4]. The two-component Weil field $\nu_L(x)$ is the simplest possibility for massless neutrino: Two degrees of freedom (instead of four in the case of four-component Dirac neutrino).[1]

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[1] Notice that for massless two-component neutrino and $V - A$ interaction there is no difference between Dirac and Majorana cases.

We know now that neutrinos have small masses. For neutrino with a mass, the Majorana field is the simplest, most economical possibility: Two degrees of freedom (left-handed and right-handed neutrinos). The Standard Model teaches us that nature chooses the simplest possibilities. It looks very plausible that in the case of neutrino with mass a simplest Majorana possibility is also realized.

Neutrino masses are many orders of magnitude smaller than masses of leptons and quarks. It is very unlikely that neutrino masses are of the same Standard Model (SM) Higgs origin as masses of other fundamental fermions. If we assume that Standard Model neutrinos are two-component massless particles, in this case neutrino masses are generated by a new, beyond the SM mechanism. The method of the effective Lagrangian allows us to describe effects beyond SM physics in the electroweak region. There exists only one $SU_L(2) \times U_Y(1)$ invariant effective Lagrangian, which generates a neutrino mass term [5]. We can build such a Lagrangian only if we assume that the total lepton number L is not conserved. After spontaneous symmetry breaking this effective Lagrangian generates Majorana neutrinos with definite masses, three-neutrino mixing, and seesaw-type suppression of neutrino masses with respect to Standard Model masses of lepton and quarks. This is the simplest and most plausible, beyond the SM possibility to generate neutrino masses and mixing.[2]

There are many models which (after heavy fields are integrated out) lead to the Weinberg effective Lagrangian (or its generalization) and the Majorana mass term (see review [6]). Despite all such models that were proposed to explain smallness of neutrino masses, values of masses can not be predicted (many unknown parameters are involved). There are, however, two general features that are common to all models, based on the assumption that SM neutrinos are massless particles and that beyond SM particles, responsible for generation of neutrino masses, are heavy:

1. Neutrinos with definite masses are Majorana particles;
2. The number of neutrinos with definite masses is equal to the number of flavor neutrinos (there are no sterile neutrinos).

The problem of sterile neutrinos, which started about 25 years with the first data of the LSND experiment [7], is still open. We will briefly discuss it later. The study of neutrinoless double β-decay of some even-even nuclei ($0\nu\beta\beta$-decay) is the most sensitive way which could allow us to discover the non conservation of the total lepton number L and to reveal the Majorana nature of neutrinos with definite masses. We will briefly discuss the phenomenological theory of this process and present the latest data.

In conclusion we would like to stress that in spite of strong arguments in favor of Majorana neutrinos, the possibility of Dirac neutrinos (of the Standard Model or beyond the SM origin) is not excluded. The observation of the $0\nu\beta\beta$-decay will allow the exclusion of this, apparently, artificial possibility.

2. On the Higgs Mechanism of the Generation of Fermion Masses

From the discovery of neutrino oscillations, measurement of neutrino masses in tritium experiments and cosmological data follows that neutrino masses are many orders of magnitudes smaller than masses of leptons and quarks. From this basic experimental fact we can conclude that it is very plausible that neutrino masses and masses of other fundamental fermions are of a different origin. We will discuss the possible origin of neutrino masses later. In this introductory section we will consider the standard Higgs mechanism of the generation of fermion masses.

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[2] "Simplicity is a guide to the theory choice" A. Einstein.

The standard Higgs mechanism of the generation of fermion masses is based on the assumption that in the total Lagrangian there are $SU_L(2) \times U_Y(1)$ invariant Yukawa interactions. For the charged leptons, the Yukawa interaction has the form:

$$\mathcal{L}_Y(x) = -\sqrt{2} \sum_{l_1,l_2} \bar{\psi}_{l_1 L}(x) Y_{l_1 l_2} l'_{2R}(x) \phi(x) + \text{h.c.}. \tag{1}$$

Here,

$$\psi_{lL}(x) = \begin{pmatrix} \nu'_{lL}(x) \\ l'_L(x) \end{pmatrix} \quad (l = e, \mu, \tau), \quad \phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix} \tag{2}$$

are lepton and Higgs doublets, $l'_R(x)$ is a right-handed lepton singlet field, and Y is a 3×3 dimensionless complex matrix. The requirements of the $SU_L(2) \times U_Y(1)$ invariance do not put any constraints on the matrix Y.

Charged lepton masses are generated after spontaneous symmetry breaking. Let us introduce the hermitian field of the neutral Higgs particles $H(x)$ and choose the Higgs doublet in the form (the unitary gauge):

$$\phi(x) = \begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix}. \tag{3}$$

Here $v = (\sqrt{2}\ G_F)^{-1/2} \simeq 246$ GeV (G_F is the Fermi constant) is the Higgs vacuum expectation value (vev). With such a choice the local $SU_L(2) \times U_Y(1)$ gauge symmetry will be spontaneously broken.

From (1) and (3) for the Yukawa Lagrangian we find the following expression:

$$\mathcal{L}_Y(x) = -\sum_{l_1,l_2} \bar{l}'_{1L}(x) Y_{l_1 l_2} l'_{2R}(x)(v + H(x)) + \text{h.c.} \tag{4}$$

The 3×3 complex matrix Y can be diagonalized by the biunitary transformation:

$$Y = V_L\, y\, V_R^\dagger, \tag{5}$$

where V_L and V_R are unitary matrices and

$$y_{l'l} = y_l\, \delta_{l'l}, \quad y_l \geq 0 \quad (l, l' = e, \mu, \tau). \tag{6}$$

From (4) and (5) we find that the Yukawa Lagrangian takes the form:

$$\mathcal{L}_Y(x) = -\sum_{l=e,\mu,\tau} m_l\, \bar{l}(x) l(x) - \sum_{l=e,\mu,\tau} y_l\, \bar{l}(x) l(x)\, H(x). \tag{7}$$

Here $l(x) = l_L(x) + l_R(x)$ is the field of the leptons $l^\pm$ ($l = e, \mu, \tau$) with the mass:

$$m_l = y_l\, v. \tag{8}$$

The fields $l_{L,R}(x)$ are connected with primed fields $l'_{L,R}(x)$ by the unitary transformations:

$$l_L(x) = \sum_{l_1} (V_L^\dagger)_{l l_1} l'_{1L}(x), \quad l_R(x) = \sum_{l_1} (V_R^\dagger)_{l l_1} l'_{1R}(x). \tag{9}$$

Thus, Yukawa interaction (1) after spontaneous symmetry breaking generates the standard mass term of the charged leptons and the Lagrangian of the interaction of charged leptons and the Higgs field. The Yukawa constants of this interaction are determined by the lepton mass and are given by:

$$y_l = \frac{m_l}{v}. \tag{10}$$

The SM masses of up and down quarks are generated in the same way as charged lepton's masses. For masses of quarks we find:

$$m_q = y_q\, v \quad (q = u, d, c, s, t, b). \tag{11}$$

The Yukawa constants of the quark-Higgs interaction are determined by quark masses and are given by:

$$y_q = \frac{m_q}{v}. \tag{12}$$

Let us notice that Yukawa constants for different channels were determined from the LHC ((Large Hadron Collider)) study of the decay of the Higgs boson into fermion-antifermion pairs (see [8]). The obtained data are in good agreement with the SM predicted values (10) and (12). From our point of view this is an important argument in favor of the Higgs mechanism of quark and lepton masses generation.

Formally neutrino masses can also be generated by the standard Higgs mechanism. In fact, let us assume that into the total Lagrangian enters the $SU_L(2) \times U_Y(1)$ invariant Yukawa interaction:

$$\mathcal{L}_Y^{\nu}(x) = -\sqrt{2} \sum_{l_1, l_2} \bar{\psi}_{l_1 L}(x) Y^{\nu}_{l_1 l_2} \nu'_{l_2 R}(x) \tilde{\phi}(x) + \text{h.c.} \tag{13}$$

where $\tilde{\phi} = i\tau_2\ \phi^*$ is the conjugated Higgs doublet and right-handed fields ν'_{lR} are singlets. Thus, we need to assume that not only left-handed flavor neutrino fields ν'_{lL} but also right-handed fields ν'_{lR} are SM fields.

After the spontaneous symmetry breaking we find:

$$\mathcal{L}_Y^{\nu}(x) = -\sum_{l_1, l_2} \bar{\nu}'_{l_1 L}(x) Y^{\nu}_{l_1 l_2} \nu'_{l_2 R}(x)(v + H(x)) + \text{h.c.}. \tag{14}$$

The proportional to v term of this expression is the neutrino mass term. After the standard diagonalization of 3×3 matrix Y^{ν} for the neutrino mass term we obtain the following expression:

$$\mathcal{L}^{\nu}(x) = -\sum_{i=1}^{3} m_i\, (\bar{\nu}_{iL}(x)\, \nu_{iR}(x) + \text{h.c.}) = -\sum_{i=1}^{3} m_i\, \bar{\nu}_i(x)\, \nu_i(x), \tag{15}$$

where $\nu_i(x) = \nu_{iL}(x) + \nu_{iR}(x)$ is the field of neutrino with the mass:

$$m_i = y_i^{\nu}\, v. \tag{16}$$

It is easy to check that the Lagrangian of Standard Model with the Yukawa interaction (14) is invariant under the global transformation:

$$\nu_{iL}(x) \to e^{i\Lambda}\nu_{iL}(x),\ \ \nu_{iR}(x) \to e^{i\Lambda}\nu_{iR}(x),\ \ l_L(x) \to e^{i\Lambda} l_L(x),\ \ l_R(x) \to e^{i\Lambda} l_R(x), \tag{17}$$

where Λ is an arbitrary constant. Thus, in the Standard Model with massive neutrinos the total lepton number L is conserved and $\nu_i(x)$ is the Dirac field of neutrinos ($L = 1$) and antineutrinos ($L = -1$).

Despite that Dirac neutrino masses can be introduced in the Standard Model, this possibility looks extremely implausible. The main reason is connected with the smallness of neutrino masses and neutrino Yukawa couplings $y_i^{\nu} = \frac{m_i}{v}$.

Absolute values of neutrino masses at present are unknown. However, from existing neutrino oscillation and cosmological data for the heaviest neutrino mass the following conservative bounds can be found:

$$5\cdot 10^{-2}~\mathrm{eV}\simeq(\sqrt{\Delta m^2_A})\leq m_3\leq(\frac{1}{3}\sum_i m_i)\simeq 3\cdot 10^{-1}~\mathrm{eV},\qquad(18)$$

where $\Delta m^2_A\simeq 2.5\cdot 10^{-3}~\mathrm{eV}^2$ is the atmospheric neutrino mass-squared difference. From (18) for the Yukawa coupling y^{ν}_3 we have:

$$2\cdot 10^{-13}\leq y^{\nu}_3\leq 10^{-12}.\qquad(19)$$

Yukawa couplings of other particles of the third generation (t, b quarks, and τ-lepton) are equal, respectfully,

$$y_t\simeq 7\cdot 10^{-1},~~y_b\simeq 2\cdot 10^{-2},~~y_{\tau}\simeq 7\cdot 10^{-3}.\qquad(20)$$

Thus, Yukawa coupling of the heaviest neutrino *is more than 10 orders of magnitude smaller* than Yukawa couplings of other particles of the third family.

Notice also that in the SM Lagrangian, which do not include Yukawa interactions, enter left-handed and right-handed fields of all *charged particles*. A generation of their masses do not require additional degrees of freedom. The $SU_L(2)\times U_Y(1)$ invariant SM can be built only with left-handed neutrino fields. The SM generation of neutrino masses *requires right-handed neutrino fields, additional degrees of freedom.* Right-handed neutrino fields are out of line of economy and simplicity of the Standard Model.

Thus, it is very unnatural and unlikely that neutrino masses and masses of leptons and quarks are of the same Standard Model origin. In the next section we will consider the most plausible (and popular) beyond the Standard Model mechanism of the generation of neutrino masses.

3. The Weinberg Effective Lagrangian Mechanism of the Neutrino Mass Generation

The Weinberg effective Lagrangian mechanism of the generation of small neutrino masses [5] is, apparently, the most popular beyond the SM mechanism. Before discussing this mechanism we will make the following remark.

In the framework of the approach, based on a neutrino mass term, neutrino masses and mixing were introduced for the first time by Gribov and Pontecorvo in 1969 [9].[3] At that time only ν_e and ν_μ were known and it was established that the lepton charged current had a $V-A$ form:

$$j^{CC}_{\alpha}=2(\bar\nu_{eL}\gamma_{\alpha}e_L+\bar\nu_{\mu L}\gamma_{\alpha}\mu_L).\qquad(21)$$

In 1967 Pontecorvo's paper [11] for the two lepton flavors all possible neutrino oscillations (between active neutrinos and active and sterile neutrinos) were considered. Gribov and Pontecorvo put forward the following question: Is it possible to introduce neutrino masses and oscillations in the case if we assume that they exist only left-handed neutrino fields ν_{eL} and $\nu_{\mu L}$ and there are no right-handed sterile fields? The authors of the paper [9] showed that it is possible to introduce neutrino masses in this case *if the total lepton number L is not conserved*.[4]

Neutrinos have masses and are mixed if a neutrino mass term enters into the total Lagrangian. The mass term is a sum of Lorenz-invariant products of left-handed and right-handed components. Authors of the paper [9] took into account that the conjugated field:

$$\nu^c_{lL}=C\bar\nu^T_{lL}\qquad(22)$$

[3] The results of this paper were generalized in [10].
[4] At that time some authors claimed that if neutrinos are left-handed, their masses had to be equal to zero. This assertion is based, however, on the assumption that the total lepton number is conserved.

is right-handed. Here C is the matrix of the charge conjugation which satisfies the relations:

$$C\gamma_\alpha^T C^{-1} = -\gamma_\alpha, \;\; C^T = -C.$$

Thus, from the left-handed neutrino fields ν_{lL} $(l = e, \mu, \tau)$ it was possible to build the following neutrino mass term:

$$\mathcal{L}^{\rm M} = -\frac{1}{2}\sum_{l',l} \bar\nu_{l'L}\, M^{\rm M}_{l'l} \nu^c_{lL} + \text{h.c.} \tag{23}$$

where $M^{\rm M}$ is a complex, non diagonal 3×3 matrix. From requirements of the Fermi-Dirac statistics it follows that $M^{\rm M} = (M^{\rm M})^T$.

Let us stress that:

1. The Majorana mass term $\mathcal{L}^{\rm M}$ is *the only possible neutrino mass term* which can be built from the left-handed neutrino fields ν_{lL}. This also means that the Majorana mass term is *the most economical general possibility* for neutrino masses and mixing (there are no right-handed neutrino fields in the Lagrangian, the number of neutrino degrees of freedom is minimal);
2. The mass term $\mathcal{L}^{\rm M}$ does not conserve the total lepton number $L = L_e + L_\mu + L_\tau$.

The mass term $\mathcal{L}^{\rm M}$ is called *the Majorana mass term.*

The symmetrical matrix M can be diagonalized by the following transformation:

$$M^{\rm M} = U\, m\, U^T. \tag{24}$$

Here $U\,U^\dagger = 1$ and $m_{ik} = m_i \delta_{ik}, \;\; m_i > 0$. From (23) and (24) we find:

$$\mathcal{L}^{\rm M} = -\frac{1}{2}\sum_{i=1}^{3} m_i\, \bar\nu_i \nu_i, \tag{25}$$

where

$$\nu_i = \sum_l U^\dagger_{il}\nu_{lL} + \sum_l (U^\dagger_{il}\nu_{lL})^c. \tag{26}$$

From (25) and (26) follows that:

- ν_i is the field of neutrino with the mass m_i;
- The field ν_i satisfies the Majorana condition:

$$\nu_i = \nu_i^c \tag{27}$$

and is the Majorana field;

- The flavor neutrino fields ν_{lL} are mixed fields:

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\nu_{iL}. \tag{28}$$

The unitary 3×3 mixing matrix U is called the PMNS matrix [12,13]. Let us stress that in the phenomenological Gribov–Pontecorvo approach *neutrino masses m_i are parameters*. There are no explanation of the smallness of these parameters.

Weinberg [5] proposed a beyond the SM mechanism of neutrino mass generation which leads to the Majorana mass term and allows for an explanation of the smallness of neutrino masses. This mechanism is based on the effective Lagrangian approach.

If SM particles interact with heavy, beyond SM particles with masses much larger than $v \simeq 246$ GeV, then in the electroweak region fields of heavy particles can be "integrated out" and this new interaction induce a nonrenormalizable interaction which is described by the effective Lagrangian. The effective Lagrangians are dimension five or more operators, invariant under $SU_L(2) \times U_Y(1)$ transformations and built from the Standard Model fields.

We are interested in the effective Lagrangian, which generates a neutrino mass term. We assume that only left-handed neutrino fields ν'_{lL}, components of the lepton doublets ψ_{lL}, enter in the SM.

Let us consider $SU_L(2) \times U_Y(1)$ invariant:

$$(\tilde{\phi}^\dagger\ \psi_{lL}) \quad (l = e, \mu, \tau), \tag{29}$$

where $\tilde{\phi} = i\tau_2\phi^*$ is the conjugated Higgs doublet. After the spontaneous symmetry breaking for the proportional to v term we have:

$$(\tilde{\phi}^\dagger\ \psi_{lL})_{\rm SSB} = \frac{v}{\sqrt{2}}\,\nu'_{lL}. \tag{30}$$

From this expression it is obvious that *the only possible effective Lagrangian* which generate neutrino mass term has the form [5]:

$$\mathcal{L}^{\rm eff}_I = -\frac{1}{\Lambda}\sum_{l',l}\overline{(\tilde{\phi}^\dagger\psi_{l'L})}\,X'_{l'l}\,(\tilde{\phi}^\dagger\psi_{lL})^c + \text{h.c.} = -\frac{1}{\Lambda}\sum_{l',l}\bar{\psi}_{l'L}\tilde{\phi}\,X'_{l'l}\,\tilde{\phi}^T\,\psi^c_{lL} + \text{h.c.}. \tag{31}$$

where X' is a symmetrical, non-diagonal matrix.

The operator in (31) has a dimension M^5. As Lagrangian has dimension M^4, the parameter Λ, which has a dimension M, is introduced in (31). The parameter Λ characterizes a scale of new physics (at $\Lambda \to \infty$ effects of a new physics disappear). We could expect that: $\Lambda \gg v$.

Let us stress again that the operator in (31) *is the only dimension five effective Lagrangian.* Other effective Lagrangians have dimension six and higher. In such effective Lagrangians enter coefficients $\frac{1}{\Lambda^n}$ with $n \geq 2$. This means that the *investigation of effects of neutrino masses and mixing*, (neutrino oscillations, neutrinoless double β-decay, etc.) *is the most sensitive way to probe a beyond the SM new physics.*

Let us return back to the effective Lagrangian (31). After the spontaneous symmetry breaking we find the following neutrino mass term:

$$\mathcal{L}^{\rm M} = -\frac{v^2}{2\Lambda}\sum_{l',l}\bar{\nu}'_{l'L}\,X'_{l'l}\,\nu'^c_{lL} + \text{h.c.}. \tag{32}$$

We can present $\mathcal{L}^{\rm M}$ in the standard form in which flavor neutrino fields ν_{lL} enter. The SM leptonic charged current has the following form:

$$j^{CC}_\alpha = 2\sum_l \bar{\psi}_{lL}\frac{1}{2}(\tau_1 + i\tau_2)\gamma_\alpha\psi_{lL} = 2\sum_l \bar{\nu}'_{lL}\gamma_\alpha l'_L. \tag{33}$$

Taking into account (9) from (33) we find:

$$j^{CC}_\alpha = 2\sum_{l,l_1}\bar{\nu}'_{lL}\gamma_\alpha(V_L)_{ll_1}l_{1L} = 2\sum_l \bar{\nu}_{lL}\gamma_\alpha l_L. \tag{34}$$

Here l is the field of the charged lepton with mass m_l and the flavor (current) neutrino field ν_{lL} is given by:

$$\nu_{lL} = \sum_{l_1}(V^\dagger_L)_{ll_1}\nu'_{l_1L}. \tag{35}$$

From (32) and (35), we find the following Majorana mass term:

$$\mathcal{L}^{\rm M} = -\frac{1}{2}\sum_{l',l}\bar{\nu}_{l'L}\,M^{\rm M}_{l'l}\nu^c_{lL} + \text{h.c.} \tag{36}$$

in which the Majorana matrix $M^{\rm M}$ is given by the expression:

$$M^{\rm M} = \frac{v^2}{\Lambda}\, X, \tag{37}$$

where: $X = V_L^\dagger\, X'\, (V_L^\dagger)^T = X^T$.

The symmetrical, dimensionless matrix X can be presented in the form:

$$X = U\, x\, U^T, \tag{38}$$

where $U\, U^\dagger = 1$ and $x_{ik} = x_i \delta_{ik},\ \ x_i > 0$.

From (36)–(38) we find:

$$\mathcal{L}^{\rm M} = -\frac{1}{2} \sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i. \tag{39}$$

Here,

$$\nu_i(x) = \nu_i^c(x) = C\, \bar{\nu}_i^T(x) \tag{40}$$

is the field of the Majorana neutrino with the mass:

$$m_i = \frac{v}{\Lambda}(v x_i) \tag{41}$$

and

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\, \nu_{iL}, \tag{42}$$

where U is the unitary PMNS mixing matrix.

Generated by the standard Higgs mechanism masses of quarks and leptons (and also $W^\pm$, Z^0, and Higgs bosons) are proportional to the Higgs vacuum expectation value v. This is obviously connected with the fact that v is the only SM parameter that has dimension of mass. The Weinberg mechanism of the neutrino mass generation is characterized by two parameters with the dimension of mass: v and Λ. We have $m_i \to 0$ at $\Lambda \to \infty$. It is also obvious that at $v \to 0$ neutrino masses disappear. Thus, for neutrino masses, generated by the effective Lagrangian mechanism, we naturally come to the expression (41) from which it follows that generated by this mechanism neutrino masses are suppressed with respect to SM masses of quarks and leptons by the factor:

$$\frac{v}{\Lambda} = \frac{\text{EW scale}}{\text{scale of a new physics}}, \tag{43}$$

which is naturally much smaller than one.

From (41) we can try to estimate the parameter Λ, which characterize the scale of a new physics. In accordance with latest neutrino oscillation and cosmological data let us assume hierarchy of neutrino masses. In this case we have:

$$m_3 \simeq (\sqrt{\Delta m_A^2}) \simeq 5 \cdot 10^{-2}\ \text{eV}. \tag{44}$$

The parameters x_i in (41) are unknown. If we assume (by analogy with Yukawa couplings of the particles of the third family) that $x_3 \lesssim 1$ we obtain the following estimate:

$$\Lambda \lesssim 10^{15}\ \text{GeV}. \tag{45}$$

The effective Lagrangian (31) does not conserve the total lepton number L. Notice that the global invariance and conservation of L (and the baryon number B) are not proper symmetries of the Quantum Field Theory (constant phases are not dynamical variable, etc.). In the Standard Model, local gauge symmetry and renormalizable Yukawa interactions ensure conservation of L and B [14]. We could expect that beyond the SM theory, it does not conserve L and B (see recent discussion in [15]). This is an additional general argument in favor of Majorana nature of beyond the SM neutrino masses.

In conclusion, let us stress again that we assumed (and this is our basic assumption) that in the Lagrangian of the minimal, renormalizable Standard Model there is no neutrino mass term. Then in the framework of the non-renormalizable, beyond the Standard Model effective Lagrangian approach the residual $SU_L(2) \times U(1)_R$ symmetry naturally ensure the smallness of neutrino masses (via the additional factor $\frac{v}{\Lambda}$ in the expression for the neutrino mass (41)).

4. On the Origin of the Weinberg Effective Lagrangian

Let us consider the lepton number violating, $SU_L(2) \times U_Y(1)$ invariant interaction [5,14]:

$$\mathcal{L}_I = -\sqrt{2}\sum_{li}(\bar{\psi}_{lL}\tilde{\phi})y_{li}\, N_{iR} + \text{h.c.} \tag{46}$$

Here,

$$N_i = N_i^c = C(\bar{N}_i)^T, \quad i = 1, 2, ...n \tag{47}$$

is the field of the Majorana heavy leptons, $SU_L(2) \times U_Y(1)$ singlet, ψ_{lL} and $\tilde{\phi}$ are SM lepton and conjugated Higgs doublets and y_{li} are Yukawa couplings.

If masses of the heavy leptons M_i are much larger than v the Lagrangian (46) in the second order of the perturbation theory generates the Weinberg effective Lagrangian. In fact, for the S-matrix we find:

$$\begin{aligned} S^{(2)} &= \frac{(-i)^2}{2!}2\int T(\sum_{l',l,i,k}\bar{\psi}_{l'L}(x_1)\tilde{\phi}(x_1)y_{l'i}\, N_{iR}(x_1) \\ &\times \; N^T_{kR}(x_2)y_{lk}\tilde{\phi}^T(x_2)\psi^T_{lL}(x_2))d^4x_1 d^4x_2 + ... \end{aligned} \tag{48}$$

In the electroweak region $Q^2 \ll M_i^2$ for the heavy lepton's propagator we have:

$$\langle 0|T(N_{iR}(x_1)N^T_{kR}(x_2))|0\rangle \simeq i\frac{1}{M_i}\delta(x_1 - x_2)\frac{1+\gamma_5}{2}C\delta_{ik}. \tag{49}$$

From (48) and (49) we obtain the effective Lagrangian (31) in which:

$$\frac{1}{\Lambda}X'_{l'l} = \sum_i y_{l'i}\frac{1}{M_i}y_{li}. \tag{50}$$

From this relation it follows that a scale of a new physics Λ is determined by masses of heavy Majorana leptons N_i.[5]

Thus, the Weinberg effective Lagrangian and (after spontaneous symmetry breaking) Majorana neutrino mass term can be generated by the exchange of heavy virtual Majorana leptons, $SU_L(2) \times U_Y(1)$ singlets, between lepton-Higgs pairs. This mechanism of the generation of Majorana neutrino masses and mixing is called type-I seesaw mechanism [19–23].

The interaction (46) is not, however, the only possible interaction that generates effective Lagrangian (31) in the tree-approximation. The Weinberg effective Lagrangian can also be generated in

[5] Heavy Majorana leptons could explain the baryon asymmetry of the Universe. In fact, decays of N_i's, produced in the early Universe, into Higgs-lepton pairs could create a lepton asymmetry, if the interaction (46) violates CP. The lepton asymmetry via QCD sphaleron processes can generate the baryon asymmetry. This mechanism of the generation of the baryon asymmetry of the Universe is called leptogenesis (see reviews [16–18]).

the tree-approximation by interaction of heavy triplet scalar boson field with a pair of lepton doublet and pair of Higgs doublet fields. This scenario is called the type-II seesaw mechanism.

Finally, the Weinberg effective Lagrangian can be generated in the tree approximation by the exchange of heavy virtual Majorana triplet leptons between lepton-Higgs pairs. This mechanism is called the type-III seesaw mechanism.

5. General Remarks on the Neutrino Mass Generation

The interaction (46) and other seesaw interactions are minimal (in a sense of new degrees of freedom) beyond the SM, lepton number violating possibilities of generation of the effective Weinberg Lagrangian and small Majorana neutrino masses. However, the scale Λ of a new physics is naturally very large and unreachable in laboratory experiments ($\Lambda \lesssim 10^{15}$ GeV). This circumstance-inspired creations of many non minimal radiative neutrino mass models with new physics at much lower scales. Detailed discussions of such models and their classification can be found in the review [6] where references to numerous original papers can be found.

A wide class of radiative models is based on the assumption that neutrinos in the Standard Model are massless, left-handed particles and that there are more new heavy fields and more interactions than in the tree-level seesaw cases. The Weinberg effective Lagrangian is generated by interactions of SM fields and beyond the SM heavy fields via one or more loops. Smaller Λ than in the the seesaw cases are usually needed to explain values of neutrino masses. [6]

In many radiative neutrino mass models dimension five Weinberg effective Lagrangian (31) is generated. Some models lead to a dimension $5 + 2n$ effective Lagrangian:

$$\mathcal{L}_I^{\rm eff} = -\frac{1}{\Lambda}\,(\bar{\psi}_L\tilde{\phi})\;X'^{n}\;(\tilde{\phi}^T\;\psi_L^c)\frac{(\phi^\dagger\phi)^n}{\Lambda^{2n}} + \mathrm{h.c.} \tag{51}$$

Here $n = 1, 2, ...$ For the Majorana neutrino masses we have in this case:

$$m_i = v\,(\frac{v}{\Lambda})^{2n+1}x_i^n. \tag{52}$$

From this expression it follows that a scale of a new physics Λ, much smaller than in the classical Weinberg case, is required in order to ensure smallness of neutrino masses.

Let us now try to extract some *general conclusions* from a wide class of neutrino mass models, tree-level and radiative, which generate the Weinberg effective Lagrangian or its generalization and are based on the following assumptions:

- The Standard Model neutrinos ν_l are massless left-handed particles (there are no right-handed neutrino fields in the SM);
- Small neutrino masses are generated by a L-violating, beyond a SM interactions and a scale of a new physics is much larger than the electroweak scale v.

In the electroweak region, effects of beyond the SM particles are integrating out and the effective Weinberg Lagrangian (or its generalizations) and, correspondingly, the Majorana neutrino mass term:

$$\mathcal{L}^{\rm M} = -\frac{1}{2}\sum_{l',l=e,\mu,\tau}\bar{\nu}_{l'L}\,M^{\rm M}_{l'l}\nu^c_{lL} + \mathrm{h.c.} \tag{53}$$

is generated.

Basic assumptions, listed above, ensure smallness of Majorana neutrino masses with respect to the SM masses of leptons and quarks. However, values of neutrino masses depend on unknown

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[6] One of the reason is that with every loop enters a suppression factor $\frac{1}{16\pi^2} \simeq 0.006$.

parameters and can not be predicted. *The common features of all models, in which the Weinberg effective Lagrangian is generated*, are the following:

1. Neutrinos with definite masses ν_i are Majorana particles;
2. The number of neutrinos with definite masses ν_i is equal to the number of the lepton flavors (three).

The most sensitive experiments, which allow us to probe the Majorana nature of ν_i, are experiments on the search for neutrinoless double β- decay of some even-even nuclei. In the next sections we will briefly consider this process.

If the number of massive neutrinos is equal to three, THERE will be no *sterile neutrinos*, neutrinos which do not have the Standard Model weak interaction. As it is well known, indications in favor of sterile neutrinos were obtained in different short baseline neutrino experiments: in the appearance $\bar\nu_\mu \to \bar\nu_e$ LSND [7] and $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ MiniBooNE [24] experiments, in the disappearance $\bar\nu_e \to \bar\nu_e$ reactor experiments [25], and in the disappearance $\nu_e \to \nu_e$ source Gallium experiments [26].

The existing data can be explained by neutrino oscillations if we assume that fields of three flavor neutrinos $\nu_{e,\mu,\tau}$ and one sterile neutrino ν_s are mixtures of the fields of four massive neutrinos $\nu_{1,2,3,4}$ with $m_{1,2,3} \ll m_4$ (so called $3+1$ scheme). From analysis of the data follows that heavy mass m_4 is in the range $(10^{-1} \lesssim m_4 \lesssim 10)$ eV.

Several new short baseline reactor, accelerator, and source neutrino experiments are going on or in preparations at present. Analysis of the status of the light sterile neutrino was done in [27,28]. The latest update of the status of experiments on the search for sterile neutrinos can be found in talks presented at the NEUTRINO 2020 conference (see http://nu2020.fnal.gov). From existing data it is not possible to make a definite conclusion on the existence of sterile neutrinos.

I will limit myself by the following remarks:

1. In new reactor experiments DANSS [29] (with reactor-detector distances 10.7, 11.7, and 12.7 m) and PROSPECT [30] (with reactor-detector distances in the range 6.7–9.2 m), no indications in favor of short baseline neutrino oscillations were found. Best-fit point of previous reactor experiments is excluded in both experiments (at 5σ in the DANSS experiment);
2. Combined analysis of the data, obtained in the reactor Daya Bay and Bugey-3 experiments and accelerator MINOS+ experiment, allows to exclude at 90% CL LSND and MiniBooNE allowed regions for $\Delta m^2_{14} < 5$ eV^2 [31];
3. In the framework of $3+1$ oscillation scheme with one mass-squared difference for all oscillation channels there exist a relation between oscillation amplitudes that allows one to predict from LSND and old reactor data the oscillation amplitude of the $\nu_\mu \to \nu_\mu$ transition [32,33]. This prediction is in strong tension with negative results of the search for short-baseline $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu$ oscillations in MINOS+ and IceCube experiments (see [27,34]). Notice that this problem does not disappear if the data are analyzed in the framework of schemes with two or more sterile neutrinos;
4. Sensitive searches for light sterile neutrinos will be performed in the nearest future. In the SBN experiment at Fermilab [35] three liquid argon TPC detectors at the distances 110 m, 470 m, and 600 m from the target will be used. In the same experiment the search for oscillations in appearance $\nu_\mu \to \nu_e$ and disappearance $\nu_\mu \to \nu_\mu$ channels will be performed. It is planned that the full 99% LSND allowed region will be covered at more than 5 σ. In the $JSNS^2$ experiment at J-PARC (Japan) [36] the direct test of the LSND result will be performed. Much better than in the LSND experiment beam and better detector (Gd-loaded liquid scintillator) will be used. It is expected that a background in the $JSNS^2$ experiment will be much lower than in the LSND experiment. The experiment with one detector would allow the exclusion of a larger part of the LSND-allowed region. In a future experiment with two detectors all LSND-allowed region will

be covered. There are all reasons to believe that the sterile neutrino anomalies will be resolved in the nearest years.

6. On the Phenomenological Theory of the $0\nu\beta\beta$-decay

Experiments on the search for the lepton number violating $0\nu\beta\beta$-decay of some even-even nuclei:

$$(A, Z) \to (A, Z+2) + e^- + e^- \tag{54}$$

ensure a unique probe of the Majorana nature of neutrinos with definite masses. These experiments have many advantages with respect to other possible L-violating experiments: Large targets (in future experiments about 1 ton or more), low backgrounds, high energy resolutions, etc. However, $0\nu\beta\beta$-experiments are extremely difficult and challenging. This is connected with the fact that the expected probabilities of the neutrinoless double β-decay of different nuclei (in the case if ν_i are Majorana particles) are extremely small. The main reasons are the following:

- The $0\nu\beta\beta$-decay is the second order of the perturbation theory in the Fermi constant $G_F \simeq 1.166 \cdot 10^{-5} \frac{1}{\text{GeV}^2}$ process;
- Because of the $V - A$ nature of the weak interaction, the matrix element of the $0\nu\beta\beta$-decay is proportional to the effective Majorana mass $m_{\beta\beta} = \sum_k U_{ek}^2 m_k$. Smallness of the neutrino masses is a reason for the additional severe suppression of the probability of the decay;
- There are two possibilities for neutrino mass spectra: Normal Ordering or Inverted Ordering (see later). Existing neutrino oscillation data favor Normal Ordering. In the case of the Normal Ordering the effective Majorana mass is much smaller than in the case of the Inverted Ordering.

The standard theory of the neutrinoless double β-decay is based on the following assumptions (see reviews [37–40]):

1. The CC interaction is the SM interaction:

$$\mathcal{L}_I^{CC}(x) = -\frac{g}{2\sqrt{2}} j_\alpha^{CC}(x) W^\alpha(x) + \text{h.c.}. \tag{55}$$

Here,

$$j_\alpha^{CC}(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\gamma_\alpha l_L(x) + j_\alpha^{CC\text{quark}}(x). \tag{56}$$

Here $j_\alpha^{CC\text{quark}}(x)$ is the quark charged current, $W^\alpha(x)$ is the field of the charged $W^\pm$ vector bosons. and g is the constant of the electroweak interaction.

2. The flavor neutrino field $\nu_{lL}(x)$ is given by the mixing relation:

$$\nu_{lL}(x) = \sum_{k=1}^{3} U_{lk}\nu_{kL}(x), \tag{57}$$

where

$$\nu_k(x) = \nu_k^c(x) = C\bar{\nu}_k^T(x), \quad (k = 1, 2, 3) \tag{58}$$

is the field of *the Majorana neutrino* with mass m_k and U is the unitary 3×3 PMNS mixing matrix.[7]

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[7] We assume that a beyond the SM physics contributes to the matrix element of the $0\nu\beta\beta$-decay via Majorana neutrino masses. If neutrinos with definite masses are Majorana particles their masses definitely contribute to the matrix element of the $0\nu\beta\beta$-decay. For discussion of possible additional contribution of effective Lagrangians to the $0\nu\beta\beta$-decay see, for example, [41].

Existing weak interaction data are in perfect agreement with (55). All atmospheric, accelerator, solar, and long baseline reactor neutrino oscillation data are in agreement with the three neutrino mixing (57).

The effective Hamiltonian of the β-decay is given by the expression:

$$\mathcal{H}_I(x) = \frac{G_F}{\sqrt{2}} 2\, \bar{e}_L(x)\gamma_\alpha\, \nu_{eL}(x)\, j^\alpha(x) + \text{h.c.} \tag{59}$$

where $j^\alpha(x)$ is $\Delta S = 0$ hadronic charged current and the field of electron neutrinos $\nu_{eL}(x)$ is given by (57).

In the second order of the perturbation theory in G_F the matrix element of the $0\nu\beta\beta$-decay is given by the following expression:

$$\begin{aligned} \langle f|S^{(2)}|i\rangle &= 4\frac{(-i)^2}{2\,!}\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\int \bar{u}_L(p_1)e^{ip_1x_1}\gamma_\alpha\, \langle 0|T(\nu_{eL}(x_1)\,\nu_{eL}^T(x_2)|0\rangle \\ &\times \gamma_\beta^T\, \bar{u}_L^T(p_2)e^{ip_2x_2}\langle N_f|T(J^\alpha(x_1)J^\beta(x_2))|N_i\rangle\, d^4x_1 d^4x_2 - (p_1 \leftrightarrows p_2). \end{aligned} \tag{60}$$

Here p_1 and p_2 are electron momenta, $J^\alpha(x)$ is the weak charged current in the Heisenberg representation, N_i (N_f) are states of the initial (final) nuclei with 4-momenta $P_i = (E_i, \vec{p}_i)$ ($P_f = (E_f, \vec{p}_f)$), and $N_p = \frac{1}{(2\pi)^{3/2}\sqrt{2p^0}}$ is the standard normalization factor.

From the Majorana condition (58) follows that:

$$\nu_k^T(x) = \bar{\nu}_k(x)C^T = -\bar{\nu}_k(x)C. \tag{61}$$

Using this relation, for the neutrino propagator we have:

$$\begin{aligned} \langle 0|T(\nu_{eL}(x_1)\,\nu_{eL}^T(x_2)|0\rangle &= -\sum_k U_{ek}^2 \frac{1-\gamma_5}{2}\langle 0|T(\nu_k(x_1)\,\bar{\nu}_k(x_2)|0\rangle\frac{1-\gamma_5}{2}C \\ &= -\frac{i}{(2\pi)^4}\sum_k U_{ek}^2 \int \frac{m_k\, e^{-iq(x_1-x_2)}}{q^2 - m_k^2} d^4q\, \frac{1-\gamma_5}{2}C. \end{aligned} \tag{62}$$

Furthermore, taking into account that:

$$\bar{u}_L(p_1)\gamma_\alpha\frac{1-\gamma_5}{2}\gamma_\beta C\, \bar{u}_L^T(p_2) = -\bar{u}_L(p_2)\gamma_\beta\frac{1-\gamma_5}{2}\gamma_\alpha C\, \bar{u}_L^T(p_1) \tag{63}$$

and

$$T(J^\alpha(x_1)J^\beta(x_2)) = T(J^\beta(x_2)J^\alpha(x_1)) \tag{64}$$

from (60) and (62) for the matrix element of $0\nu\beta\beta$-decay we obtain the following expression:

$$\begin{aligned} \langle f|S^{(2)}|i\rangle &= -4\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\int \bar{u}_L(p_1)e^{ip_1x_1}\gamma_\alpha\left(\frac{i}{(2\pi)^4}U_{ek}^2\sum_k\int\frac{m_k\, e^{-iq\,(x_1-x_2)}}{q^2-m_k^2}d^4q\right) \\ &\times\gamma_\beta\frac{1+\gamma_5}{2}C\, \bar{u}_L^T(p_2)e^{ip_2x_2}\langle N_f|T(J^\alpha(x_1)J^\beta(x_2))|N_i\rangle\, d^4x_1d^4x_2. \end{aligned} \tag{65}$$

In this expression we can perform integration over x_1^0, x_2^0, and q^0. The matrix element of the $0\nu\beta\beta$-decay takes the form:

$$\langle f|S^{(2)}|i\rangle = 2i\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\bar{u}(p_1)\gamma_\alpha\gamma_\beta(1+\gamma_5)C\bar{u}^T(p_2)\int d^3x_1 d^3x_2 e^{-i\vec{p}_1\vec{x}_1-i\vec{p}_2\vec{x}_2}$$
$$\times\sum_k U_{ek}^2 m_k \frac{1}{(2\pi)^3}\int\frac{e^{i\vec{q}\,(\vec{x}_1-\vec{x}_2)}}{q_k^0}d^3q\,[\sum_n\frac{\langle N_f|J^\alpha(\vec{x}_1)|N_n\rangle\langle N_n|J^\beta(\vec{x}_2))|N_i\rangle}{E_n+p_2^0+q_k^0-E_i-i\epsilon}$$
$$+\sum_n\frac{\langle N_f|J^\beta(\vec{x}_2)|N_n\rangle\langle N_n|J^\alpha(\vec{x}_1))|N_i\rangle}{E_n+p_1^0+q_k^0-E_i-i\epsilon}]\,2\pi\delta(E_f+p_1^0+p_2^0-E_i). \tag{66}$$

Here $q_k^0 = \sqrt{\vec{q}^2+m_k^2}$ and $|N_n\rangle$ is the vector of the state of the intermediate nucleus with 4-momentum $P_n = (E_n, \vec{p}_n)$. In (66) the sum over the total system of the states $|N_n\rangle$ is assumed. Notice that we used the relation:

$$J^\alpha(x) = e^{iHx^0}J^\alpha(\vec{x})e^{-iHx^0}, \quad J^\alpha(\vec{x}) = J^\alpha(0,\vec{x}). \tag{67}$$

The Equation (66) is an exact expression for the matrix element of the $0\nu\beta\beta$-decay in the second order of the perturbation theory. The following approximations are standard ones:

1. Small neutrino masses m_k in the expression for the neutrino energy can be neglected ($q_k^0 \simeq |\vec{q}| = q$). In fact, an average neutrino momentum is given by $\bar{q} \simeq \frac{1}{r}$, where $r \simeq 10^{-13}$ cm is the average distance between nucleons in a nucleus. We have $\bar{q} \simeq 100$ MeV $\gg m_k$;
2. Long-wave approximation $e^{-i\vec{p}_i\vec{x}_i} \simeq 1$ $(i = 1, 2)$. In fact, we have $|\vec{p}_i \cdot \vec{x}_i| \leq p_i R$, where $R \simeq 1.2\cdot 10^{-13}\ A^{1/3}$ cm is the radius of a nucleus. Taking into account that $p_i \lesssim 1$ MeV we find $|\vec{p}_i\cdot\vec{x}_i| \ll 1$;
3. Closure approximation: The energy of neutrino in an intermediate state $q \simeq 100$ MeV is much larger than the excitation energy $(E_n - E_i)$. This means that energies of intermediate states E_n in the denominators of Equation (66) can be replaced by an average energy $\bar{E}$. In this approximation we can sum over intermediate states $|N_n\rangle$ in (66).

In the laboratory frame for the energy denominators in (66) we have:

$$E_n + p_{1,2}^0 + q_k^0 - E_i \simeq \bar{E} + (\frac{p_1^0+p_2^0}{2}) \pm (\frac{p_1^0-p_2^0}{2}) + q - M_i \simeq \bar{E} + q - \frac{M_i+M_f}{2}, \tag{68}$$

where $M_i(M_f)$ is the mass if initial (final) nucleus. In (68) we neglect nuclear recoil and take into account that $(\frac{p_1^0-p_2^0}{2}) \ll q$. For the matrix element of the $0\nu\beta\beta$-decay we find the following expression:

$$\langle f|S^{(2)}|i\rangle \simeq 2i\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\bar{u}(p_1)\gamma_\alpha\gamma_\beta(1+\gamma_5)C\bar{u}^T(p_2)m_{\beta\beta}$$
$$\times\int d^3x_1 d^3x_2\frac{1}{(2\pi)^3}\int\frac{e^{i\vec{q}\,(\vec{x}_1-\vec{x}_2)}}{q(\bar{E}+q-\frac{M_i+M_f}{2})}d^3q\,[\langle N_f|(J^\alpha(\vec{x}_1)J^\beta(\vec{x}_2)$$
$$+J^\beta(\vec{x}_2)J^\alpha(\vec{x}_1))|N_i\rangle]\,2\pi\delta(E_f+p_1^0+p_2^0-E_i), \tag{69}$$

where

$$m_{\beta\beta} = \sum_{i=1}^{3} U_{ek}^2\, m_k \tag{70}$$

is *the effective Majorana mass*.

Thus, due to the smallness of neutrino masses the matrix element of the $0\nu\beta\beta$-decay is a product the effective Majorana mass $m_{\beta\beta}$ (which depends on absolute values of neutrino masses with unknown lightest neutrino mass and on (known) θ_{13} and θ_{12} and on (unknown) Majorana phases) and nuclear part (which include the propagator of virtual neutrino). As a result of this factorization, the total decay rate of the $0\nu\beta\beta$-decay has the following general form:

$$\Gamma^{0\nu} = \frac{\ln 2}{T^{0\nu}_{1/2}} = |m_{\beta\beta}|^2 \, |M^{0\nu}|^2 \, G^{0\nu}(Q, Z). \tag{71}$$

Here $M^{0\nu}$ is the Nuclear Matrix Element (NME) and $G^{0\nu}(Q, Z)$ is the known phase-space factor which includes effects of the column interaction of electrons and daughter nucleus.

The calculation of the nuclear matrix elements of the $0\nu\beta\beta$-decay is a complicated many-body nuclear problem. Different approximate methods are used in such calculations. At the moment the results of different calculations of NME differ by 2–3 times. Discussion of these calculations is out of the scope of this review (see reviews [38,42,43]).

7. Effective Majorana Mass

In the case of the three neutrino mixing there are two independent mass-squared differences. From an analysis of experimental data it follows that one mass-squared difference is about 30 times smaller than the other one. The small (large) mass-squared difference is usually called solar (atmospheric) and is denoted by Δm^2_S (Δm^2_A).

Neutrino masses are labeled in such a way that the solar mass-squared difference is given by:[8]

$$\Delta m^2_S \equiv \Delta m^2_{12}. \tag{72}$$

From the MSW effect [44,45], observed in solar neutrino experiments, follows that[9]

$$\Delta m^2_{12} > 0. \tag{73}$$

For the mass of the third neutrino m_3 there are two possibilities:

1. Normal Ordering (NO):

$$m_3 > m_2 > m_1 \tag{74}$$

2. Inverted Ordering (IO):

$$m_2 > m_1 > m_3. \tag{75}$$

Atmospheric mass-squared difference Δm^2_A can be determined as follows:[10]

$$\Delta m^2_A = \Delta m^2_{23} \; (NO), \quad \Delta m^2_A = |\Delta m^2_{13}| \; (IO). \tag{76}$$

In the NO case for the neutrino masses m_2 and m_3 we obviously have:

$$m_2 = \sqrt{m^2_1 + \Delta m^2_S}, \;\; m_3 = \sqrt{m^2_1 + \Delta m^2_S + \Delta m^2_A}. \tag{77}$$

In the case of IO we find:

$$m_1 = \sqrt{m^2_3 + \Delta m^2_A}, \quad m_2 = \sqrt{m^2_3 + \Delta m^2_A + \Delta m^2_S}. \tag{78}$$

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8 We will use the following definition: $\Delta m^2_{ki} = m^2_i - m^2_k$.

9 In fact, the MSW resonance condition has a form $\Delta m^2_{12} \cos 2\theta_{12} = 2\sqrt{2} G_F n_e E > 0$ (n_e is the electron number density). From this condition follows that $\Delta m^2_{12} > 0$ under the standard assumption that the principal values of the mixing angle θ_{12} are in the range $0 \leq \theta_{12} \leq \frac{\pi}{2}$.

10 Notice that other definitions of Δm^2_A are also used in the literature (see [46]).

If neutrinos with definite masses ν_i are Dirac particles the PNMS mixing matrix is characterized by three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$, and one phase δ. In the standard parametrization the mixing matrix has the form:

$$U^{\rm D} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}. \tag{79}$$

Here $c_{ik} = \cos\theta_{ik}$, $s_{ik} = \sin\theta_{ik}$. If in the lepton sector CP is conserved $U^{\rm D*} = U^{\rm D}$ and $\delta = 0$.

If ν_i are Majorana particles, the mixing matrix is characterized by three angles and three phases and has the form:

$$U^M = U^{\rm D}\, S^M(\alpha), \tag{80}$$

where,

$$S^M(\alpha) = \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{81}$$

Values of Δm_S^2 and Δm_A^2, three neutrino mixing angles and CP phase, obtained from the global fit of the neutrino oscillation data, are presented in the Table 1.

Table 1. Values of neutrino oscillation parameters, obtained from the global fit of the data [47].

Parameter	Normal Ordering	Inverted Ordering
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.304^{+0.013}_{-0.012}$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.575^{+0.017}_{-0.021}$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02240^{+0.00062}_{-0.00062}$
δ (in °)	(195^{+51}_{-25})	(286^{+27}_{-32})
Δm_S^2	$(7.42^{+0.21}_{-0.20}) \cdot 10^{-5}$ eV^2	$(7.42^{+0.21}_{-0.20}) \cdot 10^{-5}$ eV^2
Δm_A^2	$(2.514^{+0.028}_{-0.027}) \cdot 10^{-3}$ eV^2	$(2.497^{+0.028}_{-0.028}) \cdot 10^{-3}$ eV^2

Notice that from data of all the latest neutrino oscillation experiments follows that the preferable neutrino mass spectrum is the NO spectrum (see talks at Neutrino 2020, http://nu2020.fnal.gov).

The effective Majorana mass strongly depends on the character of the neutrino mass spectrum. For illustration we will consider the following viable neutrino mass spectra:

Hierarchy of the neutrino masses

$$m_1 \ll m_2 \ll m_3. \tag{82}$$

In this case we have:

$$m_2 \simeq \sqrt{\Delta m_S^2}, \quad m_3 \simeq \sqrt{\Delta m_A^2} \quad m_1 \ll \sqrt{\Delta m_S^2}. \tag{83}$$

Thus, in the case of the neutrino mass hierarchy neutrino masses m_2 and m_3 are determined by the solar and atmospheric mass-squared differences and the lightest mass m_1 is very small. Neglecting its contribution and using the standard parametrization of the PMNS mixing matrix for the effective Majorana mass we have:

$$|m_{\beta\beta}| \simeq \left| \cos^2\theta_{13}\sin^2\theta_{12}\sqrt{\Delta m_S^2} + e^{2i\alpha}\sin^2\theta_{13}\sqrt{\Delta m_A^2} \right|, \tag{84}$$

where α is a phase difference.

The first term in (84) is small because of the smallness of Δm_S^2. The contribution to $|m_{\beta\beta}|$ of the "large" atmospheric mass-squared difference Δm_A^2 is suppressed by the smallness of $\sin^2\theta_{13}$. As a result, absolute values of the first and second terms in (84) are of the same order of magnitude. From (84) and Table 1 we find the 3σ upper bound:

$$|m_{\beta\beta}| \leq 4.29 \cdot 10^{-3}\text{eV}, \tag{85}$$

which is much smaller than upper bounds of $|m_{\beta\beta}|$ reached in modern experiments on the search for the $0\nu\beta\beta$-decay (see later).

Inverted hierarchy of neutrino masses

$$m_3 \ll m_1 < m_2. \tag{86}$$

In this case from (78) we have:

$$m_1 \simeq \sqrt{\Delta m_A^2}, \;\; m_2 \simeq \sqrt{\Delta m_A^2}\,(1 + \frac{\Delta m_S^2}{2\,\Delta m_A^2}) \simeq \sqrt{\Delta m_A^2}, \;\; m_3 \ll \sqrt{\Delta m_A^2}. \tag{87}$$

The lightest mass m_3 in the expression for the effective Majorana mass is multiplied by the small parameter $\sin^2\theta_{13}$. Neglecting the contribution of this term, we find:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m_A^2}\cos^2\theta_{13}\,(1 - \sin^2 2\,\theta_{12}\,\sin^2\alpha)^{\frac{1}{2}}, \tag{88}$$

where $\alpha = \alpha_2 - \alpha_1$ is the Majorana phase difference, the only unknown parameter in expression (88).

From (88) we find the following upper and lower bounds for the effective Majorana mass:

$$\cos^2\theta_{13}\;\cos 2\,\theta_{12}\,\sqrt{\Delta m_A^2} \leq |m_{\beta\beta}| \leq \cos^2\theta_{13}\,\sqrt{\Delta m_A^2}, \tag{89}$$

which are realized if there is CP invariance in the lepton sector.

From (89) and Table 1 we find the following $3\,\sigma$ range for the effective Majorana mass in the case of the inverted hierarchy of neutrino masses:

$$1.44 \cdot 10^{-2}\text{ eV} \leq |m_{\beta\beta}| \leq 5.01 \cdot 10^{-2}\text{ eV}. \tag{90}$$

As we will see in the next section, future experiments will be sensitive to the inverted hierarchy region.

8. Experiments on the Search for $0\nu\beta\beta$-decay

Up to now neutrinoless double β-decay was not observed. The results of some recent experiments on the search for $0\nu\beta\beta$-decay are presented in the Table 2 (see [48,49]). In the third column of Table 2, the 90% CL lower bounds for the half-life of the decay of different elements are given. In the fourth column, upper bound ranges for the effective Majorana mass $|m_{\beta\beta}|$ are presented. It was assumed that $g_A = 1.27$ (g_A is the axial constant) and nuclear matrix elements, calculated in different models, were taken from [42].

Table 2. Lower limits of half-lives and upper limits for the effective Majorana mass, obtained in recent experiments on the search for the $0\nu\beta\beta$-decay.

Experiment	Nucleus	$T_{1/2}(10^{25}$ yr)	$\lvert m_{\beta\beta}\rvert$ (eV)
GERDA [49]	^{76} Ge	9	0.10–0.23
KamLAND-Zen [50]	^{136} Xe	10.7	0.08-0.24
EXO-200 [51]	^{136} Xe	1.8	0.09–0.29
CUORE [52]	^{130} Te	3.2	0.08–0.35
CUPID-0 [53]	^{82} Se	0.24	0.39–0.81

Many new experiments on the search for the $0\nu\beta\beta$-decay are in preparation (see [48,54]). In these experiments the inverted hierarchy region and, possibly, part of the normal hierarchy region will be probed. We will mention only a few of them. In the KamLAND-Zen experiment (^{136} Xe) after 5 years of running the sensitivity $T_{1/2} > 2 \cdot 10^{27}$ yr will be reached. In the SNO+ (^{130} Te), LEGEND (^{76} Ge), n-EXO (^{136} Xe), CUPID (^{100} Mo), and NEXT-HD (^{136} Xe) sensitivities $T_{1/2} > 1 \cdot 10^{27}$ yr, $T_{1/2} > 2 \cdot 10^{28}$ yr, $T_{1/2} > 5.7 \cdot 10^{27}$ yr, $T_{1/2} > 1.1 \cdot 10^{27}$ yr, and $T_{1/2} > 3 \cdot 10^{27}$ yr will be reached after 10 years of running.

Future experiments on the search for the $0\nu\beta\beta$-decay are planned to solve the most fundamental problem of modern neutrino physics: *Are neutrinos with definite masses Majorana or Dirac particles?* Neutrinos are the only fundamental fermions that can be Dirac or Majorana particles. If neutrino masses and mixing are generated by the Standard Higgs Mechanism, in this case into the SM Lagrangian enter right-handed neutrino fields ν_{lR}, the total lepton number L is conserved and neutrino with definite masses are Dirac particles. We know, however, that neutrino masses are many orders of magnitude smaller than masses of other fundamental fermions. It is very unlikely that neutrinos masses are generated by the same Higgs mechanism as masses of leptons and quarks. *It is very plausible that Standard Model neutrinos are massless particles and neutrino masses are generated by a beyond the SM mechanism.*

The most economical and general possibility to introduce neutrino masses and mixing is based on the assumption that there exist only left-handed flavor neutrino fields ν_{lL} ($l = e, \mu, \tau$). Neutrino masses can be introduced in this case only if the total lepton number L is not conserved and neutrinos with definite masses ν_i ($i = 1, 2, 3$) are Majorana particles (Majorana mass term).

The Weinberg effective Lagrangian (and its generalizations), which are based on numerous beyond the SM models, lead to the Majorana neutrino mass term, the three-neutrino mixing

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\nu_{iL}, \quad \nu_i = \nu_i^c$$

and neutrino masses m_i, which are naturally suppressed with respect to SM masses of leptons and quarks, by a ratio of the electroweak scale v and a scale of a new, beyond the SM physics Λ.

Non conservation of the lepton number, which is the basics of such an explanation of the smallness of neutrino masses, is an attractive and natural feature of a beyond the SM physics (see [15]).

In spite of very strong arguments in favor of the Majorana nature of neutrinos, a possibility of the Dirac neutrinos is not excluded *experimentally*. Notice that even a beyond the SM seesaw mechanism for the Dirac neutrinos, which could explain the smallness of neutrino masses, was proposed (see [6]). *Observation of the neutrinoless double β-decay* and a proof that neutrinos with definite masses are Majorana particle would be a unique and crucial test of our basic understanding of the phenomenon of small neutrino masses.

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