Article

Constraining CP4 3HDM with Top Quark Decays

Igor P. Ivanov [1,*] and Semyon A. Obodenko [2]

[1] School of Physics and Astronomy, Sun Yat-sen University, Zhuhai 519082, China
[2] Institute of Physics, Kazan Federal University, 16a Kremlyovskaya St., 420008 Kazan, Russia; semeon.obodenko@gmail.com
[*] Correspondence: ivanov@mail.sysu.edu.cn

Abstract: CP4 3HDM is a unique three-Higgs-doublet model equipped with a higher-order *CP*-symmetry in the scalar and Yukawa sector. Based on a single assumption (the minimal model with a *CP*-symmetry of order 4 and no accidental symmetry), it leads to a remarkable correlation between its scalar and Yukawa sectors, which echoes in its phenomenology. A recent scan of the parameter space of CP4 3HDM under the assumption of scalar alignment identified a few dozens of points which passed many flavor constraints. In the present work, however, we show that almost all of these points are now ruled out by the recent LHC searches of $t \to H^+ b$ with subsequent hadronic decays of H^+. Apart from a few points with charged Higgses heavier than the top quark, only one point survives all the checks, the model with an exotic, non-2HDM-like generation pattern of H^+ couplings with quarks. One can expect many more points with exotic H^+ couplings to quarks if the scalar alignment assumption is relaxed.

Keywords: multi-Higgs models; 3HDM; charged Higgs bosons

1. Introduction

The Standard Model (SM), despite its lasting success, does not provide any clue on the origin of the fermion masses, their mixing patterns, and *CP*-violation. Many models beyond the SM (bSM) address these issues by postulating new fields, symmetries, or interactions. A conservative approach is to assume that, around the electroweak energy scale, bSM physics manifests itself only via a non-minimal Higgs sector. Although the LHC measurements show that the discovered 125 GeV Higgs is very SM-like [1–3], it is well possible that there exist (many) other scalar fields, which have escaped collider searches due to decoupling from the main Higgs production channels.

Among various options for non-minimal Higgs sectors [4,5], *N*-Higgs-doublet models (NHDM) remain among the simplest and most attractive frameworks. Based on the simple idea that Higgs doublets can come in generations, NHDM can accommodate various new symmetries, discrete [6,7] or continuous [8], with phenomenologically interesting consequences.

The two-Higgs-doublet model (2HDM), by far the most explored version, has had limited success in linking the observed fermion properties with patterns of the scalar sector [9]. Very few symmetries are possible with two Higgs doublets [10–15]. Although they can be used to shape the Yukawa sectors (see the recent developments [16–18] beyond the classical 2HDM Type-I and Type-II models), they are insufficiently constraining or they predict highly peculiar properties not seen in experiment [15,19].

The three-Higgs doublet model (3HDM) can accommodate many more symmetry groups [6–8], some of which were used back in early 1980s to deduce the fermion masses and mixing hierarchies from symmetries, see a historical overview in Ref. [4]. Despite significant efforts, the overall situation is similar to the 2HDM. If one imposes a large discrete symmetry group, which would strongly constrain the scalar and Yukawa sectors, one predicts features which are in conflict with experiment, such as massless or mass-degenerate quarks, insufficient mixing, or the absence of *CP*-violation. The pedagogical

insight into the anatomy of this mismatch in A_4 and S_4-symmetric 3HDMs was given in Ref. [20]. In Ref. [21], improving the earlier observations of Ref. [22], it was shown that this failure stems from inability of sufficiently large symmetry groups to break down completely upon minimization of the Higgs potential. If, instead, one builds a 3HDM with a small symmetry group, or if one assumes that a symmetry of softly broken, one can successfully reproduce fermion masses and mixing at the expense of losing the symmetry control over the flavor properties. Each sector (scalar, up quark, and down quark) comes with its own free parameters, and one typically loses predictivity.

1.1. CP4 3HDM

Multi-Higgs-doublet models allow one to incorporate not only new Higgs or flavor symmetry groups but also exotic forms of CP-symmetries, which would not be possible within the SM. Recently, a version of the 3HDM was proposed in Ref. [23], which combines the minimality of assumptions with a surprising degree of control over the Yukawa sector. The model is based on a single symmetry, the CP-symmetry of order 4 (CP4), without any other accidental symmetry. Although this symmetry seems exotic, we remind the reader that quantum field theory does not uniquely specify how P or C transformations act on fields; see more details in the classical papers by Ref. [24,25] and textbooks of Ref. [26,27]. In particular, the action of CP transformation on Higgs doublets ϕ_a can be defined not only as the usual conjugation $\phi_a(t, \vec{r}) \xrightarrow{CP} \phi_a^*(t, -\vec{r})$ but also as the conjugation accompanied by a unitary rotation in the Higgs family space: $\phi_a(t, \vec{r}) \xrightarrow{CP} X_{ab}\phi_b^*(t, -\vec{r})$. It may happen that, when this CP transformation is applied twice, one does not get the identity transformation, and one would need to apply it four times for that. A CP transformation with this property can be called CP of order 4 and labeled as CP4.

The model proposed in Ref. [23] is CP4 invariant. Despite not respecting the usual CP-symmetry, the model is nevertheless CP-conserving [23,28,29]. It also offers the first example of a *physically distinct* form of CP-symmetry, with observables specific only to CP4 [29]. Therefore, it can, in principle, be distinguished from all other CP-conserving models constructed so far.

The CP4 symmetry was extended to the quark Yukawa sector in Ref. [30], leading to unusually strong constraints on the Yukawa matrices. In order to avoid mass-degenerate quarks, the initial CP4 symmetry must be spontaneously broken, which can be easily arranged by the scalar potential. After symmetry breaking, the Yukawa sector still contains enough free parameters to accommodate quark masses and mixing, as well as the appropriate amount of CP-violation. The numerical scan performed in Reference [30] gave many points in the parameter space which satisfy the electroweak precision tests and do not violate the kaon and B-meson oscillation parameters. After that, there remains very little freedom, and the model exhibits correlations which can be tested with further experimental results.

In short, the parameter space scans performed in Ref. [30] demonstrated that the CP4 3HDM can accomplish unexpectedly much for a 3HDM based on a single symmetry. Therefore, it is interesting to check if additional phenomenological constraints can be satisfied in this minimalistic setting.

1.2. Light Charged Higgses in Top Quark Decays

Due to the intrinsic relations between the scalar and Yukawa sectors of the CP4 3HDM, one expects scalar-induced flavor-changing neutral currents (FCNC). The FCNC generated by the SM-like Higgs can be eliminated by the simplifying assumption of an exact alignment in the scalar sector, the approach used in Ref. [30]. However, one still expects FCNCs in additional neutral Higgses, as well as non-trivial generation patterns in the charged Higgs couplings with quarks.

In this paper, we investigate a particular feature which emerged from the numerical scan of Ref. [30]. The majority of the parameter space points which passed the constraints of Ref. [30] led to one or both charged Higgses H_1^+ and H_2^+ being lighter than the top

quark. This allows for the decays $t \to H_{1,2}^+ d_i$, where $d_i = (d, s, b)$, with the subsequent decays of the charged Higgses $H_{1,2}^+ \to u_i \bar{d}_j$, with $u_i = (u, c, s)$. Such decays have been studied theoretically in the 2HDM [9,31,32], as well as in various versions of the 3HDM equipped with natural flavor conservation (NFC) [33–38]. These decays were also searched for at the LHC [39,40]. After Ref. [30] appeared, two more searches were published by the CMS collaboration [41,42]. All of them gave negative results, with the upper limits on the corresponding branching ratios at the subpercent level (see details below).

These searches required an isolated lepton, a large missing E_T, and the presence of at least two b-jets indicative of the $t\bar{t}$ production. To separate the signal from the SM background, at least one additional heavy-quark jet, b or c, arising from the $H^+ \to c\bar{s}$ or $H^+ \to c\bar{b}$ decays was required. These H^+ decay channels naturally arise in 2HDM or 3HDMs with natural flavor conservation. However, in CP4 3HDM, the flavor preferences of the off-diagonal couplings $\bar{u}_i d_j H_{1,2}^+$ do not necessarily follow the this pattern. In particular, the largest off-diagonal couplings do not always link the heaviest fermions. Thus, it remains an open question whether the existing ATLAS and CMS constraints rule out the CP4 3HDM examples with light charged Higgses. This is the question we address in the present paper.

Before going into the details, let us mention that charged Higgs bosons can of course manifest themselves through other processes, such as the direct pair production, Drell-Yan process, and charged Higgs loop contributions to the $h_{SM} \to \gamma\gamma$ decay, to name a few. These signatures have been studied at length for the 2HDM [9,31,32], and the negative results of their searches were used to constrain the 2HDM parameter space. However, care should be taken when extending these constraints beyond the two-Higgs-doublet models, in particular, to 3HDM. Just to give an example, it is well known that $B_s \to X\gamma$ decays rule out charged Higgs bosons of the 2HDM Type II for masses below about 600 GeV [32,43]. However, this limit is much weaker for the 2HDM Type I [43,44], 2HDM Type III [45], and for the so-called Aligned 2HDM [34], as well as in models with more than two doublets equipped with NFC [34,35,38].

Since CP4 3HDM does *not* possess the NFC property, it will be interesting to see, eventually, whether the light charged Higgses could be ruled out by $B_s \to X\gamma$ or other collider searches. This requires substantial work. What we point out in this paper is that a much more direct check can first be done before plunging into the radiative meson decay machinery: comparing the CP4 3HDM predictions for the top quark decay chains involving charged Higgses with the LHC searches. As we will show, a combination of three experimental checks rules out almost all the viable points identified in Ref. [30]. However, a detailed analysis also reveals a few highly exotic patterns of charged Higgs interaction with quarks, which will be interesting to check in new scans of the CP4 3HDM parameter space.

The structure of the paper is the following. In the next section, we give a brief reminder of the model CP4 3HDM and discuss the role of light charged Higgs bosons in top quark decays. In Section 3, we study three observables: the total top quark decay width Γ_t and the two decay chains investigated experimentally at the LHC: $t \to H^+ b$ followed by $Br(H^+ \to c\bar{s})$ or $Br(H^+ \to c\bar{b})$. We show that almost all points of Ref. [30] fail in at least one of these tests. We discuss and summarize our results in the last section. Appendix A provides some details on the possible Yukawa sectors of the CP4 3HDM.

2. Charged Higgses in CP4 3HDM

2.1. CP4 3HDM Scalar Sector

The 3HDMs make use of three Higgs doublets ϕ_a, $a = 1, 2, 3$ with identical quantum numbers. CP4 is a transformation which maps Higgs doublets to their conjugates with a simultaneous rotation in the doublet space. Following Ref. [23,30], we use the following form of the CP4:

$$\phi_a \xrightarrow{CP} X_{ab} \phi_b^*, \quad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}. \tag{1}$$

Applying this transformation twice leads to the Higgs family transformation with the matrix $XX^* = \text{diag}(1, -1, -1) \neq \mathbf{1}$. In order to get the identity transformation, one must apply CP4 four times, hence order-4 transformation. It is known that any CP-type transformation of order 4 acting in the space of three complex fields can be turned into (1) by a suitable basis change [27].

The most general renormalizable 3HDM potential respecting this symmetry [23] can be written as $V = V_0 + V_1$, where

$$
\begin{aligned}
V_0 &= -m_{11}^2(\phi_1^\dagger \phi_1) - m_{22}^2(\phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3) + \lambda_1(\phi_1^\dagger \phi_1)^2 + \lambda_2\Big[(\phi_2^\dagger \phi_2)^2 + (\phi_3^\dagger \phi_3)^2\Big] \\
&+ \lambda_3(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3) + \lambda_3'(\phi_2^\dagger \phi_2)(\phi_3^\dagger \phi_3) \\
&+ \lambda_4\Big[(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + (\phi_1^\dagger \phi_3)(\phi_3^\dagger \phi_1)\Big] + \lambda_4'(\phi_2^\dagger \phi_3)(\phi_3^\dagger \phi_2),
\end{aligned}
\tag{2}
$$

with all parameters being real, and

$$
V_1 = \lambda_5(\phi_3^\dagger \phi_1)(\phi_2^\dagger \phi_1) + \lambda_8(\phi_2^\dagger \phi_3)^2 + \lambda_9(\phi_2^\dagger \phi_3)(\phi_2^\dagger \phi_2 - \phi_3^\dagger \phi_3) + h.c.,
\tag{3}
$$

with real λ_5 and complex λ_8, λ_9.

Minimization of this potential and the resulting scalar bosons mass matrices were studied in Ref. [30]. The minimum breaks CP4, thus leading to a spontaneously broken *CP*-symmetry. Expansion of potential near the minimum produces five neutral scalar bosons and two pairs of charged Higgses $H_{1,2}^\pm$. For a generic setting, all neutral Higgs bosons can couple to WW and ZZ pairs. However, if one fixes $m_{11}^2 = m_{22}^2$, the model displays the scalar alignment property: one of the neutral Higgses h couples to the WW and ZZ exactly as in the SM, while the other four neutral boson decouple from these channels. In this case, the additional Higgses cannot be produced in gauge-boson fusion, and the only way to produce them would be through their couplings to quarks, which may help these bosons escape the present day LHC searches.

A particular feature which emerged from the numerical scan of Ref. [30] was that the additional Higgses turned out rather light, with masses of few hundred GeV. Thus, this particular version of the model *does not* possess the decoupling limit [46] (in fact, the results of Ref. [47] indicate that a spontaneously broken CP4 3HDM cannot possess this limit). Even more impressive, almost all parameter space points emerging from the numerical scan of Reference [30] contained one or two charged Higgses lighter than the top quark. This feature opens up new channels for the top decay to light quarks and the charged Higgses $H_{1,2}^\pm$, with subsequent quark decays of $H_{1,2}^\pm$.

These channels offers a robust check of the model. Indeed, we do not need to estimate direct production of the charged Higgses in $q\bar{q}$ collisions at the LHC, which may be subject to uncertainties. We simply need to analyze the top decay properties and search for $t \to d_j H_{1,2}^+(\to u_i \bar{d}_k)$, where d_j stands for (d, s, b) and $u_i = (u, c)$. However, this signal may differ substantially from the 2HDM pattern because there is no guarantee that the preferred decay chains involve the heaviest quarks. Thus, it is not immediately clear whether light charged Higgses are in conflict with the LHC searches.

2.2. CP4 3HDM Yukawa Sector

In order to describe charge Higgs coupling preferences, we briefly recapitulate the structure of the CP4 3HDM Yukawa sector, explored in detail in Ref. [30]. The quark Yukawa Lagrangian

$$
-\mathcal{L}_Y = \bar{q}_L \Gamma_a d_R \phi_a + \bar{q}_L \Delta_a u_R \tilde{\phi}_a + h.c.,
\tag{4}
$$

where $\tilde{\phi}_a = i\sigma_2\phi_a^* = (\phi_a^{0*}, -\phi_a^-)^T$ can be made CP4 invariant if we assume that CP4 acts non-trivially not only on the scalar doublets but also on fermions

$$
\psi_i \xrightarrow{CP} Y_{ij}\psi_j^{CP}, \quad \text{where} \quad \psi^{CP} = \gamma^0 C \bar{\psi}^T.
\tag{5}
$$

The Yukawa matrices Γ_a and Δ_a can only be of special types, producing cases A, B_1, B_2, B_3 in the up and down quark sectors; see details in the Appendix A. These cases can be combined: one can pick up one case for the down quarks and another for the up quarks, provided the left-handed doublets transform in the same way. However, several combinations were ruled out since they induced a way too strong meson oscillations. As a result, only three pairs were found possible in Ref. [30]: cases (B_1, B_1), (B_2, B_2), and (B_1, B_3), for the down and up quarks, respectively.

Once the scalar potential and the Yukawa sector are constructed, one minimizes the scalar potential and obtains the vacuum expectation values (vevs): $\langle \phi_a^0 \rangle = v_a / \sqrt{2}$, which are in general complex. One then substitutes them into the Yukawa sector and obtains the quark mass matrices:

$$\bar{d}_L M_d d_R + \bar{u}_L M_u u_L + h.c., \quad M_d = \frac{1}{\sqrt{2}} \sum_a \Gamma_a v_a , \quad M_u = \frac{1}{\sqrt{2}} \sum_a \Delta_a v_a^* . \tag{6}$$

As usual, we switch to the physical quark fields, $d_L = V_{dL} d_L^{\text{phys.}}$, $d_R = V_{dR} d_R^{\text{phys.}}$, $u_L = V_{uL} u_L^{\text{phys.}}$, $u_R = V_{uR} u_R^{\text{phys.}}$, so that the mass matrices M_d and M_u become diagonal. The $\bar{u}_L d_L W^+$ interaction then becomes non-diagonal leading to the CKM matrix $V = V_{uL}^\dagger V_{dL}$. The fitting procedure used in Ref. [30] made sure that all quark masses and mixing parameters, as well as the amount of CP violation, coincide with the experimental results.

2.3. Charged Higgs Bosons Couplings

We are interested here in finding the physical charged Higgs interactions with physical quarks. We use (4) to extract the charged scalar interaction matrices

$$- \mathcal{L}_{ch.} = (\bar{u}_L \Gamma_a d_R - \bar{u}_R \Delta_a^\dagger d_L) \phi_a^+ + h.c. \tag{7}$$

Then, we perform the rotations in the quark spaces as outlined above, as well as the rotation in the charged scalars space:

$$\phi_a^+ = R_{ab} H_b^+ , \tag{8}$$

where the index $b = 0, 1, 2$, so that $H_b^+ = (G^+, H_1^+, H_2^+)^T$, with $b = 0$ corresponding to the charged would-be Goldstone boson and $b = 1, 2$ corresponding to the physical charged Higgs bosons. Notice that the rotation matrix R_{ab} not only diagonalizes the charged scalar sector, but also brings us to a Higgs basis: $v_a = R_{a0} v$. Then, in terms of the physical fields, the interactions have the form

$$- \mathcal{L}_{ch.} = \bar{u}^{\text{phys.}} \left(\tilde{\Gamma}_b \cdot P_R - \tilde{\Delta}_b^\dagger \cdot P_L \right) d^{\text{phys.}} H_b^+ + h.c. \tag{9}$$

Here, $P_L = (1 - \gamma^5)/2$ and $P_R = (1 + \gamma^5)/2$ are the chiral projectors and

$$\tilde{\Gamma}_b = V_{uL}^\dagger \cdot \Gamma_a R_{ab} \cdot V_{dR} , \quad \tilde{\Delta}_b^\dagger = V_{uR}^\dagger \cdot \Delta_a^\dagger R_{ab} \cdot V_{dL} . \tag{10}$$

Both matrices have the following generation structure:

$$\tilde{\Gamma}, \tilde{\Delta}^\dagger \sim \begin{pmatrix} ud & us & ub \\ cd & cs & cb \\ td & ts & tb \end{pmatrix} . \tag{11}$$

One can also explicitly factor out the CKM matrix V and represent these interaction matrices as

$$\tilde{\Gamma}_b = V \cdot V_{dL}^\dagger \Gamma_a R_{ab} V_{dR} , \quad \tilde{\Delta}_b^\dagger = V_{uR}^\dagger \Delta_a^\dagger R_{ab} V_{uL} \cdot V . \tag{12}$$

Before we proceed with CP4 3HDM, it is instructive to see how Equation (9) simplifies in models with natural flavor conservation (NFC) [33,48,49]. In these cases, only one

structure Γ is responsible for down quark mass matrix and only one structure Δ gives rise to the up-quark mass matrix. Therefore, the charged Higgs interactions become

$$-\mathcal{L}_{ch.}^{(NFC)} = \frac{\sqrt{2}}{v}\bar{u}^{\text{phys.}}(Y_b V D_d \cdot P_R + X_b D_u V \cdot P_L)d^{\text{phys.}}H_b^+ + h.c., \tag{13}$$

where $D_d = \text{diag}(m_d, m_s, m_b)$ and $D_u = \text{diag}(m_u, m_c, m_t)$. Here, X_b and Y_b, $b = 1, 2$ are *numbers*, not matrices, and they depend on the particular type of the NFC realization. If one needs to extract a specific flavor pair, then the coupling becomes proportional to the corresponding CKM matrix element. In particular, the strongest coupling is $\bar{t}bH^+$ coming from

$$-\mathcal{L}_{tbH^+}^{(NFC)} = \frac{\sqrt{2}}{v}V_{tb}\,\bar{t}^{\text{phys.}}(Y_b m_b P_R + X_b m_t P_L)b^{\text{phys.}}H_b^+ + h.c. \tag{14}$$

The coefficients X and Y depend on the models and can also be constrained from the experiment; see, for example, Ref. [33]. Within 2HDM with NFC, there is only one charged Higgs, and its coefficient can be related with the angle β. For example, Type I 2HDM leads to $X = -Y = \cot\beta$, while, in Type II 2HDM $X = \cot\beta$, $Y = \tan\beta$.

CP4 3HDM does *not* possess the NFC property. Individual Yukawa structures Γ_a and Δ_a cannot produce viable quark mass matrices. It is crucial that several structures sum up to produce the mass matrices M_d and M_u. Therefore, in CP4 3HDM, we do not expect the charged Higgs coupling matrices $\tilde{\Gamma}_b$ and $\tilde{\Delta}_b^\dagger$ to always bear the CKM structure. In fact, as we will see below, there exist parameter space points with non-2HDM-like patterns. Since existing experimental searches are partially motivated by Type I or Type II 2HDM predictions, these points may avoid existing constraints. At the same time, they will show up strong in novel, non-canonical final states and can be checked in future.

2.4. Decays $t \to H^+ d_j$ and $H^+ \to u_i \bar{d}_j$

If the charged Higgs boson H^+ is sufficiently light, the top-quark can decay as $t \to d_j H^+$. This new channel leads to two effects: a modification of the total top-quark width with respect to the SM value, and the appearance of a novel final-state signal, which depends on the H^+ decay preferences.

At tree level, the decay width $t \to d_j H^+$, where $d_j = (d, s, b)$ with masses m_j, can be written as

$$
\begin{aligned}
\Gamma_{t \to d_j H^+} &= \frac{\sqrt{\lambda(m_t^2, m_{H^+}^2, m_j^2)}}{32\pi m_t^3}\left[(m_t^2 + m_j^2 - m_{H^+}^2)(|\tilde{\Gamma}_{tj}|^2 + |\tilde{\Delta}_{tj}^\dagger|^2) - 4m_t m_j \text{Re}\left(\Gamma_{tj}\Delta_{jt}\right)\right], \\
&\approx \frac{|\tilde{\Gamma}_{tj}|^2 + |\tilde{\Delta}_{tj}^\dagger|^2}{32\pi}m_t\left(1 - \frac{m_{H^+}^2}{m_t^2}\right)^2.
\end{aligned}
\tag{15}
$$

Here, we introduced the function

$$\lambda(m_t^2, m_{H^+}^2, m_j^2) = m_t^4 + m_{H^+}^4 + m_j^4 - 2m_{H^+}^2 m_t^2 - 2m_{H^+}^2 m_j^2 - 2m_t^2 m_j^2. \tag{16}$$

The second line (15) corresponds to neglecting the light quark mass $m_j \to 0$. Notice that Γ_{tj} and Δ_{jt} denote here individual matrix entries, not the entire matrices.

To get a qualitative estimate of the importance of this channel, let us compare this contribution with the SM top decay width, which, at tree level and in the approximation $m_b = 0$, has the form

$$\Gamma_{SM} = \frac{G_F m_t^3}{8\sqrt{2}\pi}\left(1 - \frac{m_W^2}{m_t^2}\right)^2\left(1 + 2\frac{m_W^2}{m_t^2}\right). \tag{17}$$

Since $2\sqrt{2}Gm_t^2 = 2m_t^2/v^2 \approx 1$, one gets $\Gamma_{SM}/m_t \approx 1/(32\pi)$ times the brackets of Equation (17), which yields $\Gamma \sim 1$ GeV. Comparing the two decay widths, we see that the competition is essentially between $|\tilde{\Gamma}_{tj}|^2 + |\tilde{\Delta}_{tj}^\dagger|^2$ and $(1 + 2m_W^2/m_t^2)/4 \approx 0.4$, corrected by the corresponding phase space factors. So, if $\sum_j(|\tilde{\Gamma}_{tj}|^2 + |\tilde{\Delta}_{tj}^\dagger|^2)$ stays well below 0.1, the contribution of the new channels to the top-quark width can be neglected. If $\sum_j(|\tilde{\Gamma}_{tj}|^2 + |\tilde{\Delta}_{tj}^\dagger|^2) \sim 1$, we must check whether the phase space factor in Equation (15) provides sufficient suppression of the new contribution to the decay width.

Once H_b^+ are produced in top decays, they subsequently decay into pairs of lighter quarks $H^+ \to u_i \bar{d}_j$, where $u_i = (u,c)$ and $d_j = (d,s,b)$. Keeping only one of the quark masses m non-zero (which is done only to track the threshold shift in the $H^+ \to u_i \bar{b}$ decays), we calculate the corresponding decay width as

$$\Gamma_{ij} = \Gamma(H^+ \to u_i \bar{d}_j) = \frac{N_c m_{H^+}}{16\pi}\left(1 - \frac{m^2}{m_{H^+}^2}\right)^2 (|\tilde{\Gamma}_{ij}|^2 + |\tilde{\Delta}_{ij}^\dagger|^2), \tag{18}$$

where $N_c = 3$ is the number of quark colors. Since we focus on cases with sufficiently light charged Higgses, $m_{H^+} \sim 90 - 160$ GeV, which were dominant in the numerical scan of Reference [30], there are no $H^+ \to W^+ H$ decays, with H being either SM or extra neutral Higgses.

We would like to stress an important difference between the $H^+ \to q\bar{q}$ decays, for which no SM counterpart exists, and the charged Higgs channel of the t decays. Since the additional Higgses are assumed to be leptophobic, H^+ decays only to quark pairs. The branching ratios of individual channels arise not from the absolute magnitudes of the entries $\tilde{\Gamma}_{ij}$ and $\tilde{\Delta}_{ji}$ but from the competition among all the entries. Therefore, if an individual entry $\tilde{\Gamma}_{ij} \ll 1$, nevertheless, it can be the dominant decay channel H^+ if all the other entries are even smaller.

3. Numerical Results

3.1. Top Quark Width

For numerical calculations, we use the parameter space points, including the matrices $\tilde{\Gamma}_{ij}$ and $\tilde{\Delta}_{ij}^\dagger$ produced by the numerical scan of Ref. [30]. The total statistics of points passing all criteria used in Ref. [30] is: 9 points for case (B_1, B_1), 10 points for case (B_2, B_2), 48 points for case (B_1, B_3). Among these 67 points, 5 did not contain charged Higgses lighter than the top quark. These five parameter space are not constrained by the present analysis. From the remaining 62 points, one-third (21 points) contained two charged Higgses lighter than 170 GeV. When analyzing them, we will plot branching ratios for each of the two Higgses. The remaining points contained only one charged Higgs lighter than the top quark.

We start by checking the charged Higgs contributions to the total top quark decay width. We calculate $\Gamma_t = \Gamma_{SM} + \sum_j \Gamma(t \to H_{1,2}^+ d_j)$, where the NLO (EW) + NNLO (QCD) Standard Model result $\Gamma_{SM} = 1.322$ GeV is taken from Ref. [50], while $\sum_j \Gamma(t \to H_{1,2}^+ d_j)$ is computed from Equation (15) summed over all the relevant channels. The latest PDG total top-quark decay width is $\Gamma_t = 1.42_{-0.15}^{+0.19}$ GeV [51]. It is consistent with the SM calculation but also leaves some room for New Physics contributions with branchng ratios at the level of tens of percents.

The results of this comparison are shown in Figure 1, where different colors correspond to cases (B_1, B_1), (B_2, B_2), and (B_1, B_3). If a parameter space point contains two light charged Higgses, we plot two points corresponding to the two $M_{H^\pm}$ values, with the common Γ_t summed over both charged Higgs contributions. We allow for at most a 2σ upward deviation from the central experimental value, which implies that we accept a point if the partial decay width $t \to H_a^+ d_j$, when summed over both kinematically allowed charged Higgses H_a^+ and over all down-type quarks d_j, is less than 0.5 GeV. As it can be seen on this plot, many points lead to a significant charged Higgs contribution to Γ_t and

are excluded by this check (in fact, there exist a few points with $\Gamma_t > 4$ GeV). Nevertheless, roughly half of all parameter space points survive this check.
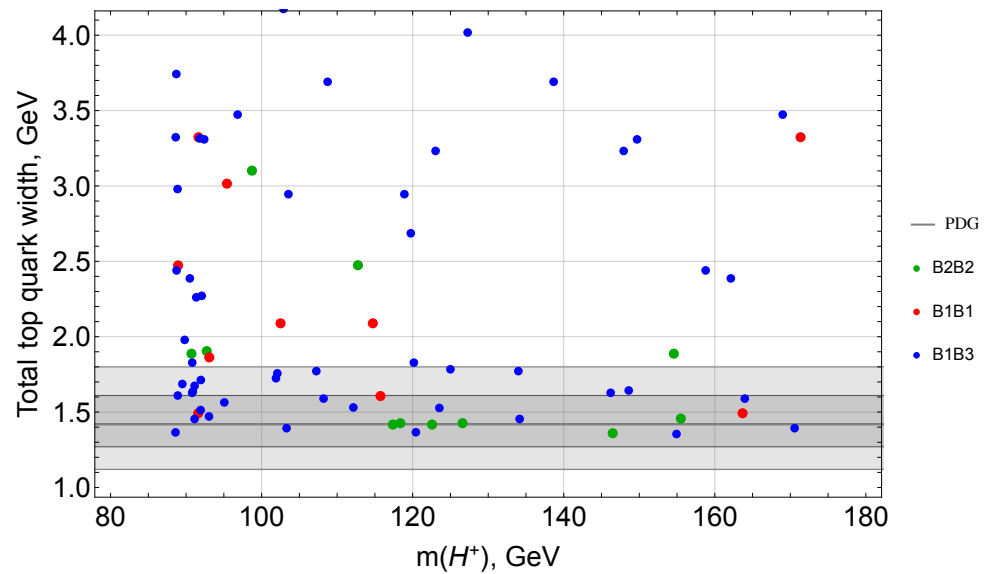

Figure 1. The total top-quark width calculated as the SM contribution plus all the $t \rightarrow H_a^+ d_j$ contributions for all the parameter space points used. Different colors correspond to the three Yukawa sector combinations. The shaded regions cover $\pm 1\sigma$ and $\pm 2\sigma$ regions around the central PDG value.

3.2. Branching Ratios

Now, we turn to the searches for light charged Higgses via their production in top-quark decays $t \rightarrow H^+ b$ and subsequent hadronic decays $H^+ \rightarrow u_i \bar{d}_j$, where $u_i \bar{d}_j = c\bar{s}$ [39,40,42] and $c\bar{b}$ [41]. The results of these searches are presented as upper limits on the top-quark branching fraction $Br(t \rightarrow bH^+) < p$ under the assumption that the corresponding H^+ decay is fully dominated by the selected hadronic channel: $Br(H^+ \rightarrow c\bar{s}) = 100\%$ in Ref. [39,40,42] and $Br(H^+ \rightarrow c\bar{b}) = 100\%$ in Ref. [41]. In our case, $H_{1,2}^+$ have several decay channels. Therefore, we present these results as upper limits on the *product* of branching ratios corresponding to production and decay of $H^\pm$ of the specific channel used in the experiment:

$$Br(t \rightarrow bH^+) \times Br(H^+ \rightarrow u_i \bar{d}_j) < p. \tag{19}$$

The value of p depends on the channel and on the charged Higgs mass. The strongest limits correspond to $p = 0.25\%$ in the $c\bar{s}$ channel [42] and $p = 0.5\%$ in the $c\bar{b}$ channel [41], both established by the CMS experiment. Although for the H^+ masses close to m_W or to m_t the limits are weaker, we used the most conservative (the strongest) limits for all the charged Higgs masses.

In Figure 2, we compare the prediction for the parameter space points found in Ref. [30] with the experimental constraints (19). We plot the points on the plane $Br(H^+ \rightarrow c\bar{s})$ (the upper plot) or $Br(H^+ \rightarrow c\bar{b})$ (the lower plot) vs $Br(t \rightarrow bH^+)$ and draw the line corresponding to the upper limit (19). Only points lying below the line on *both* plots can be considered as passing the check. If a parameter space point contains two light charged Higgses, we analyze their signals individually and plot them as separate points on these plots. In this case, in order for a model to pass the check, both charged Higgses must stay below the lines on both plots.
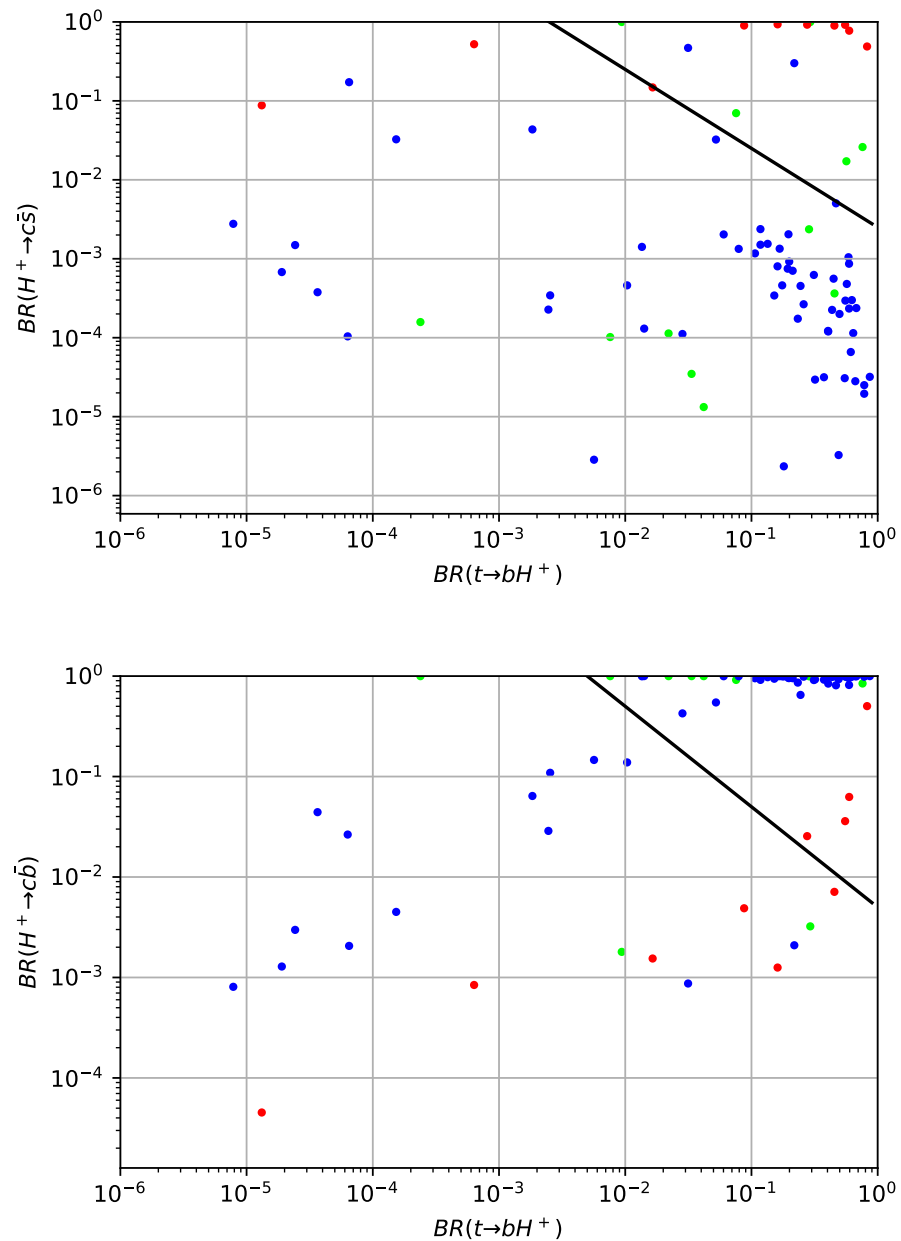
Figure 2. Branching ratios $Br(H^+ \to c\bar{s})$ (top) and $Br(H^+ \to c\bar{b})$ (bottom) vs. $Br(t \to H^+b)$. The colors encoding the Yukawa combinations (B_1, B_1), (B_2, B_2), and (B_1, B_3) are the same as in Figure 1. The oblique lines corresponds to the LHC upper limits (19) with $p = 0.25\%$ for $c\bar{s}$ [42] and $p = 0.5\%$ for $c\bar{b}$ [41].

As can be immediately seen, the vast majority of the points found in Ref. [30] fail this check for $H^+ \to c\bar{b}$ (case (B_1, B_3), blue points) or for $H^+ \to c\bar{s}$ (cases (B_1, B_1) and (B_2, B_2), red and green points). In addition, in some cases, when a point appears on both plots below the line, it corresponds to the *second* charged Higgs of the model lying close to the $t \to H_2^+ b$ decay threshold. However, the first charged Higgs of the same model usually leads to large branching ratios; therefore, the model is ruled out.

3.3. Notable Parameter Space Points

Among the 62 candidate models reported in Ref. [30] with at least one charged Higgses lighter than the top quark, only one model passed all our checks. This is a (B_1, B_3) model

with a very remarkable quark interaction patterns. This point contains only one light charged Higgs of mass $m(H_1^+) = 155$ GeV, which is only slightly lighter than the top quark. At the same time, the largest top coupling is only $(\tilde{\Delta}^\dagger)_{tb} = 0.34$, so that $Br(t \to H^+ b)$ is well below 1%. As for the subsequent decay of H_1^+, its couplings with quarks exhibit exotic patterns across generations:

$$\tilde{\Gamma} \approx \begin{pmatrix} 0.60 & 0.06 & 0.21 \\ 0.14 & 0.08 & 0.40 \\ 0.007 & 0.005 & 8.5 \end{pmatrix} \times 10^{-3}, \quad \tilde{\Delta}^\dagger \approx \begin{pmatrix} 0.0008 & 0.0011 & 0.168 \\ 0.006 & 0.0003 & 0.070 \\ 0.039 & 0.003 & 0.34 \end{pmatrix}. \qquad (20)$$

The numbers indicated refer to the absolute values of matrix entries. By inspecting the first two rows in these matrices, we conclude that the dominant decay mode is neither $H^+ \to c\bar{s}$ nor $H^+ \to c\bar{b}$ but $H^+ \to u\bar{b}$ with the branching ratio of about 90%. No experimental search exists for such final state of the charged Higgs decay. Even if the CMS results [41] can be recast in an equally strong constraint on $H^+ \to u\bar{b}$ as on $H^+ \to c\bar{b}$, this point would still pass the test thanks to the small $Br(t \to H^+ b)$.

We also found other points which were close to satisfying all our checks. Perhaps, the most intriguing example is given by a (B_1, B_1) model which contains two light charged Higgses with masses $m(H_1^+) = 103$ GeV and $m(H_2^+) = 115$ GeV. Their strongest couplings to quarks come from the following matrices $\tilde{\Delta}^\dagger$:

$$\tilde{\Delta}_1^\dagger \approx \begin{pmatrix} 2 \cdot 10^{-5} & 0.187 & 0.003 \\ 4 \cdot 10^{-5} & 0.194 & 0.008 \\ 2 \cdot 10^{-4} & 0.989 & 0.042 \end{pmatrix}, \quad \tilde{\Delta}_2^\dagger \approx \begin{pmatrix} 5 \cdot 10^{-5} & 9 \cdot 10^{-4} & 1.4 \cdot 10^{-5} \\ 0.0065 & 0.0027 & 1 \cdot 10^{-4} \\ 0.035 & 0.015 & 0.252 \end{pmatrix}. \qquad (21)$$

The two charged Higgses display very different preferences. The first one, H_1^+, is produced in the $t \to H^+ s$ decay and because of that it easily avoids experimental constraints. The second one, H_2^+, has mildly suppressed branching ratios and barely passes the decay constraints; it is visible in the top plot of Figure 2 as the red dot right on the line. However, due to the large $H_1^+ ts$ coupling, the total top quark decay width is about 2.1 GeV, which conflicts with the measurements.

These observations highlight the necessity of simultaneously checking all three observables: Γ_t, $Br(t \to bH^+(\to c\bar{b}))$ and $Br(t \to bH^+(\to c\bar{s}))$. They also confirm the initial expectations that models with very exotic, non-2HDM-like patterns of $H^+ q\bar{q}$ couplings can arise in CP4 3HDM.

4. Discussion and Conclusions

In this work, we continued exploration of a unique three-Higgs-doublet model equipped with a higher-order *CP*-symmetry, which was suggested first in Ref. [23]. We used the results of the parameter space scan performed in Ref. [30] and focused on the charged Higgs bosons. It turns out that almost all parameter space points which passed the electroweak precision and flavor constraints of Ref. [30] contain one or two charged Higgses lighter than the top quark. As a result, new top decay channels open up, such as $t \to H^+ b$ with subsequent hadronic decay of H^+.

Unlike other studies of the light charged Higgses in 3HDMs, such as Ref. [34–38], we do not—and cannot—assume the natural flavor conservation within CP4 3HDM. Thus, we can suspect that many points emerging from the scan of Ref. [30] could be in conflict with experimental data.

After Ref. [30] was published, new LHC results on light charged scalars searches appeared. In particular, the two CMS searches [41,42] of light charged Higgses emerging from top decays $t \to H^+ b$ and decaying hadronically to $Br(H^+ \to c\bar{s})$ or $Br(H^+ \to c\bar{b})$ placed subpercent level upper limits on the relevant branching ratios. In this work, we took these new data into account and checked whether the parameter space points considered viable in Ref. [30] were compatible with these new results. We also took into account

the updated value of the top quark total width, which places an upper bound on any non-standard decay of the top.

Out of 67 parameter space points borrowed from Ref. [30], five contained no charged Higgses lighter than top. These points remain viable and must be subjected to other experimental constraints. Among the remaining 62 points, only one passed all the three experimental constraints (Γ_t, $c\bar{s}$ and $c\bar{b}$ decays). This point avoided the experimental constraints because of its non-2HDM-like pattern of the H^+-quark couplings; see the matrices (20): the dominant decay channel is $H^+ \rightarrow u\bar{b}$, which was not searched for in experiment.

We also observed other examples where peculiar H^+-quark patterns allowed the charged Higgses to avoid two tests and only moderately fail the third one. Thus, such exotic patterns defying the 2HDM-based intuition are not exceptional and represent an intriguing feature of the CP4 3HDM.

It is interesting to check whether additional parameters space scans of the CP4 3HDM can identify other benchmark models with unusual charged Higgs patterns. To this end, we want to mention that the numerical scan of Ref. [30] used two additional assumptions: the 125 GeV was identified with the lightest neutral scalar, and the exact alignment was assumed in the scalar sector. The results of the present work show that these assumptions within CP4 3HDM tend to conflict with the data. By relaxing the alignment assumption, one can obtain many more viable points within the CP4 3HDM. Although there are indications that the true decoupling regime cannot be achieved within the spontaneously broken CP4 3HDM [47], one can still hope to generate benchmark models with heavier additional scalars which satisfy all present collider constraints. There remains much to explore within the CP4 3HDM.

Appendix A. CP4 Symmetric Yukawa Sectors

The quark Yukawa lagrangian (4) can be made CP4 symmetric, if the CP4 transformation acts on the fermion fields in a non-trivial way:

$$\psi_i \xrightarrow{CP} Y_{ij}\psi_j^{CP}, \quad \text{where} \quad \psi^{CP} = \gamma^0 C \bar{\psi}^T. \tag{A1}$$

For each sector, q_L, u_R, and d_R, one can use its own Y_{ij}. Within each sector, there always exists a basis in which the corresponding matrix Y takes the form

$$Y = \begin{pmatrix} 0 & e^{i\alpha} & 0 \\ e^{-i\alpha} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{A2}$$

with parameters α which can be different for the three sectors. The simultaneous solution of the consistency equations leads to one of the four possible options for these matrices labeled in Ref. [30] cases A, B_1, B_2, B_3:

Case *A*: $\alpha_L = \alpha_d = 1$, giving

$$\Gamma_1 = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{12}^* & g_{11}^* & g_{13}^* \\ g_{31} & g_{31}^* & g_{33} \end{pmatrix}, \quad \Gamma_{2,3} = 0. \tag{A3}$$

Case B_1: $\alpha_L = \pi/2$, $\alpha_d = 0$, giving

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{31} & g_{31}^* & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} -g_{22}^* & -g_{21}^* & -g_{23}^* \\ g_{12}^* & g_{11}^* & g_{13}^* \\ 0 & 0 & 0 \end{pmatrix}. \tag{A4}$$

Case B_2: $\alpha_L = 0$, $\alpha_d = \pi/2$, giving

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{13}^* \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} g_{22}^* & -g_{21}^* & 0 \\ g_{12}^* & -g_{11}^* & 0 \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}. \tag{A5}$$

Case B_3: $\alpha_L = \pi/2$, $\alpha_d = \pi/2$, giving

$$\Gamma_1 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ -g_{12}^* & g_{11}^* & 0 \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{23} \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & -g_{23}^* \\ 0 & 0 & g_{13}^* \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}. \tag{A6}$$

All parameters apart from g_{33} can be complex in each cases. Notice also that, in all cases, the matrices $\Gamma_{2,3}$ are expressed in terms of the same complex parameters and have the same textures.

The same list of cases exists for the up-quark sector. When constructing a viable model, we can combine different cases for down and up quarks, making sure that the transformation properties of the left-handed doublets (defined by α_L) are the same. Therefore, we get two series of possible CP4 3HDM Yukawa sectors:

$$\alpha_L = 0: \quad (A, A), \ (A, B_2), \ (B_2, A), \ (B_2, B_2), \tag{A7}$$

$$\alpha_L = \pi/2: \quad (B_1, B_1), \ (B_1, B_3), \ (B_3, B_1), \ (B_3, B_3). \tag{A8}$$

References

1. Aad, G.; Abajyan, T.; Abbott, B.; Abdallah, J.; Khalek, S.A.; Abdelalim, A.; Abdinov, O.; Aben, R.; Abi, B.; Abolins, M.; et al. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Phys. Lett. B* **2012**, *716*, 1. [CrossRef]
2. Chatrchyan, S.; Khachatryan, V.; Sirunyan, A.; Tumasyan, A.; Adam, W.; Aguilo, E.; Bergauer, T.; Dragicevic, M.; Erö, J.; Fabjan, C.; et al. Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. *Phys. Lett. B* **2012**, *716*, 30–61. [CrossRef]
3. Aad, G.; Abbott, B.; Abdallah, J.; Abeloos, B.; Aben, R.; AbouZeid, O.S.; Abraham, N.L.; Abramowicz, H.; Abreu, H.; Abreu, R.; et al. Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV. *JHEP* **2016**, *8*, 45. [CrossRef]
4. Ivanov, I.P. Building and testing models with extended Higgs sectors. *Prog. Part. Nucl. Phys.* **2017**, *95*, 160–208. [CrossRef]
5. Kanemura, S.; Tsumura, K.; Yagyu, K.; Yokoya, H. Fingerprinting nonminimal Higgs sectors. *Phys. Rev. D* **2014**, *90*, 075001. [CrossRef]
6. Ivanov, I.P.; Vdovin, E. Discrete symmetries in the three-Higgs-doublet model. *Phys. Rev. D* **2012**, *86*, 095030. [CrossRef]
7. Ivanov, I.P.; Vdovin, E. Classification of finite reparametrization symmetry groups in the three-Higgs-doublet model. *Eur. Phys. J. C* **2013**, *73*, 2309. [CrossRef]
8. Darvishi, N.; Pilaftsis, A. Classifying Accidental Symmetries in Multi-Higgs Doublet Models. *Phys. Rev. D* **2020**, *101*, 095008. [CrossRef]
9. Branco, G.C.; Ferreira, P.M.; Lavoura, L.; Rebelo, M.N.; Sher, M.; Silva, J.P. Theory and phenomenology of two-Higgs-doublet models. *Phys. Rep.* **2012**, *516*, 1–102. [CrossRef]
10. Ivanov, I.P. Minkowski space structure of the Higgs potential in 2HDM. *Phys. Rev. D* **2007**, *75*, 035001; Erratum in **2007**, *76*, 039902. [CrossRef]

11. Nishi, C.C. CP violation conditions in N-Higgs-doublet potentials. *Phys. Rev. D* **2006**, *74*, 036003; Erratum in **2007**, *76*, 119901. [CrossRef]

12. Ivanov, I.P. Minkowski space structure of the Higgs potential in 2HDM. II. Minima, symmetries, and topology. *Phys. Rev. D* **2008**, *77*, 015017. [CrossRef]

13. Maniatis, M.; von Manteuffel, A.; Nachtmann, O. A New type of CP symmetry, family replication and fermion mass hierarchies. *Eur. Phys. J. C* **2008**, *57*, 739–762. [CrossRef]

14. Ferreira, P.M.; Haber, H.E.; Silva, J.P. Generalized CP symmetries and special regions of parameter space in the two-Higgs-doublet model. *Phys. Rev. D* **2009**, *79*, 116004. [CrossRef]

15. Ferreira, P.M.; Silva, J.P. Abelian symmetries in the two-Higgs-doublet model with fermions. *Phys. Rev. D* **2011**, *83*, 065026. [CrossRef]

16. Cogollo, D.; Silva, J.P. Two Higgs doublet models with an S_3 symmetry. *Phys. Rev. D* **2016**, *93*, 095024. [CrossRef]

17. Alves, J.M.; Botella, F.J.; Branco, G.C.; Cornet-Gomez, F.; Nebot, M.; Silva, J.P. Symmetry Constrained Two Higgs Doublet Models. *Eur. Phys. J. C* **2018**, *78*, 630. [CrossRef]

18. Nebot, M. Bounded masses in two Higgs doublets models, spontaneous *CP* violation and $\mathbb{Z}_{\not{2}}$ symmetry. *Phys. Rev. D* **2020**, *102*, 115002. [CrossRef]

19. Maniatis, M.; Nachtmann, O. On the phenomenology of a two-Higgs-doublet model with maximal CP symmetry at the LHC. *JHEP* **2009**, *5*, 28. [CrossRef]

20. Felipe, R.G.; Serôdio, H.; Silva, J.P. Models with three Higgs doublets in the triplet representations of A_4 or S_4. *Phys. Rev. D* **2013**, *87*, 055010. [CrossRef]

21. Felipe, R.G.; Ivanov, I.P.; Nishi, C.C.; Serôdio, H.; Silva, J.P. Constraining multi-Higgs flavour models. *Eur. Phys. J. C* **2014**, *74*, 2953. [CrossRef]

22. Leurer, M.; Nir, Y.; Seiberg, N. Mass matrix models. *Nucl. Phys. B* **1993**, *398*, 319–342. [CrossRef]

23. Ivanov, I.P.; Silva, J.P. *CP*-conserving multi-Higgs model with irremovable complex coefficients. *Phys. Rev. D* **2016**, *93*, 095014. [CrossRef]

24. Feinberg, G.; Weinberg, S. On the phase factors in inversions. *Nuovo C.* **1959**, *14*, 571. [CrossRef]

25. Lee, T.D.; Wick, G.C. Space Inversion, Time Reversal, and Other Discrete Symmetries in Local Field Theories. *Phys. Rev.* **1966**, *148*, 1385. [CrossRef]

26. Branco, G.C.; Lavoura, L.; Silva, J.P. CP Violation. *Int. Ser. Monogr. Phys.* **1999**, *103*, 1. [CrossRef]

27. Weinberg, S. *The Quantum Theory of Fields. Volume 1: Foundations*; Cambridge University Press: Cambridge, UK, 1995.

28. Aranda, A.; Ivanov, I.P.; Jiménez, E. When the C in CP does not matter: anatomy of order-4 CP eigenstates and their Yukawa interactions. *Phys. Rev. D* **2017**, *95*, 055010. [CrossRef]

29. Haber, H.E.; Ogreid, O.M.; Osland, P.; Rebelo, M.N. Symmetries and Mass Degeneracies in the Scalar Sector. *JHEP* **2019**, *1*, 42. [CrossRef]

30. Ferreira, P.M.; Ivanov, I.P.; Jiménez, E.; Pasechnik, R.; Serôdio, H. CP4 miracle: shaping Yukawa sector with CP symmetry of order four. *JHEP* **2018**, *1*, 65. [CrossRef]

31. Akeroyd, A.G.; Aoki, M.; Arhrib, A.; Basso, L.; Ginzburg, I.F.; Guedes, R.; Hernandez-Sanchez, J.; Huitu, K.; Hurth, T.; Kadastik, M.; et al. Prospects for charged Higgs searches at the LHC. *Eur. Phys. J. C* **2017**, *77*, 276. [CrossRef]

32. Arbey, A.; Mahmoudi, F.; Stal, O.; Stefaniak, T. Status of the Charged Higgs Boson in Two Higgs Doublet Models. *Eur. Phys. J. C* **2018**, *78*, 182. [CrossRef]

33. Grossman, Y. Phenomenology of models with more than two Higgs doublets. *Nucl. Phys. B* **1994**, *426*, 355–384. [CrossRef]

34. Akeroyd, A.G.; Moretti, S.; Hernandez-Sanchez, J. Light charged Higgs bosons decaying to charm and bottom quarks in models with two or more Higgs doublets. *Phys. Rev. D* **2012**, *85*, 115002. [CrossRef]

35. Akeroyd, A.G.; Moretti, S.; Yagyu, K.; Yildirim, E. Light charged Higgs boson scenario in 3-Higgs doublet models. *Int. J. Mod. Phys. A* **2017**, *32*, 1750145. [CrossRef]

36. Akeroyd, A.G.; Moretti, S.; Song, M. Light charged Higgs boson with dominant decay to quarks and its search at the LHC and future colliders. *Phys. Rev. D* **2018**, *98*, 115024. [CrossRef]

37. Akeroyd, A.G.; Moretti, S.; Song, M. Light charged Higgs boson with dominant decay to a charm quark and a bottom quark and its search at LEP2 and future e^+e^- colliders. *Phys. Rev. D* **2020**, *101*, 035021. [CrossRef]

38. Chakraborti, M.; Das, D.; Levy, M.; Mukherjee, S.; Saha, I. Prospects of light charged scalars in a three Higgs doublet model with Z_3 symmetry. *arXiv* **2021**, arXiv:2104.08146.

39. Allison, L.; Barton, A.; Borissov, G.; Bouhova-Thacker, E.; Chilingarov, A.; Davidson, R.; Dearnaley, W.; Fox, H.; Grimm, K.; Henderson, R.; et al. Search for a light charged Higgs boson in the decay channel $H^+ \rightarrow c\bar{s}$ in $t\bar{t}$ events using pp collisions at $\sqrt{s}$ = 7 TeV with the ATLAS detector. *Eur. Phys. J. C* **2013**, *73*, 2465. [CrossRef]

40. Hollar, J.; Calpas, B.; Nayak, A.; Bargassa, P.; Beirão Da Cruz ESilva, C.; Di Francesco, A.; Faccioli, P.; Ferreira Parracho, P.G.; Gallinaro, M.; Leonardo, N.; et al. Search for a light charged Higgs boson decaying to $c\bar{s}$ in pp collisions at $\sqrt{s}$ = 8 TeV. *JHEP* **2015**, *12*, 178. [CrossRef]

41. Sirunyan, A.M.; Tumasyan, A.; Adam, W.; Ambrogi, F.; Asilar, E.; Bergauer, T.; Brandstetter, J.; Brondolin, E.; Dragicevic, M.; Erö, J.; et al. Search for a charged Higgs boson decaying to charm and bottom quarks in proton-proton collisions at $\sqrt{s}$ = 8 TeV. *JHEP* **2018**, *11*, 115. [CrossRef]

42. Sirunyan, A.M.; Tumasyan, A.; Adam, W.; Ambrogi, F.; Bergauer, T.; Dragicevic, M.; Erö, J.; Del Valle, A.E.; Frühwirth, R.; Jeitler, M.; et al. Search for a light charged Higgs boson in the H$^{\pm} \to$ cs channel in proton-proton collisions at $\sqrt{s} = 13$ TeV. *Phys. Rev. D* **2020**, *102*, 072001. [CrossRef]

43. Misiak, M.; Steinhauser, M. Weak radiative decays of the B meson and bounds on $M_{H^{\pm}}$ in the Two-Higgs-Doublet Model. *Eur. Phys. J. C* **2017**, *77*, 201. [CrossRef]

44. Arhrib, A.; Benbrik, R.; Harouiz, H.; Moretti, S.; Wang, Y.; Yan, Q.S. Implications of a light charged Higgs boson at the LHC run III in the 2HDM. *Phys. Rev. D* **2020**, *102*, 115040. [CrossRef]

45. Hernandez-Sanchez, J.; Moretti, S.; Noriega-Papaqui, R.; Rosado, A. Off-diagonal terms in Yukawa textures of the Type-III 2-Higgs doublet model and light charged Higgs boson phenomenology. *JHEP* **2013**, *7*, 44. [CrossRef]

46. Gunion, J.F.; Haber, H.E. The CP conserving two Higgs doublet model: The Approach to the decoupling limit. *Phys. Rev. D* **2003**, *67*, 075019. [CrossRef]

47. Carrolo, S.; Romão, J.C.; Silva, J.P.; Vazão, F. Symmetry and decoupling in multi-Higgs boson models. *Phys. Rev. D* **2021**, *103*, 075026. [CrossRef]

48. Albright, C.H.; Smith, J.; Tye, S.H.H. Signatures for Charged Higgs Boson Production in $e^{+}e^{-}$ Collisions. *Phys. Rev. D* **1980**, *21*, 711. [CrossRef]

49. Branco, G.C.; Buras, A.J.; Gerard, J.M. CP Violation in Models With Two and Three Scalar Doublets. *Nucl. Phys. B* **1985**, *259*, 306. [CrossRef]

50. Gao, J.; Li, C.S.; Zhu, H.X. Top Quark Decay at Next-to-Next-to Leading Order in QCD. *Phys. Rev. Lett.* **2013**, *110*, 042001. [CrossRef] [PubMed]

51. Zyla, P.; Barnett, R.M.; Beringer, J.; Dahl, O.; Dwyer, D.A.; Groom, D.E.; Lin, C.J.; Lugovsky, K.S.; Pianori, E.; Robinson, D.J.; et al. Review of Particle Physics. *PTEP* **2020**, *2020*, 083C01. [CrossRef]