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Consideration of Additive Quantum Numbers of Fermions and Their Conservations

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Abstract: Two new flavor quantum numbers D and U for down and up quarks, respectively, are introduced, and then quark quantum number H is proposed as the sum of the flavor quantum numbers of quarks. Moreover, lepton quark-like quantum number H_L and finally fermion quantum number F are brought forward. Old and new additive quantum numbers are conserved at three different levels in weak interaction, and F builds up a clear relationship to the electric charge of fermions.

Keywords: additive quantum number; conservation; fermion

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1. Introduction

Fermions include quarks and leptons. Six flavors of quarks are up (u), down (d), strange (s), charmed (c), bottom (b) and top (t). Quarks can be classified into three generations: (u,d), (c,s) and (t,b). Each of generation of quarks includes one u-type quark and one d-type quark. Isospin (I) and isospin projection (I_3) to neutron and proton [1,2], and then to u and d quarks, was assigned, because u and d quarks have similar mass so that they can be treated as two states of a common particle in the nucleon and pion. On the other hand, I and I_3 of the other flavors of quarks are zero, and these quarks have the flavor quantum numbers: S, C, B^* and T for s, c, b and t quarks, respectively. All quarks have the baryon quantum number $B = 1/3$. I, I_3, B and four flavor quantum numbers are additive quantum numbers. Antiquarks have additive quantum numbers with the same absolute values as quarks but the opposite sign of quarks, except that antiquarks have same I as quarks. Only B is conserved in both strong interaction and weak interaction. The other additive quantum numbers are conserved in strong interaction, but not conserved in weak interaction.

Leptons have six flavors too: electron (e), electron neutrino (ν_e), muon (μ), muon neutrino (ν_μ), tau (τ) and tau neutrino (ν_τ). Leptons can be classified into three generations: (ν_e,e), (ν_μ,μ) and (ν_τ,τ). Each of generation of leptons includes one u-type lepton and one d-type lepton. Leptons have the Lepton quantum number L which is equal to +1 for all six flavors of leptons, and -1 for all antileptons. L is an additive quantum number and conserved in weak interaction. In fact, weak interaction changes the lepton only into itself or into the other member of the same generation in the lepton family (e.g., [3]).

It has been noticed that quarks and leptons have different additive quantum numbers and, except for B, additive quantum numbers of quarks are not conserved in weak interaction. Here, some new additive quantum numbers of quarks are introduced, then leptons and finally fermions, and the corresponding conservations, are investigated.

2. Additive Quantum Numbers of Quarks

Corresponding to the other flavors of quarks, derived from I and I_3, we can define new the flavor quantum number D and U for d and u quarks, respectively, as

$$D = I_3 + (-1)^n I = \begin{cases} -1 & : \quad n = 1, \text{d quark} \\ 0 & : \quad n = 1, \text{u quark} \\ +1 & : \quad n = 0, \text{d anti-quark} \\ 0 & : \quad n = 0, \text{u anti-quark} \end{cases} \tag{1}$$

and

$$U = I_3 - (-1)^n I = \begin{cases} 0 & : \quad n = 1, \text{d quark} \\ +1 & : \quad n = 1, \text{u quark} \\ 0 & : \quad n = 0, \text{d anti-quark} \\ -1 & : \quad n = 0, \text{u anti-quark} \end{cases} \tag{2}$$

where $n = 1$ if considering a u or d quark, $n = 0$ if considering a u or d anti-quark. So I and I_3 can be expressed by U and D as

$$I_3 = (D + U)/2 \tag{3}$$

and

$$I = (-1)^n (D - U)/2, \tag{4}$$

Weight diagrams for, e.g., the hadrons including u, d and s quarks can be performed in the coordinate system among U, D and S instead of ones between S and I_3 (Figure 1). It is shown in Table 1 that U and D have the consistent character with the other four flavor quantum numbers of quarks. Moreover, U and D are same in conservations as the other flavor quantum numbers of quarks, i.e., U and D are conserved in strong interaction, for example:

$$\pi^- + p \rightarrow K^0 + \Lambda \tag{5}$$

but not conserved in weak interaction, for example:

$$K^0 \rightarrow \pi^+ + \pi^- \tag{6}$$

The relationship between the electric charge and quantum numbers of quarks was given by the Gell–Mann–Nishijima formula [4–6] (in extended form):

$$Q/e = I_3 + Y/2 \tag{7}$$

where Q is electric charge in units of e and hypercharge $Y = B + S + C + B^* + T$. After putting Equation (3) into Equation (7), the following is obtained

$$Q/e = B/2 + (D + U + S + C + B^* + T)/2 \tag{8}$$

Here, introduce the quark quantum number defined as the sum of all six flavor quantum numbers:

$$H = D + U + S + C + B^* + T \tag{9}$$

then (Table 1)

$$H = \begin{cases} -1 & : \quad \text{quark is 'd' type} \\ +1 & : \quad \text{quark is 'u' type} \end{cases} \tag{10}$$

H of each lepton is zero. Therefore,

$$Q/e = (B + H)/2 \tag{11}$$

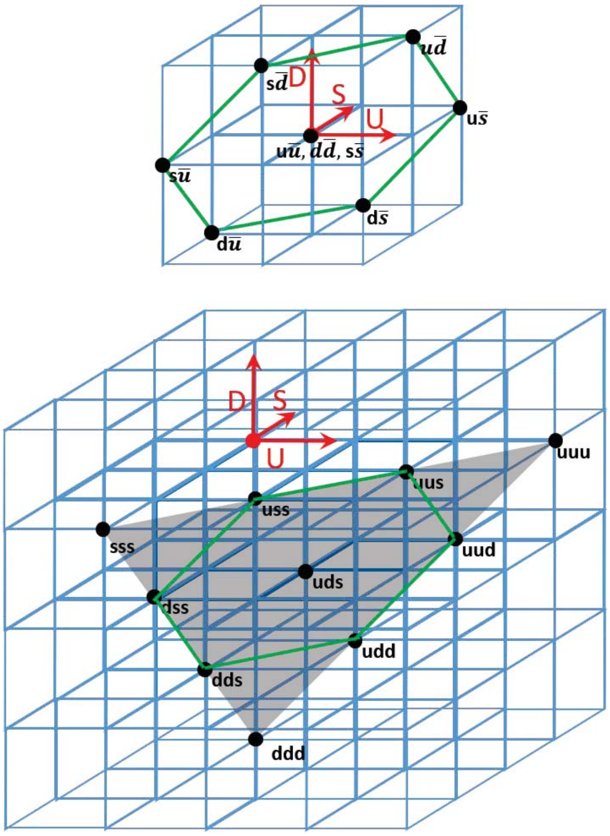

Figure 1. Weight diagrams for hadrons including u, d and s quarks. Upper panel: the meson nonet. Lower panel: the baryon octet (green lines) and decuplet (gray shadow).

Table 1. Additive quantum numbers of quarks. Antiquarks have additive quantum numbers with the same absolute values as quarks but the opposite sign of quarks except that antiquarks have same *I* as quarks.

Quark	Q/e	B	I	I_3	D	U	S	C	B^*	T	Y	H
d	−1/3	+1/3	+1/2	−1/2	−1	0	0	0	0	0	+1/3	−1
u	+2/3	+1/3	+1/2	+1/2	0	+1	0	0	0	0	+1/3	+1
s	−1/3	+1/3	0	0	0	0	−1	0	0	0	−2/3	−1
c	+2/3	+1/3	0	0	0	0	0	+1	0	0	+4/3	+1
b	−1/3	+1/3	0	0	0	0	0	0	−1	0	−2/3	−1
t	+2/3	+1/3	0	0	0	0	0	0	0	+1	+4/3	+1

Different from I, I_3, Y and six flavor quantum numbers which are conserved in strong interaction but not conserved in weak interaction, H is conserved in strong interaction and in weak interaction only including hadrons. For example, in Equation (6), I, I_3, Y, U, D and S are not conserved, but H is conserved. In weak interaction including both hadrons and leptons, H is not conserved. For example, H changes from -1 to +1 in β decay:

$$n \to p + e^- + \bar{\nu}_e \tag{12}$$

3. Additive Quantum Numbers of Leptons

In order to keep conservation in weak interaction including both hadrons and leptons, similarly as quark quantum numbers (Equation (10)), the lepton quark-like quantum number H_L is defined as:

$$H_L = \begin{cases} -1 & : \text{ lepton is 'd' type} \\ +1 & : \text{ lepton is 'u' type} \end{cases} \tag{13}$$

H_L of each quark is zero. It can be seen from Table 2 that relation between Q/e, $-L$ and H_L is same as one between Q/e, B and H of quarks (Equation (11)):

$$Q/e = (-L + H_L)/2 \tag{14}$$

Table 2. Additive quantum numbers of leptons. Antileptons have additive quantum numbers with the same absolute values as leptons but opposite sign of leptons.

Lepton	Q/e	L	H_L
e	−1	+1	−1
ν_e	0	+1	+1
μ	−1	+1	−1
ν_μ	0	+1	+1
τ	−1	+1	−1
ν_τ	0	+1	+1

It is shown that H_L is conserved in weak interaction including only leptons, for example in muon decay

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \tag{15}$$

but not conserved in weak interaction including both leptons and quarks, for example in β decay (Equation (12)).

4. Fermion Quantum Number

Finally, the fermion quantum number F for all fermions is combined from H and H_L:

$$F = H + H_L \tag{16}$$

and from Equations (10) and (13)

$$F = \begin{cases} -1 & : \text{ fermion is 'd' type} \\ +1 & : \text{ fermion is 'u' type} \end{cases} \tag{17}$$

Antifermions have F with the same absolute values as fermions but the opposite sign of fermions. F is conserved in both strong interaction and weak interaction for all fermions. For example, in β decay (Equation (12)), neither H nor H_L is conserved, but F is conserved. Equations (11) and (14) can be combined as

$$Q/e = (F_0 + F)/2 \tag{18}$$

where F_0 is:

$$F_0 = \begin{cases} 1/3 & : \text{ fermion is quark} \\ -1 & : \text{ fermion is lepton} \end{cases} \tag{19}$$

Conservations of H, H_L and F are compared with ones of the conventional additive quantum numbers in interactions in Table 3.

Table 3. Additive quantum numbers and conservations in different interactions.

	I, I_3, Y and Six Flavor Quantum Numbers	H	H_L	F
strong interaction	conserved	conserved	-	conserved
weak interaction including only quarks	not conserved	conserved	-	conserved
weak interaction including only leptons	-	-	conserved	conserved
weak interaction including all fermions	not conserved	not conserved	not conserved	conserved

5. Conclusions and Discussion

In weak interaction, there are three levels of conservation of additive quantum numbers: at the third level (the lowest level), I, I_3, Y and six flavor quantum numbers (U, D, S, C, B^* and T) are not conserved in weak interaction; at the second level, H or H_L is conserved in weak interaction including only part of fermions (hadrons or leptons, respectively); at the first level, F is conserved in weak interaction including all fermions. It is well-known that conservation of one quantum number corresponds to one type of gauge symmetry. Because conservation of F is conserved in electromagnetic interaction, strong interaction and weak interaction, including all fermions, F corresponds to fermion gauge symmetry in all interactions. From this point, F is at the same level as Q/e, B and L, but really different from I, I_3, Y and flavor quantum numbers. It can be expected to apply the new additive quantum numbers in quantum field theory, starting from the interaction Lagrangian. Moreover, the fact that F only has a value of +1 or -1 is consistent to the Pauli exclusion principle obeyed by fermions so that F can be expected to perform a different role from the old additive quantum numbers, e.g., in the theory of emergence of Pauli exclusion principle during the phase of decay to fermions at the early universe [7]. Moreover, the direct relation between Q/e, F_0 and F indicates that conservation of F is determined by conservation of Q/e or the reverse, and infers that there is a tight correlation between F (for strong interaction and weak interaction) and Q/e (for electro-magnetic interaction).

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