

Review

Moffat MODified Gravity (MOG)

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Abstract: Scalar Tensor Vector Gravity (STVG) or MODified Gravity (MOG) is a metric theory of gravity with dynamical scalar fields and a massive vector field introduced in addition to the metric tensor. In the weak field approximation, MOG modifies the Newtonian acceleration with a Yukawa-like repulsive term due to a Maxwell-Proca type Lagrangian. This associates matter with a fifth force and a modified equation of motion. MOG has been successful in explaining galaxy rotation curves, cosmological observations and all other solar system observations without the need for dark matter. In this article, we discuss the key concepts of MOG theory. Then, we discuss existing observational bounds on MOG weak field parameters. In particular, we will present our original results obtained from the X-COP sample of galaxy clusters.

Keywords: MODified Gravity; Scalar-Tensor-Vector-Gravity; alternate theories of gravity

1. Introduction

The theory of gravitation underwent a significant revolution over the last few centuries from the heliocentric model of Copernicus to discovering Kepler's laws of planetary motion to Newton's treatise Principia [1]. The idea also underwent a significant breakthrough during the early 20th century with Albert Einstein's General Theory of Relativity [2]. This was a tremendous leap considering the notion of gravity we had before Einstein. General Relativity (GR) received a warm welcome from the physics community with two immediate successes; the prediction of the advance of perihelion of mercury and the confirmation of gravitational deflection of light in 1919. Now it is beyond doubt that GR is an unavoidable tool in astrophysics. According to [3], the modern experimental relativity can be divided into four periods: Genesis, Hibernation, the Golden era and the quest for strong gravity. This quest for strong gravity over the last three decades took us to a position of detecting and studying the most violent events in the Universe such as the merging of binary blackholes, which has not only opened a new window to the universe but also supporting the validity of Einstein's equation in a strong and highly dynamical regime [4]. Over the last few decades, GR has been subjected to several other stringent tests, all confirming the validity of GR within the experimental precision. Therefore, it is quite natural that one will encounter a question—why do we need an alternative to GR?

Attempts to modify GR are motivated by one or more of the following factors. Measurements of galaxy rotation curves [5], cluster mass and precise measurement of Cosmic Microwave Background all suggest that most of the observable universe is dark. Based on our currently accepted Λ CDM cosmology, 26% of the universe is made of dark matter and around 68% is made of dark energy. Although we have plenty of theoretical models to fit most of our astrophysical observations, we still lack answers to the most important questions regarding the nature of dark stuff, their fundamental composition and properties. This so called dark matter candidate believed to be a particle in some theories beyond standard model physics. However, efforts to detect such candidate particles in experiments have not obtained any evidence yet [6,7]. Another unsolved mystery is the existence of dark energy confirmed by supernovae studies. Apart from the above mentioned difficulties, some approaches to look beyond GR were motivated by the high energy perspective, the incompatibility of general relativity with quantum mechanics and studies related to the



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nature of spacetime singularities in GR. A century ahead now, we have so many alternative theories which modify general relativity motivated by the above mentioned factors.

Modified Gravity (MOG), also known as Scalar-Tensor-Vector-Gravity (STVG), is a relativistic covariant extension of GR with a massive vector field and scalar fields. It was proposed by John. W. Moffat in 2006 [8] to address the issue of the dark matter component in a natural way. The massive vector field in MOG is sourced by a gravitational charge, which means that, in MOG, all massive objects possess a gravitational force charge proportional to its inertial mass. As a consequence of this gravitational force, massive test particles do not free fall on geodesics and are subjected to a Lorentz-like force. Moreover, in the weak field limit the equations of motion in MOG reduce to a Yukawa type repulsive potential in addition to an attractive Newtonian potential. Since its formulation in 2006, MOG was able to explain several astrophysical observations without invoking any non-baryonic models which GR employ. MOG was able to fit the galaxy rotation curves, cluster data and reproduce the acoustic peaks of the CMB power spectrum [9–12] without any non-baryonic matter. In a recent study [13] it is shown that MOG is consistent with Λ CDM cosmology and is also successful in explaining the growth of structure without dark energy and dark matter.

This paper has been organized as follows; in Section 2, we introduce the MOG action, the field equations and conservation laws. In Section 3, we explain the modified equation of motion in MOG for a test particle and show that it satisfies the weak equivalence principle. In Section 4 we discuss the gravitational waves in MOG. Finally, we show that MOG can explain the galaxy cluster data and best fit values of the running parameters α and μ are evaluated for the X-COP galaxy cluster sample in Section 5. Further we discuss some observations in MOG in Section 6 and conclude in Section 7. In this paper we use the metric signature $(+, -, -, -)$.

2. MOG Field Equations

Modified Gravity is a metric theory of gravity which has, in addition to the metric, a dynamical massive vector field which couples universally to the matter, the gravitational constant G promoted to the status of a field and the mass of the vector field μ . The gravitational interaction is mediated by a massless spin-2 graviton, a massless spin-0 graviton and a massive spin-1 graviton and hence MOG was known in the beginning as Scalar-Tensor-Vector-Gravity (STVG) [8]. MOG has undergone some changes not only in its name but also in its action in the last few years from its original form in [8], which will be discussed later this section. The most recent version of MOG has the following action:

$$S = S_G + S_\phi + S_s + S_M. \tag{1}$$

S_G is the Einstein–Hilbert action with a non-zero cosmological constant. S_ϕ denotes the action for the vector field, S_s corresponds to the scalar field which includes the gravitational coupling strength G and the mass of the vector field μ and, finally, S_M denotes the matter action. They are defined as follows:

$$S_G = \frac{1}{16\pi} \int d^4x \sqrt{-g} \frac{1}{G} (R + 2\Lambda) \tag{2}$$

$$S_\phi = \int d^4x \sqrt{-g} \left[-\frac{1}{16\pi} B^{\mu\nu} B_{\mu\nu} + \frac{1}{8\pi} \mu^2 \phi_\mu \phi^\mu - J_\mu \phi^\mu \right] \tag{3}$$

$$S_s = \int d^4x \sqrt{-g} \left[\frac{1}{G^3} \left(\frac{1}{2} g^{\mu\nu} \nabla_\mu G \nabla_\nu G \right) - V_G + \frac{1}{\mu^2 G} \left(\frac{1}{2} g^{\mu\nu} \nabla_\mu \mu \nabla_\nu \mu - V_\mu \right) \right], \tag{4}$$

where $B_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$ is the antisymmetric Faraday tensor of the vector field. In the original version, MOG [8] had an additional scalar field ω which is called the dimensionless vector field coupling parameter $\omega(x)$. This parameter has been set to unity as it does not

introduce any new physics; this is not something new but followed in most of the recent papers; [9] is one example. For a detailed discussion of this issue, we refer the reader to Section 5 of [14]. The terms $V_G(G)$, $V_\mu(\mu)$ denote the self-interacting potentials associated with the scalar field G and μ . For simplicity, we assume the self-interacting potential to vanish in the rest of our discussion. The energy momentum tensor is defined as follows:

$$T_{\mu\nu}^M = \frac{-2}{\sqrt{g}} \frac{\delta S_M}{\delta g^{\mu\nu}}, \quad T_{\mu\nu}^\phi = \frac{-2}{\sqrt{g}} \frac{\delta S_\phi}{\delta g^{\mu\nu}}, \quad T_{\mu\nu}^s = \frac{-2}{\sqrt{g}} \frac{\delta S_s}{\delta g^{\mu\nu}}. \tag{5}$$

The field equations corresponding to the action in Equation (1) can be obtained using the standard techniques, such as using first and second order Euler Lagrange equations. For more details in this regard, we refer the reader to [15]:

$$G_{\mu\nu} - \Lambda g_{\mu\nu} + Q_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{6}$$

$$\nabla_\nu B^{\mu\nu} + \mu^2 \phi^\mu = 4\pi J^\mu \tag{7}$$

$$\square G = K \tag{8}$$

$$\square \mu = L, \tag{9}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor, $Q_{\mu\nu} = \frac{2}{G^2} (\partial^\alpha G \partial_\alpha G g_{\mu\nu} - \partial_\mu G \partial_\nu G) - \frac{1}{G} (\square G g_{\mu\nu} - \nabla_\mu \partial_\nu G)$ and $T_{\mu\nu} = T_{\mu\nu}^M + T_{\mu\nu}^\phi + T_{\mu\nu}^s$. In the field equations for scalar field G and μ the terms K and L are given by:

$$K = \frac{3}{G} \left(\frac{1}{2} \nabla_\mu G \nabla^\mu G - V_G \right) - \frac{G}{\mu^2} \left(\frac{1}{2} \nabla_\mu \mu \nabla^\mu \mu - V_\mu \right) - \frac{\partial V_G}{\partial G} - \frac{G}{16\pi} R \tag{10}$$

$$L = \frac{1}{G} \nabla^\mu G \nabla_\mu \mu + \frac{\nabla_\mu \mu \nabla^\mu G}{G} - G \mu^3 \phi_\mu \phi^\mu + \frac{2}{\mu} V_\mu - \frac{\partial V_\mu}{\partial \mu}. \tag{11}$$

Using Equation (6) along with Bianchi identity $\nabla_\nu G^{\mu\nu} = 0$, it is straight forward to obtain the following conservation law:

$$\nabla_\nu T^{\mu\nu} + \frac{1}{G} \nabla_\nu G T^{\mu\nu} - \frac{1}{8\pi G} \nabla_\nu Q^{\mu\nu} = 0. \tag{12}$$

J^ν in the in-homogeneous vector field equation is the new gravitational force matter current density which acts as a source for the vector field ϕ^μ , it can be obtained by varying its action.

$$J^\nu = \frac{-1}{\sqrt{-g}} \frac{\delta S_M}{\delta \phi_\nu}. \tag{13}$$

An important feature of MOG is that all massive particles possess a new gravitational force charge $Q_g = \int J^0(x) d^3x$. The homogeneous equations that follow as a consequence of the definition of $B_{\mu\nu}$ are:

$$\nabla_\alpha B_{\mu\nu} + \nabla_\nu B_{\alpha\mu} + \nabla_\mu B_{\nu\alpha} = 0. \tag{14}$$

3. EOM and Weak Field Limit of MOG

3.1. Equation of Motion in MOG

Test particle motion in MOG is different from general relativity due to the presence of extra dynamical fields. In MOG, all massive particles possess a gravitational charge q_g proportional to its inertial mass and experiences a Lorentz like force. It is shown in [14,16] that a test particle has an action of the form:

$$S_m = - \int (m + q_g \phi_\mu u^\mu) d\tau, \tag{15}$$

where τ is the proper time along the world line of the test particle of mass m , u^μ is the four velocity and the gravitational fifth force charge is given by:

$$q_g = \int J^0(\mathbf{x}')d^3x' = \kappa M, \tag{16}$$

where $\kappa = \sqrt{\alpha G_N}$ and G_N is the Newton’s gravitational constant. The dimensionless parameter α to be discussed in Section 3.2 measures the enhancement of gravitational strength, given by $G = G_N(1 + \alpha)$. Test particle action in Equation (15) is different from the one in [16]. The vector field coupling constant ω in [16] is not considered as it does not affect the equation of motion. We can obtain the test particle equation of motion in MOG by finding the Euler Lagrange equation:

$$\frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial u^\nu} - \frac{\partial \mathcal{L}}{\partial x^\nu} = 0 \tag{17}$$

where $\mathcal{L} = -m\sqrt{g_{\alpha\beta}u^\alpha u^\beta} - q_g\phi_\mu u^\mu$ and $u^\nu = \dot{x}^\nu = \frac{dx^\nu}{d\tau}$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}^\nu} = -mg_{\nu\beta}\dot{x}^\beta - q_g\phi_\nu \tag{18}$$

$$\frac{\partial \mathcal{L}}{\partial x^\nu} = -\frac{m}{2}g_{\alpha\beta,\nu}\dot{x}^\alpha \dot{x}^\beta - q_g\phi_{\mu,\nu}\dot{x}^\mu \tag{19}$$

$$\frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial u^\nu} - \frac{\partial \mathcal{L}}{\partial x^\nu} = -mg_{\nu\beta}\dot{x}^\beta + (-mg_{\nu\beta,\alpha} + \frac{m}{2}g_{\alpha\beta,\nu})\dot{x}^\alpha \dot{x}^\beta + q_g\dot{x}^\mu(\phi_{\mu,\nu} - \phi_{\nu,\mu}) = 0 \tag{20}$$

$$m\left(\frac{d^2x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}\right) = q_g B_\alpha^\mu \frac{dx^\alpha}{d\tau}. \tag{21}$$

A careful analysis of Equation (21) shows that MOG satisfies the *Weak Equivalence Principle* as the gravitational force charge of the test particle is proportional to its mass, this makes Equation (21) as a whole independent of mass of the particle. Unlike in GR, the test particles do not free-fall along the geodesics of spacetime as it is evident from the presence of a non-zero term on the right hand side of Equation (21). It is also important to note that, for massless particles such as photons and spin-2 gravitons, the gravitational charge vanishes and they free-fall along the geodesics of the spacetime and obey the geodesic equation of general relativity.

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0. \tag{22}$$

The theory is consistent with LIGO GW170817 results [17], a binary merger of two neutron stars with an electromagnetic counterpart confirming that gravitational waves and electromagnetic waves travel at the same speed [18].

3.2. Weak Field Approximation

MOG field equations are non-linear and therefore exact solutions require efficient numerical simulations. However, most of the astrophysical systems the we are interested in do not require exact solutions and there exists a weak field approximation to MOG in which we recover an effective potential with a Yukawa-like repulsive contribution in addition to the usual attractive Newtonian term. This weak-field limit to MOG was first derived in [9]. To derive the field equations in the weak-field approximation, we perturb the metric $g_{\mu\nu}$ around the flat Minkowski metric $\eta_{\mu\nu}$ as in GR:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{23}$$

In addition to the metric there are vector and scalar fields in MOG and they can be perturbed in the following manner:

$$\phi_\mu = \phi_{\mu(0)} + \phi_{\mu(1)} \tag{24}$$

$$G = G_{(0)} + G_{(1)} \tag{25}$$

$$\mu = \mu_{(0)} + \mu_{(1)}. \tag{26}$$

The numbers in the subscripts denote the order of perturbation, where $\phi_{\mu(0)}$ denotes the zeroth order contribution and it vanishes in the Minkowski spacetime in the absence of a gravitating source. Similarly, the scalar field G and mass of the vector field μ are also perturbed around the Minkowski space. $G_{(0)}$ and $\mu_{(0)}$ are treated as constants. We follow the same notation as in [9] and treat $\mu_{(0)} = \mu$ as a constant in the weak field approximation and all higher order contributions are assumed to be negligible. The energy momentum tensor is perturbed and we assume the density of the vector field to be negligible $T_{\mu\nu}^{(\phi)} \ll T_{\mu\nu}^{(M)}$.

$$T_{\mu\nu} = T_{\mu\nu(0)} + T_{\mu\nu(1)}. \tag{27}$$

Substituting the perturbations from Equations (24)–(26) in the MOG field equations, we obtain:

$$R_{\mu\nu(1)} - \frac{1}{2}\eta_{\mu\nu}R_{(1)} = -8\pi G_0 T_{\mu\nu(1)}^{(M)}, \tag{28}$$

where $R_{(1)}$ is the first order perturbations in the Ricci scalar, for the field G we have:

$$\square G_{(1)} = -\frac{G_{(0)}}{16\pi}R_{(1)}. \tag{29}$$

Under the assumption of pressure-less dust, $T_{\mu\nu}^{(M)} = \text{diag}(\rho, 0, 0, 0)$ and using the trace of Equation (28), $R_{(1)} = 8\pi G_0\rho$, we obtain:

$$R_{00(1)} = -\frac{1}{2}\nabla^2 h_{00} \tag{30}$$

$$\frac{1}{2}\nabla^2 h_{00} = -4\pi G_0\rho. \tag{31}$$

If we assume the new gravitational force matter current to be conserved, $\nabla_\mu j^\mu = 0$, we can impose a Lorenz gauge condition on the vector field $\nabla_\nu \phi^\nu = 0$. With this assumption, we obtain the static weak-field limit of the vector field equation as:

$$(\nabla^2 - \mu^2)\phi^0 = -4\pi J^0. \tag{32}$$

The solution is given by:

$$\phi^0(x) = \int \frac{e^{-\mu|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|} J^0(\mathbf{x}') d^3\mathbf{x}'. \tag{33}$$

Using the modified equation of motion (21), it is straight forward to obtain the effective potential. Taking the divergence of the spatial component of EOM (21), along with the definition of modified acceleration $\mathbf{a} = -\nabla\Phi_{eff}$, we arrive at:

$$\nabla(\nabla\Phi_{eff} - \kappa\nabla\phi^0) = 4\pi G_0\rho. \tag{34}$$

The left hand side of the above expression can be equated to the familiar Newtonian Poisson equation:

$$\Phi_N = \Phi_{eff} - \kappa\phi^0. \tag{35}$$

Using the solution for Φ_N and the solution for ϕ^0 from Equation (33), we obtain the MOG effective potential in the required weak field limit. Substituting for the matter density $\rho(\mathbf{x}')$ and the fifth force matter current density $J^0(\mathbf{x}')$, one can study the dynamics of physical systems in the weak field limit of MOG:

$$\Phi_{eff} = - \int \frac{G_0 \rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3 \mathbf{x}' + \kappa^2 \int \frac{e^{-\mu|\mathbf{x} - \mathbf{x}'|}}{|\mathbf{x} - \mathbf{x}'|} J^0(\mathbf{x}') d^3 \mathbf{x}' \tag{36}$$

$$\Phi_{eff}(\vec{x}) = -G_N \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \left(1 + \alpha - \alpha e^{-\mu|\mathbf{x} - \mathbf{x}'|} \right) d^3 \mathbf{x}'. \tag{37}$$

In contrast to the Newtonian force, the fall in Newtonian attractive force with radial distance is counter acted by the repulsive Yukawa-like potential arising from massive vector field. Therefore, rotation curves in MOG are flatter than the Newtonian. The terms α and μ appearing in effective potential are not universal constants but depend on the mass of the physical system. The effective potential in Equation (37) reduces to GR in the limit $\alpha = 0$. A phenomenological formula for these parameters is obtained in [16] for static spherically symmetric systems.

$$\mu = \frac{D}{\sqrt{M}} \tag{38}$$

$$\alpha = \alpha_\infty \frac{M}{(\sqrt{M} + E)^2}. \tag{39}$$

D and E appearing in the above equations are universal constants in MOG and take values $D = 6.25 \times 10^3 M_\odot^{1/2} \text{kpc}^{-1}$, $E = 2.5 \times 10^4 M_\odot^{1/2}$, where α_∞ is given by $\alpha_\infty = \frac{G_\infty - G_N}{G_N}$, where G_∞ is the asymptotic limit of G for very large mass concentrations. For the Milky Way galaxy, estimates obtained from galaxy rotation curves suggest that the parameters take the following values $\alpha_{MW} = 8.89$ and $\mu_{MW} = 0.04 \text{kpc}^{-1}$. Using the above expression for α and μ it is easy to show that MOG can produce the Tully–Fisher relation [19]. There exists an empirical relation between mass and rotational velocity obtained from observations, which suggests that $v^4 \approx M$. Assuming a circular orbit, one can equate MOG radial acceleration to centripetal acceleration of the particle and obtain the rotational velocity:

$$v = \sqrt{[1 + \alpha - \alpha(1 + \mu r)e^{-\mu r}] \frac{G_N M}{r}}. \tag{40}$$

At scales $r = \mu^{-1}$ and using the phenomenological formula for μ given in Equation (38) one obtains:

$$v = \sqrt{[1 + \alpha(1 - 2e^{-1})] D G_N \sqrt{M}}. \tag{41}$$

This is in agreement with observation.

4. Gravitational Waves

The LIGO Science Collaboration and Virgo Collaboration observed the first GW event in 2015 [20]. This has opened a new window for observing the universe and has also provided us with a tool for testing general relativity like never before. The gravitational waves in general relativity have two polarization states, which are ‘plus’ and ‘cross’, but other theories of gravity could have more degrees of freedom which could be detected using gravitational waves. It has been shown in [21] that MOG has five polarization states—two tensor polarization states same as in GR due to the presence of a metric tensor, two vector polarization states due to the presence of a vector field and a scalar mode of the field G . In [21], the mass of the vector field is neglected as it has been known from experiments [18] that the mass of vector graviton is approximately of the same order of the experimental bound of photon mass $2.8 \times 10^{-28} \text{ eV}$. However in the presence of a non-vanishing μ , the vector field could excite a longitudinal mode increasing the polarization states to six. The spin-2 mass-less gravitons travel along the same null- geodesics due to a vanishing

gravitational charge ($q_g = 0$). In this section we derive the linearized gravitational wave equations and also discuss the polarization states in MOG. The field equations can be linearized by using the perturbations (23)–(26), and the indices are raised and lowered using the Minkowski metric $\eta_{\mu\nu}$. In this discussion, we set the cosmological constant $\Lambda = 0$ and obtain the field equation for the metric as:

$$G_{\mu\nu} + Q_{\mu\nu} = 8\pi GT_{\mu\nu}, \tag{42}$$

which, after substituting the metric perturbation (23), we arrive at the following equations for the Einstein tensor $G_{\mu\nu}$ and $Q_{\mu\nu}$.

$$G_{\mu\nu} = \frac{1}{2}(\partial_\mu\partial_\alpha h_\nu^\alpha + \partial_\nu\partial_\alpha h_\mu^\alpha - \partial_\mu\partial_\nu h - \square h_{\mu\nu} + \eta_{\mu\nu}(\square h - \partial_\alpha\partial_\beta h^{\alpha\beta})) \tag{43}$$

$$Q_{\mu\nu} = -\eta_{\mu\nu}\square\psi + \partial_\mu\partial_\nu\psi, \tag{44}$$

where $\psi = \frac{G^{(1)}}{G^{(0)}}$ is the scalar perturbation. The field equation can be written in a more compact form by re-writing in terms of the following tensor $\gamma_{\mu\nu}$ introduced in [21] and its trace $\gamma = 4\psi - h$, where h is the trace of first order metric perturbation.

$$\gamma_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu} + \psi\eta_{\mu\nu}. \tag{45}$$

Hence, we obtain the field equation as:

$$\partial_\mu\partial^\alpha\gamma_{\alpha\nu} + \partial_\nu\partial^\alpha\gamma_{\alpha\mu} - \partial^\alpha\partial^\beta\gamma_{\alpha\beta} - \square\gamma_{\mu\nu} = 16\pi GT_{\mu\nu}. \tag{46}$$

Now exploiting the gauge freedom we have by imposing Hilbert gauge condition $\partial^\mu\gamma_{\mu\nu} = 0$, we obtain the final tensor field equation, which is the same as in general relativity.

$$\square\gamma_{\mu\nu} = -16\pi GT_{\mu\nu} \tag{47}$$

In the absence of source $T_{\mu\nu} = 0$ Equation (47) becomes

$$\square\gamma_{\mu\nu} = 0. \tag{48}$$

The solution to the above equation can be expressed as a plane wave with wave vector k^μ .

$$\gamma^{\mu\nu} = A^{\mu\nu} \exp(ik_\mu x^\nu), \tag{49}$$

where $A^{\mu\nu}$ is a complex tensor. Using the above solution in the vacuum field Equation (48) and the Hilbert gauge condition we obtain:

$$\eta_{\mu\nu}k^\mu k^\nu = 0 \tag{50}$$

$$A^{\mu\nu}k_\nu = 0. \tag{51}$$

This shows that gravitational waves corresponding to the tensor fields travel along the null surface, i.e., we do not observe any difference between speed of light and speed of this component of the gravitational waves in MOG. This has been tested with the nearly simultaneous detection of electromagnetic signals and gravitational waves produced by the merger of binary neutron stars [18] GW170817/GRB170817A. For the vector field, using the perturbation (24) in the field equation of vector field (6) and imposing the Lorenz gauge condition $\partial_\nu\phi^\nu = 0$ we obtain:

$$(\square + \mu^2)\phi^\nu = 4\pi J^\nu. \tag{52}$$

In the absence of a source and neglecting the mass μ of the vector graviton owing to its negligible value [21] we get:

$$\square\phi^\nu = 0. \tag{53}$$

Similarly, for the perturbations of scalar field G given in (25), the equations of motion reduce to $\square\Psi = 0$. The immediate solutions are plane waves:

$$\psi = A_\psi \exp(il_\mu x^\mu) \tag{54}$$

$$\phi^\alpha = A^\alpha \exp(ip_\mu x^\mu). \tag{55}$$

Therefore, MOG has five polarization states which are plus, cross, Vector-X, Vector-Y and a breathing mode excited by the metric, vector field and the scalar field G respectively. If the mass μ of the vector field is taken into account, it would give rise to a longitudinal mode of polarization. A rigorous study of gravitational waves in MOG and a test with LIGO data are yet to be performed.

5. Hydrostatic Mass Profiles from X-ray Observations

Clusters are the most massive gravitationally bound structures in the universe. In the current understanding of structure formation, galaxy clusters are formed by the hierarchical sequence of mergers and the accretion of smaller systems driven by gravity with a dominant role of dark matter. This causes the Intra Cluster Medium (ICM) to heat up to a temperature $T \sim (2 - 100) \times 10^6$ K and the electrons in the ICM radiate in the X-ray band via thermal bremsstrahlung. Therefore, one of the most robust methods for studying their properties is based on X-ray data. The success of such an approach lies in the ability of the modern instruments to spatially resolve gas temperature and density profiles which helps in the reconstruction of total mass of the cluster. In particular, measuring the X-ray surface brightness integrated along the line of sight provides the total gravitating mass. Considering the Inter Cluster Medium (ICM) matter to be a perfect gas in hydrostatic equilibrium with the gravitational potential of the cluster, one can use the acceleration experienced by the gravitating mass $g = GM_{tot}(< r)/r^2$ in the equation of hydrostatic equilibrium:

$$\frac{dP_{gas}}{dr} = -\rho(r)g(r), \tag{56}$$

to obtain:

$$M_{tot}(< r) = \frac{r^2}{G\rho_{gas}(r)} \frac{dP_{gas}}{dr}, \tag{57}$$

where $M_{tot}(< r)$ is the mass of the cluster measured within a given radius r . The assumption that ICM behaves as a perfect gas obeying the equation of state of the form $P_{gas} = n_{gas}k_B T$, leads to

$$M_{tot}(< r) = -\frac{k_B}{\mu m_\mu} \frac{r T_{gas}(r)}{G} \left[\frac{d \ln \rho_{gas}(r)}{d \ln r} + \frac{d \ln T_{gas}(r)}{d \ln r} \right]. \tag{58}$$

It is convenient to describe a galaxy cluster as a spherical region of radius R_Δ with mean density Δ times the critical density $\rho_{c,z}$ at the clusters redshift z , where $\rho_{c,z} = 3H_z^2/8\pi G$ and $H_z = H_0[\Omega_\Lambda + \Omega_m(1+z)^3]^{0.5}$ is the expansion rate at the redshift z . Then the convenient quantity M_Δ is defined as:

$$M_\Delta \equiv M_{tot}(< R_\Delta) = \frac{4}{3}\pi\Delta\rho_{c,z}R_\Delta^3. \tag{59}$$

Similarly, the total mass of the cluster within the virial radius R_Δ in MOG can be obtained by using the expression for the modified acceleration with the parameters α and μ . The most general expression for the total mass within the radius r in MOG theory is obtained in [22] and the best fit values of the running parameters are obtained using X-COP galaxy cluster data.

$$M_{MOG}(< r) = \frac{M_{tot}(< r)}{1 + \alpha - \alpha e^{-\mu r}(1 + \mu r)}, \tag{60}$$

where $M_{tot}(< r)$ is the total mass given by the Newtonian potential (58).

Hydrodynamical simulations predict that a certain amount of energy content in the galaxy clusters may not be thermalized and could be present in the form of turbulence and bulk motions. Therefore masses estimated under the assumption that kinetic energy is fully thermalized might be biased and need to account for non-thermal pressure support to estimate total cluster mass. Although non-thermal pressure support is a difficult quantity to calculate, there exist some promising approaches to this problem. For example, total baryon fraction can be used to estimate the integrated non-thermal pressure support [23]. Another approach could be to use the Sunyaev–Zel’dovich effect. Its essence is that high energy ICM electrons change the temperature distribution of the Cosmic Microwave Background observed in the cluster direction through inverse Compton scattering: $\frac{\Delta T_{CMB}}{T_{CMB}} = f(x)y$, where y is the Compton parameter, i.e., average fractional energy per collision multiplied by the average number of collisions (hence it is proportional to the integrated pressure), $f(x) = \left(x \frac{e^x+1}{e^x-1} - 4\right)(1 + \Delta_{SZ}(x, T_e))$, with $x = hv/k_B T_{CMB}$ being the dimensionless photon frequency and $\Delta_{SZ}(x, T_e)$ the relativistic correction. The sample comprises 12 massive galaxy clusters with redshifts in the range $0.04 < z < 0.1$ observed in X-rays on the XMM-Newton telescope in combination with the SZ effect observed within the Planck all-sky survey [24,25]. The Planck SZ signal for these clusters has been recorded with the highest signal-to-noise ratio [25] translating into the especially good quality of $M_{tot}(< r)$ estimates with relative uncertainties around 5%. Hence, the X-COP galaxy cluster sample provided high-confidence total mass measurements. The best fit values of the running parameters for the X-COP galaxy sample are given in Table 1 and their average value is found to be $\alpha = 9.1$ and $\mu = 0.196 \text{ Mpc}^{-1}$. They can be obtained using the phenomenological formula given in (38,39) after estimating α_∞ given in Table 2.

Table 1. Best fit values for α_{200} and μ_{200} estimated for X-COP cluster sample.

Name	M_{200} ($10^{14} M_\odot$)	R_{200} (Mpc)	α_{200}	μ_{200} (Mpc^{-1})
A1644	6.58 ± 0.66	1.778 ± 0.051	9.102 ± 2.715	0.244 ± 0.012
A1795	6.76 ± 0.36	1.755 ± 0.021	9.102 ± 2.715	0.240 ± 0.006
A2029	13.29 ± 0.69	2.173 ± 0.034	9.108 ± 2.716	0.171 ± 0.004
A2142	16.37 ± 0.89	2.224 ± 0.027	9.109 ± 2.717	0.154 ± 0.004
A2255	10.70 ± 0.68	2.033 ± 0.081	9.106 ± 2.716	0.191 ± 0.006
A2319	20.11 ± 1.23	2.040 ± 0.035	9.110 ± 2.717	0.139 ± 0.004
A3158	7.34 ± 0.41	1.766 ± 0.035	9.103 ± 2.715	0.231 ± 0.006
A3266	14.49 ± 2.70	2.325 ± 0.074	9.108 ± 2.716	0.164 ± 0.015
A644	8.35 ± 0.61	1.847 ± 0.059	9.104 ± 2.715	0.216 ± 0.008
A85	9.56 ± 0.49	1.921 ± 0.027	9.105 ± 2.716	0.202 ± 0.005
RXC1825	6.87 ± 0.59	1.719 ± 0.024	9.103 ± 2.715	0.238 ± 0.010
ZwC11215	13.03 ± 1.25	2.200 ± 0.069	9.107 ± 2.716	0.173 ± 0.008

Table 2. Best fit values for α estimated for the modified cluster mass in Equation (60).

R_Δ	α_∞	χ^2/\mathcal{N}	σ_1
R_{200}	9.12	1.13	2.72
R_{500}	9.99	1.13	3.40

6. Observations in MOG

The weak field limit of MOG has two running parameters—(1) α , which measures the magnitude with which the attractive Newtonian force is enhanced; (2) the mass of the vector field μ —both of these are discussed in Section 3.2. These parameters can be constrained from observations. In a recent study [26], the motion of S2- star around the super massive black hole at the center of our Milk Way galaxy has been used to constrain

the parameter to $\alpha \lesssim 0.410$. Gravitational lensing of light is another important prediction of GR and it has now become an unavoidable tool in dark matter studies. Light bending in MOG was first studied in [16] and later recalculated in [27]. MOG has also been used in the study of lensing in Abell 3827 [28] and the Einstein ring of it has been estimated to be $\theta_E = 10''$. The velocity dispersion in the Globular Cluster (GC) located in our Milky Way galaxy provides us with another opportunity to test theories such as MOND and MOG [29]. The GCs experience gravity at varying strengths, measured from the galactic center, which also depends on the mass and size of the GC in question. The velocity dispersion of GC in the context of MOG has been studied in [30] and found to be in agreement with the observed data. It is also claimed in the literature that MOG needs large stellar mass-to-light ratios in the range of 10 to 100; one such issue in this regard is found in fitting the velocity dispersion of dwarf Spheroidal galaxies in the Milky Way [31]. Such high mass-to-light ratios are unacceptable according to population synthesis models. Another issue reported in the literature includes the inability of MOG to reproduce the dynamics of the very diffuse Low Surface Brightness galaxy Antlia II [32]. However, in a recent study [33] it has been reported that dwarf Spheroidal galaxies such as Antlia II show strong evidence for tidal disruption; this may inflate the sizes of dynamical masses and affect the measurement of dispersion velocities. The tidally disrupted galaxies significantly influence predictions of alternate gravity theories such as MOG.

7. Conclusions

MOG is a covariant modification of General Relativity with a massive vector field ϕ^μ and two scalar fields G and μ . MOG was developed as an alternative to dark matter models, which believe them to be some hypothetical particles whose presence has not been reported so far in any particle physics experiments whose primary objective is to detect them. The main ingredient of MOG is the presence of a massive vector field which is sourced by the gravitational force charge q_g , which all matter possess in addition to its mass. This new gravitational force charge q_g is proportional to the inertial mass of the particle and as a consequence the weak field approximation, MOG reduces to an effective potential which has a contribution from attractive Newtonian term and a repulsive Yukawa-like interaction. The effect of this Yukawa potential becomes negligible far away from the gravitating source. Unlike in GR, the equation of motion of a test particle bearing non-zero mass does not free fall along the geodesics of space-time due to a non zero Lorentz like interaction arising from the massive vector field. However, massless particles such as spin-2 gravitons and photons obey geodesic equation and hence follow the same null geodesics as in GR. MOG is successful in explaining galaxy rotation curve, bullet cluster data and CMB data. We have shown here that MOG is also consistent with X-COP galaxy cluster data and estimated the best fit values of the two running parameters α and μ for 12 X-COP galaxy clusters. MOG has to be subjected to more stringent tests to confirm its validity. A test of MOG with strong gravitational lensing systems and gravitational wave observations would help us shed light in this regard.

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