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Field-Theoretical Representation of Interactions between Particles: Classical Relativistic Probability-Free Kinetic Theory

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Abstract: It was proven that the class of stable interatomic potentials can be represented exactly as a superposition of Yukawa potentials. In this paper, an auxiliary scalar field was introduced to describe the dynamics of a system of neutral particles (atoms) in the framework of classical field theory. In the case of atoms at rest, this field is equivalent to the interatomic potential, but in the dynamic case, it describes the dynamics of a system of atoms interacting through a relativistic classical field. A relativistic Lagrangian is proposed for a system consisting of atoms and an auxiliary scalar field. A complete system of equations for the relativistic dynamics of a system consisting of atoms and an auxiliary field was obtained. A closed kinetic equation was derived for the probability-free microscopic distribution function of atoms. It was shown that the finite mass of the auxiliary field leads to a significant increase in the effect of interaction retardation in the dynamics of a system of interacting particles.

Keywords: classical relativistic dynamics; stable interatomic interactions; retarded interactions; irreversibility phenomenon; probability-free kinetics

MSC: 82C05; 82B21; 70E55; 80A10; 82C03; 82B03

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1. Introduction

At present, theoretical studies of both the thermodynamic properties and kinetic processes of many-body systems are being carried out mainly on the basis of statistical mechanics within the framework of the non-relativistic approximation. In this approximation, the interaction between particles is given by using the potential energy, which depends on the instantaneous configuration of the system. Then, a system consisting of a finite number of particles has a finite number of degrees of freedom. The microscopic dynamics of such a system is described by the deterministic equations of classical mechanics, in which there is no difference between past and future. However, such a picture fundamentally contradicts the real thermodynamic behaviors of systems and therefore cannot serve as a convincing basis for thermodynamics and kinetics. Boltzmann found a way out of this impasse by proposing the molecular chaos hypothesis. This hypothesis was retained even after the creation of the theory of relativity, although back in 1909, Ritz [1] suggested that the cause of the irreversibility phenomenon could be a retardation in interactions (i.e., a purely relativistic effect).

Numerous attempts at relativistic generalizations of both kinetic theory and statistical mechanics were undertaken shortly after the creation of the theory of relativity and continue to the present.

The non-existence of absolute time within the framework of the relativistic theory leads to the well-known yet unsolved problem of describing the interactions between particles,

which cannot be represented as a function depending on the instantaneous positions of the particles [2–9].

The study of systems with retarded interactions leads to a need to use the mathematical apparatus of functional differential equations. Due to the insufficient development of this mathematical apparatus, today, it has been possible to study only an extremely limited number of model problems—as a rule, two-body problems [10–14]. In such works, it was established that the retardation in interactions leads to the irreversibility of the dynamics of even two-body systems. Thus, the use of the concept of probability to explain the physical nature of the phenomenon of irreversibility is not necessary.

For a correct quantitative description of the classical dynamics of a system of interacting particles within the framework of the classical relativistic approach, it is necessary to construct a theory that includes both the dynamics of particles and the evolution of the field through which the particles interact. Such a theory in the case of a system consisting of charged particles is classical electrodynamics, within which the complete system of equations contains both the relativistic equations of particle dynamics and the Maxwell's equations.

A detailed study of the dynamics of a system of atoms, the interaction between which, at rest, can be described by given interatomic potentials, requires the construction of a relativistic theory of the corresponding field.

In this paper, we propose a method for the classical field theoretical description of the dynamics of a system consisting of neutral particles (atoms).

2. Transition from Instantaneous Potentials to a Field Picture of Interactions

To describe the dynamics of a system of neutral particles in terms of the classical field theory, we introduce the auxiliary scalar field $\varphi(\mathbf{r}, t)$, which in the static case (i.e., in the case of particles at rest), is equivalent to the central interatomic potential $v(\mathbf{r})$, and in the dynamic case, allows us to describe the dynamics of a system of atoms in terms of the relativistic classical field theory.

Let us assume that the interatomic potential for particles at rest admits the Fourier transform

$$v(\mathbf{r}) = \int \frac{d\mathbf{k}}{(2\pi)^3}\, \tilde{v}(\mathbf{k})\, e^{-i\mathbf{kr}}. \tag{1}$$

Let us denote $f(x)$ as an arbitrary function of one variable x and apply the corresponding operator $f(\Delta)$ (Δ is the Laplace operator) on the function $v(\mathbf{r})$:

$$f(\Delta)\{v(\mathbf{r})\} = \int \frac{d\mathbf{k}}{(2\pi)^3}\, f\left(-k^2\right) \tilde{v}(\mathbf{k})\, e^{-i\mathbf{kr}} \tag{2}$$

where $k = |\mathbf{k}|$. We find the explicit form of the function $f(x)$ from the condition (the case of a point source of the field):

$$f(\Delta)\{v(\mathbf{r})\} = \delta(\mathbf{r}). \tag{3}$$

This implies that

$$f\left(-k^2\right) = \frac{1}{\tilde{v}(\mathbf{k})}. \tag{4}$$

This relation establishes a connection between the Fourier transform of the static potential $\tilde{v}(\mathbf{k})$ and the differential Equation (3), which describes the corresponding static field. In particular, from the Fourier transforms of the Coulomb $\tilde{v}_1(\mathbf{k}) = \frac{4\pi}{k^2}$ and the Yukawa $\tilde{v}_2(\mathbf{k}) = \frac{4\pi}{k^2+\mu^2}$ potentials follow the Poisson equation and the static Klein–Gordon equation, respectively.

3. General Form of Stable Interatomic Potentials

Within the framework of classical statistical mechanics, only *stable* interatomic potentials can be used that satisfy the criterion of Ruelle [15], Fischer [16], and Dobrushin [17]. In terms of the Fourier transform, this condition has the form [18]

$$\tilde{v}(\mathbf{k}) > 0 \tag{5}$$

for all values of $\mathbf{k}$. Thus, the static equation of the scalar field $v(\mathbf{r})$, through which interactions between *resting* atoms are carried out, satisfies the Equation (3), where $f\left(-k^2\right)$ is the function determined by using the interatomic potentials using the relation (4).

Consider the class of central interatomic potentials $v(r)$, for which the Fourier transform $\tilde{v}(\mathbf{k})$ satisfies the stability condition (5). If $\tilde{v}(\mathbf{k})$ is a rational function of k^2, then it has the form:

$$\tilde{v}(\mathbf{k}) = \frac{Q_m(k)}{P_n(k)}, \quad (m < n), \tag{6}$$

where $Q_m(k)$ and $P_n(k)$ are polynomials of the degrees $2m$ and $2n$, respectively:

$$P_n(k) = \sum_{s=0}^{n} C_s\, k^{2s}, \quad Q_m(k) = \sum_{s=0}^{m} D_s\, k^{2s}, \tag{7}$$

C_s, D_s are real coefficients. From the condition (5), it follows that the polynomials $Q_m(k)$ and $P_n(k)$ have no real roots.

If all roots of the polynomial $P_n(k)$ are single, then the Fourier transform of the interatomic potential can be represented as the sum of n partial fractions

$$\tilde{v}(\mathbf{k}) = \sum_{s=1}^{n} \frac{g_s}{k^2 + \mu_s^2}. \tag{8}$$

Consequently, the interatomic potential in the case when the Fourier transform of the interatomic potential is a rational function of k^2 is a sum of Yukawa-type potentials:

$$v(\mathbf{r}) = \frac{1}{4\pi r} \sum_{s=1}^{n} g_s\, e^{-\mu_s r}. \tag{9}$$

The continual generalization of the discrete expansion (8) has the following form:

$$\tilde{v}(\mathbf{k}) = \int_0^{\infty} d\mu\, \frac{\Psi(\mu)}{k^2 + \mu^2}. \tag{10}$$

To find a direct relationship between the function $\Psi(\mu)$ and the interatomic potential $v(\mathbf{r})$, we perform the Fourier transform and obtain

$$\Psi(\mu) = \frac{2}{i} \int_{\gamma - i\infty}^{\gamma + i\infty} r v(r)\, e^{r\mu}\, dr, \quad \gamma > a, \tag{11}$$

where $r = |\mathbf{r}|$, a is the abscissa of the absolute convergence of the Laplace transform

$$|rv(r)| \leq A\, e^{ar}. \tag{12}$$

Note that for the discrete superposition of the Yukawa potentials (8), function $\Psi(\mu)$ has the following form:

$$v(\mathbf{r}) = \frac{1}{4\pi r} \sum_{s=1}^{n} g_s\, e^{-\mu_s r} \Leftrightarrow \Psi(\mu) = \sum_{s=1}^{n} g_s\, \delta(\mu - \mu_s). \tag{13}$$

In the general case, the support of the function $\Psi(\mu)$ contains both the discrete part of the "spectrum" with a zero Lebesgue measure and the continuous part of the "spectrum" with a non-zero Lebesgue measure. Certainly, in the general case, $\tilde{v}(\mathbf{k})$ can well be a transcendental function with an infinite set of singular points on the complex plane k, but all singular points lie outside the real axis.

Composite Field Dynamics Equations

For the first time, the transition from the static equations of Laplace and Poisson to equations relativistic in form was carried out (in 1867) by Lorenz and Riemann [19,20]. The result was as follows:

$$\Delta = \sum_{j=1}^{3} \frac{\partial^2}{\partial x_j^2} \to \Box = \sum_{j=1}^{3} \frac{\partial^2}{\partial x_j^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}. \tag{14}$$

Applying this procedure to the Yukawa potential leads to the Klein–Gordon–Fock Equation [21–23]:

$$\left(\Box - \mu^2\right) \varphi(\mathbf{r}, t) = 0. \tag{15}$$

This equation is the only option of a relativistic real scalar field described by using a linear partial differential equation of the second order.

If the Fourier transform of the potential $\tilde{v}(\mathbf{k})$ is a rational algebraic function, then the interaction between atoms described by the field $\varphi(\mathbf{r}, t)$ consists of the n elementary fields $\varphi_s(\mathbf{r}, t)$. Each of these elementary fields is characterized by the single parameter μ_s and obeys the Klein–Gordon equation

$$\hat{L}_s \varphi_s(\mathbf{r}, t) = 0, \tag{16}$$

where

$$\hat{L}_s = \Box - \mu_s^2. \tag{17}$$

4. Retarded Potentials of Yukawa Fields and Their Superpositions

The fundamental solution (Green's function in physical terminology) of the Klein–Gordon operator of the elementary field $\varphi_s(\mathbf{r}, t)$ is determined by using the equation

$$\left(\Box - \mu_s^2\right) G_s\left(\mathbf{r} - \mathbf{r}', t - t'\right) = -\delta\left(\mathbf{r} - \mathbf{r}'\right) \delta\left(t - t'\right) \tag{18}$$

and has the well-known form [24]

$$\begin{aligned} G_s\left(\mathbf{r} - \mathbf{r}', t - t'\right) = & \frac{\delta\left(t - t' - \frac{|\mathbf{r} - \mathbf{r}'|}{c}\right)}{4\pi|\mathbf{r} - \mathbf{r}'|} \\ -\theta\left(t - t' - \frac{|\mathbf{r} - \mathbf{r}'|}{c}\right) & c\mu_s \frac{J_1\left(\mu_s \sqrt{c^2(t - t')^2 - |\mathbf{r} - \mathbf{r}'|^2}\right)}{4\pi\sqrt{c^2(t - t')^2 - |\mathbf{r} - \mathbf{r}'|^2}}, \end{aligned} \tag{19}$$

where $\theta(t)$ is the Heaviside step function and $J_1(x)$ is the Bessel function.

Hence, the retarded potential of the Klein–Gordon field follows [25]:

$$\begin{aligned} \varphi_s(\mathbf{r}, t) = & \int d\mathbf{r}' \left[\frac{\rho\left(\mathbf{r}', t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}\right)}{4\pi|\mathbf{r} - \mathbf{r}'|} \right. \\ -\mu_s & \left. \int_0^\infty \rho\left(\mathbf{r}', t - \frac{1}{c}\sqrt{\xi^2 + |\mathbf{r} - \mathbf{r}'|^2}\right) \frac{J_1(\mu_s \xi)}{4\pi\sqrt{\xi^2 + |\mathbf{r} - \mathbf{r}'|^2}} d\xi \right], \end{aligned} \tag{20}$$

where $\rho(\mathbf{r}, t)$ is the instantaneous microscopic density of the number of particles (atoms):

$$\rho(\mathbf{r}, t) = \sum_a \delta(\mathbf{r} - \mathbf{r}_a(t)). \tag{21}$$

Thus, the field $\varphi_s(\mathbf{r}, t)$ consists of two contributions.

1. The first contribution determines the one-to-one relationship between the distance $|\mathbf{r} - \mathbf{r}'|$ and the retardation of interactions τ_1 between points $\mathbf{r}$, $\mathbf{r}'$:

$$\tau_1 = \frac{|\mathbf{r} - \mathbf{r}'|}{c}. \tag{22}$$

This contribution corresponds to a d'Alembert wave propagating from a source at the speed of light c.

2. The second contribution contains a whole spectrum of retardations in interaction $\tau_2(\xi)$ between points $\mathbf{r}$ and $\mathbf{r}'$ depending on the parameter ξ:

$$\tau_2(\xi) = \frac{\sqrt{\xi^2 + |\mathbf{r} - \mathbf{r}'|^2}}{c}, \tag{23}$$

where

$$0 < \xi < \infty, \ \tau_1 < \tau_2(\xi) < \infty. \tag{24}$$

This contribution corresponds to a set of Klein–Gordon waves propagating from a source with all velocities $\tilde{c}(\xi)$ from 0 to c:

$$\tilde{c}(\xi) = c\left(1 + \frac{\xi^2}{|\mathbf{r} - \mathbf{r}'|^2}\right)^{-1/2}, \quad 0 < \tilde{c}(\xi) < c. \tag{25}$$

Therefore, the presence of the non-zero field parameter μ_s increases the retardation in interactions between particles; the function $\tau_2(\xi)$ is unbounded above.

Using Relation (9), we find the connection between the dynamics of a system of atoms and the auxiliary field $\varphi(\mathbf{r}, t)$, through which the atoms interact:

$$\varphi(\mathbf{r}, t) = \sum_{s=1}^{n} g_s \, \varphi_s(\mathbf{r}, t). \tag{26}$$

This expression has an elementary generalization to the general case when the support of the function $\Psi(\mu)$ is not limited to the discrete spectrum:

$$\varphi(\mathbf{r}, t) = \int d\mu \, \Psi(\mu) \, \varphi_\mu(\mathbf{r}, t). \tag{27}$$

Thus, the instantaneous configuration of both the auxiliary relativistic field $\varphi(\mathbf{r}, t)$ and its components $\varphi_s(\mathbf{r}, t)$ at time t depend on all configurations of the atomic system described by the function $\rho(\mathbf{r}, t')$ under the condition $-\infty < t' \leq t$ (i.e., on the whole, the prehistory of the mechanical component of the complete system, which includes both atoms and an auxiliary field created by atoms). This implies that the field at the point $(\mathbf{r}, t)$ is determined by using the whole history of the world line inside Hilbert's causality cone.

However, only the Equations (20) and (27), describing the effects of atoms on the field, are not enough to fully describe the dynamics of the system as a whole; we also need equations that describe the effects of the field on the particles.

5. Equations of Motion of Particles and Auxiliary Field

To determine the equations of the evolution of a system consisting of particles and the auxiliary field created by them, we turn to the variational formulation of dynamics. The action of the system under consideration has the form

$$\begin{aligned} S = &-\sum_a m_a c \int ds_a - \sum_{s=1}^{n} \sum_a \frac{\gamma_s}{c} \int \varphi_s(x_a)\, ds_a \\ &+ \sum_{s=1}^{n} \frac{\varkappa_s}{2c} \int d^4x \left(\partial_\nu \varphi_s(x)\, \partial^\nu \varphi_s(x) - \mu_s^2 \varphi_s^2(x) \right), \end{aligned} \tag{28}$$

where γ_s are the coupling constants between the particles and the fields and $\varkappa_s$ are dimensional constants.

The complete system of equations for the dynamics of particles and fields has the form

$$\begin{cases} \dfrac{\partial}{\partial x^\nu} \dfrac{\partial \mathcal{L}}{\partial(\partial_\nu \varphi_s(x))} - \dfrac{\partial \mathcal{L}}{\partial \varphi_s(x)} = 0; \\ \dfrac{d}{d\tau_a} \dfrac{\partial L}{\partial \dot{x}_a^\nu} - \dfrac{\partial L}{\partial x_a^\nu} = 0. \end{cases} \tag{29}$$

For the convenience of further calculations, we transform the first two terms on the right side of the expression (28):

$$\begin{aligned} &-\sum_a m_a c \int ds_a - \sum_{s=1}^{n} \sum_a \frac{\gamma_s}{c} \int \varphi_s(x_a)\, ds_a \\ &= -c \sum_a m_a \int d\tau_a \sqrt{\dot{x}_a^\nu \dot{x}_{\nu a}} \\ &- \sum_{s=1}^{n} \sum_a \frac{\gamma_s}{c} \int d\tau_a \int d^4x\, \varphi_s(x) \sqrt{\dot{x}^\nu \dot{x}_\nu}\, \delta^4(x - x_a(\tau_a)). \end{aligned} \tag{30}$$

Substituting expressions (30) and (28) into Equation (29), we obtain the following system of equations for the dynamics of the fields

$$\left(\Box - \mu_s^2 \right) \varphi_s(x) = \frac{\gamma_s}{\varkappa_s} \sum_a \int d\tau_a \sqrt{\dot{x}^\nu \dot{x}_\nu}\, \delta^4(x - x_a(\tau_a)) \equiv \frac{\gamma_s}{\varkappa_s} \rho(x) \tag{31}$$

and particles

$$\begin{aligned} &\left(1 + \frac{1}{m_a c^2} \sum_{s=1}^{n} \gamma_s \varphi_s(x_a) \right) \frac{dp_{\mu a}}{d\tau_a} \\ &= \frac{\partial}{\partial x_a^\mu} \left(\sum_{s=1}^{n} \gamma_s\, \varphi_s(x) \right) - \frac{p_a^\nu p_{\mu a}}{m_a^2 c^2} \frac{\partial}{\partial x_a^\nu} \left(\sum_{s=1}^{n} \gamma_s\, \varphi_s(x) \right), \end{aligned} \tag{32}$$

where $p_{\nu a} = m_a u_{\nu a}$.

Solutions of Equation (31) in accordance with (18) and (20) have the following form:

$$\begin{aligned} \phi_s(\mathbf{r}, t) = \frac{\gamma_s}{\varkappa_s} \int d^3\mathbf{r}' \Bigg[&\frac{\rho\left(\mathbf{r}', t - \frac{|\mathbf{r} - \mathbf{r}'|}{c} \right)}{|\mathbf{r} - \mathbf{r}'|} \\ -\mu_s \int_0^\infty &\rho\left(\mathbf{r}', t - \frac{1}{c} \sqrt{\xi^2 + |\mathbf{r} - \mathbf{r}'|^2} \right) \frac{J_1(\mu_s \xi)}{\sqrt{\xi^2 + |\mathbf{r} - \mathbf{r}'|^2}} d\xi \Bigg]. \end{aligned} \tag{33}$$

As a result, we have

$$\varphi(\mathbf{r},t) = \sum_{s=1}^{n} g_s\,\varphi_s(\mathbf{r},t) = \sum_{s=1}^{n} \frac{g_s\gamma_s}{\varkappa_s}\int d^3\mathbf{r}'\left[\frac{\rho\left(\mathbf{r}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c}\right)}{|\mathbf{r}-\mathbf{r}'|} - \mu_s\int_0^\infty \rho\left(\mathbf{r}',t-\frac{1}{c}\sqrt{\xi^2+|\mathbf{r}-\mathbf{r}'|^2}\right)\frac{J_1(\mu_s\xi)}{\sqrt{\xi^2+|\mathbf{r}-\mathbf{r}'|^2}}d\xi\right]. \tag{34}$$

Equation (32) describes the dynamics of the a-th particle for a given evolution of the fields $\varphi(x)$ given by using the right-hand side of this equation. However, instead of describing the dynamics of a system of identical particles using a system of equations, each of which corresponds to a separate particle, it is more expedient to use the kinetic equation for the Klimontovich microscopic distribution function of the system of particles as a whole:

$$f(\mathbf{r},\mathbf{p},t) = \sum_a \delta(\mathbf{r}-\mathbf{r}_a(t))\,\delta(\mathbf{p}-\mathbf{p}_a(t)). \tag{35}$$

Following Klimontovich [26], we introduce the relativistic microscopic distribution function of particles

$$\mathcal{F}(x,p) = \sum_a \int d\tau_a\,\delta^4(x-x_a(\tau_a))\,\delta^4(p-p_a(\tau_a)). \tag{36}$$

As shown in [27], this distribution function $\mathcal{F}(x,p)$ obeys the relativistic kinetic equation in a covariant form:

$$\left(\frac{p^\nu}{m}\frac{\partial}{\partial x^\nu} + F^\nu(x,p)\frac{\partial}{\partial p^\nu} + \frac{\partial F^\nu(x,p)}{\partial p^\nu}\right)\mathcal{F}(x,p) = 0, \tag{37}$$

where F^ν is the four-vector of force and m is the mass of each of the particles.

Using the expression in [27],

$$\mathcal{F}(x,p) = \frac{1}{mp^0}\delta\left(p^0-\sqrt{\mathbf{p}^2+m^2c^2}\right)f(\mathbf{r},\mathbf{p},t). \tag{38}$$

We integrate the Equation (37) over p^0, taking into account the Equation (32). As a result, the covariant kinetic Equation (37) for a system of particles interacting through the scalar field $\varphi(\mathbf{r},t)$ is transformed into the following kinetic equation in terms of the Klimontovich distribution function (35):

$$\left(\frac{\partial}{\partial t} + \frac{c\mathbf{p}}{\sqrt{\mathbf{p}^2+m^2c^2}}\frac{\partial}{\partial \mathbf{r}} + \mathbf{F}(\mathbf{r},\mathbf{p},t)\frac{\partial}{\partial \mathbf{p}}\right)f(\mathbf{r},\mathbf{p},t) = \frac{3k(\varphi(\mathbf{r},t))}{mc^2}\left(\frac{\partial\varphi(\mathbf{r},t)}{\partial t} + \frac{c\mathbf{p}}{\sqrt{\mathbf{p}^2+m^2c^2}}\frac{\partial\varphi(\mathbf{r},t)}{\partial \mathbf{r}}\right)f(\mathbf{r},\mathbf{p},t), \tag{39}$$

where

$$\mathbf{F}(\mathbf{r},\mathbf{p},t) = -\frac{mc\,k(\varphi(\mathbf{r},t))}{\sqrt{\mathbf{p}^2+m^2c^2}}\left[\frac{\partial}{\partial \mathbf{r}} + \frac{\mathbf{p}}{m^2c^2}\left(\mathbf{p}\frac{\partial}{\partial \mathbf{r}} + \frac{\sqrt{\mathbf{p}^2+m^2c^2}}{c}\frac{\partial}{\partial t}\right)\right]\varphi(\mathbf{r},t), \tag{40}$$

$$k(\varphi(\mathbf{r},t)) = \frac{1}{1+\frac{1}{mc^2}\sum_{s=1}^{n}\gamma_s\varphi_s(x_a)}. \tag{41}$$

$$\varphi(\mathbf{r},t)=c\sum_{s=1}^{n}\frac{g_s\gamma_s}{\varkappa_s}\int\frac{d\mathbf{p}'}{m\sqrt{\mathbf{p}'^2+m^2c^2}}\left[\frac{f\left(\mathbf{r}',\mathbf{p}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c}\right)}{4\pi|\mathbf{r}-\mathbf{r}'|}\right.$$
$$\left.-\mu_s\int_0^\infty f\left(\mathbf{r}',\mathbf{p}',t-\frac{1}{c}\sqrt{\xi^2+|\mathbf{r}-\mathbf{r}'|^2}\right)\frac{J_1(\mu_s\xi)}{4\pi\sqrt{\xi^2+|\mathbf{r}-\mathbf{r}'|^2}}d\xi\right]. \quad (42)$$

The kinetic Equation (39) with account notations (40)–(42) describes the evolution of a system of particles interacting through the auxiliary relativistic scalar field $\varphi(\mathbf{r},t)$. In the static mode, the field $\varphi(\mathbf{r},t)$ is equivalent to interatomic potentials, and in the dynamic mode, it is a mediator of interatomic interactions.

6. Discussion and Conclusions

The main principles underlying this work are as follows.

1. The dynamics of a system of interacting particles is described in terms of an exact microscopic distribution function (35), which has no probabilistic interpretation and contains the dynamics of all the particles that make up the system. Unlike statistical distribution functions, exact microscopic functions describe the dynamics of a system but not the evolution of probabilities.
2. The description of particle dynamics is based on the relativistic equations of motion and the principle of causality. In contrast to classical non-relativistic mechanics, in the theory of relativity, there is asymmetry between the past and the future due to the principle of causality. This asymmetry can provide a link between the principles of relativity and the laws of thermodynamics.
3. Interactions between atoms within the framework of the theory of relativity are possible only on the basis of field concepts. In view of the neutrality of atoms, an auxiliary scalar field is used to describe interatomic interactions. In the case of atoms at rest, this field is equivalent to interatomic potentials. Thus, within the framework of the relativistic theory, the description of the dynamics of a system of interacting atoms includes both the equations of motion of particles and the equations of the evolution of an auxiliary scalar field that transmits interactions between particles.
4. It has been proven that in the case of stable interatomic interactions, the auxiliary scalar field is a superposition of Yukawa fields, the parameters of which are uniquely expressed in terms of the interatomic potentials of atoms at rest. As a result, the system of interacting particles consists of two subsystems, one of which is the particles and the other if which is the field. The Hamiltonians of these subsystems do not exist since the subsystems of even Hamiltonian systems, generally speaking, are not Hamiltonian [28].
5. The closed exact probability-free kinetic Equation (39) is derived within the framework of the theory of relativity for a system of interacting particles.
   It should be noted that a preliminary version of the relativistic kinetic equation for a system of particles interacting through the field $\varphi(\mathbf{r},t)$ with a zero mass and without considering the field dynamics, Equation (18), was proposed in our paper [27] . A preliminary version of the relativistic kinetic equation for a system of particles interacting through the field $\varphi(\mathbf{r},t)$ with the zero mass $\mu_s=0$ is proposed in our paper [27]. The finiteness of the field mass $\mu_s\neq 0$ leads to a very significant enhancement of the effect of interaction retardation on the dynamics of the system of interacting particles (23).
6. The relativistic kinetic Equation (39) is equally applicable to both many-body and few-body systems. At the same time, in both cases, the system's dynamics have signs characteristic of its thermodynamic behavior, including the property of irreversibility [14,27,29] and the implementation of the microscopic equilibration mechanism [30,31]. Therefore, the relativistic field approach to describing the dynamics

of systems can be used as a probability-free basis for constructing the microscopic thermodynamics of both macroscopic and "small" systems, including nano-systems.

Thus, the following are planned as future areas of research:

- The possible exclusion of the concept of probability as a source of the thermodynamic behavior of a system of interacting particles and the corresponding microscopic substantiation of the laws of thermodynamics and kinetics.
- The investigation of the dynamics of a system of particles in terms of exact microscopic distribution functions and a subsequent analysis of the qualitative properties of the solutions of the resulting equations.
- A search for methods for constructing the microscopic thermodynamics and kinetics of small systems for which the statistical approach is inapplicable in principle.
- A full analysis of the relationships between our approach and other options.

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