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Significance of Charge on the Dynamics of Hyperbolically Distributed Fluids

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Abstract: This manuscript is devoted to analyze hyperbolically symmetric non-static fluid distribution incorporated with heat flux and electromagnetic field. We have developed a general framework in order to examine the dynamic regime of the matter configuration which eventually results in the static spacetime. With the aim of doing this, we constructed the Einstein-Maxwell (EM) field equations and obtained the conservation equation. Furthermore, the formulation of mass function indicates the presence of the negative energy density, which leads towards the significant quantum implications. Taking into account the transport equation, we have observed the thermodynamical attributes of the fluid. Additionally, quasi- homologous constraint has been utilized to construct several models. We have deduced the worthwhile applications of the astrophysical objects by evaluating several analytical solutions in terms of the kinematical variables.

Keywords: dissipative fluid; hyperbolically symmetric; electromagnetic field; anisotropic pressure

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1. Introduction

Gravitational lensing is proving to be a valuable technique in astronomy, from exoplanets to cosmology. This single phenomenon results in the diversity of applications that can be derived from a bending of light by curved spacetime. Due to the fact that it is a geometrical phenomenon, its concepts are simple to grasp and explain. The large clusters of galaxies (which are both massive and centrally concentrated) are excellent candidates for gravitational lens. The field of gravitational lensing has been steadily expanding in its nearly two decades as an observational area of astronomy. As new understanding of the phenomenon emerges from time to time, there are several articles on gravitational lensing [1–3].

All information about the cosmos, with the exception of the solar system, which we may directly examine, is carried by radiation or particles that reach on Earth after voyages spanning millions or even billions of years. Even though meteorites and cosmic rays contribute extremely significant data, the majority of our knowledge is derived from the radiation emitted by celestial bodies, particularly the portion of this radiation accessible to the naked eye and referred to as visible light. As a result, astronomical advancement is dependent on the detection of this radiation and the development of tools to detect it. To examine an astronomical object, whether it is a planet, a star, a massive structure such as a big molecular cloud, or a galaxy, we must gather and study the radiation it emits.

Spherical symmetry [4] and hyperbolic symmetry [5] have experimentally meaningful ramifications in both classical and quantum physics. The phrase "spherical symmetry" suggests to the fact that an observer at the center of the matter distribution would see the identical physical picture regardless of direction, making two of the coordinates cyclic. The term "hyperbola" refers to a particular type of curve. It is defined as a trajectory of the moving particle in which the distance between a stationary point and a fixed vertical line is constant and greater than one. In both the geometries, a straight line acts as a connection between any two points. A circle may be created using any straight line segment as the radius and one endpoint as the center. Moreover, right angles are all the same. The difference between the two geometries is that the spherical one has the sum of right angles greater than 180 degree. Its greatest possible path while keeping the initial and terminal points same is a piece of a great circle. In it the parallel lines converge and its curvature is positive. Contrary to that, in hyperbolic geometry, the sum of right angles are less than 180 degree. Its greatest possible path while keeping the initial and terminal points same is a piece of a hyperbola. In it the parallel lines diverge and its curvature is negative.

After Einstein's special theory of relativity (SR) was published in 1905, the immediate step was to extend his theory to incorporate non-inertial reference frames, i.e., gravity and acceleration. In 1907, Einstein wrote an article in which he described how he was attempting to apply his theory of relativity to gravity for the first time. As a result, few gravitational effects were investigated, including light bending, gravitational redshift (GvR) and gravitational time dilation (GvT). In the sense that the frequency of light as it rises out of a gravitational potential becomes redshifted as it undergoes time dilation, Einstein demonstrated that GvT is the causal mechanism of GvR and bending of light. As a result, observational and experimental confirmations of GvR are indirect confirmations of GvT.

Eventually, he published the proper version of GR on 25 November 1915 [6]. Over a hundred years after its publication in 1915, GR has remained unchanged, and it is crucial to astrophysics and cosmology. This theory has never failed a single test, making it one of the most thoroughly tested theory in physics.

Since a long time, black holes and its alternative have been a very prominent topic as end-states of stellar collapse [7–15]. In the late 1960s and early 1970s, new formal breakthroughs in the area of GR, such as global approaches and Hawking and Penrose's singularity theorems, sparked renewed interest in black hole research. Quantum mechanics and field theoretical approaches were also applied to black holes, resulting in groundbreaking concepts such as Hawking Radiation and black hole entropy calculation. Black holes have been intensively explored in both higher dimensions theories and lower dimensional gravity. The event horizon, a null hypersurface, the interior and exterior of the black hole, are the most essential characteristic of a black hole spacetime.

Karl Schwarzschild developed a precise solution to Einstein's field equations (EFE) in 1915, which was published in 1916 [16]. Unless otherwise noted, the Schwarzschild solution is utilized in most GR experiments. He also calculated the "Schwarzschild radius" (R_s), which is the radius of a sufficiently massive object at which all particles, including photons, will descend into the central region of massive object. Reissner-Nordström (RN) [17,18] for charged static black holes, Kerr for rotating black holes [19], and Kerr-Newman [20] for charged rotating black holes were later proposed as black hole solutions to the EFE. Harrison [21] was the first one to derive alternative solution to Einstein equations of the sort characterized by hyperbolic symmetry, it has since become the subject of investigation in several disciplines [22–35].

Gravitational collapse is known as highly dissipative phenomena [36–38]. Gaudin et al. [39] extensively analyzed the substantial parameters of specific case of hyperbolically distributed matter. Furthermore, they investigated the several solutions of static Einstein equation in the presence of a massless scalar field and established their relationship to Kantowski-Sachs cosmological solutions via duality transformations. Herrera et al. [40] conducted a thorough investigation on the hyperbollically symmetric matter configuration by taking into account the non-static regime. They evaluated several solutions using quasi-homologous, vanishing of complexity factor condition and the supplementary constraints for the dissipative as well as non-dissipative systems. Moreover, they [41] also did the aforementioned work by using the approach of Lemaître-Tolman-Bondi spacetime endorsed with hyperbolic symmetry. Oikonomou and his collaborators [42–46] analyzed various issue of stellar and cosmic evolution, in particularly the role of cosmological attractors on a slowly rotating celestial bodies. The articles [47–50] addressed the significant issues of cosmology and inflation in the context of teleparallel and tetrad theory of gravity.

Yousaf et al. [51–54] examined the anisotropic static fluid distributed hyperbolically in the framework of different modified theories. They carried out this analysis both in the presence as well as in the absence of charge. Along with this, they have evaluated structure scalars and several analytical solutions. Malik et al. [55] gave a few numerical solutions after solving the associated differential equations to represent the distribution of hyperbolically symmetric matter throughout the cylindrical geometry. Cao and Wu [56] analyzed the occurrence of strong hyperbolicity under certain gauge restrictions by taking into consideration metric $f(R)$ gravity.

This manuscript presents the consequence of electromagnetic force on the Herrera et al. [40] work. With the aim of doing this, we assess the impact of charge on the dynamics of hyperbolically symmetric matter distribution. Section 2 describes the general formalism for computing the EM field equations. Section 3 specifies the field equations together with the conservation equations for the dynamical system. The evaluation of the kinematical variables, mass function and the collapsing velocity are included in Section 4. Section 5 deals with Conformal tensor and complexity factor. Sections 6 and 7 are devoted for the calculation of transport equation and quasi-homologous condition, respectively. In section 8, the possible non-static solutions for the anisotropic charged hyperbolically symmetric fluid distribution have been offered by evaluating various conditions for both the dissipative and non-dissipative systems. Section 9 is occupied for the discussion and final remarks of the manuscript.

2. Basic Formalism

In the framework of GR, the consequence of charge on the 4-dimensional gravitational action is expressed as

$$A_{G(R)} = \frac{1}{2\kappa} \int d^4x \sqrt{-g}[R + \mathcal{L}^e + \mathcal{L}^m], \tag{1}$$

where κ, g, $\mathcal{L}^e$ and $\mathcal{L}^m$ depict the coupling constant, the metric tensor's magnitude, electromagnetic field and matter distribution, respectively. Due to the fact we use normalized units in our scenario ($G = c = 1$), the coupling constant is reduced to 8π. The Ricci scalar, symbolized as R, is the portion of spacetime curvature that defines the amount of change in surface area caused by matter. The tensor characterizing the electromagnetic field can be defined as

$$T^{\rm em}_{\zeta\xi} = \frac{1}{4\pi}(F^{\alpha}_{\zeta} F_{\xi\alpha} - \frac{1}{4} F^{\alpha\eta} F_{\alpha\eta} g_{\zeta\xi}). \tag{2}$$

The electromagnetic field tensor $F_{\xi\zeta}$ is expressed as $F_{\xi\zeta} = \phi_{\zeta,\xi} - \phi_{\xi,\zeta}$. The four-potential is formulated as $\phi_\zeta = \phi(t,r)\delta^0_\zeta$ via the scalar potential $\phi(t,r)$. We chose the aforementioned form of potential as we consider that the charge is at rest and as a result, the magnetic field vanishes. Next, we consider a suitable interior metric to study the evolving fluid distributed hyperbolically, given below

$$ds^2 = -J^2(t,r)dt^2 + K^2(t,r)dr^2 + H^2(t,r)d\theta^2 + H^2(t,r)\sinh^2\theta d\phi^2, \tag{3}$$

where $K^2(t,r)$, $J^2(t,r)$ and $H^2(t,r)$ are assumed positive. As, metric coefficients are non-static they depend on t and r. In order to evaluate the electromagnetic stress tensor's non-vanishing components, we utilize Equations (2) and (3) and get

$$T^{\rm em}_{00} = \frac{s^2 J^2}{8\pi H^2}; \qquad T^{\rm em}_{11} = -\frac{s^2 J^2}{8\pi H^2}; \qquad T^{\rm em}_{22} = \frac{s^2}{8\pi H^2}; \qquad T^{\rm em}_{33} = T^{\rm em}_{22}\sinh^2\theta,$$

where s and θ illustrate the charge and angle, respectively. The relationship between the geometry and the matter of the spacetime is accomplished as

$$G_{\zeta\xi} = 8\pi T_{\zeta\xi} = 8\pi\left(T^{m}_{\zeta\xi} + T^{\text{em}}_{\zeta\xi}\right), \tag{4}$$

where $G_{\zeta\xi}$ denotes the Einstein tensor and $T^{m}_{\zeta\xi}$ is expressed as

$$T^{m}_{\zeta\xi} =(\mu + P)V_{\zeta}V_{\xi} - Pg_{\zeta\xi} + \Pi_{\zeta\xi} + q(V_{\zeta}\chi_{\xi} + V_{\xi}\chi_{\zeta}), \tag{5}$$

where $T^{m}_{\zeta\xi}$ portrays the stress-energy tensor for the locally anisotropic hyperbolically symmetric evolving fluid configuration. Moreover, μ, V_{ζ}, P and $\Pi_{\zeta\xi}$ are the energy density, four velocity, anisotropic pressure, and anisotropic tensor of the evolving matter, respectively. They are characterized as

$$\begin{aligned} P &= \frac{2P_{\perp} + P_{r}}{3}, \quad \Pi_{\zeta\xi} = \Pi\left(K_{\zeta}K_{\xi} - \frac{h_{\zeta\xi}}{3}\right), \\ h_{\zeta\xi} &= g_{\zeta\xi} + V_{\xi}V_{\zeta}, \quad \Pi = P_{r} - P_{\perp}, \end{aligned}$$

where K_{ζ}, $P_{\perp}$ and P_{r} represent the four vector, tangential and radial components of pressure, respectively. In the case of comoving observers, we take $V^{\zeta} = J^{-1}\delta^{\zeta}_{0}$, $q^{\zeta} = qK^{-1}\delta^{\zeta}_{1}$, and $\chi^{\zeta} = K^{-1}\delta^{\zeta}_{1}$. It is notable that the reformulation of tangential and the radial pressures can involve the bulk viscosity. Since, we are examining the influence of the electromagnetic force, it will be worthy to derive the Maxwell's equations, which are formulated as

$$F^{\zeta\xi}_{;\xi} = \mu_{0}J^{\zeta}, \qquad F_{[\zeta\xi;\alpha]} = 0. \tag{6}$$

where $J^{\zeta} = \rho(t,r)V^{\zeta}$ is the four-current and $\rho(t,r)$ shows charge density. The differential equations evaluated from the tensorial forms of Maxwell's equations are

$$\phi'' + \left(\frac{J'}{J} + \frac{K'}{K} - 2\frac{H'}{H}\right)\phi' = 4\pi\rho JK^{2}, \tag{7}$$

$$\dot{\phi}' - \left(\frac{\dot{J}}{J} + \frac{\dot{K}}{K} - 2\frac{\dot{H}}{H}\right)\phi' = 0 \tag{8}$$

where primes and dots indicate the derivative with respect to radial and temporal coordinates. Equation (7) on integration produces

$$\phi' = \frac{JKs}{H^{2}}, \tag{9}$$

where the electric charge (s), interior to the radius is expressed as

$$s = 4\pi\int_{0}^{r}\rho KH^{2}dr. \tag{10}$$

In the rotation of plasma resulted by an external force, the concept of global charge is indubitably important. The stars could be made of highly ionized matter, they are expected to have a charge on it.

3. Einstein-Maxwell and Conservation Equation

In this section, we will evaluate the field equations as well as the conservation equations to further proceed our work. The field equations are determined using Equations (3) and (4) and the non-zero Einstein and electromagnetic field tensors, as follows

$$8\pi\left(\mu+\frac{s^2}{8\pi H^4}\right)=-\frac{1}{H^2}-\frac{1}{K^2}\left[\left(\frac{H'}{H}\right)^2-\frac{2K'}{K}\frac{H'}{H}+\frac{2H''}{H}\right]+\frac{1}{J^2}\left(\frac{2\dot{K}\dot{H}}{HK}+\frac{\dot{H}^2}{H^2}\right), \tag{11}$$

$$4\pi q=-\frac{1}{JK}\left(\frac{H'}{H}\frac{\dot{K}}{K}+\frac{J'}{J}\frac{\dot{H}}{H}-\frac{\dot{H}'}{H}\right), \tag{12}$$

$$8\pi\left(P_r-\frac{s^2}{8\pi H^4}\right)=\frac{1}{H^2}+\frac{1}{K^2}\left[\frac{2H'J'}{JH}+\left(\frac{H'}{H}\right)^2\right]+\frac{1}{J^2}\left(\frac{2\dot{H}\dot{J}}{JH}-\frac{\dot{H}^2}{H^2}-\frac{2\ddot{H}}{H}\right), \tag{13}$$

$$8\pi\left(P_\perp+\frac{s^2}{8\pi H^4}\right)=\frac{1}{K^2}\left(\frac{J'H'}{JH}-\frac{J'K'}{JK}-\frac{H'K'}{HK}+\frac{J''}{J}+\frac{H''}{H}\right)+\frac{1}{J^2}\left(\frac{\dot{J}}{J}\frac{\dot{K}}{K}+\frac{\dot{J}}{J}\frac{\dot{H}}{H}-\frac{\dot{K}}{K}\frac{\dot{H}}{H}-\frac{\ddot{K}}{K}-\frac{\ddot{H}}{H}\right). \tag{14}$$

Next, we evaluate the dynamical equations in the presence of charge. There exists only two independent components of the conservation laws $(T_{\zeta}^{\xi(m)}+T_{\zeta}^{\xi(em)})_{;\xi}=0$, which are as follows

$$\dot{\mu}+2\left(P_\perp+\mu\right)\frac{\dot{H}}{H}+\left(\mu+P_r\right)\frac{\dot{K}}{K}+q'\frac{J}{K}+2q\frac{J}{K}\left(\frac{J'}{J}+\frac{H'}{H}\right)=0 \tag{15}$$

$$P_r'-\frac{ss'}{4\pi H^4}+\left(\mu+P_r\right)\frac{J'}{J}+2\Pi\frac{H'}{H}+\dot{q}\frac{K}{J}+2q\frac{K}{J}\left(\frac{\dot{K}}{K}+\frac{\dot{H}}{H}\right)=0. \tag{16}$$

In Equation (15) the effects of charge has been cancelled during mathematical work. Besides, one can observe the presence of charge in Equation (16). In latter equation the first term, i.e., P_r' is the gradient of the pressure which acts as the opposite to gravity. The anisotropy factor is indicated by Π which measures the influence of anisotropy in the system.

4. Kinematical Variables, Mass Function and Collapsing Velocity

Kinematics is used to describe the motion of an object that is under consideration, neglecting the forces that cause that motion. These variables are those significant variables that are used to describe the motion of the fluid while the mass of the object has not been taken into account. The first variable is the four acceleration which gives the information about the inertial forces and is defined as

$$a_\gamma=V_{\gamma;\xi}V^{\xi}. \tag{17}$$

Utilizing the definition of four velocity and the line element, we can obtain the non-zero components of four acceleration as follows

$$a_1=\frac{J'}{J},\quad a=\sqrt{a^{\xi}a_{\xi}}=\frac{J'}{JK}, \tag{18}$$

where a depicts the scalar of four acceleration. The second essential variable to illustrate the fluid motion is expansion tensor. The general formula for the expansion tensor that appears in the non static fluid distribution is

$$\Theta_{\gamma\xi}=V_{(\beta;\nu)}h_{\gamma}^{\beta}h_{\xi}^{\nu} \tag{19}$$

The trace of the tensor is named as expansion scalar. The expansion scalar describes us about the amount by which the volume of the fluid element increases with respect to time. It can be written as $\Theta = V^{\xi}_{;\xi}$. Utilizing the line element and the four velocity, we obtain

$$\Theta = \left(2\frac{\dot{H}}{H} + \frac{\dot{K}}{K}\right)\frac{1}{J}. \tag{20}$$

The third variable is the stress tensor which calculates the distortion in timelike curves without changing the volume. It depicts the prospect of a geodesic sphere becoming warped into an ellipsoidal shape. The non-zero components of shear tensors are evaluated as

$$\sigma_{11} = \frac{2}{3}K^2\sigma, \quad \sigma_{22} = \frac{\sigma_{33}}{\sinh^2\theta} = -\frac{1}{3}H^2\sigma. \tag{21}$$

The trace of the shear tensor is formulated and delineated as

$$\sigma^{\gamma\xi}\sigma_{\gamma\xi} = \frac{2}{3}\sigma^2, \; \sigma = \left(\frac{\dot{K}}{K} - \frac{\dot{H}}{H}\right)\frac{1}{J}. \tag{22}$$

The consequences of charge on the mass of the object defined by Misner-Sharp [57] is evaluated as

$$m = -\frac{H}{2}R^3_{232} = \left[1 + \left(\frac{H'}{K}\right)^2 - \left(\frac{\dot{H}}{J}\right)^2 + \frac{s^2}{H^2}\right]\frac{H}{2}. \tag{23}$$

Now, we look at the dynamics of the celestial objects by taking into account the proper radial derivative ($D_H = \frac{1}{H'}\frac{\partial}{\partial t}$) and the proper time derivative ($D_T = \frac{1}{J}\frac{\partial}{\partial t}$), respectively. The proper derivative operator can be used to calculate the relativistic velocity of the interior of the collapsing star. Collapsing fluid velocity (U) is defined as a change in areal radius with proper time, i.e., $U = D_T H$. This collapsing velocity of the fluid should be taken as negative. The connection between collapsing velocity and mass of the object under the influence of charge, can also be observed with the help of following equation

$$E^2 \equiv \left(\frac{H'}{K}\right)^2 = \left(\frac{2m}{H} - 1 + U^2 - \frac{s^2}{H^2}\right). \tag{24}$$

The mass function varies with respect to the proper derivative operators as

$$D_T(m) = 4\pi\left(P_r - \frac{s^2}{8\pi H^4}\right)UH^2 - \frac{s^2}{JH^2} - \frac{s^2\dot{H}}{2JH^2} + 4\pi qEH^2, \tag{25}$$

$$D_H m + \frac{s^2}{H^2} = -4\pi\left[\mu + \frac{s^2}{8\pi H^4} + q\frac{U}{E}\right]H^2 + \frac{ss'}{HH'}. \tag{26}$$

Equation (26) upon integration yields

$$m - \frac{s^2}{2H} = -\frac{4\pi\mu H^3}{3} + 4\pi\int_0^r \frac{H^3}{3}\frac{\partial\mu}{\partial r}dr - 4\pi\int_0^r \frac{qUH^2H'}{E}dr - \int_0^r \frac{ss'}{H}dr. \tag{27}$$

Integration of Equation (27) produces

$$\frac{3m}{H^3} \quad = \quad -4\pi\mu + \frac{4\pi}{H^3}\int_0^r H^3\left[D_H\mu - \frac{3D_H s^2}{2H} - \frac{3qUH^2}{E}\right]H'dr + \frac{3s^2}{H^4}. \tag{28}$$

Equation (27) satisfies the regular condition $m(t,0) = 0$. Due to the fact that any causal transport equation is formed on the notion that the fluid is close to thermal equilibrium, i.e., $q << |\mu|$. Hence, Equation (27) indicates that μ is inevitably negative with the restriction that $H' > 0$ along with $\frac{s^2}{2H} - \int_0^r \frac{ss'}{H}dr > 0$ while keeping in mind that E is a regular function and the mass of the fluid distribution cannot be negative. The use of Equations (12) and (13) give

$$D_T U = \frac{m}{H^2} - 4\pi H P_r + aE, \tag{29}$$

$$D_H\left(\frac{U}{H}\right) = \frac{4\pi q}{E} + \frac{\sigma}{H'}, \tag{30}$$

which transform Equations (15) and (16) as

$$D_T\mu + \frac{1}{3}\left(3\mu + P_r + 2P_\perp\right)\Theta + \frac{2}{3}\left(P_r - P_\perp\right)\sigma + ED_H q + 2q\left(a + \frac{E}{H}\right) = 0 \tag{31}$$

$$ED_H P_r - \frac{ss'}{4\pi K H^4} + \left(\mu + P_r\right)a + 2\left(P_r - P_\perp\right)\frac{E}{H} + D_T q + \frac{2}{3}q(2\Theta + \sigma) = 0. \tag{32}$$

The combination of Equations (29) and (32) yield

$$(P_r + \mu)D_T U = -E^2\left[D_H P_r + \frac{2}{H}\Pi\right] + \frac{ss'}{4\pi K H^4} - \left(\mu + P_r\right)\left(4\pi P_r H^3 - m\right)\frac{1}{H^2} - E\left[D_T q + \frac{2}{3}q(2\Theta + \sigma)\right], \tag{33}$$

where $(\mu + P_r)$ and $(4\pi P_r H^3 - m)$ depict the passive gravitational mass and active gravitational mass, respectively.

5. Conformal Tensor and Complexity Factor

In this section, the conformal scalar will be evaluated by means of Conformal tensor. The Conformal tensor is the conformally invariant part of the Curvature tensor [58]. While moving along a geodesic a body feels a tidal force which can be expressed with the help of Conformal tensor. Moreover, we will discuss about the structure scalar Y_{TF} which is chosen to be the complexity factor of the fluid configuration. Eventually, the complexity factor will be stated in the context of metric coefficients and their derivatives, which would aid to construct different stellar models. The first step is to evaluate the conformal scalar with the help of Conformal tensor which is designated as

$$W^{(e)}_{\zeta\xi} = \varepsilon\left(\chi_\zeta\chi_\xi - \frac{h_{\xi\zeta}}{3}\right) \tag{34}$$

where $W^{(e)}_{\zeta\xi}$ is the electric part of the Conformal tensor as the magnetic part vanishes in our scenario. Here, the conformal scalar is denoted by ε and is computed in terms of our line element as

$$\varepsilon = \frac{1}{2K^2}\left[-\frac{J'}{J}\frac{H'}{H} + \frac{K'}{K}\left(\frac{H'}{H} - \frac{J'}{J}\right) + \left(\frac{H'}{H}\right)^2 + \frac{J''}{J} - \frac{H''}{H}\right] + \frac{1}{2J^2}\left[\frac{\dot{J}}{J}\frac{\dot{K}}{K} - \frac{\dot{J}}{J}\frac{\dot{H}}{H} + \frac{\dot{H}}{H}\frac{\dot{K}}{K} - \left(\frac{\dot{H}}{H}\right)^2 - \frac{\ddot{K}}{K} + \frac{\ddot{H}}{H}\right] + \frac{1}{2H^2}. \tag{35}$$

In GR, when the Ricci tensor is zero, the Conformal tensor imparts curvature to the spacetime. The Ricci tensor in GR is derived from the energy-momentum of the local fluid distribution. The Ricci tensor will be zero if the matter distribution is zero, but spacetime is not always flat in this instance due to the fact that Conformal tensor adds curvature to the Riemann curvature tensor. Thus, the gravitational field in spacetime vacuum scenarios is not always zero. This phrase permits gravity to propagate in areas with no matter or energy.

Next, we are going to compute the structure scalar Y_{TF} named as the complexity factor. There will be five structure scalars that can be found for our fluid configuration by implementing the method of orthogonal splitting of the Reimann tensor [59]. Our focus is on the Y_{TF} which is helpful to estimate the extent of complexity in celestial objects and hence named as complexity factor. To gain more knowledge about the structure scalars one can see [60–63]. We will cover the main steps to reach our goal. In order to evaluate the complexity factor, the first step is to consider the tensor [60]

$$Y_{\zeta\xi} = R_{\zeta\eta\xi\delta}V^{\eta}V^{\delta},$$

where V^{η} depicts the four velocity. The previously mentioned tensor can be divided in term of its trace and trace free portion as

$$Y_{\zeta\xi} = \frac{Y_T h_{\xi\zeta}}{3} + Y_{TF}\left(\chi_\zeta \chi_\xi - \frac{h_{\xi\zeta}}{3}\right) \tag{36}$$

With the help of the field equations and Equation (35), one can easily procure

$$Y_T = 4\pi(\mu - 2\Pi + 3P_r) + \frac{s^2}{4\pi H^4},$$
$$Y_{TF} = -4\pi\Pi + \varepsilon + \frac{s^2}{H^4}. \tag{37}$$

The combination of Equations (13), (14), (23) and (35) result in the following equation

$$\frac{3m}{H^3} = 4\pi\Pi - 4\pi\mu + \varepsilon - \frac{3s^2}{2H^4} \tag{38}$$

Making use of Equations (28) and (37), the following expression is achieved

$$Y_{TF} = -8\pi\left(\Pi - \frac{s^2}{4\pi H^4}\right) + \frac{4\pi}{H^3}\int_0^r H^3\left(D_R\mu - \frac{3D_R s^2}{2H} - \frac{3qUH^2}{E}\right)H' dr + \frac{3s^2}{H^4}. \tag{39}$$

The use of field equations and the expression for ε turns Equation (39) into

$$Y_{TF} = \frac{1}{K^2}\left(\frac{J''}{J} - \frac{J'}{J}\frac{H'}{H} - \frac{J'}{J}\frac{K'}{K}\right) + \frac{1}{J^2}\left(\frac{\dot{J}}{J}\frac{\dot{K}}{K} - \frac{\dot{J}}{J}\frac{\dot{H}}{H} - \frac{\ddot{K}}{K} + \frac{\ddot{H}}{H}\right). \tag{40}$$

The scalar function Y_{TF} has been chosen as a measure of the complexity of the fluid structure. The motivation behind it is that, it highlights the maximum information about the matter distribution by measuring the anisotropic pressure and inhomogeneity in the energy density. In addition to that, Y_{TF} is the same as in the static case, ensuring that we recover the correct complexity factor expression in the limit to the static regime. The dissipative variables are also included in it. The condition $Y_{TF} = 0$ is helpful to develop different models as well as it can also tell us about the effects of charge on the physical attributes of the astrophysical objects.

6. Transport Equation

In thermodynamics, the radiative flow within a fluid is governed by the transport equation. This equation is utilized in diffusion approximation during a dissipative gravitational collapse. We will use a heat transfer equation [64] derived from a well-known dissipation theory [65,66] to describe the thermodynamical effects. The temperature of the dynamically collapsing stellar fluid is taken into account in this equation. It is a general partial differential equation that describes transportation phenomena including mass transfer, heat transmission and fluid dynamics, etc, and is formulated as

$$\tau h^{\zeta\xi}V^{\gamma}q_{\xi;\gamma}+q^{\zeta}=-\kappa h^{\zeta\xi}\left(T_{,\xi}+Ta_{\xi}\right)-\frac{1}{2}\kappa T^{2}\left(\frac{\tau V^{\xi}}{\kappa T^{2}}\right)_{;\xi}q^{\zeta}, \tag{41}$$

where T,κ and τ indicate the temperature, thermal conductivity and the relaxation time, respectively. The non-vanishing component of Equation (41) is

$$\tau D_{T}q=-q-\frac{\kappa}{JK}(JT)'-\frac{1}{2}\tau\Theta q-\frac{1}{2}\kappa T^{2}D_{T}\left(\frac{\tau}{\kappa T^{2}}\right)q. \tag{42}$$

The truncated version of transport equation can be written as

$$\tau h^{\zeta\xi}V^{\gamma}q_{\xi;\gamma}+q^{\zeta}=-\kappa h^{\zeta\xi}\left(T_{,\xi}+Ta_{\xi}\right), \tag{43}$$

whose non-vanishing component is

$$\tau\dot{q}+qJ=-\frac{\kappa}{K}(TJ)'. \tag{44}$$

The temperature gradient is related to the four acceleration as

$$(TJ)'=0\Rightarrow T'=\frac{TJ'}{J}=-TaK. \tag{45}$$

Because of the equivalence principle, thermal energy will seek to shift to places with lower gravitational potential, changing the thermal equilibrium condition in the presence of a gravitational field. To put it another way, a temperature gradient is now required to maintain thermal equilibrium [67]. Equation (45) shows that if $a<0$ then the gravitational force will be repulsive in nature and the temperature gradient will be positive in this case to keep the system in thermal equilibrium.

7. Quasi- Homologous Constraint

In this section, we determine the condition which is designed to meet the need of least intricate form of evolution. To do so, the first step is to write Equation (12) as

$$\left(\frac{U}{H}\right)'=4\pi qK+\sigma\frac{H'}{H}. \tag{46}$$

The solution of the differential Equation (46) is obtained with the aid of Equation (24), is

$$U=\tilde{j}(t)H+H\int_{0}^{r}\left(\frac{4\pi q}{E}+\frac{\sigma}{H}\right)H'dr, \tag{47}$$

where $\tilde{j}(t)$ denotes the integration constant. The boundedness of fluid distribution by a surface Σ^{e} is defined with the help of expression $r=r_{\Sigma^{e}}=constant$. Implementing the previously stated constraint on Equation (47), one can attain

$$U=H\frac{U_{\Sigma^{e}}}{H_{\Sigma^{e}}}-H\int_{r}^{r_{\Sigma^{e}}}\left(\frac{4\pi q}{E}+\frac{\sigma}{H}\right)H'dr. \tag{48}$$

The quasi-homologous constraint suggests

$$U=H\frac{U_{\Sigma^{e}}}{H_{\Sigma^{e}}}, \tag{49}$$

which entails

$$\frac{4\pi q}{E} + \frac{\sigma}{H} = 0. \tag{50}$$

In classical astrophysics, this condition has wide applications as it is the relativistic interpretation of homologous condition. This condition will be used as a supplementary condition with the condition $Y_{TF} = 0$ to construct several specific models. In order to have a better understanding for this condition one can see [68–71].

8. Stellar Models

In this section, we will give several precise solutions representing dissipative or non-dissipative systems that meet the diminishing complexity factor criterion and evolve quasi-homologously. Moreover, to completely describe the models, some realistic conditions will also be used.

8.1. Vanishing Dissipation

In spite of the fact that in this manuscript our concern is with the dissipative systems still, to complete the picture, we will discuss what will be the scenario when the heat flux nullifies. Consider the quasi homologous condition and put heat flux equals to zero in it, we obtain

$$q = 0 \Rightarrow \sigma = 0 \Rightarrow \frac{\dot{K}}{K} = \frac{\dot{H}}{H} \Rightarrow H = rK. \tag{51}$$

Utilizing Equation (47), we get

$$U = \frac{\dot{H}}{J} = \frac{r\dot{K}}{J} = \tilde{j}(t) rK. \tag{52}$$

The implementation of the condition of vanishing complexity factor gives

$$\frac{J''}{J} - \frac{J'K'}{JK} - \frac{J'H'}{JH} = 0 \tag{53}$$

Equation (53) shows that in non-dissipative system there is also no influence of charge on the complexity factor, in order to keep it zero.

8.1.1. Isotropic Pressure and Conformal Flatness

Here, we impose a few supplementary constraints that the surface is conformally flat ($\varepsilon = 0$) and the pressure is isotropic ($\Pi = 0$), in order to provide particular solution. The previously stated constraints along with $Y_{TF} = 0$ nullifies the effect of density inhomogeneity, i.e., $\mu' = 0$. Utilizing the condition of conformal flatness and pressure isotropy, we achieve the following equation

$$\frac{1}{H^2} + \frac{1}{K^2}\left[\left(\frac{H'}{H}\right)^2 + \frac{K'}{K}\frac{H'}{H} - \frac{H''}{H}\right] - \frac{1}{J^2}\left(\frac{\dot{H}^2}{H^2} - \frac{\dot{K}}{K}\frac{\dot{H}}{H}\right) = 0. \tag{54}$$

Equations (53) and (54), with the help of Equation (51), turns out into

$$1 + r^2\left[2\left(\frac{H'}{H}\right)^2 - \frac{1}{r}\frac{H'}{H} - \frac{H''}{H}\right] = 0 \tag{55}$$

and

$$\frac{J''}{J} - \frac{J'}{J}\left(\frac{2H'}{H} - \frac{1}{r}\right) = 0. \tag{56}$$

The solution of the system of differential Equations (55) and (56) give

$$H = \frac{\tilde{H}(t)}{\cos\left[a_1(t) + \ln r\right]}, \tag{57}$$

$$K = \frac{\tilde{H}(t)}{r\cos\left[a_1(t) + \ln r\right]}, \tag{58}$$

$$J = \gamma(t)\tilde{H}^2(t)\tan\left[a_1(t) + \ln r\right] + k(t), \tag{59}$$

where $\tilde{H}(t)$, $a_1(t)$, $\gamma(t)$, $k(t)$ are arbitrary functions of time. One can also accomplish the above mentioned solutions using *Maple or Mathematica*. In order to further specify the solution, we introduce some functions

$$\dot{a}_1 = \frac{\dot{\tilde{H}}}{\tilde{H}}, \quad k(t) = \gamma(t)\tilde{H}^2, \tag{60}$$

which generates

$$\frac{\dot{H}}{H} = \frac{\dot{H}}{\tilde{H}}(1 + \tan u), \tag{61}$$

$$J = \gamma(t)\tilde{H}^2(1 + \tan u), \quad \Rightarrow \quad J = \frac{\tilde{j}\dot{H}}{H}, \tag{62}$$

where $u = a_1(t) + \ln r$ and $\tilde{j} = \frac{\gamma(t)\tilde{H}^3}{\dot{\tilde{H}}}$. With the aid of above functions one can easily evaluate the physical parameters and the mass functions as

$$8\pi\left(\mu + \frac{s^2}{8\pi H^4}\right) = -\frac{3}{\tilde{H}^2} + \frac{3}{\tilde{j}^2}, \tag{63}$$

$$8\pi\left(P_r - \frac{s^2}{8\pi H^4}\right) = -\frac{3}{\tilde{j}^2} + \frac{3\tan u + 1}{\tilde{H}^2(\tan u + 1)} + \frac{2\tilde{H}\dot{\tilde{j}}}{\tilde{j}^3\dot{H}(\tan u + 1)}, \tag{64}$$

$$8\pi\left(P_\perp + \frac{s^2}{8\pi H^4}\right) = -\frac{3}{\tilde{j}^2} + \frac{3\tan u + 1}{\tilde{H}^2(\tan u + 1)} + \frac{2\tilde{H}\dot{\tilde{j}}}{\tilde{j}^3\dot{H}(\tan u + 1)} \tag{65}$$

$$\left(m - \frac{s^2}{2H}\right) = \frac{\tilde{H}}{2\cos^3 u}\left(1 - \frac{\tilde{H}^2}{\tilde{j}^2}\right). \tag{66}$$

If we set $\tilde{H}(t)$, $a_1(t)$ and $\gamma(t)$ to constants as t approaches to infinity, the foregoing solution tends to the incompressible isotropic solution in [26], which is a special case of the hyperbolically symmetric Bowers-Liang solution in [28].

Due to the fact that this model share several aspects (i.e., $\varepsilon = \sigma = \Pi = \mu' = 0$), the above solution might be regarded as the version of the Friedman-Lemaître-Robertson-Walker spacetime (FLRW) for the charged hyperbolically symmetric situation. However, unlike the spherically symmetric instance, it is not geodesic. As a result, we will look for another hyperbolically symmetric FLRW spacetime that also meets the geodesic criterion in the presence of charge.

8.1.2. Geodesic Solutions

In this section, we will impose the additional geodesic condition on the fluid and then evaluate the physical parameters together with mass function under the influence of charge. To do so, we put $J = 1$ and the quasi homologous condition gives

$$\frac{H_I}{H_{II}} = constant \tag{67}$$

where H_I and H_{II} are the areal radii of two shells (I, II), illustrating with help of expressions $r = r_I = constant$ and $r = r_{II} = constant$, respectively. Putting $J = 1$ and $q = 0$ in Equation (12), we get

$$\frac{\dot{K}}{K} = \frac{\dot{H}'}{H'}. \tag{68}$$

Equation (67) is the indication that H is a separable function. As a result, K is also separable function utilizing $H = rK$. Therefore, according to simple reparametrization of r, K depends on t only, i.e.,

$$H = rK(t). \tag{69}$$

Equation (68) is also satisfied using Equation (69). Moreover, $Y_{TF} = 0$, as verified by Equation (40). The physical attributes of this model are found to be

$$8\pi\left(\mu + \frac{s^2}{8\pi H^4}\right) = -\frac{2}{r^2K^2} + \frac{3\dot{K}^2}{K^2}, \tag{70}$$

$$8\pi\left(P_r - \frac{s^2}{8\pi H^4}\right) = \frac{2}{r^2K^2} - \frac{\dot{K}^2}{K^2} - \frac{2\ddot{K}}{K}, \tag{71}$$

$$8\pi\left(P_\perp + \frac{s^2}{8\pi H^4}\right) = -\frac{2\dot{K}^2}{K^2} - \frac{2\ddot{K}}{K}, \tag{72}$$

$$\left(m - \frac{s^2}{2H}\right) = \frac{rK}{2}\left(2 - r^2\dot{K}^2\right), \tag{73}$$

Equation (47) in the absence of heat flux and shear becomes $U = \tilde{j}(t)H$. If the fluid is geodesic then the expression $U = \tilde{j}(t)H$ suggests (67) in the background of relativistic regime. While in non relativistic regime, the proportionality between areal radius and velocity always suggests (67).

In short, the fluid is shear-free, conformally flat, geodesic, develops homologously, and meets the criterion of diminishing complexity factor even in the presence of electromagnetic force. In this way, it might be thought of as a hyperbolically symmetric form of the FRW spacetime under the influence of charge. But it is anisotropic, and the energy density is inhomogeneous, unlike the spherically symmetric case.

Eventually , it is informative to construct a toy model using the aforementioned results given in Equations (70)–(73) by selecting a particular form for the function H that leads to a static regime asymptotically. Therefore, suppose

$$K = \omega(e^{-\alpha t} + 1) \tag{74}$$

where ω and α are positive constants. It can be straightforwardly verified that as t approaches to infinity we get

$$8\pi\left(\mu + \frac{s^2}{8\pi H^4}\right) = -\frac{2}{r^2\omega^2}, \quad 8\pi\left(P_r - \frac{s^2}{8\pi H^4}\right) = \frac{2}{r^2\omega^2} \tag{75}$$

$$8\pi\left(P_\perp + \frac{s^2}{8\pi H^4}\right) = 0, \quad \left(m - \frac{s^2}{2H}\right) = r\omega. \tag{76}$$

As a result, the static solution corresponding to the stiff equation of state $(P_r = |\mu|)$ reported in [72] converges to our toy model.

8.2. Dissipative Case with $K = 1$

Dissipation is the concept of the dynamical system which experiences the loses in energy by virtue of the frictional forces. In this subsection, we will discuss about the dissipative solutions whose velocity between the neighboring layers of matter $(D_T(\delta l))$ is zero but the areal velocity (U) is not zero. The condition $D_T(\delta l) = 0$ produces $H = H(r)$ and by simple reparametrization and without the loss of generality, we can consider

$$K = 1, \quad J = \frac{\dot{H}}{\tilde{j}(t)H}. \tag{77}$$

Utilizing Equation (77) in the field equations, the consequence of charge on the physical variables for this specific model is evaluated as

$$8\pi\left(\mu + \frac{s^2}{8\pi H^4}\right) = -\frac{1}{H^2} - \frac{2H''}{H} - \left(\frac{H'}{H}\right)^2 + \tilde{j}^2, \tag{78}$$

$$4\pi q = \frac{\tilde{j}(t)H'}{H}, \tag{79}$$

$$8\pi\left(P_r - \frac{s^2}{8\pi H^4}\right) = \frac{1}{H^2} - \left(\frac{H'}{H}\right)^2 + \frac{2\dot{H}'H'}{\dot{H}H} - 2\dot{\tilde{j}}\tilde{j}\frac{H}{\dot{H}} - 3\tilde{j}^2, \tag{80}$$

$$8\pi\left(P_\perp + \frac{s^2}{8\pi H^4}\right) = \frac{\dot{H}''}{\dot{H}} - \frac{\dot{H}'}{\dot{H}}\frac{H'}{H} + \left(\frac{H'}{H}\right) - \dot{\tilde{j}}\tilde{j}\frac{H}{\dot{H}} - \tilde{j}^2. \tag{81}$$

As we described earlier that there is a loss of energy in the process of dissipation. This loss in energy becomes the cause for the rise in temperature by transforming it into heat. Hence, one can say that the thermal energy releases in the dissipation processes of the system. The temperature for this particular model is calculated using Equations (44) and (77) as

$$T(t,r) = \frac{\tilde{j}H}{\dot{H}}\left(f(t) - \frac{\tau\dot{\tilde{j}}}{4\pi\kappa}\ln H - \frac{1}{4\pi\kappa}\int\frac{\dot{H}}{H}\frac{H'}{H}dr\right) - \frac{\tau\tilde{j}^2}{4\pi\kappa}, \tag{82}$$

where $f(t)$ is arbitrary function of t and treated as function of integration. The combination of quasi-homologous condition and vanishing complexity factor generate

$$J'' - J'\frac{H'}{H} + J\sigma^2 = \dot{\sigma}, \tag{83}$$

$$-\frac{\dot{H}}{\sigma H} = J. \tag{84}$$

Inserting variables (Y, Z), we get

$$J = Y + \frac{\dot{\sigma}}{\sigma^2} \qquad \mathit{and} \qquad R = Y'Z. \tag{85}$$

Equations (83) and (84) turn out to be

$$-\frac{Y'Z'}{YZ} + \sigma^2 = 0, \tag{86}$$

$$\frac{\dot{Y}'}{Y'} + \frac{\dot{Z}}{Z} = -\sigma Y - \frac{\dot{\sigma}}{\sigma}. \tag{87}$$

Due to the fact that these models include a vacuole surrounding the center, we should not be concerned with regularity conditions at the center. Thereby, we will choose certain solutions among the family of dissipative solutions. Implementation of supplementary restrictions on Equations (86) and (87) will permit us to integrate these equations.

8.2.1. Y Is a Separable Function

Now, we integrate the Equations (86) and (87) by treating Y as a separable function and compute the metric coefficients as

$$J = \frac{\dot{\sigma}}{2\omega^2\sigma^2}\left[2\omega^2 - \sigma^2(\omega r + a_1)^2\right], \tag{88}$$

$$H = \frac{\tilde{H}_0}{\sigma} e^{\frac{\sigma^2}{4\omega^2}(\omega r + a_1)^2}(\omega r + a_1), \tag{89}$$

$$\tilde{j} = -\sigma. \tag{90}$$

The substantial parameters are obtained with the help of Equations (88)–(90) as

$$8\pi\left(\mu + \frac{s^2}{8\pi H^4}\right) = -\frac{\sigma^2 e^{-\frac{\sigma^2}{2\omega^2}(\omega r + a_1)^2}}{\tilde{H}_0^2(\omega r + a_1)^2} - \frac{\omega^2}{(\omega r + a_1)^2} - \frac{3\sigma^4}{4\omega^2}(\omega r + a_1)^2 - 3\sigma^2, \tag{91}$$

$$4\pi q = -\frac{\sigma[2\omega^2 + \sigma^2(\omega r + a_1)^2]}{2\omega(\omega r + a_1)}, \tag{92}$$

$$8\pi\left(P_r - \frac{s^2}{8\pi H^4}\right) = \frac{\sigma^2 e^{-\frac{\sigma^2}{2\omega^2}(\omega r + a_1)^2}}{\tilde{H}_0^2(\omega r + a_1)^2} - \frac{4\sigma^2\omega^2}{2\omega^2 - \sigma^2(\omega r + a_1)^2} + \frac{\omega^2}{(\omega r + a_1)^2} + \frac{\sigma^4}{4\omega^2}(\omega r + a_1)^2, \tag{93}$$

$$8\pi\left(P_\perp + \frac{s^2}{8\pi H^4}\right) = -\frac{\sigma^2[2\omega^2 + \sigma^2(\omega r + a_1)^2]^2}{4\omega^2(2\omega^2 - \sigma^2(\omega r + a_1)^2)}, \tag{94}$$

$$\left(m - \frac{s^2}{2H}\right) = \frac{\tilde{H}_0(\omega r + a_1)}{2\sigma} e^{\frac{\sigma^2}{4\omega^2}(\omega r + a_1)^2}\left\{1 + \frac{\tilde{H}_0^2}{4\sigma^2\omega^2}\left[4\omega^4 + \sigma^4(\omega r + a_1)^4\right] e^{\frac{\sigma^2}{2\omega^2}(\omega r + a_1)^2}\right\}. \tag{95}$$

Equations (91) and (94) show the fall in the density and the tangential pressure under the influence of electric force. Contrary to that, Equations (93) and (95) show that the radial pressure and mass function increase in the presence of charge. The heat flux remains unchanged under the charge distribution. In this case, the temperature is computed as

$$\begin{aligned} T(t,r) \quad &= \frac{2\omega^2\sigma^2}{\dot{\sigma}\left[2\omega^2 - \sigma^2(\omega r + a_1)^2\right]}\left\{f(t) + \frac{\dot{\sigma}\tau}{4\pi\kappa}\left[\frac{\sigma^2}{4\omega^2}(\omega r + a_1)^2 + \ln\left[\frac{\tilde{H}_0}{\sigma}(\omega r + a_1)\right]\right]\right. \\ &\left. + \frac{\dot{\sigma}}{4\pi\sigma\kappa}\ln(\omega r + a_1) - \frac{\dot{\sigma}\sigma^3}{64\pi\omega^4\kappa}(\omega r + a_1)^4\right\} - \frac{\tau\sigma^2}{4\pi\kappa}, \end{aligned} \tag{96}$$

where $f(t)$ is an arbitrary function associated with T. It is found that there is no effect of charge distribution on the temperature of the fluid.

8.2.2. $J = J(r)$

Next, we derive the solutions of Equations (86) and (87) by treating J as a function of r only

$$J = \frac{1}{4}(\sqrt{2b_0}r + a_1)^2, \quad \tilde{j} = b_0 t - b_1, \tag{97}$$

$$H = \tilde{H}(r) e^{-\frac{1}{4}(\sqrt{2b_0}r + a_1)^2(-\frac{b_0}{2}t^2 + b_1 t)}, \tag{98}$$

where b_0, b_1 and a_1 are constants. Next, the physical parameters are obtained by utilizing a restriction, i.e., $\tilde{H} = \tilde{H}_0 = constant$

$$8\pi\left(\mu+\frac{s^2}{8\pi H^4}\right)=b_1^2-\frac{3b_0}{2}(\sqrt{2b_0}r+a_1)^2\left(-\frac{b_0}{2}t^2+b_1t\right)^2-\frac{1}{\tilde{H}_0^2}e^{\frac{1}{2}(\sqrt{2b_0}r+a_1)^2(-\frac{b_0}{2}t^2+b_1t)}, \tag{99}$$

$$4\pi q=\frac{\sqrt{2b_0}}{2}(\sqrt{2b_0}r+a_1)(-\frac{b_0}{2}t^2+b_1t)(-b_0t+b_1), \tag{100}$$

$$8\pi\left(P_r-\frac{s^2}{8\pi H^4}\right)=\frac{1}{\tilde{H}_0^2}e^{\frac{1}{2}(\sqrt{2b_0}r+a_1)^2(-\frac{b_0}{2}t^2+b_1t)}-t^2b_0^2+2tb_0b_1-3b_1^2-\frac{8b_0}{(\sqrt{2b_0}r+a_1)^2}$$
$$+\frac{b_0}{2}(\sqrt{2b_0}r+a_1)^2\left(-\frac{b_0}{2}t^2+b_1t\right)^2, \tag{101}$$

$$8\pi\left(P_\perp+\frac{s^2}{8\pi H^4}\right)=\frac{1}{2}b_0^2t^2-tb_0b_1-b_1^2+\frac{b_0}{2}(\sqrt{2b_0}r+a_1)^2\left(-\frac{b_0}{2}t^2+b_1t\right)^2, \tag{102}$$

$$\left(m-\frac{s^2}{2H}\right)=\frac{\tilde{H}_0}{2}e^{-\frac{1}{4}(\sqrt{2b_0}r+a_1)^2(-\frac{b_0}{2}t^2+b_1t)}\left\{1+\tilde{H}_0^2\left[\frac{b_0}{2}(\sqrt{2b_0}r+a_1)^2\left(-\frac{b_0}{2}t^2+b_1t\right)^2\right.\right.$$
$$\left.\left.-(b_0t-b_1)^2\right]e^{-\frac{1}{2}(\sqrt{2b_0}r+a_1)^2(-\frac{b_0}{2}t^2+b_1t)}\right\}. \tag{103}$$

The decrease in the density and tangential pressure has been noticed in the presence of charge from Equations (99) and (102). While the involvement of charge shows the increase in radial pressure and mass function as one can witness it from Equations (101) and (103). Also, there is no effect of charge on the heat flux. The expression of temperature for this particular model is obtained as

$$T(t,r)=\frac{4}{(\sqrt{2b_0}r+a_1)^2}\left\{f(t)-\frac{\tau b_0}{4\pi\kappa}\left[\ln\tilde{H}_0-\frac{1}{4}\left(-\frac{b_0}{2}t^2+b_1t\right)(\sqrt{2b_0}r+a_1)^2\right]\right\}$$
$$-\frac{\left(-\frac{b_0}{2}t^2+b_1t\right)(-b_0t+b_1)(\sqrt{2b_0}r+a_1)^2}{32\pi\kappa}-\frac{\tau(-b_0t+b_1)^2}{4\pi\kappa}, \tag{104}$$

where $f(t)$ is an arbitrary function of time associated with the temperature. It is found that temperature remains unaltered in the presence of charge.

8.3. Constant Shear Scalar

Here, we use the approach of constant shear scalar in order to obtain the solutions of Equations (86) and (87), as

$$J=\omega r-\frac{\omega^2}{\sigma}t+\omega_0,\quad \tilde{a}=-\sigma=\text{ const}, \tag{105}$$

$$H=\tilde{H}_0\omega e^{\left(\frac{\sigma^2}{2}r^2-\sigma\omega tr+\frac{\sigma^2\omega_0}{\omega}r+\frac{\omega^2}{2}t^2-\sigma\omega_0t\right)}. \tag{106}$$

Making use of the Equations (105) and (106), one can compute the substantial parameters for this particular model as

$$8\pi\left(\mu+\frac{s^2}{8\pi H^4}\right)=-\sigma^2-3\left[\sigma^2\left(r+\frac{\omega_0}{\omega}\right)-\sigma\omega t\right]^2-\frac{e^{-2\left(\frac{\sigma^2}{2}r^2-\sigma\omega tr+\frac{\sigma^2\omega_0}{\omega}r+\frac{\omega^2}{2}t^2-\sigma\omega_0 t\right)}}{\tilde{H}_0^2\omega^2}, \quad (107)$$

$$4\pi q=-\sigma^3\left(r-\frac{\omega}{\sigma}t+\frac{\omega_0}{\omega}\right), \quad (108)$$

$$8\pi\left(P_r-\frac{s^2}{8\pi H^4}\right)=-\sigma^2+\sigma^4\left(r-\frac{\omega}{\sigma}t+\frac{\omega_0}{\omega}\right)^2+\frac{e^{-2\left(\frac{\sigma^2}{2}r^2-\sigma\omega tr+\frac{\sigma^2\omega_0}{\omega}r+\frac{\omega^2}{2}t^2-\sigma\omega_0\right)}}{\tilde{H}_0^2\omega^2}, \quad (109)$$

$$8\pi\left(P_\perp+\frac{s^2}{8\pi H^4}\right)=\sigma^2+\left[\sigma^2\left(r+\frac{\omega_0}{\omega}\right)-\sigma\omega t\right]^2, \quad (110)$$

$$\left(m-\frac{s^2}{2H}\right)=\frac{\tilde{H}_0\omega}{2}e^{\left(\frac{\sigma^2}{2}r^2-\sigma\omega tr+\frac{\sigma^2\omega_0}{\omega}r+\frac{\omega^2}{2}t^2-\sigma\omega_0 t\right)}\left\{1+\tilde{H}_0^2\omega^2\sigma^2\left[\frac{\sigma^2}{\omega^2}\left(\omega r-\frac{\omega^2 t}{\sigma}+\omega_0\right)^2-1\right]e^{2\left(\frac{\sigma^2}{2}r^2-\sigma\omega tr+\frac{\sigma^2\omega_0}{\omega}r+\frac{\omega^2}{2}t^2-\sigma\omega_0 t\right)}\right\}. \quad (111)$$

The decrease in the density and tangential pressure has been noticed in the presence of charge from Equations (107) and (110). While in the presence of electric force the radial pressure and mass function increases as one can verify it from Equations (109) and (111). The temperature for the particular model is evaluated as

$$T(t,r)=\frac{f(t)}{\left(\omega r-\frac{\omega^2}{\sigma}t+\omega_0\right)}+\frac{\sigma^3}{12\omega^2\pi\kappa}\left(\omega r-\frac{\omega^2}{\sigma}t+\omega_0\right)^2-\frac{\tau\sigma^2}{4\pi\kappa}, \quad (112)$$

here $f(t)$ depicts the arbitrary function of time associated with temperature. There is no influence of charge on the temperature as one can see from Equation (112).

9. Discussion and Final Remarks

The effects of electric field inside the massive star allowed by a particular charge fraction has been investigated in this manuscript. The quantity of charge stored in a dense system like compact star can be quite immense, several orders of magnitude greater than those estimated by classical balance of forces at the star's surface. The high density of the system is primarily responsible for this quantity of charge. Highly compact stars, whose radius is on the cusp of producing an event horizon, can be balanced by massive amounts of net charge, allowing the massive gravitational attraction to be balanced. A star can have a large quantity of charge if the charge to mass ratio of the particles that make up the star is low, say one or of order one. The matter inside the stars is large in density and pressure. Also, the gravitational field is very strong. This shows the presence of an electric charge as well as a strong electric field.

In this paper, we thoroughly examined the substantial determinants of the non-static anisotropic matter configuration endorsed with the hyperbolically symmetry in the presence of charge. To do so, we have constructed a fundamental scheme that permit us to gain insight into the dynamics of aforementioned fluid configuration, which eventually leads towards the static spacetime. All the results accomplished are quite extensive that it is expected they could be applied to further scenarios associated with hyperbolic symmetry with an electric charge. The presence of vacuole in the central region and the negative energy density are notable characteristics of both static and non-static hyperbolically distributed fluids. The negative energy density point towards the violation of the weak energy condition as well as the involvement of the quantum effects. As a consequence, we can say that the fluid which is taken into account in this manuscript can be useful in the study where quantum effects are intended to be relevant. The presence of the vacuole is the indication of the zero expansion scalar. In this scenario, the relativistic fluid evolve without experiencing any compression. Moreover, the zero expansion condition suggests the occurrence of pressure anisotropy as well as the energy density inhomogeneity. Whereas, $\Theta \gtrless$ depicts the expanding and decelerating features of the universe. It should be noted that in the scenario when the fluid fills the whole sphere, including the center $(r = 0)$, the regularity requirement $r = 0$ must be applied. However, we do not need such a condition because we are considering the availability of a cavity encircling the center.

We pay attention to both non-dissipative and dissipative systems. Taking into account, the quasi-homologous constraint for the former systems, we are able to construct a large number of models that meet the diminishing complexity factor criteria. We implemented a transport equation in the later scenario, which allowed us to obtain the explicit temperature expressions for each model. These expressions contains the term for relaxation time (τ) which refers to transiant processes that occur prior to relaxation. They play a critical role for time scales on the order of $\tau \; or < \tau$, although their contributions are applicable for all time scales. Furthermore, the terms where $\tau = 0$ are linked with the stationary dissipative regime. Consequently, this expression encompasses whole thermal history of the astrophysical objects, including the era earlier to relaxation.

As a measure of the fluid structure's complexity, the scalar function Y_{TF} was used. It is motivated by the fact that it emphasizes the most information about the matter distribution by measuring anisotropic pressure and inhomogeneity in the energy density. Furthermore, the complexity factor is the same as in the static case, ensuring that we recover the right complexity factor expression. It also includes the dissipative variables. If we take a system that begins its evolution from rest $(\sigma = 0)$, it will stay shearfree if the fluid is geodesic and $Y_{TF} = 0$. This is another argument in favor of using Y_{TF} as the complexity factor. The quasi-homologous condition implies the disappearance of Y_{TF} in the non-dissipative situation. Additionally, the condition $Y_{TF} = 0$ is useful for developing alternative models and can also give insight about the impact of charge on the physical properties of astrophysical objects.

Under the charge distribution, the results that are achieved here are

- Due to the fact that any causal transport equation is formed on the notion that the fluid is close to thermal equilibrium, i.e., $q << |\mu|$. Hence, Equation (27) indicates that μ is inevitably negative with the restriction that $H' > 0$ along with $\frac{s^2}{2H} - \int_0^r \frac{ss'}{H} dr > 0$ while keeping in mind that E is a regular function and the mass of the fluid distribution cannot be negative.
- The charged fluid is unable to occupy the cental region. This indicates the presence of cavity over there.
- The temperature gradient is required to maintain the system in thermal equilibrium [67]. Equation (45) shows that if $a < 0$ then the gravitational force will be repulsive in nature and the temperature gradient will be positive in this case to keep the system in thermal equilibrium.
- The quasi-homologous condition implies the disappearance of Y_{TF} in the non-dissipative situation.
- The condition $Y_{TF} = 0$ is useful for developing alternative models and can give insight about the impact of charge on the physical properties of astrophysical objects.
- The fluid become less dense and the tangential pressure decreases in the presence of charge as one can verify it from Equations (91), (94), (99), (102), (107) and (110) .
- The system become massive and more radial pressure exerts on it in the presence of charge as one can witness it from Equations (93), (95), (101), (103), (109) and (111).
- The temperature and the heat flux remain unchanged in the presence of charge as one can observe it from Equations (92), (96), (100), (104), (108) and (112).
- All the results obtained in this manuscript will be reduced in GR on substituting $s = 0$.

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