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Cubic–Quartic Optical Soliton Perturbation for Fokas–Lenells Equation with Power Law by Semi-Inverse Variation

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Abstract: The current work addresses cubic–quartic solitons to compensate for the low count of the chromatic dispersion that is one of the major hindrances of soliton transmission through optical fibers. Thus, the present paper handles the cubic–quartic version of the perturbed Fokas–Lenells equation that governs soliton communications across trans-oceanic and trans-continental distances. The model is also considered with the power-law form of nonlinear refractive index that is a sequel to the previously reported result. This is a tremendous advancement to the previously known result that was only with the Kerr-law form of nonlinear refractive index. The present paper mainly contributes by generalizing the Kerr law of nonlinearity to the power law of nonlinearity. The prior results therefore fall back as a special case to the results of this paper. The semi-inverse variational principle yields a bright 1-soliton solution that is imperative for the telecommunication engineers to carry out experimental investigation before the rubber meets the road. Hamiltonian perturbation terms are included that come with maximum intensity. The soliton amplitude–width relation is retrievable from a polynomial equation with arbitrary degree. The parameter constraints are also identified for the soliton to exist.

Keywords: solitons; power law; Fokas–Lenells; semi-inverse; integrability; bright

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1. Introduction

The optical soliton is a treasure trove of engineering marvels, during modern times, in the field of telecommunications. It has its applications all across the board [1–20]. For high-bandwidth services that range from home-based PCs to large business and research firms, telecommunication industries worldwide are using light waves traveling through optical fibers as the dominant transmission mode. Other examples consist of services such as database queries and modern shopping, video-on-demand, online classes and meetings, video conferencing and other web-based courses. Therefore, it is imperative to achieve performance enhancement, at its best, with such a modern technological marvel. There are several issues that need to be addressed for soliton communications across trans-oceanic and trans-continental distances.

One of the major hindrances of soliton transmission through optical fibers is the low count of the chromatic dispersion (CD). To remedy this issue, several measures have been taken and these include implementing fiber Bragg gratings, introducing additional dispersion terms and many more. One such approach was proposed by incorporating a nonlinear dispersion term that is now modeled with the Fokas–Lenells equation (FLE) [1–11]. Another approach is to completely eliminate CD and replenish it with the collective count of third-order dispersion (3OD) and fourth-order dispersion (4OD). This combination of 3OD and 4OD constitutes the CQ–FLE. Our focus is to analyze this CQ–FLE along with its perturbation terms. The self-phase modulation (SPM) effect is first generalized from Kerr law to power law. Hamiltonian perturbative effects are considered with maximum allowable intensity. Thus, the full-blown mathematical model is the perturbed CQ–FLE with full nonlinearity. The paper recovers a bright 1-soliton to the newly formulated mathematical model by the application of the semi-inverse variational principle (SVP). The parameter constraints naturally emerge from the mathematical analysis of the model. The velocity of the soliton, in presence of the perturbation terms, also emerges from the analysis. A surface plot of the bright 1-soliton solution is also included to supplement the analytical result for the soliton solution. The details of the derivation are addressed in the rest of the present study, and the constraint relations that naturally emerge from the mathematical analysis are also enumerated and exhibited.

To step back in time, the cubic–quartic (CQ) version of the Fokas–Lenells equation was first integrated in 2021 by Zayed et al. [10]. Prior to that, FLE with CD was integrated for chirped solitons by Triki et al. [6]. The Lie symmetry analysis has also been applied to study this model but in birefringent fibers earlier this year by Zhang et al. [11]. The long-time asymptotic analysis was also carried out earlier this year by Cheng et al. [2]. The rogue wave solutions to FLE were also studied in 2012 by He et al. [3]. Subsequently Kudryashov applied the first integral and reported the general solution of the model in 2019 [4]. Recently, Lashkin studied the perturbed FLE by inverse scattering transform in 2021 [5]. Prior to that in 2019, Ye et al. reported rogue wave solutions to the coupled FLE with the usage of the Darboux transform [7]. Earlier this year, Yildirim et al. reported soliton solutions to CQ–FLE with maximum intensity using a different approach. Also earlier this year, Yue et al. studied the generalized coupled FLE and located its conservation laws and modulational instability analysis [9]. The prequel to this paper by Biswas et al. studied the Kerr law of nonlinearity in 2021 [1].

Governing Model

The perturbed mathematical model with the power-law form of SPM is introduced as below:

$$iq_t + iaq_{xxx} + bq_{xxxx} + |q|^{2n}(cq + idq_x) = i\left[\lambda\left(|q|^{2n}q\right)_x + \mu\left(|q|^{2n}\right)_x q\right]. \tag{1}$$

Here, a, b, c, d, μ and λ stand for constraint parameters, whereas $q(x,t)$ comes from the complex-valued function. The coefficient a is from 3OD, whilst $q(x,t)$ stems from the wave profile and the coefficient b is from 4OD. The temporal evolution comes from the first term, where $i^2 = -1$, whereas x depicts the distance variable and d is the nonlinear dispersion. t denotes the time variables, while the parameter c represents the power-law form of SPM. λ accounts for the self-steepening effect, while μ gives the self-frequency shift. The power-law parameter n is from the maximum allowable intensity.

The unperturbed version of (1), where $\lambda = \mu = 0$, admits the 1-soliton solution [8]

$$q(x,t) = A\mathrm{sech}^{\frac{2}{n}}[B(x - vt)]e^{i(-\kappa x + \omega t + \theta_0)}. \tag{2}$$

Here A, B, n, θ_0, ω and κ are real-valued parameters. κ is the soliton frequency, while ω is the soliton wave number and A is the soliton amplitude. θ_0 is the phase constant, whereas v is the speed and B is the soliton inverse width.

The paper is organized as thus. Section 2 conducts the mathematical analysis and reduces the governing partial differential equation to an ordinary differential equation by the aid of the traveling wave hypothesis. The parameter constraints naturally emerge from the analysis. Section 3 introduces the SVP and applies it to the model. The soliton solution is subsequently retrieved along with additional parameter constraints. Section 4 discusses the results and gives some concluding remarks.

2. Mathematical Set-up

Equation (1) satisfies the fundamental solution

$$q(x,t) = g(s)e^{i(-\kappa x+\omega t+\theta_0)} = g(x-vt)e^{i(-\kappa x+\omega t+\theta_0)}. \tag{3}$$

Substituting (3) into (1) yields the imaginary part:

$$(a-4b\kappa)g'' - \left(v+2a\kappa^2-4b\kappa^3\right)g' + [d-(2n+1)\lambda-2n\mu]g^{2n}g' = 0, \tag{4}$$

which leads to the conclusion

$$a = 4b\kappa, \tag{5}$$

$$d = (2n+1)\lambda + 2n\mu, \tag{6}$$

and

$$v = -3a\kappa^2 + 4b\kappa^3. \tag{7}$$

The relations (5) and (6) are the parameter constraints, while the speed of the solitons is given by (7). Therefore, one can observe that the coefficients of perturbation terms are not free parameters. Such a connectivity between the parameters is essential for the integrability of the model which is the focus of the current paper. Without such parameter constraints, the model would not be rendered integrable even numerically. Therefore, such constraints are mandated for soliton solutions to exist: a necessity in nonlinear optics for telecommunication purposes.

The real part, after integrating once, with the integration constant K gives:

$$\begin{aligned} b(g'')^2 - 3\kappa(a-2b\kappa)(g')^2 + \left(\omega + a\kappa^3 - b\kappa^4\right)g^2 \\ -\frac{1}{n+1}\{c+(d-\lambda)\kappa\}g^{2n+2} = K. \end{aligned} \tag{8}$$

Equation (8), as it stands, is not integrable by the usage of the traveling wave hypothesis or by any other algorithms known thus far. Therefore, one must resort to a specific principle, known as SVP, to recover a bright 1-soliton solution to the model of study although this principle would not lead to an exact 1-soliton solution.

3. Analysis of SVP

The stationary integral J is now presented as below:

$$J = \int_{-\infty}^{\infty}\left[\begin{array}{c} b(g'')^2 - 3\kappa(a-2b\kappa)(g')^2 + \left(\omega + a\kappa^3 - b\kappa^4\right)g^2 \\ -\frac{1}{n+1}\{c+(d-\lambda)\kappa\}g^{2n+2} \end{array}\right]dx. \tag{9}$$

The soliton amplitude is recovered as

$$\frac{\partial J}{\partial A} = 0, \tag{10}$$

while the soliton inverse width is extracted as

$$\frac{\partial J}{\partial B} = 0. \tag{11}$$

SVP states that the soliton solution of the perturbed model will be the same as that of its unperturbed version. However, the amplitude and the width of the perturbed soliton can be recovered after uncoupling the set (10) and (11) [16–18].

Therefore, substituting the 1–soliton solution (2) into (9) yields:

$$J = \left[\frac{16b(2n+3)A^2B^3}{(3n+4)(n+4)n^2} - \frac{12\kappa(a-2b\kappa)A^2B}{(n+4)n} + \left(a\kappa^3 + \omega - b\kappa^4\right)\frac{A^2}{B} + \frac{8\{c+(d-\lambda)\kappa\}(n+2)}{(1+n)(4+3n)(4+n)}\frac{A^{2n+2}}{B} \right] \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{2}{n}\right)}{\Gamma\left(\frac{2}{n}+\frac{1}{2}\right)}. \quad (12)$$

Therefore, Equations (10) and (11) transform to:

$$\frac{16b(2n+3)B^4}{(4+3n)(4+n)n^2} - \frac{12\kappa(a-2b\kappa)B^2}{n(4+n)} + \left(\omega + a\kappa^3 - b\kappa^4\right) + \frac{8(n+2)\{c+(d-\lambda)\kappa\}A^{2n}}{(4+3n)(4+n)} = 0, \quad (13)$$

and

$$\frac{48b(2n+3)B^4}{(4+3n)(4+n)n^2} - \frac{12\kappa(a-2b\kappa)B^2}{n(4+n)} - \left(\omega + a\kappa^3 - b\kappa^4\right) - \frac{8(n+2)\{c+(d-\lambda)\kappa\}A^{2n}}{(4+3n)(4+n)(n+1)} = 0. \quad (14)$$

Uncoupling (13) and (14) leads to

$$16b(2n+3)B^4 - 12n(n+2)\kappa(a-2b\kappa)B^2 - n^2(n+2)(n+4)\left(\omega + a\kappa^3 - b\kappa^4\right) = 0. \quad (15)$$

Solving Equation (15) gives:

$$B = \left[\frac{3n(n+2)\kappa(a-2b\kappa) + \sqrt{9(a-2b\kappa)^2n^2\kappa^2(2+n)^2 + 4n^2(2+n)(3+2n)(n+4)\left(\omega + a\kappa^3 - b\kappa^4\right)b}}{8b(2n+3)^2} \right]^{\frac{1}{2}}. \quad (16)$$

This is subject to the constraints that are given by:

$$9(n+2)\kappa^2(a-2b\kappa)^2 + 4(n+4)(2n+3)b\left(\omega + a\kappa^3 - b\kappa^4\right) \geq 0, \quad (17)$$

and

$$b\left[3(n+2)\kappa(a-2b\kappa) + \sqrt{9(n+2)^2\kappa^2(a-2b\kappa)^2 + 4(n+2)(2n+3)(n+4)b\left(\omega + a\kappa^3 - b\kappa^4\right)}\right] > 0. \quad (18)$$

In order to recover the amplitude (A) of the soliton in terms of its width (B), again, uncoupling (13) and (14) gives the relation

$$A = \left[\frac{(n+1)B^2\{3n(3n+4)(a-2b\kappa) - 8b(2n+3)B^2\}}{8n^3(n+2)\{c+(d-\lambda)\kappa\}} \right]^{\frac{1}{2n}}, \quad (19)$$

which introduces the constraint condition

$$\{c+(d-\lambda)\kappa\}\left\{3n(3n+4)(a-2b\kappa) - 8b(2n+3)B^2\right\} > 0. \quad (20)$$

Hence, finally, the bright 1-soliton of the perturbed mathematical model with the power-law form of SPM (1) is still represented by (2), provided the parameter constraints (5), (6), (17) and (18) hold true. Alternatively, the soliton amplitude can be directly written in terms of its inverse width as given by (19) together with the parameter constraint as in (20). The velocity of the soliton is depicted in (7).

The following Figures 1 and 2, obtained with the aid of Mathematica software (www.wolfram.com/solutions/industry/software-engineering/, accessed on 20 June, 2022), show the surface plots of a bright 1-soliton (2) to the mathematical model Equation (1) for $\kappa = 1$, $b = 1$, $\omega = 1$, $c = 1$ and $d = 1$. The power-law parameter n is addressed within the stability region $0 < n < 1$. The effects of the power-law parameter n on bright 1-soliton behavior are indicated in Figures 1 and 2.

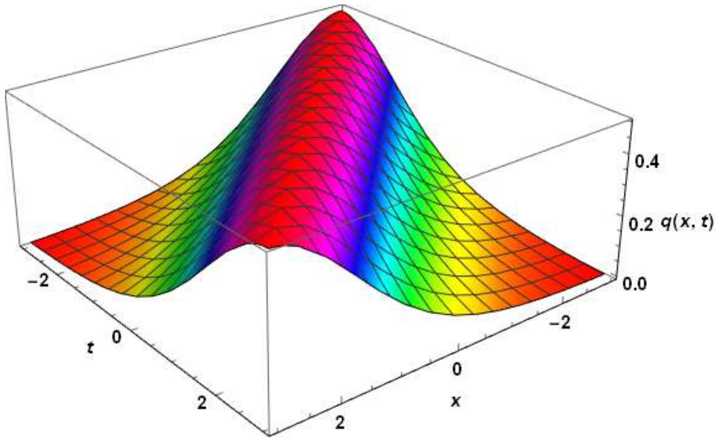

Figure 1. The surface plot of a bright 1-soliton (2) for $n = \frac{1}{2}$.

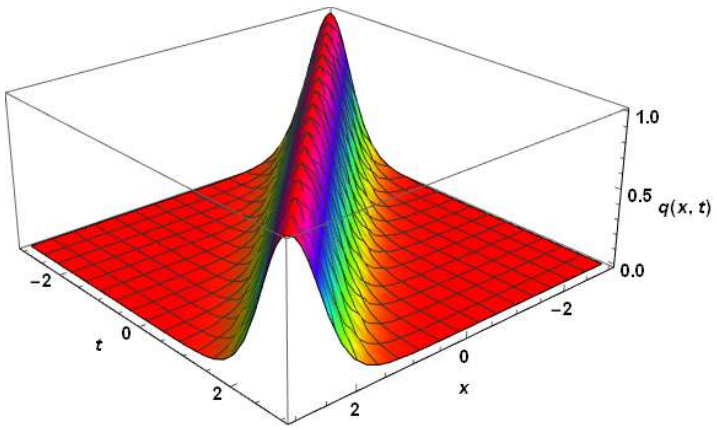

Figure 2. The surface plot of a bright 1-soliton (2) for $n = \frac{1}{6}$.

4. Discussion and Conclusions

The current study obtains the bright 1-soliton of the perturbed CQ–FLE through maximum allowable intensity. This is a tremendous advancement to the previously known result that was only with the Kerr-law form of SPM [1]. The current paper, a sequel to the previously reported result, mainly contributes by generalizing the Kerr law of nonlinearity to the power law of nonlinearity. The prior results therefore fall back as a special case to the results of this paper. Thus, the primary results of this work is an improvement to the generalized version of the result that previously appeared with the Kerr law of nonlinear refractive index in 2021 [1]. The derived results from the generalized version of the previously reported results are now displayed. This would lead to a better perspective than the previously reported result during 2021 that was confined to the Kerr law. While the modeling is not quite advanced, the generalization from the Kerr law to the power law and the replacement of CD with CQ dispersion to replenish the low count of dispersion gives a better perspective to the model for addressing the soliton dynamics.

It must be noted that the nonlinear ODE, recovered from the real part of the model, is not integrable by the traveling hypothesis or any other known algorithms; SVP is therefore to the rescue. The retrieval of a bright 1-soliton solution to such an electromagnetic model is imperative for telecommunication engineers to carry out experimental investigation before the rubber meets the road.

There are several pros and cons to the adopted SVP as a tool to perform the integration of the model studied in this paper. One advantage is that SVP can yield an analytical bright soliton solution when all of the known integration schemes fail to retrieve an exact soliton solution [21–28]. A sure disadvantage is that SVP fails to recover soliton radiation which can only be recovered from the inverse scattering transform, provided the model passes the Painleve test of integrability. Another notable disadvantage of SVP is its failure to recover dark or singular soliton solutions or even multiple soliton solutions. Thus, no integration scheme outperforms any other!

The advantage of the present methodology is that the model recovers a CQ bright 1-soliton solution even though the model may not be integrable by any known integration algorithms. Thus, an analytical solution would do the job just as an exact 1-soliton solution would. The results of the current work are applicable to various applications of the model such as optical couplers, Bragg gratings, optical metamaterials and others. Later, this model can be handled further along to locate its conserved quantities and also to include stochastic effects: additive and/or multiplicative. The results of this paper thus form a strong footing to explore various additional avenues as indicated.

The recovered results are very important to conduct further analysis with CQ–FLE. These would include the computation of the collision–induced timing jitter, extension to birefringent fibers and dispersion-flattened fibers as well as in meta-materials, meta-surfaces, magneto-optic waveguides and optical couplers. Later, the topic of conservation laws for FLE, along with the quasi-monochromatic dynamics of perturbed solitons, can be addressed for the Kerr law as well as the power law of nonlinear refractive indexes as a separate marathon project. The extended version of the model can be taken up, in this context, to include additional perturbation terms such as Raman scattering, saturable amplifiers, nonlinear dissipation, quasi-solitons and several others during the study of soliton perturbation theory. These results are being studied and will be subsequently aligned according to the previously reported results and sequentially disseminated elsewhere [12–15]. This would mean that in future the Lakshmanan–Porsezian–Daniel model will be studied with a different form of SPM [12]. The complex Ginzburg–Landau equation can later seek highly dispersive optical solitons for additional forms of SPM [13,14]. Later additional forms of the nonlinear evolution equation can also be addressed that appear in additional areas of physics apart from optics [15,19,20].

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