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Essay

On Extra Top Yukawa Couplings of a Second Higgs Doublet

George Wei-Shu Hou

Department of Physics, National Taiwan University, Taipei 10617, Taiwan; wshou@phys.ntu.edu.tw

Abstract: A very likely New Physics in plain sight, but that the community does not see, is a second Higgs doublet that has a second set of Yukawa couplings. The extra *tt* and *tc* couplings can each drive baryogenesis, with O(1) Higgs quartic couplings providing a first order electroweak phase transition. A natural cancellation mechanism can tame electron EDM, if extra *ee*, *tt* couplings "know" the known fermion mass and mixing hierarchies. Colliding *c* with *g* produces *tH/A*, bH^+ via extra *tc* coupling, and together with extra *tt* coupling give *ttc*(bar), *ttt*(bar), and *btb*(bar) signatures at the LHC. Extra *tu* coupling can also be probed, but more definitive would be the B to $\mu\nu$ and $\tau\nu$ decay rate ratio. Myriad extra Yukawa couplings can make an impact on flavor physics and CP violation, including on muon g-2. The opening to the prelude of a new physics Higgs and flavor era may unfold before us.

Keywords: second Higgs doublet; Yukawa couplings; baryogenesis; Higgs quartic couplings; phase transition; electric dipole moment (EDM); mass and mixing hierarchy; LHC; flavor physics; *CP* violation; muon g-2

1. Introduction: Historical Development of Extra Top Yukawa Couplings

Mass is central to physics, energy alone is not enough. Without mass, an electron will not bind to a nucleus to form an atom, nor planets to its sun to form solar systems. For elementary particles with no known structure, mass is dynamically generated by coupling to the Higgs field. For the SU(2) gauge theory of the weak interactions of the standard model (SM), Weinberg introduced [1] a complex Higgs doublet to spontaneously generate the vacuum expectation value $v \cong 246$ GeV, giving mass to vector bosons via the Higgs mechanism [2–4]: $2m_V = gv$, with g the gauge coupling. As left-handed fermions come in weak doublets while right-handed fermions in singlets, Weinberg introduced [1] the Yukawa interaction Lagrangian by balancing the gauge charge of the left–right fermion bilinear with the Higgs doublet. Thus, charged fermion mass generation is analogous to the Higgs mechanism: $\sqrt{2}m_f = \lambda_f v$, with λ_f the Yukawa coupling. This linear relation of Higgs couplings vs mass was spectacularly demonstrated recently [5] by ATLAS and CMS, from $\lambda_t \cong 1$ for top, down to λ_μ for the muon [6] that is over a thousand times weaker! Yukawa couplings are now measured dynamically.

With Yukawa couplings now "real", they are also "complex". The chiral nature of fermions in the above discussion means that the Yukawa couplings involved in fermion mass generation are complex. Kobayashi and Maskawa demonstrated [7] that it takes three generations of u- and d-type quarks to yield a single *CP* violating (CPV) phase; all laboratory-verified CPV effects so far [5] can be accounted for by this unique phase.

With $\lambda_t \cong 1$, the strongest coupling we know, we turn to the possibility of extra top Yukawa couplings. To explain the absence of antimatter in the Universe is one of the greatest challenges facing particle physics, which provides the biggest motivation for exploring extra top Yukawa couplings. The fourth generation (4G) naturally comes to mind [8], which, besides $\lambda_{t'}$, $\lambda_{b'}$ themselves, there are their mixings with the top quark. Indeed, around 2010 there was some hope [9] that 4G may provide enough CPV for electroweak baryogenesis (EWBG [10]), although a sufficient first order phase transition seemed lacking. However, except discovering [5] the expected SM-like $h(125)$, no new physics has emerged at the LHC, and 4G is also absent. Furthermore, $h(125)$ production through gluon–gluon fusion

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is consistent with only the top quark in the triangle loop, rather than having t, t', b' all contributing [8], which would have given an order of magnitude larger cross section.

Next would be vector-like quarks (VLQ), where left- and right-handed quarks have the same gauge charge, but can mix with SM quarks. For example, when the top remained unseen in the early 1990s, a left–right singlet Q was invoked [11] to hide the top via $t \to cH$ (H here is SM Higgs boson) by t–Q mixing, i.e., extra top Yukawa couplings. It was refuted, however, that t–Q mixing cannot generate sufficiently large c–t mixing [12], as it would still be suppressed by m_c/m_Q. There is further the issue of m_Q scale: be it singlet or doublet, the gauge invariant m_Q is arbitrary, making its plausibility dubious, as we have never seen one [5]. Since the VLQ representations do not follow the SM pattern, they are seen as more exotic than 4G. However, there were still some interest in VLQs, e.g., in the LEP2 era [13].

With $h(125)$ discovery at the LHC, however, at the same time that it spelled the demise of 4G [14,15], VLQ became in vogue. For example, in the context of "composite Higgs" models [16–20], to protect the lightness of $h(125)$, one "engineers" a cancellation of top-loop corrections by additional VLQ loops. This bears analogy to SUSY where there are "scalar top partners" (stop), so these VLQs are also called vector-like "top partners". The cancellation mechanism points m_Q to the TeV scale. Before long, one had handbooks and hunter's guides [21,22]. However, a decade after $h(125)$ discovery, experimental bounds from various singlet/doublet T and B VLQ searches have reached the TeV scale, but they still have not been seen [5]. Are VLQ top partners going the way of SUSY particles? The pursuit certainly continues.

In this article, we comment on another type of extra top Yukawa couplings, those that ought to be present if a second Higgs doublet (2HDM) exists [23] in *Nature*. Like 4G, the existence of a second Higgs doublet is a conservative extension of SM. The "traditional" approach, however, is to enforce the Natural Flavor Conservation (NFC) condition of Glashow and Weinberg [24], that only one doublet can generate fermion masses, which forbids the extra Yukawa couplings altogether by fiat! Though clearly ad hoc, the NFC condition is usually implemented via a Z_2 symmetry, hence come in two types [23]: in Model I, both u- and d-type quarks receive mass from the same doublet; for Model II, they receive mass from separate doublets. The latter arises automatically in SUSY, making 2HDM II the most popular. Dropping NFC, however, there is a third type, 2HDM III [25], one that possesses extra Yukawa couplings. Such couplings were used to stress $t \to ch$ [25] as a decay mode to watch, where h is some Higgs boson lighter than top.

So how does one address Glashow's concern of flavor changing neutral Higgs (FCNH) couplings? As pointed out by Cheng and Sher [26], by capitalizing on fermion mass and especially mixing hierarchies that emerged after the NFC paper—and never predicted by SM— *Nature* has spoken her mind! It was further clarified [25] that one need not assume a specific mass-mixing "Ansatz", as in Ref. [26], but the known mass-mixing hierarchies may suffice to hide the effect of the second doublet. Just before the top finally emerged, it was noted that $t \to ch$ for some light h boson could actually still hide the top below M_W [27], but *Nature* soon unveiled the surprisingly heavy top at Fermilab.

Fast forward to LHC, the lightness of $h(125)$ itself indeed immediately prompted [25] the experimental pursuit of $t \to ch$, first done by ATLAS [28], and the clarification [29] that, aside from the extra FCNH coupling ρ_{tc}, there is a mixing factor $\cos(\beta - \alpha)$ in SUSY notation, and later called plainly $\cos\gamma$ in 2HDM III. Surprisingly, the emergent "alignment" phenomenon [30], that h so resembles the SM Higgs boson, helped clarify the situation further: the mixing angle $c_\gamma \equiv \cos\gamma$ of h with the exotic CP-even Higgs, H, is small, which can explain the absence [5] of $t \to ch$ so far. The emerging picture is that the mass-giving doublet Φ, to which h belongs, does not mix much with the exotic second doublet Φ' that is not involved in generating v. The absence so far in searches [5] for the H, pseudoscalar A and H^+ bosons from Φ' means the Φ and Φ' doublets are somewhat separate in mass scale. We shall argue that one need not send the exotic Higgs bosons to multi-TeV, as in the minds of some SUSY advocates, but they should be sub-TeV in mass for reasons [31] we shall discuss. Such a mass range should be fully explored at the LHC in any case.

Reflecting our times, with no new physics found, it is common to assume that the scale Λ of beyond SM (BSM) physics is high, hence a popular approach is [32] effective field theory (EFT), i.e., expanding in powers of $1/\Lambda$ with higher dimension operators. We do not take this approach, however, as we do not think dimension-4 operators are exhausted yet.

Thus, this article proceeds as follows. We promote 2HDM III and call it the general 2HDM (g2HDM), where an extra Higgs doublet exists but without NFC, and let *Nature* speak her design. Descending down this "Road Not Taken" by most, we present in Section 2 two sets of dimension-4 dynamical operators, namely the extra Higgs quartic couplings and the extra Yukawa couplings. In Section 3, we assert that the extra top Yukawa couplings ρ_{tt} and ρ_{tc} can separately drive EWBG, while the current bound on the electron electric dipole moment (eEDM) can be satisfied with finite ρ_{ee}, where intriguingly its ratio with ρ_{tt} must reflect the ratio of λ_e/λ_t with specific phase correlation. That is, the two types of Yukawa matrices "know" each other. For the sake of EWBG, we argue that the extra Higgs bosons ought to be sub-TeV in mass, hence ripe for the LHC to explore. In Section 4, we recount the main production processes at the LHC, all with ρ_{tc} as crux, but also comment on accessing ρ_{tu}. Turning to flavor, we explore in Section 5 the special ratio of $\mathcal{B}(B \to \mu\nu)/\mathcal{B}(B \to \tau\nu)$, that if it is found to deviate from 0.0045 by Belle II, it not only would rule out both SM and 2HDM II, but point to $\rho_{tu} \neq 0$ in g2HDM, which would provide impetus for collider study. We then generalize and make clear that the pursuit of extra Higgs bosons with extra Yukawa couplings (not just top-related) is an experimental question, altogether involving over 60 new parameters. A new era on Higgs and flavor could unfold before us, and we offer the prospects and conclusions in Section 6.

This article is written in essay style, giving a narrative on the physics but avoiding formulas as much as possible. A companion article with considerably more detail on the new Higgs/flavor era can be found in Ref. [33].

2. Two Sets of Dimension-4 Dynamical Operators

The general 2HDM, or g2HDM, has an extra Higgs doublet that possesses extra Yukawa couplings, i.e., without the NFC condition imposed. It thus provides two sets of dimension-4 operators to be scrutinized at the LHC and by flavor experiments: extra Higgs quartics, and extra Yukawa couplings. This is in contrast with the present EFT trend [32], that there is a high scale Λ for BSM operators, or else one has particles with weaker than SM couplings. Actually, g2HDM does involve weaker couplings, in general, but is not part of the dark sector, with coupling strengths to be determined by experiment.

Generalizing the familiar Higgs potential of SM, $V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4$ with $\mu^2 < 0$, the Higgs potential of g2HDM, i.e., with no Z_2 symmetry imposed, is [31,34]

$$\begin{aligned} V(\Phi, \Phi') = {} & \mu_{11}^2|\Phi|^2 + \mu_{22}^2|\Phi'|^2 - \left(\mu_{12}^2\Phi^\dagger\Phi' + \text{h.c.}\right) + \frac{1}{2}\eta_1|\Phi|^4 + \frac{1}{2}\eta_2|\Phi'|^4 \\ & + \eta_3|\Phi|^2|\Phi'|^2 + \eta_4|\Phi^\dagger\Phi'|^2 + \left[\frac{1}{2}\eta_5(\Phi^\dagger\Phi')^2 + \left(\eta_6|\Phi|^2 + \eta_7|\Phi'|^2\right)\Phi^\dagger\Phi' + \text{h.c.}\right], \end{aligned} \tag{1}$$

in notation of Ref. [31], with Φ the mass-giving doublet that is responsible for v, and Φ' the exotic doublet with $\langle\Phi'\rangle = 0$, i.e., $\mu_{22}^2 > 0$. This natural separation is called the Higgs basis [35]. With $\mu_{11}^2 < 0$, v is generated in the usual way, except for a slight change in convention [31]. A second minimization condition $\mu_{12}^2 = \eta_6 v^2/2$ eliminates μ_{12}^2, and η_6 is the sole parameter for Φ-Φ' mixing. Note that η_6, η_7 would be absent under usual Z_2 to impose NFC. Thus, g2HDM has two inertial mass parameters $\mu_{11}^2 < 0$ (transferred to v) and $\mu_{22}^2 > 0$, plus seven quartics η_i. It is more intuitive than 2HDM I and II, where both μ_{11}^2, $\mu_{22}^2 < 0$ (transferred to v_1, v_2), with μ_{12}^2 playing the dual role of inertial mass and Φ-Φ' mixing, while η_6 and η_7 are absent. In SUSY, μ_{12}^2 is now often considered at several TeV^2.

The extra Yukawa couplings for charged fermions are [34,36,37]

$$
-\frac{1}{\sqrt{2}}\sum_{f=u,d,\ell}\bar{f}_i\Big[\big(-\lambda^f_i\delta_{ij}\,s_\gamma+\rho^f_{ij}\,c_\gamma\big)\,h+\big(\lambda^f_i\delta_{ij}\,c_\gamma+\rho^f_{ij}\,s_\gamma\big)\,H-i\,\mathrm{sgn}(Q_f)\,\rho^f_{ij}\,A\Big]R\,f_j
$$
$$
-\bar{u}_i\Big[(V\rho^d)_{ij}\,R-(\rho^{u\dagger}V)_{ij}\,L\Big]d_j\,H^+-\bar{\nu}_i\,\rho^\ell_{ij}\,R\,\ell_j\,H^+ +\mathrm{h.c.}, \tag{2}
$$

in notation of Ref. [37], where i and j are summed over generations, $L, R = (1 \mp \gamma_5)/2$ are projections, and V the Cabibbo–Kobayashi–Maskawa matrix; the lepton matrix is taken as unity due to vanishing neutrino mass. The h–H mixing angle c_γ is known to be small [30], but the value in g2HDM would be harder to extract than 2HDM II. The smallness of c_γ means that h largely arises from Φ, while Φ' gives exotic Higgs bosons H, A, and H^+. As stressed in the Introduction, the fermion mass hierarchies $m_1^2 \ll m_2^2 \ll m_3^2$ across generations plus $m_b^2 \ll m_t^2$, together with the mixing hierarchy $|V_{ub}|^2 \ll |V_{cb}|^2 \ll |V_{us}|^2 \ll |V_{tb}|^2 \cong 1$, if reflected also in the ρ^f_{ij} Yukawa matrices, may be *Nature*'s mechanism in hiding the exotic bosons so far. None of these relations were predicted, and constitute together the flavor enigma, with the non-flavor $c_\gamma^2 \ll 1$ thrown in as bonus. For example, the bound on $t \to ch$ is now below 10^{-3}, but this can be largely absorbed by c_γ^2, rather than small ρ_{tc}^2.

Equation (1) has 8 parameters (including c_γ) besides v, and Equation (2) has $9 \times 3 \times 2 = 54$ parameters, but the latter extend from similar SM parameters, which are already plenty. Linking to CPV in the Heavens (baryon asymmetry of the Universe) and on Earth (electron EDM constraint), one finds profound implications, as discussed in the next section:

- CPV for EWBG calls for $\mathcal{O}(1)$ extra top Yukawa couplings, while first order phase transition calls for $\mathcal{O}(1)$ Higgs quartics. The latter, in turn, suggests sub-TeV exotic Higgs masses, as we shall see.
- For the electron EDM constraint, the diagonal extra electron Yukawa coupling ρ_{ee} needs to correlate with extra top Yukawa coupling ρ_{tt} that echoes the known Yukawa coupling pattern.

3. Driving EWBG and Facing eEDM: Extra tt, tc and ee Couplings

A main reason to pursue extra Yukawa couplings is for *CP* violation: Kobayashi–Maskawa phase [7] can account for all CPV measured so far on Earth, but the CPV needed for baryogenesis is tremendously larger. As a Belle member, we keenly recall the time when the detector was under construction, there was the sense that, even if we demonstrated the Kobayashi–Maskawa phase, the baryon asymmetry of the Universe was far out of reach. That agony was only uplifted when the Belle direct CPV result [38] pointed to a possible fourth generation solution for baryogenesis [9]. However, with 4G now out of favor, as there are no clear hints of new physics whatsoever, the extra Yukawa couplings for each type of charged fermions in g2HDM should be kept and scrutinized experimentally.

In this vein, it was found [39] that the product

$$
\lambda_t\,\mathrm{Im}\,\rho_{tt}, \tag{3}
$$

can robustly account for EWBG: with $\lambda_t \simeq 1$ measured, a best guess for $|\rho_{tt}|$ is also $\mathcal{O}(1)$ and applicable to the imaginary part, making the product—interfering Φ and Φ'—rather transparent and more convincing than the previous 4G work [9]. Interestingly, if ρ_{tt} turns out accidentally small, then ρ_{tc} could kick in as a back-up option. However, it would need to be [39] close to 1 with near maximal CPV phase, hence is less robust than via ρ_{tt}, where strength at 0.1 is more than enough. Note that ρ_{bb} has also been employed [40,41] for EWBG, but whether the estimate of scattering off the expanding bubble wall is trustworthy enough has been questioned [42] for such light quarks. Furthermore, the strength of ρ_{bb}, by analogy with λ_b, is likely much weaker, and strongly constrained [36] by $b \to s\gamma$.

One prerequisite (of Sakharov) for EWBG [10] is a first order electroweak phase transition, which is possible if [43] quartic couplings η_i are also $\mathcal{O}(1)$. It was argued [31] that this implies H, A, and H^+ are sub-TeV in mass, or else perturbation theory starts to fail for exotic Higgs scatterings, which can be tested in principle at the LHC. Although it is not in itself a failure, one loses control of predictions. Similarly, μ_{22}^2/v^2 should [31] also be $\mathcal{O}(1)$, otherwise a large inertial mass would damp the needed scattering off the bubble wall of our very early Universe, hence quench EWBG.

Worthy of mention [31] is the η_1 coupling, the analog of $\lambda\Phi^4$ of SM. It could be only slightly less than 1 in strength, with $\mathcal{O}(1)$ η_6 coupling (h–H mixing) helping to push m_h down to the observed value by level repulsion. Thus, one may discover considerably larger hh production than SM expectation and harbinger [44] first order phase transition.

It is therefore interesting that, for the sake of EWBG, the exotic H, A, and H^+ bosons would be sub-TeV in mass, and the dimension-4 operators of Equations (1) and (2) are just right for the LHC to probe, which we shall turn to in the next section. However, before that, one needs to face the challenge of low energy precision measurements such as electron EDM.

Although $\lambda_t \operatorname{Im}\rho_{tt}$ being $\mathcal{O}(1)$ can achieve the lofty goal of baryogenesis, it does "expose" one to the current frontier of eEDM, d_e. The ACME experiment already pushed the frontier in 2014 [45], so in our EWBG work, we made [39] a simplified estimate by turning off ρ_{ee}, the extra diagonal electron Yukawa coupling. The point was to make a projection for the ACME upgrade to check. To our surprise, the ACME2018 update excluded [46] our "prediction" altogether, and we were forced to put ρ_{ee} back for a more complete study. However, the study turned out fruitful [47]: we found a (still) simplified ansatz for a cancellation mechanism, if *Nature* follows the pattern that (r depends on loop functions)

$$\frac{\operatorname{Im}\rho_{ff}}{\operatorname{Im}\rho_{tt}} = r\,\frac{\lambda_f}{\lambda_t}, \quad \frac{\operatorname{Re}\rho_{ff}}{\operatorname{Re}\rho_{tt}} = -r\,\frac{\lambda_f}{\lambda_t}, \tag{4}$$

i.e., extra Yukawa couplings echo the known mass-mixing hierarchy pattern of SM, with particular phase correlation! Even values that are two orders of magnitude below [47] ACME2018 bound can be entertained. However, we think the current bound of $\sim 1 \times 10^{-29}\ e\,\mathrm{cm}$ [46] should be scrutinized carefully, hopefully by several experiments using different approaches. A discovery at, or not far below, this bound would be exciting.

Equation (4) confirms our implicit assumption that the ρ^f matrices of Equation (2) "know" the mass and mixing hierarchies that *Nature* has revealed through SM Yukawa couplings long ago. We remark that the ρ_{tc} mechanism for baryogenesis evades eEDM constraint altogether [47], but the ρ_{tt} mechanism is more robust.

4. Crux of Production at Hadron Colliders: Extra tc Coupling

Gluon–gluon fusion production of H and A proceed via sizable ρ_{tt}, with H, A subsequently decaying to $t\bar{t}$, $t\bar{c}$ and also $\tau\mu$, $\tau\tau$. The $t\bar{t}$ final state is of interest, interfering with the enormous QCD production of $t\bar{t}$ pairs, with associated difficulty in analysis due to the rise-dip "signal" [48]. Experimental studies are ongoing; for example, a CMS study with partial Run 2 data reported some global excess [49] of 1.9σ at 400 GeV, with local excess higher. We await experimental progress. The $t\bar{c}$ final state is naively simple, but the catch is the unknown $t\bar{c}$ mass resolution. For H, A decaying to lepton pairs, if one takes $\rho_{\tau\mu}$, $\rho_{\tau\tau} \sim \lambda_\tau$, one suffers from branching ratio suppression as $t\bar{c}$, $t\bar{t}$ thresholds [50] turn on.

Thus, we advocate exotic Higgs production in association with a top or a bottom quark, with three main production processes: a gluon excites a charm quark, which emits H/A (H^+) via basically the ρ_{tc} coupling and turn into top (bottom). At the parton level, the processes are (we refer to Ref. [51] for a brief review):

1. $cg \to tH/tA \to tt\bar{c}$: Same-Sign Top plus c-jet [52];
2. $cg \to tH/tA \to tt\bar{t}$: Triple-Top [52];

3. $cg \to bH^+ \to bt\bar{b}$: Single-Top plus two b-jets [53].

4.1. Top-Associated Neutral Higgs Production

The production relies on the not well-constrained ρ_{tc} coupling. If ρ_{tt} vanishes or $m_{H,A} < 2m_t$, the H, A decay only to $t\bar{c}$ ($t\bar{t}c$ signature suffers high backgrounds). For finite ρ_{tt} and above $t\bar{t}$ threshold, the $tt\bar{t}$ final state would mutually dilute $tt\bar{c}$ cross-section.

Thus, top-associated H and A production lead to the two signatures of $tt\bar{c}$ and $tt\bar{t}$. For the former "Same-Sign-Top plus jet" production, the hard (charm) jet provides additional signature for discriminating against background. For the latter "Triple-Top" production, the cross-section can be several hundred times larger [51,52] than SM expectation [54], providing exquisite signature for detailed study at the high luminosity LHC (HL-LHC).

4.2. Bottom-Associated Charged Higgs Production

A simple but subtle extension of the above is $cg \to bH^+ \to bt\bar{b}$, i.e., the H^+ boson decays to $t\bar{b}$, while production is through H^+ emission from $c \to b$ transition. Here lies the subtlety: if one thinks in 2HDM II mindset, the process would be suppressed by V_{cb}. However, following Equation (2) carefully and working out the $V\rho^d$ or $\rho^{u\dagger}V$ products, one finds [53] that the $\bar{c}bH^+$ vertex is not V_{cb}-suppressed in g2HDM, hence receives V_{tb}/V_{cb} enhancement over 2HDM II! The $\bar{c}bH^+$ vertex is on equal footing with $\bar{t}bH^+$ vertex, which governs $H^+ \to t\bar{b}$ decay, that an undiluted ρ_{tc} coupling also governs $cg \to bH^+$ production in g2HDM. Thus, not only $cg \to bH^+$ is on equal footing as tH/tA associated production, it is favored in phase space by only requiring a light accompanying b quark instead of top.

One may think single top [5] production should provide stringent constraint. However, once again the two hard b and $\bar{b}$ jets are unusual for single top production, and discriminating against QCD-produced $b\bar{b}$ should be straightforward. Our collider study of the "Single Top plus two b-jets" signature did not spot [53] particularly worrisome backgrounds.

The $t\bar{t}t\bar{t}$ ($4t$) production at the LHC has a SM cross-section at 12 fb^{-1} level [54,55], larger than triple-top in SM and has been pursued [5] by both ATLAS and CMS. Lacking a similar pursuit for triple-top, we utilized [56] the $4t$ results to constrain ρ_{tc}–ρ_{tt} parameter space. However, we believe a genuine effort on triple-top should be pushed as the signature is quite different, and with possibility of discovering a much larger cross-section than $4t$, which may in turn provide extra backgrounds to some approaches of studying $4t$.

4.3. Enter the tu Coupling

So far we have considered ρ_{tt}, ρ_{tc} couplings but not ρ_{tu}. This is prudent, given that mass-mixing hierarchies imply ρ_{tu} should be considerably weaker. However, with two u quarks per proton, one should be careful. For instance, in the latest $t \to ch$ search by CMS [57] with $h \to \gamma\gamma$, one does incorporate single-top production off a valence u quark, and the constraint on $t \to uh$ is indeed more stringent than on $t \to ch$.

We have done a collider study [58] for $4t$ feed down to $ug \to tH/tA$, keeping ρ_{tu} but not ρ_{tc}. As in the $t \to qh$ study, we find stronger constraint on ρ_{tu} than for the analogous $cg \to tH/tA$ study, but certainly not as stringent as by $\sqrt{m_u/m_c}$, as implied by Cheng–Sher ansatz [26]. A dedicated search [58] comparing significance of positively versus negatively charged same-sign dilepton events could be fruitful, especially at the HL-LHC. Note that keeping both ρ_{tu} and ρ_{tc} in such a study may not be fruitful, since there is no good tool in separating c from u jets. This brings us to the next topic: a unique probe of ρ_{tu} through the ratio of $B \to \mu\nu$ and $B \to \tau\nu$ decay rates.

5. Turning to Flavor: Ratio of B to Muon+Neutrino vs. Tau+Neutrino

The subtlety of $cg \to bH^+$ being enhanced by V_{tb}/V_{cb} [53] in g2HDM compared with 2HDM II was originally uncovered through the study of $B \to \mu\nu$ vs $\tau\nu$ rate ratio. As we noted earlier in a flavor physics and CP violation (FPCP) review [59], the ratio

$$\frac{\mathcal{B}(B \to \mu\nu)}{\mathcal{B}(B \to \tau\nu)} = \frac{m_\mu^2(m_B^2 - m_\mu^2)}{m_\tau^2(m_B^2 - m_\tau^2)} \cong 0.0045, \tag{5}$$

holds for both SM and 2HDM II, because of the m_ℓ-independent correction factor for the latter [60] that we uncovered long ago. However, thanks to a referee remark, we stressed [59] that the SM ratio need not hold for g2HDM because more Yukawa couplings enter, hence provides a very interesting probe for Belle II. It turned out [61] rather interesting.

For $B \to \ell\nu$ decay, $\bar{u}$ annihilates the b quark, the decay constant f_B accounts for the disappearance of the meson, and the virtual H^- turns into $\ell\bar{\nu}_{\ell'}$ in g2HDM; this ℓ' index is the first subtlety [61]. As $\nu_{\ell'}$ goes undetected, according to Equation (2), the ℓ' flavor should be summed over, bringing in the extra lepton Yukawa couplings $\rho_{\ell\ell'}$ in g2HDM! That is, $\rho_{\mu\tau}$ for $B \to \mu\nu_\tau$, and $\rho_{\tau\mu}$ for $B \to \tau\nu_\mu$, besides the diagonal $\rho_{\mu\mu}$, $\rho_{\tau\tau}$, respectively.

The quark side is even more subtle [61], for both $V\rho^d$ and $\rho^{u\dagger}V$ in Equation (2). The former gives $\sum_i \rho_{ib}V_{ui} = \rho_{bb}V_{ub} + \rho_{sb}V_{us} + \rho_{db}V_{ud} \cong \rho_{bb}V_{ub}$, with ρ_{sb} and ρ_{db} constrained severely at tree level by B_s and B_d mixings. The smallness of $\rho_{bb} \sim \lambda_b$ and V_{ub} suppression means this term, the one that is operative for 2HDM II [60] by simple replacement, is small compared with $\rho^{u\dagger}V$ in g2HDM. This latter sum gives $\sum_i \rho^*_{iu}V_{ib} = \rho^*_{tu}V_{tb} + \rho^*_{cu}V_{cb} + \rho^*_{uu}V_{ub} \cong \rho^*_{tu}V_{tb}$, as ρ_{cu} is constrained by D^0 mixing and ρ_{uu} is suppressed by mass-mixing hierarchy, with both terms further CKM-suppressed. Since ρ_{tu} is poorly known experimentally while receiving $|V_{tb}/V_{ub}| \sim 300$ enhancement (!) compared to $\rho_{bb}V_{ub}$, this makes $B \to \ell\nu$ very interesting for Belle and Belle II [61]. Let us not analyze further, but note that $B \to \mu\nu$ can still deviate from SM expectation (but f_B-dependent), while $B \to \tau\nu$ is expected to be SM-like, which is the case observed [5], taking ratio cancels common factors, such as f_B.

The subtleties on the neutrino and/or the quark side were missed by earlier studies, such as Refs. [62,63], but could become definitive for g2HDM in the near future. Additionally, it was through clarifying the $V\rho^d$ and $\rho^{u\dagger}V$ products that led subsequently [53] to the $cg \to bH^+ \to bt\bar{b}$ process, where production is V_{tb}/V_{cb} enhanced in amplitude compared with 2HDM II thinking.

The current Belle result [64] for $\mathcal{B}(B \to \mu\nu) = (5.3 \pm 2.0 \pm 0.9) \times 10^{-7}$ is consistent with SM but with errors still sizable, so this ratio can be measured in the near future when Belle II accumulates just a factor of two or three times Belle data. If any deviation from 0.0045 is observed, it would not only be BSM, but beyond 2HDM II. We would then know $\rho_{tu} \neq 0$ in g2HDM, which should stimulate LHC studies. It furthermore points to finite $\rho_{\tau\mu}$ ($\rho_{\mu\tau}$), which can also be studied at the LHC via $gg \to H, A \to \tau\mu$ [50,65].

Put simply, Equation (5) probes the extra Yukawa coupling product $\rho_{tu}\rho_{\tau\mu}$ in g2HDM, receiving V_{tb}/V_{ub} enhancement in amplitude and thereby provides a sophisticated probe [61] of g2HDM through subtle H^+ effects. Any deviation from the SM value of 0.0045 would also rule against [59] 2HDM II that is automatic in SUSY.

We started with extra top Yukawa couplings because they are likely the strongest and least constrained, and can drive [39,47] baryogenesis. However, by now it should be clear that the issue of extra Yukawa couplings in 2HDM— if an extra scalar doublet actually exists—is an experimental question. The NFC condition of Glashow and Weinberg [24] is plainly ad hoc, dated, and ought to be retired, as it is best left for experiment to arbitrate.

We have barely touched upon the issue of flavor. A more detailed survey finds [66] that $\mu \to e\gamma$, $\mu N \to eN$ conversion and $\tau \to \mu\gamma$ are rather interesting via ρ_{tt} enhancement through the two-loop mechanism [67]. Furthermore, $\bar{e}\mu\bar{q}q$ operators may turn $\mu N \to eN$ into sophisticated probes of diagonal ρ_{qq} couplings by utilizing many different nuclei, and with help from nuclear physics in evaluating nuclear matrix elements. For rare B

decays [66], on one hand g2HDM effects quite often hide themselves and may be hard to probe, but especially $B \to \mu\nu$, $\tau\nu$ as we have advocated [61], and $B_{s,d} \to \mu\mu$ that are hotly pursued at the LHC [5], these modes with SM expectations can probe g2HDM effects through interference, and appear rather promising [66].

For the "B anomalies", the effects are so large that they cannot arise from, but can coexist [66] with, g2HDM. As to the recently confirmed [68] muon $g-2$ anomaly, it can [69] be explained by a known [70–75] one-loop mechanism in g2HDM, if $\rho_{\tau\mu} \sim \rho_{\mu\tau} \sim 20$ times larger than $\lambda_\tau \sim 0.01$. What was surprising was that [69] CMS $gg \to H, A \to \tau\mu$ [65] search provided more stringent bound than $\tau \to \mu\gamma$ by Belle [76]. Given the mass-mixing hierarchy, we do not particularly favor such large $\rho_{\tau\mu}$ strength, but there should be no doubt that *Nature* "reigns" over all things. If the one-loop mechanism is behind muon $g-2$, it would be a great boon [77] to muon physics, making the aforementioned leptonic processes far more interesting, including perhaps [78] the observation of μEDM in the not too distant future! We would see a renaissance of muon-related physics.

6. Prospects and Conclusions

In addition to the broadened impact on the flavor front as discussed in the previous section, on LHC and future colliders side, our three main processes of $cg \to tt\bar{c}$, $tt\bar{t}$ [52], $bt\bar{b}$ [53] in Section 4 are only starting points. If there exists a second Higgs doublet that is sub-TeV in mass, the myriad extra couplings promise rich phenomena for future collider studies, which we refer to Ref. [51] for a little more discussion. For the one-loop mechanism behind muon $g-2$ in g2HDM, one could have spectacular [69] signatures, such as $pp \to b\tau\mu W$ or $btcW$, with $\tau\mu$ or tc descending from a neutral H, A scalar, while bW does not come from top but arises from a combination of bH^+ production and H^+ weak decay. Our sub-TeV exotic Higgs masses were argued based on EWBG, but we do not yet know the actual spectrum. Once we learn the spectrum at the LHC, the extra dimension-4 couplings of Equations (1) and (2) should lead to very rich phenomena that await us at the LHC, and at future colliders.

One remark we would like to make is in regards the extra ρ^d Yukawa matrix. The K^0, B_d^0, and B_s^0 systems are the most sensitive probes of FPCP that we have, in particular, to the FCNH ρ_{ds}, ρ_{db} and ρ_{sb} couplings: we could have observed spectacular "BSM" effects in meson mixings and rare decays since long ago. The fact that all three systems behave according to SM should have implications. It is our conjecture that *Nature* somehow deactivated this sector, that the ρ^d matrix is close to diagonal; otherwise there would be arbitrary tuning space with ρ_{tt} loop effects. This enhances our doubt that ρ_{bb} is behind baryogenesis, inasmuch as it can carry a CPV phase. Perhaps it traces back to *Nature*'s choice of $\lambda_b \ll \lambda_t$, where both couplings are now experimentally measured, and the astonishing hierarchy confirmed.

With $\mathcal{O}(1)$ Higgs quartics, and with possibly $\mathcal{O}(1)$ extra top Yukawa couplings ρ_{tc} and ρ_{tt}, together with 50 more (likely weaker) flavor parameters in the form of extra Yukawa couplings, we may be just at the opening to the "prelude" of a new Higgs and flavor era that could start to unfold before us. We have dubbed this prospect "the Decadal Mission" [33].

In conclusion, *Nature* may, or may not, have g2HDM in store for us, but we must walk the walk to probe these sub-TeV extra Higgs bosons and carry out this "mission" towards unveiling a possible new physics Higgs and Flavor era.

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