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Thermal Effects in Ising Cosmology

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Abstract: We consider a real scalar field in de Sitter background and compute its thermal propagators. We propose that in a dS/CFT context, nontrivial thermal effects as seen by an 'out' observer can be encoded in the anomalous dimensions of the $d = 3$ Ising model. One of these anomalous dimensions, the critical exponent η, completely fixes a number of cosmological observables, which we compute.

Keywords: field theory; thermal field theory; cosmology; de Sitter space

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1. Introduction

The rapidly expanding phase of the universe can be modelled by de Sitter (dS) space, and the simplest form of matter by a real scalar. It is believed that basic effects that have left an imprint on the Cosmic Microwave Background (CMB) were of a thermal nature. Therefore, a real scalar field ϕ in the expanding Poincare patch of dS space [1–3] represents a simple model that can explain observed features of the CMB as formulated in the context of thermal quantum field theory [4]. The action is

$$\mathcal{S} = \int d^4x \, \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(m^2 + \xi\mathcal{R})\phi^2\right], \tag{1}$$

which we quantize while taking into account finite temperature effects. Here, m is the mass of the scalar field, ξ is the non-minimal coupling of the field to gravity, and $\mathcal{R}$ is the scalar curvature.

Consider a $d+1$ dimensional FRW spacetime with the metric

$$ds^2 = a^2\left(d\tau^2 - d\mathbf{x}^2\right), \tag{2}$$

with τ being the conformal time and $a(\tau)$ the scale factor. The expanding Poincare patch of de Sitter space corresponds to $a = -\frac{1}{H\tau}$, where $H = \frac{a'}{a^2}$ is the Hubble constant with $a' = \frac{da}{d\tau}$. The expanding Poincare patch of dS space is parameterized by $\tau \in (-\infty, 0]$. The scalar field mode in d-dimensional momentum space $\phi_{|\mathbf{k}|} = \frac{\chi_{|\mathbf{k}|}}{a}$ in this background yields the classical Klein–Gordon equation of motion (with $k = (k^0, \mathbf{k})$ being the four-momentum and the prime being derivative with respect to τ):

$$\chi''_{\mathbf{k}} + \omega^2_{|\mathbf{k}|}\chi_{\mathbf{k}} = 0\,, \tag{3}$$

where $\omega^2_{|\mathbf{k}|} = |\mathbf{k}|^2 + m^2_{\rm dS}$ and the time-dependent mass is provided by $m^2_{\rm dS} = \frac{1}{\tau^2}(M^2 - \frac{d^2-1}{4})$. The dS mass parameter is $M^2 = \mu_H^2 + 12\xi$, $\mu_H^2 = \frac{m^2}{H^2}$, and H is the inverse curvature parameter of dS space satisfying $\mathcal{R} = 12H^2$. The solutions to Equation (3) are linear combinations of the Hankel function $H_{\nu_{\rm cl}}(\tau, |\mathbf{k}|)$ and its complex conjugate of weight $\nu_{\rm cl}$ with

$$\nu_{\rm cl} = \frac{d}{2}\sqrt{1 - \frac{4M^2}{d^2}} \tag{4}$$

and as we will revisit later on, $\nu_{\rm cl}$ is a part of the classical scaling dimensions of the bulk and boundary operators.

Focusing on a given vacuum at a specific time, we can use Taylor expansion in the usual way for the scalar field and calculate the Hamiltonian [2]:

$$\mathcal{H} = \frac{1}{4}\int d^3k \Big[\Omega_{|\mathbf{k}|}\Big(2\alpha^\dagger_{\mathbf{k}}\alpha_{\mathbf{k}} + \delta^{(3)}(0)\Big) + \Lambda_{|\mathbf{k}|}\alpha^\dagger_{\mathbf{k}}\alpha^\dagger_{-\mathbf{k}} + \Lambda^*_{|\mathbf{k}|}\alpha_{\mathbf{k}}\alpha_{-\mathbf{k}}\Big] \tag{5}$$

where $\alpha_{\mathbf{k}}, \alpha^\dagger_{\mathbf{k}}$ are the annihilation and creation operators that satisfy the commutation relations

$$\Big[\alpha_{\mathbf{k}}, \alpha^\dagger_{\mathbf{q}}\Big] = \delta(\mathbf{k} - \mathbf{q}), \qquad \Big[\alpha_{\mathbf{k}}, \alpha_{\mathbf{q}}\Big] = \Big[\alpha^\dagger_{\mathbf{k}}, \alpha^\dagger_{\mathbf{q}}\Big] = 0. \tag{6}$$

In addition, $\Omega_{|\mathbf{k}|}$ and $\Lambda_{|\mathbf{k}|}$ are defined as

$$\Omega_{|\mathbf{k}|} = |u'_{|\mathbf{k}|}|^2 + \omega^2_{|\mathbf{k}|}|u_{|\mathbf{k}|}|^2, \qquad \Lambda_{|\mathbf{k}|} = u'^2_{|\mathbf{k}|} + \omega^2_{|\mathbf{k}|}u^2_{|\mathbf{k}|} \tag{7}$$

where $u_{|\mathbf{k}|}, u^*_{|\mathbf{k}|}$ are the mode functions that pair with the ladder operators $\alpha_{\mathbf{k}}, \alpha^\dagger_{|\mathbf{k}|}$. The frequency $\omega^2_{|\mathbf{k}|}$ is defined below, and carries the time dependence. One of the main key points of a QFT in a curved spacetime is that there is not a single choice for a vacuum state, while different choices lead to different ladder operators $\beta_{\mathbf{k}}, \beta^\dagger_{\mathbf{k}}$ and mode functions that are connected by the Bogolyubov Transformation (BT) [2]:

$$\beta_{\mathbf{k}} = c_{|\mathbf{k}|}\alpha_{\mathbf{k}} + d^*_{|\mathbf{k}|}\alpha^\dagger_{-\mathbf{k}}, \qquad \beta^\dagger_{\mathbf{k}} = c^*_{|\mathbf{k}|}\alpha^\dagger_{\mathbf{k}} + d_{|\mathbf{k}|}\alpha_{-\mathbf{k}} \tag{8}$$

with $c_{|\mathbf{k}|}, d_{|\mathbf{k}|}$ being the Bogolyubov coefficients. Using the commutation relations defined above, it can be shown in a straightforward manner that the Hamiltonian (5) is hermitian for a given ground state defined at a given time τ. An analysis concerning the full time range that spans the entire dS space, instead of only its expanding patch, as we consider here, is more delicate, and hermiticity may be lost.

The work presented in this paper follows the related work of [5], where a possible connection between a bulk dS theory and a boundary Ising model was examined along with its implications for the value of the cosmological spectral index n_S. In particular, experiments [6,7] have found that n_S deviates slightly from unity, which indicates that the CMB is nearly scale-invariant. Although it is true that other endeavors to explain this deviation exist, e.g., [8–10], the idea of using the critical exponent of a boundary Ising field in order to predict cosmological observables is new. Furthermore, it is important to note that the calculations and results in the current paper are model-independent, meaning that we only take for granted the experimental value of n_S and the assumption that the inflation era of the universe can be explained by the expanding Poincare patch of dS spacetime.

In Section 2, we compute thermal propagators in dS spacetime by generalizing methods first produced in flat spacetime (as discussed in [4]), i.e., the Schwinger–Keldysh (SK) path integral and the Thermofield Dynamics (TFD). To the best of our knowledge, while there are cases in which the SK path integral has been used in dS before, e.g., [11], the use of the TFD formalism in a general FRW spacetime is novel. Moreover, a plain though non-trivial connection between the two formalisms is presented, which can hold for other spacetime choices other than dS, enabling the ambiguities rising in the usual SK construction to be avoided. In Section 3, we make use of the dS thermal propagators in order to incorporate the thermal corrections that arise when proceeding to calculate the scalar spectral index n_S. In contrast to previous works [12], where the spectral index to the leading order was found to be equal to unity, we argue here that thermal effects actually slightly break scale invariance, resulting in $n_S \neq 1$. This leads to a parametric freedom which can be fixed by an RG flow argument that has its origins in the $d = 3$ Ising model. In this way, it is possible to match the deviation of n_S from unity to the experimental data. Finally, using the newly formed thermal dS propagator, it is possible to extract more cosmological observables determined by the scalar metric fluctuations, namely, the running of the spectral index $n_S^{(1)}$ and the non-Gaussianity parameter f_{NL}.

2. Propagators and Temperature

Quantization of this system results in the notion of a time-dependent vacuum state and a doubled Hilbert space. Regarding the vacua, we are concerned with the so called "in" vacuum defined at $\tau = -\infty$ and the "out" vacuum defined at the boundary (i.e., the horizon) of the expanding patch at $\tau = 0$. These are empty vacua from the perspective of corresponding local (in conformal time) observers. The $|\mathrm{in}\rangle$ is chosen to be the maximally symmetric Bunch–Davies (BD) vacuum [13,14]. The two vacua are related via the BT $\langle J|\, \mathbf{\Phi}^I = \langle I|\, \mathbf{\Phi}^J$, where $I, J = \mathrm{in}, \mathrm{out}$ is a label of the vacuum and $\mathbf{\Phi}^I$ is the field operator with mode function $\chi^I_{|\mathbf{k}|}$. Note that the field is the same in both vacua, with the mode functions and the creation and annihilation operators inside it being the vacuum dependent quantities. Common notation is $\chi^{\mathrm{in}}_{|\mathbf{k}|} = u_{|\mathbf{k}|}$ and $\chi^{\mathrm{out}}_{|\mathbf{k}|} = v_{|\mathbf{k}|}$.

The doubled Hilbert space can be understood in the context of the SK path integral as being related to a $+$ (or forward) branch and a $-$ (or backward) branch in conformal time evolution. The field propagator $\mathcal{D}$ in such a basis has a 2×2 matrix structure and is ($\mathcal{T}$ ($\mathcal{T}^*$), denoting time (anti-time) ordering, while $\langle 0|$ is a generic vacuum):

$$\begin{aligned} \langle 0|\, \Phi^+(\tau_2)\Phi^-(\tau_1)\, |0\rangle &= \mathcal{D}_{-+}(\tau_1;\tau_2) \\ \langle 0|\, \Phi^-(\tau_1)\Phi^+(\tau_2)\, |0\rangle &= \mathcal{D}_{+-}(\tau_1;\tau_2) \end{aligned} \tag{9}$$

and

$$\begin{aligned} \langle 0|\, \mathcal{T}[\Phi^+(\tau_1)\Phi^+(\tau_2)]\, |0\rangle &= \mathcal{D}_{++}(\tau_1;\tau_2) \\ \langle 0|\, \mathcal{T}^*[\Phi^-(\tau_1)\Phi^-(\tau_2)]\, |0\rangle &= \mathcal{D}_{--}(\tau_1;\tau_2) \end{aligned} \tag{10}$$

where $\mathcal{D}_{+-}(\tau_1;\tau_2) = \mathcal{D}^*_{-+}(\tau_1;\tau_2)$, $\mathcal{D}_{--}(\tau_1;\tau_2) = \mathcal{D}^*_{++}(\tau_1;\tau_2)$ and $\mathcal{D}_{-+}(\tau_1;\tau_2) = \chi_{|\mathbf{k}|}(\tau_1)\chi^*_{|\mathbf{k}|}(\tau_2)$, $\mathcal{D}_{++}(\tau_1;\tau_2) = \theta(\tau_1 - \tau_2)\mathcal{D}_{-+}(\tau_1;\tau_2) + \theta(\tau_2 - \tau_1)\mathcal{D}_{+-}(\tau_1;\tau_2)$. Note that because the vacuum state $|0\rangle$ is time-dependent, for now we construct the above formulas without choosing a specific vacuum. In addition, there is no need for a time ordered product in Equation (9) because the two fields commute, as they are defined in different parts of the SK contour.

The above matrix elements satisfy the relation

$$\mathcal{D}_{++} + \mathcal{D}_{--} - \mathcal{D}_{+-} - \mathcal{D}_{-+} = 0. \tag{11}$$

Hidden in these expressions is the $i\varepsilon$ shift, implementing the projection on the vacuum at $\tau = -\infty$. It can be chosen such that in the flat limit the propagator becomes diagonal with $\mathcal{D}_{++} = \frac{-i}{k^2 - m^2 + i\varepsilon}$. The above construction of the propagator at zero temperature in dS spacetime has been recently studied in [11].

The thermal generalization of the propagator components in Equations (9) and (10) is our next goal. If the Hamiltonian of the system were time-independent, it would be possible to simply follow the process described in Appendix A and show that the propagator satisfies the KMS condition [15,16], which ensures that it is a good thermal propagator. Here, however, we are dealing with a time-dependent Hamiltonian, and this is not straightforward. Instead, we use the method introduced in [17], which takes advantage of the SK contour by adding an extra "thermal" leg to it. In particular, if $\mathcal{C}_+$ is the forward branch, where time evolution follows the path $\tau_{\mathrm{in}} \to \tau_{\mathrm{out}}$, then $\mathcal{C}_-$ is the backward branch, where $\tau_{\mathrm{out}} \to \tau_{\mathrm{in}}$; we attach an extra part to the contour $\mathcal{C}_3$, where $\tau_{\mathrm{in}} \to \tau_{\mathrm{in}} - i\frac{\beta}{2}$ and $\beta = 1/T$ is the inverse temperature parameter Figure 1:

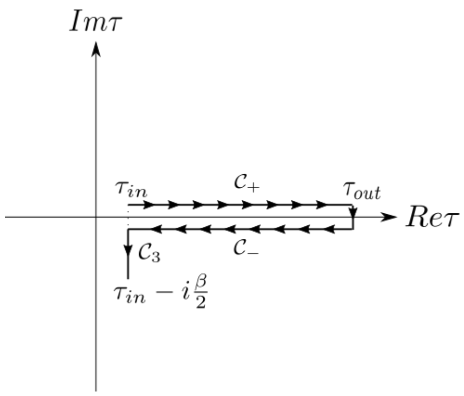

Figure 1. The Schwinger-Keldysh (closed path) contour realized as the coexistence of a forward (+) and a backward (−) branch in conformal time evolution. The thermal generalization of this contour is constructed by adding an extra "thermal" leg to it or, in other words, by attaching the extra part $\mathcal{C}_3$ at the contour such that $\tau_{\rm in} \to \tau_{\rm in} - i\frac{\beta}{2}$.

Furthermore, we introduce the propagators

$$\begin{aligned} \langle 0|\, \mathcal{T}[\Phi^3(\tau_1)\Phi^3(\tau_2)]\,|0\rangle &= \mathcal{D}_{33}(\tau_1;\tau_2) \\ \langle 0|\, \Phi^+(\tau_1)\Phi^3(\tau_2)\,|0\rangle &= \mathcal{D}_{3+}(\tau_1;\tau_2) \\ \langle 0|\, \Phi^-(\tau_1)\Phi^3(\tau_2)\,|0\rangle &= \mathcal{D}_{3-}(\tau_1;\tau_2) \end{aligned} \tag{12}$$

where $\Phi^3(\tau)$ is the field operator on $\mathcal{C}_3$ and $\tau_1, \tau_2 \in \mathbb{C}$; in addition, we demand that the following junction conditions for $a \in \{+,-,3\}$:

$$\begin{aligned} \mathcal{D}_{a+}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\rm out}} &= \mathcal{D}_{a-}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\rm out}} \\ \frac{\partial}{\partial\tau_2}\mathcal{D}_{a+}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\rm out}} &= \frac{\partial}{\partial\tau_2}\mathcal{D}_{a-}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\rm out}} \end{aligned} \tag{13}$$

must be satisfied at time instance $\tau = \tau_{\rm out}$, where the $\mathcal{C}_+$ and $\mathcal{C}_-$ contours meet. Moreover, the conditions

$$\begin{aligned} \mathcal{D}_{a-}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\rm in}} &= \mathcal{D}_{a3}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\rm in}} \\ \frac{\partial}{\partial\tau_2}\mathcal{D}_{a-}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\rm in}} &= \frac{\partial}{\partial\tau_2}\mathcal{D}_{a3}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\rm in}} \end{aligned} \tag{14}$$

need to be satisfied at $\tau = \tau_{\rm in}$, where $\mathcal{C}_-$ and $\mathcal{C}_3$ meet. Finally, in order for the SK analogue of the KMS condition to hold, we need to sew together $\mathcal{C}_+$ and $\mathcal{C}_3$, which results in the conditions

$$\begin{aligned} \mathcal{D}_{a+}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\rm in}} &= \mathcal{D}_{a3}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{in}-i\beta/2} \\ \frac{\partial}{\partial\tau_2}\mathcal{D}_{a+}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\rm in}} &= \frac{\partial}{\partial\tau_2}\mathcal{D}_{a3}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\rm in}-i\beta/2} \end{aligned} \tag{15}$$

that ensure the consistency of the deformed contour and yield a good thermal propagator.

The above conditions introduce corrections of a thermal nature into the propagators in Equations (9) and (10), which we compute by making two assumptions. Because the chosen contour allows for an imaginary time flow, we assume that there is no inflation in that direction. This means that the mode functions on the $\mathcal{C}_3$ leg of the contour can be taken as having the form of a plane wave. In addition, at $\tau = \tau_{in}$ we assume the BD vacuum such that the mode functions are expressed in terms of the Hankel functions of $\nu_{\rm cl} = 3/2$ order.

According to these assumptions, the solution to the conditions results in the in–in thermal propagator components [17]:

$$\begin{aligned}
\mathcal{D}_{++}^{\beta/2} &= \mathcal{D}_{++} + n_B(\beta/2)(\mathcal{D}_{++} + \mathcal{D}_{--}) \\
\mathcal{D}_{--}^{\beta/2} &= \mathcal{D}_{--} + n_B(\beta/2)(\mathcal{D}_{++} + \mathcal{D}_{--}) \\
\mathcal{D}_{+-}^{\beta/2} &= \mathcal{D}_{+-} + n_B(\beta/2)(\mathcal{D}_{++} + \mathcal{D}_{--}) \\
\mathcal{D}_{-+}^{\beta/2} &= \mathcal{D}_{-+} + n_B(\beta/2)(\mathcal{D}_{++} + \mathcal{D}_{--})
\end{aligned} \tag{16}$$

with n_B being the Bose–Einstein distribution parameter

$$n_B(\beta) = \frac{e^{-\beta\omega_{|\mathbf{k}|}}}{1 - e^{-\beta\omega_{|\mathbf{k}|}}}\,. \tag{17}$$

We can conveniently express this propagator collectively in matrix notation as

$$\mathcal{D}_{\beta/2} = \mathcal{D} + s^2(\beta/2)(\mathcal{D}_{++} + \mathcal{D}^*_{++})\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{18}$$

with

$$\mathcal{D} = \begin{pmatrix} \mathcal{D}_{++} & \mathcal{D}_{+-} \\ \mathcal{D}_{-+} & \mathcal{D}_{--} \end{pmatrix},$$

and the parametrization $s(\beta/2) \equiv \sinh\theta_{|\mathbf{k}|}(\beta/2) = \sqrt{n_B(\beta/2)}$ and $c(\beta/2) \equiv \cosh\theta_{|\mathbf{k}|}(\beta/2)$. The flat limit of this propagator is diagonal, and its $++$ component is such that the $i\varepsilon$ shift of the zero-temperature propagator denominator becomes $iE = i\varepsilon\coth(\beta\omega_{|\mathbf{k}|}/2)$ in the thermal state. It is easy to see that this thermal propagator satisfies a condition such as Equation (11).

Here, we are actually interested in the out–out thermal propagator. We first derive the result using a novel shortcut, then show that it yields the correct result. The shortcut uses the TFD formalism, where the doubled Hilbert space is seen as the tensor product of the Hilbert spaces of positive and negative momenta, respectively $\mathcal{H}$ and $\tilde{\mathcal{H}}$. The corresponding fields residing in these Hilbert spaces are Φ and $\tilde{\Phi}$. The validity of this strategy is based on the fact that the SK structure can be read as a TFD structure, in which case the passage to finite temperature is via the transformation $\mathcal{D}_{\beta'} = U_{\beta'}\,\mathcal{D}\,U^T_{\beta'}$ and [18]

$$U_{\beta'} \equiv \begin{pmatrix} \cosh\theta_{|\mathbf{k}|}(\beta') & \sinh\theta_{|\mathbf{k}|}(\beta') \\ \sinh\theta_{|\mathbf{k}|}(\beta') & \cosh\theta_{|\mathbf{k}|}(\beta') \end{pmatrix}. \tag{19}$$

That this is an allowed operation on dS propagators is supported by the fact that a transformation by the matrix $U_{\beta'}$ is a BT with coefficients $\sinh\theta_{|\mathbf{k}|}(\beta') = \frac{e^{-\frac{\beta'}{2}\omega_{|\mathbf{k}|}}}{\sqrt{1-e^{-\beta'\omega_{|\mathbf{k}|}}}}$ and $\cosh\theta_{|\mathbf{k}|}(\beta') = \frac{1}{\sqrt{1-e^{-\beta'\omega_{|\mathbf{k}|}}}}$. Hence, we essentially calculate the thermal corrections that the BT has on the propagator via the TFD formalism. The result of the rotation provides us with the out–out thermal propagator

$$\mathcal{D}_{\beta'} = \mathcal{D} + (s^2(\beta') + s(\beta')c(\beta'))(\mathcal{D}_{++} + \mathcal{D}^*_{++})\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{20}$$

It is immediately noticeable that the two expressions in Equations (18) and (20) disagree in the thermal correction, as the latter has an extra term along $\sinh\theta_{|\mathbf{k}|}\cosh\theta_{|\mathbf{k}|}$. While this might seem troublesome at first, they both contain the same physical information. Taking advantage of the trivial identity

$$\frac{e^{-\beta\omega_{|\mathbf{k}|}}}{1-e^{-\beta\omega_{|\mathbf{k}|}}}+\frac{e^{-\frac{\beta}{2}\omega_{|\mathbf{k}|}}}{1-e^{-\beta\omega_{|\mathbf{k}|}}}=\frac{e^{-\frac{\beta}{2}\omega_{|\mathbf{k}|}}}{1-e^{-\frac{\beta}{2}\omega_{|\mathbf{k}|}}}\,, \tag{21}$$

the propagators in Equations (18) and (20) are seen to be equal for $\beta' = \beta$. Note that the above identity does hold in the $\sinh\theta_{|\mathbf{k}|}$ and $\cosh\theta_{|\mathbf{k}|}$ parameterization, where it reads $s^2(\beta) + s(\beta)c(\beta) = s^2(\beta/2)$. Therefore, we have proved that the known form of the dS thermal propagator of [17] can be equivalently obtained via TFD rotation of the zero-temperature SK propagator of the half-thermal parameter. Of course, the equivalence of the two expressions reflects the universal nature of the dS temperature as measured at an arbitrary time instance by the in and out observers. The advantage of the TFD rotation operation is that it is very simple and can be easily generalized to any background. Thus, we use this point of view in what follows.

The result of all allowed thermal transformations of $\mathcal{D}$ are correlators of the form

$$\mathcal{D}^I_{J,\gamma} = \langle J;\gamma|\,\mathcal{T}[\mathbf{\Phi}^I(\mathbf{\Phi}^I)^T]\,|J;\gamma\rangle\ . \tag{22}$$

The doublet field, now in the language of TFD, is $(\mathbf{\Phi}^I)^T = (\Phi^I, \tilde{\Phi}^I)$, while γ is a thermal index associated with any combination of thermal transformations of the form Equation (19). The label (not index) I on the field is a reminder of the vacuum state to which the mode functions belong. The two types of thermal transformations that are relevant to us here are the insertion of an explicit density matrix, resulting in a transformation by a unitary operator U, as $|I;\beta\rangle = U\,|I\rangle$, where the eigenvalue of U is $U_\beta(\theta)$ and the Gibbons–Hawking (GH) effect [19] (for which we momentarily use the parameter δ to distinguish it from β) is expressed as $|I\rangle = |J;\delta\rangle$ with $I \neq J$. The only temperature that dS space can sustain is the GH temperature, which means that $1/\beta_{\mathrm{dS}} = T_{\mathrm{dS}} = H/2\pi = 1/\delta$. As such, it is sufficient to know the form of the thermal dS–scalar propagator for some generic temperature and then set $\beta = \beta_{\mathrm{dS}}$.

3. The Spectral Index with Thermal Effects

The propagators in Equations (18) and (20) determine several important observables. At equal space-time points and at the time of horizon exit, defined as $|\tau|H = 1$ and concentrating on horizon exiting modes specified by $|\mathbf{k}\tau| \lesssim 1$, they determine various cosmological indices derived from the following thermal scalar power spectrum [12] (here, $\mathbf{1}$ is the 2×2 matrix with unit elements):

$$P_{S,\beta}\mathbf{1} = \mathcal{D}_\beta\mathbf{1}|_{\tau_1=\tau_2}\,, \tag{23}$$

in terms of a single parameter (when the temperature takes its natural value $T = T_{\mathrm{dS}}$):

$$\kappa \equiv \omega_{|\mathbf{k}|}|\tau|\Big|_{|\mathbf{k}\tau|=1} = \sqrt{\left(|\mathbf{k}|^2 + m^2_{\mathrm{dS}}\right)|\tau|^2}\Big|_{|\mathbf{k}\tau|=1} = \sqrt{\frac{5-d^2}{4} + M^2} \tag{24}$$

where $\omega_{|\mathbf{k}|}$ is the frequency defined under Equation (3). This parameter can be traded for the weight of the Hankel function, as determined by the Klein–Gordon equation, in Equation (4). Of special importance in $d = 3$ is the choice of $M = 0$, or $\kappa = i$, which is known to generate a scale invariant CMB spectrum. This corresponds to $\nu_{\mathrm{cl}} = \frac{3}{2}$ and decaying modes at the time of exit.

A particularly useful point of view [20] is to recognize the system at $\tau = -\infty$ as being related to a UV Conformal Field Theory (CFT) labeled by the weight ν_{cl} and associated with the Gaussian fixed point of the $d = 3$ real scalar theory, which flows towards an interacting IR fixed point and the corresponding CFT at $\tau = 0$. It is clear that in the present context exact scale invariance is realized in the $|\mathrm{in}\rangle$ vacuum, with the deviations generated by a spontaneous shift in M that, according to Equation (20), should have a finite temperature origin. Deviations can be encoded in general by a shift of the weight $\nu_{\rm cl} \to \nu = \nu_{\rm cl} + \nu_{\rm q}$, which can be interpreted as a shift in the scaling dimension of a dS scalar field

$$\Delta_- = \frac{d}{2} - \nu = \frac{d}{2} - \nu_{\rm cl} - \nu_{\rm q} = \Delta_{{\rm cl},-} - \nu_{\rm q}\,. \tag{25}$$

There is a corresponding shadow partner solution to this with $\Delta_+ = \frac{d}{2} + \nu$. In this paper, we are concerned with $(\Delta_-, \Delta_+)_{\rm cl} = (0, 3)$.

In order to understand $\nu_{\rm q}$ (which turns out to be a non-trivial zero), we first point out that the $|{\rm out}; \beta\rangle$ $(\beta > \beta_{\rm dS})$ state is a BT of the Bunch–Davies vacuum. The mode functions before and after the transformation solve the same Bessel equation with frequency $\omega_{|\mathbf{k}|}$. Upon a time-dependent BT however, the frequency that an observer sees for a time other than his own is [21]

$$\Omega_{|\mathbf{k}|} = \omega_{|\mathbf{k}|}(|c|^2 + |s|^2)\,. \tag{26}$$

As a result, the horizon exit parameter is transformed as

$$\kappa \to \Lambda = \kappa\left(1 + 2\frac{e^{-2x\kappa}}{1 - e^{-2x\kappa}}\right) = \kappa \coth(x\kappa)\,, \tag{27}$$

where we have defined the dimensionless temperature parameter $x = \frac{\pi H}{2\pi T}$, which takes values in $[\pi, \infty]$. The transformed state in general has a reduced isometry with respect to the Bunch–Davies state. This can be seen by the fact that the BT introduces a non-zero mass term $(\mu_H^2 + \xi\frac{\mathcal{R}}{H^2})a^2H^2\phi^2$ in the Lagrangian with exit parameter $\Lambda^2 = |k\tau|^2 + a^2\left[\mu_H^2 + (\xi - \frac{1}{6})\frac{\mathcal{R}}{H^2})\right]$ and that the late time equations of motion

$$\phi'' + 2aH\phi' + \left(\mu_H^2 + \xi\frac{\mathcal{R}}{H^2}\right)a^2H^2\phi = 0\,, \qquad H' = -\frac{1}{2a}\phi'^2 \tag{28}$$

have no non-trivial solution with $H = $ const. and a mass term that is non-zero and finite.

The two limiting values of x are interesting. Its natural value $x = \pi$, where $T = T_{\rm dS}$, provides $\Lambda = \infty$ for $\kappa = i$. This is a special case in which we recover a dS solution of maximal isometry that corresponds to $|{\rm out}; \beta_{\rm dS}\rangle$. As in the BD vacuum, no modes are seen to exit the horizon, this time due to their ultra-short wavelength. In the limit $x \to \infty$, on the other hand, the out observer sees modes of any wavelength as exiting modes, as in this limit the time of exit approaches the horizon. This means that if the observer calls their frequencies $\Omega_{|\mathbf{k}|}$, then their horizon exit parameter is forced to be $\Lambda_0 \equiv \lim_{\tau\to 0}(\Omega_{|\mathbf{k}|}\tau) \to 0$ (in this limit, x becomes an odd multiple of $\pi/2$). This suggests the construction of a trajectory from $(\Lambda, x) \sim (\infty, \pi)$ to $(0, \infty)$ along which the value of some yet to be defined thermal effect is kept non-zero and constant, starting from a position somewhat shifted away from the scale invariant limit (∞, π). Deviations from exact dS isometry due to finite temperature effects can be encoded in the shift of the spectral index of scalar curvature fluctuations [22]:

$$n_{S,\beta} = 1 + \frac{d\ln\left(|\mathbf{k}|^3 P_{S,\beta}\right)}{d\ln|\mathbf{k}|} \tag{29}$$

where $P_{S,\beta}$ is the thermal scalar power spectrum defined in Equation (23).

In the previous section, we have shown that the SK and TFD formalisms result in equivalent propagators. Consequently, from Equation (23), they both determine the same thermal deviation

$$\delta n_S \equiv n_{S,\beta} - 1 = -\frac{2x}{\Lambda}\left[\frac{e^{-x\Lambda}}{1 - e^{-2x\Lambda}}\right], \tag{30}$$

of n_S away from unity. Observe that in $|{\rm out}; \beta_{\rm dS}\rangle$, where $x = \pi$ and $\Lambda = \infty$, δn_S vanishes and we see a scale-invariant spectrum. Moving somewhat away from it, to $x \gtrsim \pi$ (here, it is implicitly assumed that moving away from $T_{\rm dS}$ is a result of spontaneous breaking of scale invariance, which is expected to lower the temperature), the state is $|{\rm out};\beta\rangle$ and δn_S becomes a one-parameter expression of Λ. We can fix this freedom by determining the value $n_{S,\beta}$ by interpreting its deviation from unity as an anomalous dimension in the dual field theory in the spirit of the dS/CFT correspondence. Then, we can reach $x = \infty$ along a trajectory that keeps this value constant for all temperatures.

In [5], it was proposed that within the dual field theory on the horizon, the anomalous dimension that shifts the spectral index is the critical exponent η, for which the non-perturbative value is around 0.036. Thus, near the horizon we have

$$n_S \simeq 1 - \eta = 0.964 \tag{31}$$

while the experimentally measured value is equal to [6]

$$n_{S,\rm exp} = 0.9649 \pm 0.0042. \tag{32}$$

A known fact about the dual field theory of dS is that it is expected to be non-unitary [8]. Thus, the argument could be made that the Ising model (or its large N relatives), which is unitary, is not a good candidate as the complement to dS theory. The suggestion made here is that the complementary field theory to the AdS version of the model under discussion, which could be in the universality class of the Ising model (or its large N relatives), is an analytic continuation of dS. This is supplemented by the fact that Equation (31) is invariant under an analytic continuation; see [5] for a more detailed justification.

This is a constraining statement that leaves us with no free parameters. In [5], it was shown that the quantity by which $\Delta_{+,\rm cl}$ shifts is the operator anomalous dimension of the trace of the Ising stress energy tensor Θ, which is an exact zero, realized as the cancellation $\Gamma_\Theta = \gamma_{\mathcal{O}} - 2\gamma_\sigma = 0$, where $\gamma_{\mathcal{O}}$ is the "total" operator anomalous dimension and γ_σ is the field anomalous dimension (or the so-called wave function renormalization). Therefore, it is in this sense that ν_q is a non-trivial zero, being related to the vanishing anomalous dimension of a special operator that is the energy–momentum tensor. In [5], it was further demonstrated that it is the total anomalous dimension $\gamma_{\mathcal{O}}$ that ends up shifting the spectral index n_S. Of course, outside of the fixed point where the Ising field is massive, M deviates from zero in the bulk, and the solution to Equation (28) is not dS. It is important, however, to understand that the main effect on n_S comes from the critical value η of $2\gamma_\sigma$ and that the deviation from the critical value is small as long as the system sits in the vicinity of the fixed point. For this reason, the leading order results are independent of the source of the breaking. In a sense, the only assumption here is that there is a mechanism of spontaneous breaking of scale invariance. From the point of view of the boundary, this could (for example) be justified as some sort of Coleman–Weinberg mechanism.

4. Line of Constant Physics and Other Observables

A line of constant physics (LCP) is a set of points on the phase space upon which the value of a physical quantity remains fixed. What we demonstrate now is that, in the bulk, there is an LCP labelled by the fixed value $\delta n_S = -\eta$ along which the system is heated up from zero temperature, where $\Lambda_0 = 0$ and $x = \infty$, up to the dS temperature. A few points on this line and a picture of the LCP can be seen in Figure 2.

We stress that for a given x the corresponding value of Λ is fixed by the label of the LCP. Thus, near the endpoint of the LCP, where $x \simeq \pi$, the value $\Lambda_\pi \simeq 1.5117$ is a fixed output. It is important to emphasize that the LCP is really meaningful up to just outside of its two limiting points. Up to around $x \simeq \pi$, it is characterized by a non-zero δn_S; however, at exactly $x = \pi$ this becomes equal to zero, as the trace of the boundary stress–energy tensor to which the bulk scalar couples vanishes. Analogously, the interpretation of each point on it as a dS space of the same $T_{\rm dS}$ is possible everywhere except at $x = \infty$, where the intrinsic temperature must become abruptly unobservable.

Λ	x
$\to 0$	$\to \infty$
10^{-6}	$3.5 \cdot 10^7$
0.01	1600
0.5	14.8
$\to 1.5117$	$\to \pi$

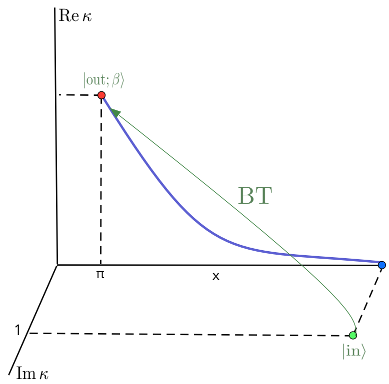

Figure 2. (**Left**) A few points of the nearly conformal LCP defined by $\delta n_S = -\eta$ and (**Right**) the Bogolyubov transformation $|\text{in}\rangle \to |\text{out}; \beta\rangle$ and the LCP on the complex plane, where $\kappa = \Lambda + i\,\text{Im}\kappa$.

Because there are no free parameters, several other observables that are determined by $P_{S,\beta}$ are expected to be fixed. For example, we can define the moment

$$n^{(1)}_{S,\beta} = \frac{dn_{S,\beta}}{d\ln|\mathbf{k}|} \tag{33}$$

and compute it using $n_S^{(1)} = 0$. The result, evaluated under the same conditions as $n_{S,\beta}$, is

$$n^{(1)}_{S,\beta} = \delta n_S \left[2 - \frac{1}{\Lambda^2} - \frac{x}{\Lambda}\left(1 + \frac{2e^{-2x\Lambda}}{1 - e^{-2x\Lambda}}\right)\right], \tag{34}$$

which, substituting $x \simeq \pi$ and $\Lambda = \Lambda_\pi \simeq 1.5117$, provides us with

$$n^{(1)}_{S,\beta} = 0.0186 \tag{35}$$

for the running of the index. The constraints in [6] are

$$n^{(1)}_{S,\text{exp}} = 0.013 \pm 0.012. \tag{36}$$

Finally, the universal contribution to the non-Gaussianity parameter [23] can be expressed in terms of $N = \int_{t_i}^{t_f} dtH$ and its derivatives in the in-vacuum as [24]

$$f^{\text{un}}_{NL} = \frac{5}{6}\frac{N_{\rho\rho}}{N_\rho^2}, \tag{37}$$

with $N_\rho = \frac{\partial N}{\partial \rho}$, $N_{\rho\rho} = \frac{\partial^2 N}{\partial \rho^2}$ and $\rho \equiv P_{S,\beta}$. (The parameter f_{NL} is defined by a more general expression [8]. The expression we use is provided in [24] for the special case of a single scalar field.) It is computed as being

$$f^{\text{un}}_{NL} = -\frac{5\left[x(-1+\Lambda^2)^2\left(1 + x\Lambda\cot(\frac{x\Lambda}{2})\right) + 2\Lambda^3\sinh(x\Lambda)\right]}{6\Lambda^2\left[x(-1+\Lambda^2) + \Lambda\sinh(x\Lambda)\right]}, \tag{38}$$

which for $x \simeq \pi$ and $\Lambda = \Lambda_\pi \simeq 1.5117$ provides us with

$$f^{\text{un}}_{NL} = -1.7138, \tag{39}$$

while one of the experimental results for f^{un}_{NL} in a certain analysis is [7]

$$f^{\text{un}}_{NL,\text{exp}} = -1.7 \pm 5.2. \tag{40}$$

5. Conclusions

In this paper, we have considered a thermal scalar in de Sitter background. Starting from the Bunch–Davies $|\text{in}\rangle$ vacuum, the Bogolyubov transformation places us in the interior of the finite temperature phase diagram in a thermal state $|\text{out};\beta\rangle$. This state can be connected through holography to the vicinity of an interacting IR fixed point in the universality class of the 3D Ising model. The system in this state is rather special, in the sense that the boundary operator that couples to the scalar curvature perturbations in the bulk has a classical scaling dimension. The critical exponent η is the order parameter for breaking the scale-invariant spectrum of curvature fluctuations, and a simple argument from the dS/CFT correspondence fixes the parametric freedom in the dS scalar theory, yielding the prediction $n_S = 0.964$. In the same context, we compute a number of additional cosmological observables, such as the first moment of the scalar spectral index and the non-Gaussianity bispectrum parameter f_{NL}, then evaluate them numerically. Our predicted values of n_S, $n^{(1)}_{S,\beta}$ and f_{NL} are well within current experimental bounds [6,7].

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Appendix A

In this Appendix, we discuss the real time construction in the Hamiltonian formulation. First, we provide a shortcut derivation of the thermal propagator Equation (18) that starts from flat space and the definitions

$$\begin{aligned} \mathcal{D}^{\beta}_{+-}(\tau_1,\tau_2) &= W_2(\tau_1,\tau_2) + W_1(\tau_1,\tau_2) \\ \mathcal{D}^{\beta}_{-+}(\tau_1,\tau_2) &= W_1(\tau_2,\tau_1) + W_2(\tau_2,\tau_1) \end{aligned} \tag{A1}$$

with the Wightman functions defined as

$$\begin{aligned} W_1(\tau_1,\tau_2) &\equiv \frac{\mathrm{Tr}\{a^\dagger(\tau_1)a(\tau_2)\,\rho\}}{\mathrm{Tr}\{\rho\}} = n_B e^{i\omega(\tau_1-\tau_2)} \\ W_2(\tau_1,\tau_2) &\equiv \frac{\mathrm{Tr}\{a(\tau_1)a^\dagger(\tau_2)\,\rho\}}{\mathrm{Tr}\{\rho\}} = (1+n_B)e^{-i\omega(\tau_1-\tau_2)} \end{aligned} \tag{A2}$$

where $\rho = e^{-\beta\mathcal{H}}$ is the thermal density matrix, $\mathcal{H}$ is the (harmonic oscillator) Hamiltonian, and the second equalities show the result of the trace computations. Now, because the time-dependent part of the mode function in flat space is $u(\tau) = e^{i\omega\tau}$, we can write $e^{i\omega(\tau_1-\tau_2)} = u(\tau_1)u^*(\tau_2)$ and pass to dS space via the substitution $u(\tau) \to \chi_{|\mathbf{k}|}(\tau)$. Then, we have

$$\begin{aligned} \mathcal{D}^{\beta}_{+-}(\tau_1,\tau_2) &= \chi^*_{|\mathbf{k}|}(\tau_1)\chi_{|\mathbf{k}|}(\tau_2) + n_B(\beta)\Big(\chi_{|\mathbf{k}|}(\tau_1)\chi^*_{|\mathbf{k}|}(\tau_2) + \chi^*_{|\mathbf{k}|}(\tau_1)\chi_{|\mathbf{k}|}(\tau_2)\Big) \\ \mathcal{D}^{\beta}_{-+}(\tau_1,\tau_2) &= \chi_{|\mathbf{k}|}(\tau_1)\chi^*_{|\mathbf{k}|}(\tau_2) + n_B(\beta)\Big(\chi_{|\mathbf{k}|}(\tau_1)\chi^*_{|\mathbf{k}|}(\tau_2) + \chi^*_{|\mathbf{k}|}(\tau_1)\chi_{|\mathbf{k}|}(\tau_2)\Big) \end{aligned} \tag{A3}$$

and by imposing $\mathcal{D}^{\beta}_{++}(\tau_1;\tau_2) = \theta(\tau_1-\tau_2)\mathcal{D}^{\beta}_{-+}(\tau_1;\tau_2) + \theta(\tau_2-\tau_1)\mathcal{D}^{\beta}_{+-}(\tau_1;\tau_2)$, $\mathcal{D}^{\beta}_{--}(\tau_1;\tau_2) = \mathcal{D}^{\beta *}_{++}(\tau_1;\tau_2)$ and applying for $\beta/2$, we arrive again at Equation (18).

A thermal propagator has to satisfy a variant of the KMS condition. The KMS condition originates from the definition

$$\langle \phi(t_1, x_1)\phi(t_2, x_2)\rangle_\beta = \frac{\mathrm{Tr}\{\phi(t_1, x_1)\phi(t_2, x_2)\rho\}}{\mathrm{Tr}\{\rho\}} \tag{A4}$$

which in principle leads to the thermally corrected dS propagator. However, the direct computation of the trace is not obvious for the time-dependent Hamiltonians.

The condition takes a simple form though near $\tau \to -\infty$, which can be shown explicitly. In Thermofield Dynamics, the form of the KMS condition depends on a gauge parameterized by a real number, say, α. It is a well known fact that TFD propagators satisfy such a condition in any of these α-gauges [25,26]. The condition holds due to the relation [18]

$$a^-_{\mathbf{k}} |0;\beta\rangle = e^{-\alpha\beta\omega_{|\mathbf{k}|}} \tilde{a}^+_{\mathbf{k}} |0;\beta\rangle\,, \qquad \langle 0;\beta| a^+_{\mathbf{k}} = \langle 0;\beta| \tilde{a}^-_{\mathbf{k}} e^{-(1-\alpha)\beta\omega_{|\mathbf{k}|}} \tag{A5}$$

between the standard annihilation operator acting on the vacuum of the Hilbert space $\mathcal{H}$ and the tilded creation operator acting on the vacuum of $\tilde{\mathcal{H}}$. The thermal vacuum $|0;\beta\rangle$ is defined by the action of a unitary operator on the tensor product of the vacuum states in $\mathcal{H} \times \tilde{\mathcal{H}}$. Using the above relations, it can be straightforwardly shown that the Wightman function between the fields $\Phi, \tilde{\Phi}$ for $\alpha = 1/2$ satisfies the following condition (with the mode functions of the scalar field near $\tau \to -\infty$ reducing to plane waves):

$$\langle 0;\beta|\Phi(\tau_1, \mathbf{x})\tilde{\Phi}(\tau_2, \mathbf{y})|0;\beta\rangle = \langle 0;\beta|\Phi(\tau_2 + i\frac{\beta}{2}, \mathbf{y})\tilde{\Phi}(\tau_1 - i\frac{\beta}{2}, \mathbf{x})|0;\beta\rangle\,, \tag{A6}$$

which is the KMS condition in the $\alpha = 1/2$ gauge. This is relevant for our case, as the transformation matrix Equation (19) is in this gauge [25]. A different gauge choice is to take $\alpha = 1$, where

$$a^-_{\mathbf{k}} |0;\beta\rangle = e^{-\beta\omega_{|\mathbf{k}|}} \tilde{a}^+_{\mathbf{k}} |0;\beta\rangle\,, \qquad \langle 0;\beta| a^+_{\mathbf{k}} = \langle 0;\beta| \tilde{a}^-_{\mathbf{k}}\,. \tag{A7}$$

Then, the same Wightman function as above needs to satisfy

$$\langle 0;\beta|\Phi(\tau_1, \mathbf{x})\tilde{\Phi}(\tau_2, \mathbf{y})|0;\beta\rangle = \langle 0;\beta|\Phi(\tau_2, \mathbf{y})\tilde{\Phi}(\tau_1 - i\beta, \mathbf{x})|0;\beta\rangle\,, \tag{A8}$$

that is, the KMS condition in the $\alpha = 1$ gauge. Note that this is a relation in which the usual form of the KMS condition of thermal field theory can be recognised. These two gauges can, however, be readily seen to correspond to the equivalent Wightman functions by $\tau_{1,2} \longrightarrow \tau_{1,2} - i\frac{\beta}{2}$, as they can be related by a shift in the imaginary time. From this shift freedom, it can additionally be seen that the diagonal elements of the propagator do not satisfy any non-trivial constraint. In conclusion, to the extent that Equation (A4) applies to dS space and the trace is computable, the thermal propagator that it defines satisfies a KMS condition.

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