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Superluminal Local Operations in Quantum Field Theory: A Ping-Pong Ball Test

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Abstract: It is known that, in quantum field theory, localized operations, e.g., given by unitary operators in local observable algebras, may lead to non-causal, or superluminal, state changes within their localization region. In this article, it is shown that, both in quantum field theory as well as in classical relativistic field theory, there are localized operations which correspond to "instantaneous" spatial rotations (leaving the localization region invariant) leading to superluminal effects within the localization region. This shows that "impossible measurement scenarios" which have been investigated in the literature, and which rely on the presence of localized operations that feature superluminal effects within their localization region, do not only occur in quantum field theory, but also in classical field theory.

Keywords: quantum field theory; classical field theory; local operations; faster-than-light communication; superluminal signalling; impossible measurements

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1. Introduction

There are some scenarios, usually set within the framework of special relativity, in which it is argued that superluminal effects are related to effects that are akin to traveling backward in time (see, e.g., Section 4.3 in [1]; see, however, also [2] and the references given therein for more critical considerations on this issue). Taken for granted that a compelling connection between superluminal effects and time travel can be established, our present contribution fits into the theme of this volume.

Recently, some attention has been given to the idea that there are "local unitary operations" in relativistic quantum field theory which can act in a "superluminal" fashion within their localization region [3–5]. This has, in fact, been observed much earlier by Sorkin [6], who employed it to argue that relativistic quantum field theory was lacking a well-defined approach to measurement comparable to the theory of measurement in non-relativistic quantum mechanics. To illustrate his point, he considers three spacetime regions, O_{Alice}, O_{Bob} and O_{Charlie}, wherein and during which the observers Alice, Bob and Charlie can carry out operations and measurements on a state of a quantum field they jointly have access to. The spacetime regions O_{Alice} and O_{Charlie} are causally separated, but there is causal contact of O_{Bob} with both O_{Alice} and O_{Charlie}. (See Figure 1 in Section 3 for an illustration. In some publications, like [5] and [3], the roles of O_{Bob} and O_{Charlie} are interchanged; our labeling coincides with that in [6] and [7].) Sorkin then argues that there are certain combinations of unitary operations carried out by Alice in O_{Alice} and by Bob in O_{Bob} so that, if Charlie measures the resulting state in O_{Charlie}, it can be determined whether Alice has carried out her operation, despite the fact that O_{Alice} and O_{Charlie} are not in causal contact. Notice that, if Bob does not carry out any operation, then Charlie cannot decide by measurements in O_{Charlie} if Alice has carried out a unitary operation in O_{Alice}.

We will describe the set-up of [6] in more detail below (in a version given by [7]), and will show that there are indeed local unitary operations with the properties just described. In response to the apparent superluminal transformations of states by local unitary operations and the ensuing difficulties regarding measurement in relativistic quantum field theory according to Sorkin, it has been shown in [7] that these difficulties do not occur in a recently proposed, covariant approach to local measurement in quantum field theory [8] (see also [9–11] for additional discussion). In the present paper, we wish to point out that certain "superluminal localized operations" are not specific to relativistic quantum field theory (and therefore, relativistic quantum field theory is not suffering from any particular conceptual defect or inconsistency), but that they appear also in classical relativistic theories. For instance, they are related to (local) symmetries that a theory, quantum or classical, may possess, but which cannot be performed "instantaneously" as they violate the principles of special or general relativity both on kinematical as well as dynamical grounds. These are occasionally (but perhaps not systematically) referred to as "passive" symmetry transformations. Therefore, the present paper provides a "ping-pong ball test" in regard to the occurrence of "superluminal localized operations". The said test is a concept which, according to the present authors' knowledge, goes back to Reinhard Werner [12], and we paraphrase it here as follows: *When someone presents a paradox as being rooted in quantum physics, replace the term 'quantum mechanical particle' by 'ping-pong ball' everywhere. If the paradox persists, it is unrelated to quantum physics.*[1] As a matter of fact, the application of the ping-pong ball test in other scenarios claiming that acausal effects may occur by means of quantum physics has already proven useful [13].

The present article is structured as follows. In Section 2, we summarize the properties of relativistic quantum field theories on $1 + 3$-dimensional Minkowski spacetime in the operator–algebraic framework. The assumption of the "split property" implies that global symmetries, such as space rotations, have unitary implementers in the local operator algebras. In Section 3, we revisit the "impossible measurements scenario" presented in [6], and we show how it can be realized by "instantaneous space rotations", which have unitary implementing operators that are contained in local algebras. The quantized Klein–Gordon field is used as a special, simple example. We show in Section 4 that, in a recent proposal for an algebraic description of classical field theory in terms of local Poisson algebras, there are also local symmetries corresponding to "instantaneous space rotations", thus the considerations leading to the "impossible measurements scenario" apply for classical field theory as well. We discuss the conclusions that can be drawn from these results in Section 5.

2. Algebraic Quantum Field Theory Setting

We start by considering relativistic quantum field theory on 1 + 3 dimensional Minkowski spacetime (represented as $\mathbb{R}^{1+3}$) in vacuum representation. This is mainly for convenience; generalizations of the arguments given below are with respect to the case of more general (globally hyperbolic) spacetimes, or spacetime dimensions $\geq 1 + 2$, and these are not difficult.

Thus, the standard assumptions are made (cf. [14,15]): there is a Hilbert space $\mathcal{H}$ on which a continuous representation U_L, $L \in \mathcal{P}_+^\uparrow$, of the proper, orthochronous Poincaré group operates; there is a (up to phase) unique unit vector $\Omega \in \mathcal{H}$ that does not change under the Poincaré transformations, i.e., $U_L\Omega = \Omega$. If the translations in $\mathcal{P}_+^\uparrow$ are denoted as $a \in \mathbb{R}^4$, and their unitary representers as U_a, then, for any future-directed, timelike unit vector e, the unitary group $t \mapsto U_{te}$ $(t \in \mathbb{R})$ is assumed to have a self-adjoint generator with a non-negative spectrum: This is the *spectrum condition*.

Moreover, it is assumed that there is a family $\mathsf{A}(O)$ of von Neumann subalgebras of $\mathsf{B}(\mathcal{H})$ indexed by the open, relatively compact subsets O of $\mathbb{R}^{1+3}$, subject to the conditions of *isotony*: $O_1 \subset O_2 \Rightarrow \mathsf{A}(O_1) \subset \mathsf{A}(O_2)$, and *locality*: $\mathsf{A}(O_2) \subset \mathsf{A}(O_1)'$ if $O_2 \subset O_1^\perp$. Here, $\mathsf{A}(O_1)' = \{C \in \mathsf{B}(\mathcal{H}) : CA = AC \text{ for all } A \in \mathsf{A}(O_1)\}$ is the commutant algebra (or simply *commutant*) of $\mathsf{A}(O_1)$, and we recall that any von Neumann algebra A in $\mathsf{B}(\mathcal{H})$ is characterized by the property that $\mathsf{A}'' = \mathsf{A}$. Furthermore, for any open subset O of $\mathbb{R}^{1+3}$, we denote by $O^\perp$ the causal complement of O, i.e., the largest open set in $\mathbb{R}^{1+3}$ such that there is no pair of points $p \in O$ and $p^\perp \in O^\perp$ which can be connected by any smooth, causal curve.

The algebra $\mathsf{A}(O)$ is viewed as the algebra of (in the sense of "generated by") the observables that can be measured within the spacetime region O. The unitary representation of $\mathcal{P}_+^\uparrow$ acts covariantly on the collection of local observable algebras,

$$\alpha_L(\mathsf{A}(O)) = \mathsf{A}(L(O)) \quad \text{where} \quad \alpha_L(A) = U_L A U_L^* \,. \tag{1}$$

It is assumed that the von Neumann algebra generated by all of the $\mathsf{A}(O)$ coincides with $\mathsf{B}(\mathcal{H})$.

We recall that for any subset S of Minkowski spacetime, the domain of dependence $D(S)$ is the set of all points p in the spacetime such that all past-directed or all future-directed causal rays emanating from p intersect S. In Minkowski spacetime, an open subset O is called *causally complete* if it has the property $O = (O^\perp)^\perp$, which also entails that $O = D(O)$.

In addition to the standard properties for a quantum field theory in the operator algebraic setting just stated, we will make a few additional assumptions that are known to hold, e.g., in models of linear quantized fields. The first property is the *local time-slice property*: $\mathsf{A}(O) = \mathsf{A}(D(O))$ (sometimes also called *primitive causality*). This is demanded to hold for spacetime regions that are of the form $O = D(B) \cap N$, where B is an open subset of an arbitrary Cauchy surface, and N is any open neighborhood of the Cauchy surface. The second property is *additivity*: if O is an open, relatively compact spacetime region, and O_i, $i \in J$, is any covering of O by open, relatively compact spacetime regions for an arbitrary index set J, then $\mathsf{A}(O)$ is contained in the von Neumann algebra generated by all the $\mathsf{A}(O_i)$. Together with the previously stated conditions, this entails the Reeh–Schlieder property of the local von Neumann algebras with respect to the vacuum vector Ω, meaning that $\mathsf{A}(O)\Omega$ is dense in $\mathcal{H}$ if O is any (non-void) open, relatively compact subset of $\mathbb{R}^{1+3}$. The third property, which is actually very relevant to our discussion, is the *split property*: Assume that O_1 and O_2 are relatively compact, causally complete open subsets of $\mathbb{R}^{1+3}$, such that $\overline{O_1} \subset O_2$, then there is a type I factor von Neumann algebra N such that

$$\mathsf{A}(O_1) \subset \mathsf{N} \subset \mathsf{A}(O_2) \,. \tag{2}$$

We will not discuss this property here, except for remarking that the local von Neumann algebras $\mathsf{A}(O)$ are typically type III, and that the type classification, roughly speaking, gives information about what kind of projection operators a von Neumann algebra possesses. The reader is referred to [14,16,17] and the references therein for a considerably more discussion. One consequence, as shown in [16], is that global symmetries of quantum field theory can be localized. Here, we are interested in a special case of that consequence, and we now introduce a suitably adapted notation. An inertial system is assumed to be chosen, and the coordinates (x^0, x^1, x^2, x^3) of $\mathbb{R}^{1+3}$ are the corresponding inertial coordinates. We consider the centered ball of radius $r > 0$ at $x^0 = 0$,

$$B(r) = \{(x^0 = 0, x^1, x^2, x^3) : (x^1)^2 + (x^2)^2 + (x^3)^2 < r^2\}, \tag{3}$$

and its domain of dependence (coinciding with its causal completion)

$$O(r) = D(B(r)) \,. \tag{4}$$

If $R \in \mathrm{SO}(3)$ denotes any space rotation in the $x^0 = 0$ hyperplane around $x^j = 0$ ($j = 1, 2, 3$), whereby it is canonically identifiable with an element in $\mathcal{P}_+^\uparrow$, then $RB(r) = B(r)$ and $RO(r) = O(r)$. Consequently, denoting by U_R the unitary implementer of R, we have

$$U_R \mathsf{A}(O(r)) U_R^* = \mathsf{A}(O(r)) \,. \tag{5}$$

For any positive numbers $r_1 < r_2$, we have $\overline{O(r_1)} \subset O(r_2)$. Since the split property (2) holds for $O_j = O(r_j)$ ($j = 1, 2$), the results of [16] show that there is a continuous unitary representation $\check{U}_R$, $R \in \mathrm{SO}(3)$ with the properties

$$\check{U}_R \in \mathsf{A}(O(r_2)) \quad \text{and} \quad \check{U}_R A_1 \check{U}_R^* = U_R A_1 U_R^* \tag{6}$$

for all $A_1 \in \mathsf{A}(O(r_1))$ and all $R \in \mathrm{SO}(3)$. The $\check{U}_R$ are therefore "localized versions" of the unitary implementers U_R of space rotations R.

Note that a space rotation R by any finite angle acts instantaneously and therefore with superluminal speed. To illustrate this to its extreme, let $R_{\pi,3}$ be a rotation by the angle π around e_3, where e_j denotes the space unit vector along the x^j coordinate axis in the $x^0 = 0$ hyperplane. We then obtain, e.g., for positive numbers s and λ such that $s + \lambda < r_1$, and defining

$$O^{(\pm)} = D(B(s) \pm \lambda e_1)\,, \tag{7}$$

that $O^{(\pm)} \subset O(r_1)$ and

$$R_{\pi,3}(O^{(\pm)}) = O^{(\mp)}\,, \tag{8}$$

thus implying

$$\check{U}_{R_{\pi,3}} \mathsf{A}(O^{(\pm)}) \check{U}_{R_{\pi,3}}^* = \mathsf{A}(O^{(\mp)}), \tag{9}$$

which means that the adjoint action of $\check{U}_{R_{\pi,3}}$ rotates the observables localized in $O^{(\pm)}$ "instantaneously" to the localization in $O^{(\mp)}$.

3. Superluminal Localized State Transformations in Quantum Field Theory

Let us recall some further concepts that are relevant to our discussion. In what follows we will consider the density matrix states for the quantum field theory described before. That means, if ϱ is a density matrix operator on $\mathcal{H}$, then

$$\omega(A) = \omega_\varrho(A) = \mathrm{Tr}(\varrho A) \qquad (A \in \mathsf{B}(\mathcal{H})) \tag{10}$$

is the *expectation value functional*—which is synonymously the *state*—induced on $\mathsf{B}(\mathcal{H})$ by ϱ. Since every local observable algebra $\mathsf{A}(O)$ is contained in $\mathsf{B}(\mathcal{H})$, any density matrix state ω as above induces a—partial—state $\omega_{[\mathsf{A}(O)]}(A) = \omega(A)$ $(A \in \mathsf{A}(O))$ on $\mathsf{A}(O)$. It is not convenient to write the subscript to indicate a partial state; thus, we generally will not use it unless ambiguity might arise.

For the concepts we summarize next, we largely adhere to [12,18]. A linear, completely positive map $T : \mathsf{B}(\mathcal{H}) \to \mathsf{B}(\mathcal{H})$ such that $T(\mathbf{1}) = \mathbf{1}$, where $\mathbf{1}$ denotes the unit operator in $\mathsf{B}(\mathcal{H})$, is called a *channel*. (Occasionally, to emphasize the property $T(\mathbf{1}) = \mathbf{1}$, it is called a non-selective channel.) Here, we are exclusively interested in channels of the form

$$T(A) = \sum_{j=1}^{N} V_j A V_j^*\,, \quad V_j \in \mathsf{B}(\mathcal{H})\,, \ \sum_{j=1}^{N} V_j V_j^* = \mathbf{1}\,. \tag{11}$$

for any $A \in \mathsf{B}(\mathcal{H})$, where N is a finite number. A special case is a *unitary channel* $T(A) = UAU^*$ with unitary $U \in \mathsf{B}(\mathcal{H})$. For a causally complete spacetime region $O = (O^\perp)^\perp$, we call a channel *localized in* O if the V_j are contained in $\mathsf{A}(O)$, which entails $T(A) \in \mathsf{A}(O)$ for all $A \in \mathsf{A}(O)$, as well as $T(A') = A'$ for all $A' \in \mathsf{A}(O')$ with $O' \subset O^\perp$. (We caution the reader that this is not necessarily canonical terminology.) The dual of a channel, $\tau : \omega \mapsto \tau(\omega)(\,.\,) = \omega(T(\,.\,))$, is called a (non-selective) *operation*; more generally, an operation maps states to states under a preservation of convex sums. In this paper, we only consider operations that arise as the dualities of channels, thereby mapping density matrix states to density matrix states. An operation is called *unitary* if it is the dual of a unitary channel, and it is called *localized* in a causally complete spacetime region O if the channel to which it is dual is localized in O. Thus, if an operation τ is localized in O, then for any $A' \in \mathsf{A}(O')$ with $O' \subset O^\perp$, and for every density matrix state ω, it holds that $\tau(\omega)(A') = \omega(A')$.

Now, we turn to the situation considered by Sorkin [6], in the form presented in [7]. Thus, we consider three spacetime regions O_{Alice}, O_{Bob} and O_{Charlie}, wherein and during which Alice, Bob and Charlie carry out localized operations and measurements on a density matrix state ω on $\mathsf{B}(\mathcal{H})$. The regions O_{Alice} and O_{Charlie} are causally separated, i.e., $O_{\text{Alice}} \subset O_{\text{Charlie}}^{\perp}$, while the causal future of O_{Alice} as well as the causal past of O_{Charlie} intersect O_{Bob}. In fact, for our argument, we need a sufficient degree of causal overlap, although in concrete quantum field models, when using specific properties of the quantum field, this could be weakened. In greater detail, we take $O_{\text{Bob}} = O(r_2)$ together with the regions $O^{(\pm)} \subset O(r_1)$, as described in the previous section. The causal overlap of O_{Bob} with O_{Alice} and O_{Charlie} is assumed to be such that $O^{(-)}$ is contained in $O_{\text{Alice}} \cap O_{\text{Bob}}$, and $O^{(+)}$ is contained in $O_{\text{Bob}} \cap O_{\text{Charlie}}$ (see Figure 1).

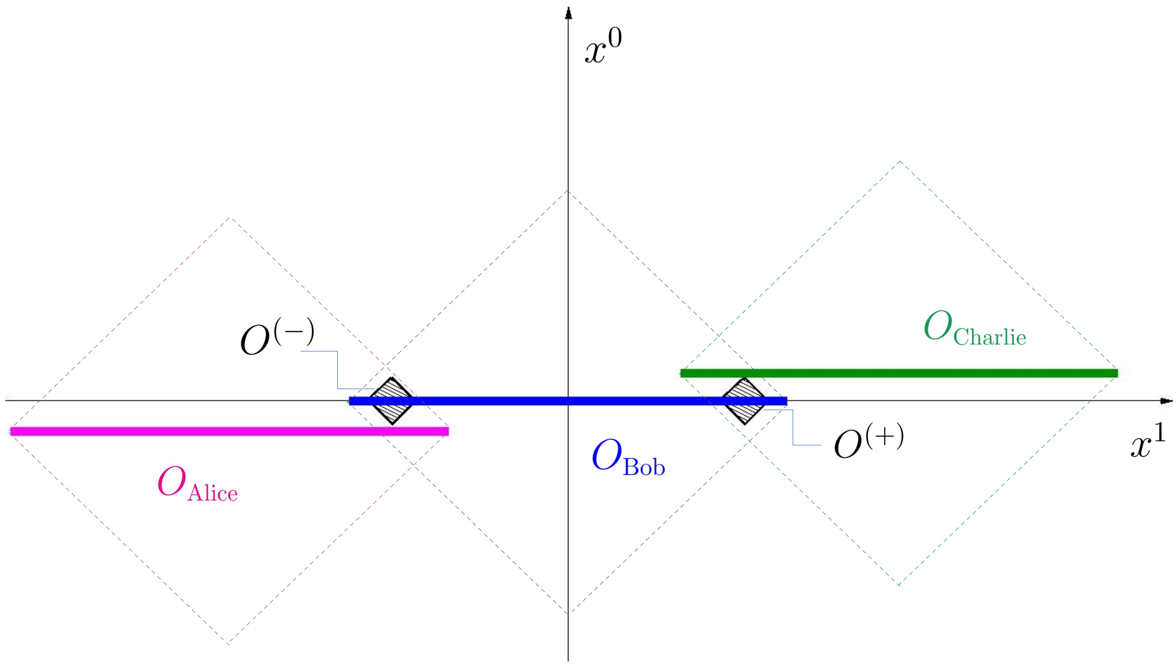

Figure 1. The figure depicts the spacetime regions and their relations described in the text.

With this set-up in place, given any density matrix state ω on $\mathsf{B}(\mathcal{H})$, we assume that Alice carries out a unitary operation τ_{Alice} localized in $O^{(-)}$—which is contained in O_{Alice}—given as

$$\tau_{\text{Alice}}(\omega)(A) = \omega(WAW^*) \tag{12}$$

with some unitary operator $W \in \mathsf{A}(O^{(-)})$. If Charlie carries out a measurement by evaluating any (symmetric) operator C in the state $\tau_{\text{Alice}}(\omega)$, the result is

$$\tau_{\text{Alice}}(\omega)(C) = \omega(WCW^*) = \omega(WW^*C) = \omega(C) \tag{13}$$

since W is unitary and $O_{\text{Alice}} \subset O_{\text{Charlie}}^{\perp}$. This means that Charlie cannot decide, by measurements in O_{Charlie}, if Alice has applied the operation τ_{Alice} localized in O_{Alice}. However, if Bob carries out operations localized in O_{Bob}, this may change. In particular, assume that Bob carries out the operation

$$\tau_{\text{Bob}}(\tilde{\omega})(B) = \tilde{\omega}(\check{U}_{\pi,3} B \check{U}_{\pi,3}^*) \tag{14}$$

on arbitrary density matrix states $\tilde{\omega}$. Since $\check{U}_{3,\pi}$ is a unitary operator in $\mathsf{A}(O_{\text{Bob}})$, τ_{Bob} is a unitary operation localized in O_{Bob}. Thus, for any operator $C \in \mathsf{A}(O^{(+)})$—recall that

$O^{(+)}$ is contained in $O_{\text{Bob}} \cap O_{\text{Charlie}}$—we have $\check{U}_{\pi,3} C \check{U}^*_{\pi,3} \in \mathsf{A}(O^{(-)})$. Consequently, if on any density matrix state ω, Alice first carries out operation τ_{Alice}, and then Bob carries out operation τ_{Bob}, then a measurement by Charlie with $C \in \mathsf{A}(O^{(+)})$ yields

$$(\tau_{\text{Bob}} \circ \tau_{\text{Alice}}\, \omega)(C) = (\tau_{\text{Alice}}\, \omega)(\check{U}_{\pi,3} C \check{U}^*_{\pi,3}) \quad (15)$$
$$= \omega(W \check{U}_{\pi,3} C \check{U}^*_{\pi,3} W^*)\,. \quad (16)$$

Since $C \mapsto \check{C} = \check{U}_{\pi,3} C \check{U}^*_{\pi,3}$ maps the von Neumann algebra $\mathsf{A}(O^{(+)})$ onto the von Neumann algebra $\mathsf{A}(O^{(-)})$, Charlie can, by conducting measurements in $O^{(+)}$, determine if Alice has carried out the operation τ_{Alice} once Bob has carried out the "instantaneous rotation by 180 degrees around the x^3-axis" operation τ_{Bob}—barring the trivial case that W commutes with all operators in $\mathsf{A}(O^{(-)})$—however, for a proper quantum field theory, the local von Neumann algebras are non-commutative, so there is a rich supply of unitary W and self-adjoint $\check{C}$ in $\mathsf{A}(O^{(-)})$ that do not commute. In other words, even if ω is the vacuum state, we will in general have many unitary $W \in \mathsf{A}(O^{(-)})$ and self-adjoint $C \in \mathsf{A}(O^{(+)})$, such that

$$(\tau_{\text{Bob}} \circ \tau_{\text{Alice}}\, \omega)(C) = \omega(W \check{U}_{\pi,3} C \check{U}^*_{\pi,3} W^*) \neq \omega(C)\,. \quad (17)$$

In fact, such unitary operators W and C are guaranteed to exist whenever $\mathsf{A}(O^{(-)})$ is non-commutative. In turn, this is a obviously a consequence of the additivity property of the local von Neumann algebras $\mathsf{A}(O)$ that we have formulated above, and the implicit assumption that we truly have a quantum field theory, i.e., that $\mathsf{B}(\mathcal{H})$ is non-commutative.

We may quickly illustrate the non-commutativity of the local algebras, leading to (17), by means of a simple example related to the linear scalar Klein–Gordon field ([19]). Here, the local von Neumann algebras $\mathsf{A}(O)$ are generated by unitary operators $W(f) = \mathrm{e}^{i\Phi(f)}$, where the real-valued, smooth test-functions f have support in O. The field operators $\Phi(f)$ are the self-adjoint extensions of symmetric operators, defined on the Wightman domain (cf. [19]), fulfilling $\Phi((\Box + m^2)f) = 0$ for some fixed mass term $m \geq 0$, where $\Box$ denotes the d'Alembert operator in Minkowski spacetime. Further properties are

$$W(f)W(h) = \mathrm{e}^{-i\mathscr{G}(f,h)/2} W(h)W(f) \quad (18)$$

for any smooth, compactly supported, real-valued test-functions f, h on Minkowski spacetime $\mathbb{R}^{1+3}$. Here, $\mathscr{G}$ is the causal Green function (or causal propagator) of the Klein–Gordon operator $\Box + m^2$. It arises as

$$\mathscr{G}(f,h) = \int f(x)(\mathcal{G}h)(x)\, d^4x \quad (19)$$

with the causal Green operator $\mathcal{G} = \mathcal{G}^+ - \mathcal{G}^-$ mapping that is a real-valued, compactly supported, smooth test-function f to solutions of the Klein-Gordon equation, i.e., $(\Box + m^2)\mathcal{G}f = 0$, such that the Cauchy data of $\mathcal{G}f$, on any Cauchy-surface, are compactly supported. The causal Green operator is given as the difference of the retarded minus the advanced Green operators, denoted as $\mathcal{G}^\pm$. The vacuum vector Ω can be characterized through

$$(\Omega, W(f)\Omega) = \mathrm{e}^{-w_2(f,f)} \quad (20)$$

with the two-point function

$$w_2(f,h) = (\Phi(f)\Omega, \Phi(h)\Omega) = \frac{1}{2\pi} \int_{\mathbb{R}^3} \overline{\hat{f}(\omega_{\mathbf{p}}, \mathbf{p})}\, \hat{h}(\omega_{\mathbf{p}}, \mathbf{p})\, \frac{d^3\mathbf{p}}{\omega_{\mathbf{p}}}, \quad (21)$$

where the hat denotes a Fourier transform and $\omega_{\mathbf{p}} = \sqrt{|\mathbf{p}|^2 + m^2}$ $(\mathbf{p} \in \mathbb{R}^3)$. The property (18) implies, writing $[A, B] = AB - BA$ for the commutator,

$$\Phi(f) - W(h)\Phi(f)W(h)^* = [\Phi(f), W(h)]W(h)^* = -\mathscr{G}(f,h)\mathbf{1} \quad (22)$$

on the Wightman domain, as can be easily checked. Hence, with $\omega(\,.\,) = (\Omega, \,.\,\Omega)$ denoting the vacuum state, we have the following:

$$\omega([\Phi(f), W(h)]W(h)^*) = -\mathscr{G}(f,h). \tag{23}$$

It is not difficult to verify that, given any open subset O of Minkowski spacetime, there are smooth, real-valued test-functions f and h, having support in O, such that the right-hand side of the last equation is different from 0. Then one can replace $\Phi(f)$ by the sequence of bounded symmetric operators $T_n = (\mathbf{1} + \frac{1}{n}\Phi(f)^2)^{-1}\Phi(f)$ to conclude that, for sufficiently large $n \in \mathbb{N}$, one has

$$\omega([T_n, W(h)]W(h)^*) \neq 0\,. \tag{24}$$

Consequently, if we choose especially $O = O^{(-)}$, and set $W = W(h)$ and $\check{C} = \check{U}_{\pi,3}C\check{U}^*_{\pi,3} = T_n$, we obtain

$$\omega([\check{C}, W]W^*) = \omega(\check{C}) - \omega(W\check{C}W^*) \neq 0\,. \tag{25}$$

On the other hand, we note that, according to the definition of the operators $\check{U}_{\pi,3}$, it holds that $\check{U}_{\pi,3}C\check{U}^*_{\pi,3} = U_{\pi,3}CU^*_{\pi,3}$ for all $C \in \mathsf{A}(O^{(+)})$. Therefore,

$$\omega(W\check{U}_{\pi,3}C\check{U}^*_{\pi,3}W^*) \neq \omega(U_{\pi,3}CU^*_{\pi,3}) = \omega(C), \tag{26}$$

where we used the notion that the vacuum state is invariant under spatial rotations: $U_{\pi,3}\Omega = \Omega$. We have also used the fact that the quantized scalar Klein–Gordon field in a vacuum representation on Minkowski spacetime fulfills all the assumptions that we have listed previously for a quantum field theory, in particular, the split property [20].

4. Superluminal Localized State Transformations in Classical Field Theory

We now wish to demonstrate that similar superluminal localized operations with the—geometrical—significance of "instantaneous spatial rotations" are also present in classical field theory. To this end, we need a description of classical field theory in a local and covariant algebraic setting, in the spirit of the approach of Haag and Kastler [15] for quantum field theory. This has been developed in the recent literature, e.g., see [21–23] and the literature cited therein. However, we are mainly focussing on the example of the classical free Klein–Gordon field on Minkowski spacetime, so we will not need the theory laid out in the mentioned references in full generality. Therefore, we present the approach, mostly following [22] and [23], in a simplified form.

We start by defining $\mathcal{S}$ as the set of all smooth, real-valued solutions to the Klein–Gordon equation on $\mathbb{R}^{1+3}, \mathbb{R}))$. Thus, $(\Box + m^2)\varphi = 0$ holds for every $\varphi \in \mathcal{S}$. Then, we consider the set of all functions $F : \mathcal{S} \to \mathbb{C}$, which forms in the usual way a unital, commutative $*$-algebra by defining the algebraic operations pointwise, i.e., $(aF + G)(\varphi) = aF(\varphi) + G(\varphi)$, $(FG)(\varphi) = F(\varphi)G(\varphi)$, $F^*(\varphi) = \overline{F(\varphi)}$ for all $\varphi \in \mathcal{S}$ ($a \in \mathbb{C}$, overlining means complex conjugation). The algebra of functions on $\mathcal{S}$ possess a unit element, given by $\mathbf{1}(\varphi) = 1$.

In the next step, we define a $*$-subalgebra of the algebra of all functions on $\mathcal{S}$, which is denoted by $\mathscr{P}$. The algebra $\mathscr{P}$ is defined to be algebraically generated by the unit element $\mathbf{1}$ and all linear functionals of the form

$$F_f(\varphi) = \int_{\mathbb{R}^{1+3}} \varphi(x)f(x)\,d^4x, \tag{27}$$

where $f \in C_0^\infty(\mathbb{R}^{1+3}, \mathbb{C})$ is arbitrary. (One can enlarge the algebra $\mathscr{P}$ by taking suitable distributional limits of the f. In the approach presented in [22,23], this is important since it allows, e.g., to include extended algebra elements of the form of $\tilde{F}(\varphi) = \int_{\mathbb{R}^{1+3}} h(x)\varphi(x)^n\,d^4x$. At this point, however, we will not discuss these matters and refer to the references for further discussion.) Then one can also define local $*$-subalgebras by defining for any open subset O of $\mathbb{R}^{1+3}$,

$$\mathscr{P}(O) = *\text{-subalgebra of } \mathscr{P} \text{ generated by } \mathbf{1} \text{ and all } F_f \text{ with } \mathrm{supp}(f) \subset O\,. \tag{28}$$

It is obvious that $O_1 \subset O_2 \Rightarrow \mathscr{P}(O_1) \subset \mathscr{P}(O_2)$. Moreover, if $L \in \mathcal{P}_+^\uparrow$, then setting $\beta_L(F_f)(\varphi) = F_f(\varphi \circ L)$ induces automorphisms of $\mathscr{P}$ such that

$$\beta_L(\mathscr{P}(O)) = \mathscr{P}(L(O))\,. \tag{29}$$

For the functions $\varphi \mapsto P(\varphi)$ in $\mathscr{P}$, one can define the functional derivative $\delta P/\delta\varphi$ by

$$\frac{d}{ds}\bigg|_{s=0} P(\varphi + s\chi) = \int_{\mathbb{R}^{1+3}} \frac{\delta P}{\delta\varphi}(\varphi)(x)\chi(x)\, d^4x, \tag{30}$$

where φ and χ are in $\mathcal{S}$. To give some examples, we have $\delta\mathbf{1}/\delta\varphi = 0$, $\delta F_f/\delta\varphi(\varphi)(x) = f(x)$, and for $P(\varphi) = F_f(\varphi)F_h(\varphi)$, we have $\delta P/\delta\varphi(\varphi)(x) = f(x)F_h(\varphi) + F_f(\varphi)h(x)$. Note that $x \mapsto \delta P/\delta\varphi(\varphi)(x)$ is a smooth, compactly supported function on $\mathbb{R}^{1+3}$, which depends (in general and non-linearly) on φ. With the help of the functional derivative of the elements of $\mathscr{P}$, one can introduce a *Poisson bracket* (or, more appropriately, a *Peierls bracket*) on $\mathscr{P}$, given by

$$\{P_1, P_2\}_{\mathrm{PB}}(\varphi) = \int_{\mathbb{R}^{1+3}} \frac{\delta P_1}{\delta\varphi}(\varphi)(x)\, \mathcal{G}\left(\frac{\delta P_2}{\delta\varphi}(\varphi)\right)(x)\, d^4x \tag{31}$$

for $P_1, P_2 \in \mathscr{P}$. Notice that $\varphi \mapsto \{P_1, P_2\}_{\mathrm{PB}}(\varphi)$ is again in $\mathscr{P}$, and we have the following relations (see [22]):

$$\{P_1, P_2\}_{\mathrm{PB}} = -\{P_2, P_1\}_{\mathrm{PB}}\,, \quad \{P_1, P_2P_3\}_{\mathrm{PB}} = \{P_1, P_2\}_{\mathrm{PB}}P_3 + P_2\{P_1, P_3\}_{\mathrm{PB}}. \tag{32}$$

This is with the algebra product in $\mathscr{P}$, $P_2P_3(\varphi) = P_2(\varphi)P_3(\varphi)$. Additionally, the Poisson bracket also fulfills a Jacobi identity. As a consequence of the causal support properties and the covariance of the causal Green operator $\mathcal{G}$ with respect to the transformations in $\mathcal{P}_+^\uparrow$ (see, e.g., [24,25] and the references cited therein), one furthermore obtains

$$\{P_1, P_2\}_{\mathrm{PB}} = 0 \;\text{ for }\; P_j \in \mathscr{P}(O_j) \;\text{ with }\; O_1 \subset O_2^\perp \tag{33}$$

as well as

$$\{\beta_L(P_1), \beta_L(P_2)\}_{\mathrm{PB}} = \beta_L(\{P_1, P_2\}_{\mathrm{PB}}) \tag{34}$$

for all $L \in \mathcal{P}_+^\uparrow$ and $P_1, P_2 \in \mathscr{P}$.

Hence, we see that the theory of the classical Klein–Gordon field on Minkowski spacetime can be formulated in a very similar way as for the quantized field. The functions $\varphi \mapsto P(\varphi)$ ($\varphi \in \mathcal{S}$) in $\mathscr{P}$ are (simple and polynomial) functions on $\mathcal{S}$, and the space of solutions to the Klein–Gordon equation that have compactly supported Cauchy data. This space of solutions can be identified with the space of Cauchy data of solutions to the Klein–Gordon equation, as we will soon discuss in more detail. The space of Cauchy data naturally corresponds to the phase space for a classical field theory in a Hamiltonian setting, and these can be dynamically described with the help of the Poisson bracket (see [26] for further discussion). The elements in $\mathscr{P}$ are functions on the phase space; hence, if real-valued, they correspond to simple observables for the classical Klein–Gordon field. (As mentioned, the set of observables could be enlarged by taking the suitable limits of elements $P \in \mathscr{P}$.) Since it is a classical field theory, the observable algebra is commutative. In the spirit of [15]—who advocated that, in relativistic field theory, the observables should be localized and covariant—we also have the local algebras $\mathscr{P}(O)$ of observables, which can be measured within the spacetime regions O, as well as the actions of the Poincaré transformations by automorphisms with the covariance property (29).

Also, for the unital $*$-algebra $\mathscr{P}$, the states are linear functionals $\nu : \mathscr{P} \to \mathbb{C}$ which are positive, $\nu(P^*P) \geq 0$ (and commonly also normalized, $\nu(\mathbf{1}) = 1$). States may arise through suitable measures μ on $\mathcal{S}$ (assuming suitable additional structure needed for defining tmeasures has been put in place) as integrals

$$\nu(P) = \int_{\mathcal{S}} P(\varphi)\, d\mu(\varphi), \tag{35}$$

and for any arbitrarily chosen φ_0 in $\mathcal{S}$, the Dirac measure $\delta_{\varphi_0}(P) = P(\varphi_0)$ is an example. We shall, however, not discuss this matter further.

For any given Cauchy-surface Σ in $\mathbb{R}^{1+3}$, with the future-pointing unit-normal vector field n^μ along it, the Cauchy data of any $\varphi \in \mathcal{S}$ on Σ are defined by

$$u_\varphi = \varphi|_\Sigma \quad \text{and} \quad v_\varphi = n^\mu \frac{\partial}{\partial x^\mu} \varphi\bigg|_\Sigma. \tag{36}$$

We define $\mathcal{S}_0$ as the subset of all $\varphi \in \mathcal{S}$ so that the Cauchy data have compact support, meaning that u_φ and v_φ are in $C_0^\infty(\Sigma, \mathbb{R})$. One can show that this property is independent of the choice of the Cauchy-surface Σ, and that $\mathcal{S}_0$ equals $\mathcal{G}(C_0^\infty(\mathbb{R}^{1+3}, \mathbb{R}))$, the image of all smooth, compactly supported test-functions under the causal Green operator [24,25].

Furthermore, $\mathcal{S}_0$ carries a canonical symplectic form σ, which is given by

$$\sigma(\varphi, \psi) = \int_\Sigma (u_\varphi v_\psi - v_\varphi u_\psi)\, d\mathrm{vol}_\Sigma, \tag{37}$$

where $d\mathrm{vol}_\Sigma$ denotes the metric-induced volume element on Σ. It is worth noting that the symplectic form σ is independent of the choice of Σ. For a proof of these properties and for further facts, which we will use about the symplectic structure of the space of solutions to the Klein–Gordon equation and the relation to the Green operator below, see e.g., [24–26] and the references cited therein. According to the definition of $\mathcal{S}_0$, there is for every $\varphi \in \mathcal{S}_0$ some $f \in C_0^\infty(\mathbb{R}^{1+3}, \mathbb{R})$ so that $\varphi = \mathcal{G}f$. In fact, the map $C_0^\infty(\mathbb{R}^{1+3}, \mathbb{R})/\ker(\mathcal{G}) \to \mathcal{S}_0$ given by $[f] = f + \ker(\mathcal{G}) \mapsto \mathcal{G}f$, is a linear bijection, and it is also a symplectomorphism upon endowing $C_0^\infty(\mathbb{R}^{1+3}, \mathbb{R})/\ker(\mathcal{G})$ with the symplectic form

$$\kappa([f],[h]) = \int_{\mathbb{R}^{1+3}} f(x)(\mathcal{G}h)(x)\, d^4x\,. \tag{38}$$

Now let us return to the geometric situation that we have been considering in Figure 1. Our aim is to construct localized rotations of the system of local Poisson algebras $\mathscr{P}(O)$ that preserve the Poisson structure. More precisely, by choosing positive radii $r_1 < r_2$, we have $\overline{B(r_1)} \subset B(r_2)$ for the coordinate balls at x^0 defined by (3) and, similarly for their domains of dependence, defined by (4), $\overline{O(r_1)} \subset O(r_2)$. In the $x^0 = 0$ hyperplane which is a copy of $\mathbb{R}^3$, we will introduce for any $0 \leq \theta < 2\pi$ a diffeomorphism γ_θ that acts like a rotation around the x^3-axis by an angle of θ within $B(r_1)$, and like the identity outside $B(r_2)$. To this end, we consider the vector field $\boldsymbol{f}$ on $\mathbb{R}^3$ given by

$$\boldsymbol{f} = \eta(r)\left(x^1 \frac{\partial}{\partial x^2} - x^2 \frac{\partial}{\partial x^1}\right), \tag{39}$$

where $r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$ is the radius function and $\eta : \mathbb{R}_+ \to \mathbb{R}_+$ is a smooth function with $\eta(r) = 1$ for $r \leq r_1$ and $\eta(r) = 0$ for $r \geq r_2$. Then we take $\gamma_\theta : \mathbb{R}^3 \to \mathbb{R}^3$ to be the flow generated by $\boldsymbol{f}$ with a flow parameter θ (so $d\gamma_\theta/d\theta = \boldsymbol{f} \circ \gamma_\theta$). It it easy to see that γ_θ has the claimed geometric properties. In the next step, we define the linear map $S_\theta : \mathcal{S} \to \mathcal{S}$ by choosing the Cauchy-surface Σ in (36) as the $x^0 = 0$ hyperplane, and by setting

$$\begin{pmatrix} u_{S_\theta \varphi} \\ v_{S_\theta \varphi} \end{pmatrix} = \begin{pmatrix} u_\varphi \circ (\gamma_\theta)^{-1} \\ q_\theta v_\varphi \circ (\gamma_\theta)^{-1} \end{pmatrix}, \tag{40}$$

where $q_\theta = \det(J_\theta)$ with J_θ is the Jacobian matrix of γ_θ. It is plain to see that, due to the compensating factor q_θ, one has

$$\sigma(S_\theta \varphi, S_\theta \psi) = \sigma(\varphi, \psi) \tag{41}$$

for all $\varphi, \psi \in \mathcal{S}_0$; hence, S_θ is a symplectomorphism on the solution space $\mathcal{S}_0$ for the Klein–Gordon equation with the symplectic form σ. Note that $q_\theta = 1$ is on $B(r_1)$, as well as outside of $B(r_2)$.

In a further step, we wish to show that the map S_θ induces a unit-preserving $*$-algebra morphism Υ_θ of $\mathscr{P}$ through

$$(\Upsilon_\theta P)(\varphi) = P(S_\theta^{-1} \varphi) \tag{42}$$

such that the Poisson bracket is preserved,

$$\{\Upsilon_\theta(P_1), \Upsilon_\theta(P_2)\}_{\mathrm{PB}} = \Upsilon_\theta(\{P_1, P_2\}_{\mathrm{PB}})\,. \tag{43}$$

In the light of relations (32), it is enough to check the preservation of the Poisson bracket for the cases $P_j = F_{f_j}$. To this end, if $f_j \in C_0^\infty(\mathbb{R}^{1+3}, \mathbb{R})$, and if $h \in C_0^\infty(\mathbb{R}^{1+3}, \mathbb{R})$ is chosen with $\varphi = \mathcal{G}h$ on the support of f_j, then

$$F_{f_j}(\varphi) = \int_{\mathbb{R}^{1+3}} f_j(x) \mathcal{G}h(x)\, d^4x = \kappa([f_j], [h]) = \sigma(\mathcal{G}f_j, \varphi). \tag{44}$$

Hence, by setting $\psi_j = \mathcal{G}f_j$, it follows that $\Upsilon_\theta F_{f_j}(\varphi) = F_{f_j}(S_\theta^{-1}\varphi) = \sigma(\psi_j, S_\theta^{-1}\varphi) = \sigma(S_\theta \psi_j, \varphi)$ since S_θ is a symplectomorphism. On the other hand, we have

$$\{F_{f_1}, F_{f_2}\}_{\mathrm{PB}}(\varphi) = \int_{\mathbb{R}^{1+3}} f_1(x) \mathcal{G} f_2(x)\, d^4x = \kappa([f_1], [f_2]) = \sigma(\psi_1, \psi_2) \tag{45}$$

from which one can now deduce

$$\{\Upsilon_\theta F_{f_1}, \Upsilon_\theta F_{f_2}\}_{\mathrm{PB}}(\varphi) = \sigma(S_\theta \psi_1, S_\theta \psi_2) = \sigma(\psi_1, \psi_2)\,. \tag{46}$$

On the other hand, since $\{F_{f_1}, F_{f_2}\}_{\mathrm{PB}}(\varphi) = \sigma(\psi_1, \psi_2)$ is independent of φ (i.e., it is a multiple of the unit element in $\mathscr{P}$), we have $\Upsilon_\theta(\{F_{f_1}, F_{f_2}\}_{\mathrm{PB}}) = \{F_{f_1}, F_{f_2}\}_{\mathrm{PB}}$. Hence, we obtain

$$\{\Upsilon_\theta F_{f_1}, \Upsilon_\theta F_{f_2}\}_{\mathrm{PB}} = \Upsilon_\theta(\{F_{f_1}, F_{f_2}\}_{\mathrm{PB}}), \tag{47}$$

as required so as to show that Υ_θ is a $*$-algebra morphism of $\mathscr{P}$, thus preserving the Poisson structure. It is also easy to see from the geometric construction that $\Upsilon_\theta P = P$ for all $P \in \mathscr{P}(\tilde{O})$ with $\tilde{O} \subset O(r_2)^\perp$, and

$$\Upsilon_\theta(\mathscr{P}(O)) = \mathscr{P}(R_{3,\theta} O) \tag{48}$$

is for all $O \subset O(r_1)$, where $R_{3,\theta}$ denotes the space rotation around the x^3-axis by the angle θ.

Hence, for the classical Klein–Gordon field on Minkowski spacetime, as described in the algebraic setting in terms of local Poisson algebras, Υ_θ is a local channel, acting trivially in the causal complement of $O(r_2) = O_{\mathrm{Bob}}$, and it is like an "instantaneous" space rotation within $O(r_1)$. Thus, in the situation depicted in Figure 1, the operation

$$\tilde{\tau}_{\mathrm{Bob}} \nu = \nu \circ \Upsilon_\pi \tag{49}$$

on the states ν of $\mathscr{P}$ is the counterpart of τ_{Bob} in (14), which we had considered before in the quantum field theory framework. Obviously, $\tilde{\tau}_{\mathrm{Bob}}$ is not provided by the action of unitary algebra elements since the algebra $\mathscr{P}$ is commutative. Thus, whenever $G \in \mathscr{P}$ (or, for that matter, on replacing $\mathscr{P}$ by a suitable extension) fulfills $GG^* = \mathbf{1}$, then $GPG^* = P$ for all $P \in \mathscr{P}$.

This said, it should now be clear that there are also channels Υ_{Alice} for $\mathscr{P}$ which are localized in O_{Alice} (i.e., they act trivially in the causal complement of O_{Alice}), so that for their induced operations $\tilde{\tau}_{\text{Alice}}$ given by $\tilde{\tau}_{\text{Alice}}\nu = \nu \circ \Upsilon_{\text{Alice}}$, we find

$$\tilde{\tau}_{\text{Alice}}\nu(C) = \nu(C) \tag{50}$$

for all states ν of $\mathscr{P}$ and all $C \in \mathscr{P}(O_{\text{Charlie}})$, while

$$(\tilde{\tau}_{\text{Bob}} \circ \tilde{\tau}_{\text{Alice}}\nu)(C) \neq \nu(C) \tag{51}$$

for some states ν and suitable $C \in \mathscr{P}(O^{(+)})$ (cf. Figure 1). For instance, one can choose for Υ_{Alice} a rotation around some space axis in O_{Alice}, constructed in the same manner as Υ_θ with respect to O_{Bob}. In other words, we have provided, in an algebraic setting for a classical, relativistic field theory, an example of an "impossible measurement scenario" where, according to [6], the information if Alice has carried out an operation in her lab is mediated by an operation in Bob's lab with "superluminal speed" to the lab of Charlie which is causally separated from Alice.

5. Discussion

We have shown that in the algebraic framework (both in quantum field theory—under very general assumptions, as well as more concretely for the quantized Klein–Gordon field—and in classical field theory (for the classical Klein–Gordon field), "superluminal localized operations" τ_{Bob} occur. They have a geometric significance as "instantaneous space rotations" by 180 degrees, and they lead to the scenario in which [6] has been connected with the "impossible measurements scenario", where (cf. Figure 1) Charlie can tell if Alice has carried out an operation on a state ω if Bob carries out τ_{Bob} localized in O_{Bob} through the relation

$$(\tau_{\text{Bob}} \circ \tau_{\text{Alice}}\omega)(C) \neq \omega(C) \tag{52}$$

for some states ω and some observables C measured by Charlie in O_{Charlie}. In this sense, at face value, the "impossible measurements scenario" in [6] *fails the ping-pong ball test* in the sense that it is not a feature of quantum field theory only, but also occurs in classical field theory.

That is not to say, however, that the scenario presented in [6] was without interest or significance. In fact, various interesting lessons can be learned by having subjected it to our ping-pong ball test.

First, we see that, as pointed out in [6] and [3,5], localized operations, both in quantum field theory and in classical field theory, are only specified by acting trivially in the causal complement of the spacetime region wherein they are localized, but they can act superluminally within that localization region. As we have seen, this includes (unsurprisingly) "passive" transformations which are related to the (local) symmetries of (the theory of) a physical system. However, carrying them out "instantaneously" is actually impossible on kinematical or dynamical grounds. What can really be carried out in a lab on a physical system must be brought about by interaction, and in a relativistic theory, it must respect the principle that "no action on a system can proceed faster than with the velocity of light", i.e., it cannot lead to superluminal effects. In the local, algebraic setting of quantum field theory, or of classical field theory, one could think of various ways of capturing this principle. A quite strong requirement on operations τ to be physical could be that they should arise as duals of channels T which obey

$$T(\mathsf{A}(O)) \subset \mathsf{A}(J(O)) \tag{53}$$

for *all* subsets O of Minkowski spacetime, where $J(O)$ is the causal set of O, i.e., the set of all points that lie on causal curves emanating from O. In [27], successions of Fermi–Walker-transported observables $t \mapsto \alpha_{L_t}(A)$ for $A \in \mathsf{A}(O)$ have been considered, where $\{L_t\}_{t \in I}$, with I real interval, is a smooth family of Poincaré transformations such that, for every $x \in O$, $t \mapsto L_t(x)$ is a future-directed, causal curve. One could attempt to restrict the possibility of instantaneous rotations (or other instantaneous Poincaré transformations) in a similar manner. (For the discussion of other, related restrictions on local operations in order to prevent them from acting in a superluminal fashion, see e.g., [3,5].)

While investigating useful kinematical characterizations of the local operations compatible with the principles of special or general relativity is important—and may actually remove a gap in the literature on localized operations—we think that what is really of prime importance is the aspect that, in the lab, the experimenter carries out "active" operations, i.e., operations that involve interactions with the physical system under consideration. In the framework of quantum field measurement set out in [8,9], the system under consideration, described by a quantum field, interacts with another quantum field, modeling the probe. The interaction is subject to specific conditions on localization and causality that, as a consequence, avoid the impossible measurement scenario for the operations resulting from the interaction of system and probe [7]. Imposing suitable locality and causality conditions on interactions is also of importance in the construction of interacting quantum field theories (see [28,29] for a recent contribution in this direction, as well as related discussion in [22,23]).

A second lesson that may be drawn is about the status of the unitary elements U of the local algebras $\mathsf{A}(O)$ in quantum field theory as operations, or more precisely, as giving rise to the channels $A \mapsto UAU^*$ that induce local operations. As we have mentioned already, this does not match too well with how local operations arise in the algebraic framework of classical field theory because the algebras $\mathscr{P}(O)$ are commutative. However, in classical field theory, the action of the generators of (local) symmetries can be obtained with the help of the Poisson bracket and the elements G of the (local) Poisson algebras, i.e., as derivations of the form $P \mapsto \{G, P\}_{\mathrm{PB}}$ [30]. Similarly in quantum field theory, the commutator bracket with (typically unbounded) operators Q affiliated to the local algebras $\mathsf{A}(O)$ gives rise to the derivations $A \mapsto [Q, A]$, thereby generating (local) symmetries. This analogy is very familiar when discussing the passage from Hamiltonian mechanics to quantum mechanics; therefore, in comparison with the classical field theory situation, the operators affiliated with local algebras should be seen as the generators, in the commutator bracket, of local channels. The circumstance that, in quantum field theory, the corresponding channels are actually implemented by unitary operators U in the local algebras $\mathsf{A}(O)$ is perhaps more a consequence of the richness of the $\mathsf{A}(O)$ and less related to an a priori significance of unitaries in the local algebras as implementers of local channels and their associated operations.

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Note

1 In [12], the ping-pong ball test is more specifically related to Bell's inequalities, and its wording is verbatim as follows: *Take an author's explanation of Bell's inequalities, and substitute "ping-pong balls" for every quantum particle. Then if whatever the author is selling as paradoxical, remains true, he/she hasn't understood a thing.*

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