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Review

Plasmas Containing Quasimonochromatic Electric Fields (QEFs): Review of the General Principles of Their Spectroscopy and Selected Applications

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Abstract: We review the general principles of the spectroscopy of plasmas containing quasimonochromatic electric fields (QEFs). We demonstrate that the underlying physics is very rich due to the complicated entanglement of four characteristic times: the typical time required for the formation of the quasienergy states, the lifetime of the excited state of the radiator, the typical time of the formation of the homogeneous Stark broadening by the electron microfield, and the typical time of the formation of the homogeneous Stark broadening by the dynamic part of the ion microfield. We exemplified how the shape and shift of spectral lines are affected by the mutual interactions of the three subsystems. Specifically, the interaction of the radiator with the plasma can be substantially influenced by the interaction of the radiator with the QEF, and vice versa, as well as by the interaction of the QEF and the plasma with each other. We also provide some applications of these various effects. Finally, we outline directions for future research.

Keywords: quasimonochromatic electric fields; Langmuir wave-caused "dips" in hydrogenic lines;Langmuir wave-caused satellites in non-hydrogenic lines; laser field-caused satellites; electron oscillatory shift; nonlinear optical phenomena

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1. Introduction

There are two types of electric fields in laboratory and astrophysical plasmas: the fields differ in terms of the relative width $\delta\omega/\omega$ of their power spectra. These two types are broadband electric fields ($\delta\omega/\omega \gtrsim 1$) and quasimonochromatic electric fields (QEFs, $\delta\omega/\omega \ll 1$). They are also distinguished by their effects on radiating atoms and ions (hereafter, radiators) in plasmas.

From the radiator point of view, in the absence of QEFs, plasma is the source of broadband electric fields of various frequency ranges. Namely, in addition to the individual broadband electric fields, caused by the random thermal motion of plasma ions and electrons, there could also be collective broadband electric fields, as seen in electrostatic plasma turbulence. One example of the latter is ion acoustic waves, which lead to the anomalous resistivity of some laboratory and astrophysical plasmas (also, Bernstein modes can cause anomalous resistivity—see, e.g., review [1]). These waves are of relatively low frequencies in terms of the band $(0, \omega_{pi} = (4\pi e N_i/m_i)^{1/2})$, where the latter quantity is the ion plasma frequency controlled by the ion mass m_i and density $N_i = N_e/Z$ (here, N_e is the electron density and Z is the charge of plasma ions).

There are a variety of spectroscopic methods for determining the electron density N_e, the electron temperature T_e, the ion temperature T_i, and the average field of the electrostatic plasma turbulence. These are based on the Stark broadening of spectral lines by the broadband electric fields—see, e.g., books [2,3]. Typically, a significant part of the ensemble of the ion microfield, as well as the low-frequency electrostatic plasma turbulence, can be perceived by the radiator as quasistatic.

In addition, some other quasistatic electric fields can "occur" in plasmas. For example, in a tokamak plasma, as a neutral beam is injected into it perpendicular (or quasi-perpendicular) to the magnetic field **B**, the radiator perceives the Lorentz electric field $\mathbf{F} = \mathbf{v} \times \mathbf{B}/\mathbf{c}$ in the order of 10^2–10^3 kV/cm (see, e.g., papers [4,5]).

By contrast, under the QEF, the time evolution of the radiator can become dynamic, even if there are some relaxation processes. This results in new structures in the spectral line profiles, with satellites forming in the simplest cases and "bump-dip-bump" structures emerging in more complicated situations—see, e.g., books [3,6].

In more detail, the situation is as follows. The energy spectrum of the radiator is represented by a set of multiplets of the characteristic separation ω_0 from each other. The microstructure of each multiplet has the characteristic scale $\Delta \ll \omega_0$. While typically, the frequency ω of the QEF is such that $\omega \ll \omega_0$, the ratio ω/Δ can have any value: it can be much smaller than unity, of the order of unity, or much greater than unity.

There are three interacting subsystems: the plasma (P) (represented, e.g., by the quasistatic electric field), the EQF (F), and the radiator (R). We denote their interactions as V_{RF}, V_{RP}, and V_{FP}. The observed spectral lines correspond to the radiative transitions at the frequency of ~ ω_0. This electromagnetic radiation serves as a probe, interacting with the perturbed microstructure of the multiplets and carrying the information about the plasma parameters and the parameters of the QEF.

We note that the overwhelming majority of practical applications correspond to the above situation, where $\omega \ll \omega_0$. The theory of spectral line broadening in the opposite case, i.e., where $|\omega_0 - \omega|/\omega_0 \ll 1$, was applied in papers [7,8].

Structures in spectral line profiles, caused by the QEF, were observed and used for diagnostics at various plasma sources: z-pinches [9], a gas-liner pinch [10], edge plasmas of tokamaks [11], clusters subjected to intense laser radiation [12], nanosecond ([13,14]) and picosecond ([15,16]) laser plasmas, and laser plasmas acted upon by the file of an additional laser [17,18], as well as in relativistic laser–plasma interactions [19,20]. In most cases, the Langmuir wave-caused "bump-dip-bump" structures were identified and utilized for diagnostic purposes.

The QEFs of plasmas can be represented either by a *single-mode* field $\mathbf{E}(t) = \mathbf{E_0} \cos(\omega t + \varphi)$ or a *multi-mode* field $\mathbf{E}(t) = \Sigma_k \mathbf{E_{0k}} \cos(\omega t + \varphi_k)$. In the latter case, the phase φ_k is uniformly distributed in the interval $(0, 2\pi)$.

To measure parameters of the QEF in plasmas, the interactions of the three subsystems (the plasma, the radiator, and the QEF) play crucial roles. In the first theoretical publications on the subject, only the interaction V_{RF} of the subsystems R and F was analyzed. Specifically, Blochinzew [21] in 1933 and Lifshitz [22] in 1967 calculated analytically how hydrogenic spectral lines are affected by the single-mode QEF [21] or by the multi-mode QEF [22]. Baranger and Mozer [23] in 1961, and Cooper and Ringler [24] in 1969, calculated analytically how non-hydrogenic spectral lines (namely, He lines) are affected by multi-mode QEFs [23] or by the single-mode QEF [24]. The results obtained in papers [21–24] were various combinations of satellites separated from the main spectral line by the QEF frequency ω and its multiples. We note that the analytical solutions in papers [21,22] were exact (non-perturbative), while the analytical solutions in papers [23,24] were obtained within the standard time-dependent perturbation theory, so that they were valid only for the relatively weak QEF.

For non-hydrogenic spectral lines, in 1983 in [25] the authors obtained analytical results valid for the stronger QEF, for which the standard perturbation theory fails. These results, obtained by the adiabatic perturbation theory, were then used for the diagnostics of the QEF in plasmas for the first time in 1983 in a paper by Brizhinev et al. [26].

For hydrogenic spectral lines, the effect of the relatively strong QEF with allowance for the fine structure (the fine structure being disregarded in papers [21,22]) was presented in 1983 in [27]. These results were obtained analytically beyond the standard perturbation theory.

Theoretical works, where the authors analyzed the combined effect of the *quasistatic* part of the interaction V_{RP} and of the interaction V_{RF}, were restricted (in terms of the V_{RP}) to the radiator interacting with the low-frequency electrostatic plasma turbulence and/or with the quasistatic part of the ion microfield. In 1972, Cohn et al. [28] presented numerical simulations, while in 1977–1987 another group of authors [29–31] published analytical results. Specifically, in [29–31] it was shown that under the combined action of the quasistatic part of the interaction V_{RP} and of the interaction V_{RF}, there occur multifrequency nonlinear dynamic resonances resulting in the "bump-dip-bump" structures (called "Langmuir dips" for brevity) in the profiles of the hydrogenic spectral lines.

These Langmuir dips in hydrogenic spectral lines profiles were first observed and identified in experiment [29] and then in many other experiments at various plasma sources over the range of the electron density from ~ 10^{13} cm^{-3} to ~ 10^{22} cm^{-3} (see, e.g., chapter 7 of book [6] and reviews [32,33]). Moreover, it was found that from the experimental positions of the Langmuir dips, it is possible to determine the electron density just as accurately as with the help of the more complicated experimental technique based on Thomson scattering [10].

As for the *non-resonant* effects of the combined influence of the quasistatic part of the interaction V_{RP} with the interaction V_{RF}, they were studied analytically in papers [11,34]. Namely, in paper [11], it was found that the QEF can partially suppress the quasistatic Stark broadening, the effect being anisotropic. In the same paper [11], this effect was also confirmed experimentally by the example of the deuterium spectral lines. In paper [34], dealing with the satellites of the dipole-forbidden non-hydrogenic spectral lines, the outcome was a very substantial modification of satellite intensities.

The most important is the following. All of the above theoretical studies were useful for the spectroscopic diagnostics of the QEF in various plasmas, but they were based on simplified setups. In 1984, in [35] the authors performed a comprehensive study of the actual situation in its entire complexity. Namely, they expounded the *general principles* of the spectroscopy of plasmas containing the QEF, and these principles were exemplified by several applications. A more detailed paper [36] on these general principles was published in 1986.

The primary idea of papers [35,36] was reiterated in paper [37] as follows: "*The spectral line broadening, caused by the interaction of the radiator with one of the subsystems (P or F) can be significantly affected by the interaction of the radiator with the other subsystem, as well as by the interaction of P and F with each other. Generally, each of the interactions V_{RP} and V_{RF} should not be considered in isolation from the other one because their characteristic temporal parameters can "entangle" and thus couple these interactions. In other words, the conventional approach, where one would first calculate two independent profiles of the same spectral line (one—caused by V_{RP}, another—cause by V_{RF}) and then perform the convolution of the two profiles, would generally yield incorrect results.*"

The consequences of this idea were the leitmotifs of book [6]. The book presented the various ramifications of this idea in applications to spectral line profiles of hydrogen, deuterium, helium lithium, hydrogenlike ions, He-like ions, and Li-like ions, examining different schemes of interactions between the three subsystems: the QEF, the plasma, and the radiator.

The present paper is motivated by the following situation. The general principles of the spectroscopy of plasmas containing the QEF were developed in paper [35], which was published long ago (in 1984) in one of the Soviet physics journals. For these reasons, it remained largely unknown in the West, especially to the last two generations of physicists. (The more detailed paper [36] was published in a Soviet journal even less known in the West than the journal where paper [35] was published.) As a consequence, in a number of publications in recent years in the West, the authors at best tried to "reinvent the wheel" or at worst made significant conceptual errors.

Therefore, in the present paper, we offer the *most detailed description* of the general principles of the spectroscopy of plasmas containing the QEF. We also bring to the attention of the readers various *nonlinear phenomena* resulting from the application of these general principles.

2. The General Principles

The broadening of spectral lines in plasmas containing the QEF depends on a large set of characteristic times and frequencies, resulting in a very complex, rich physics. It is possible to single out seven characteristic frequencies, serving as "building blocks" for more complicated constructs.

1. The frequency ω of the QEF.
2. The width (the homogeneous width)

$$\gamma = 1/\tau_F \quad (1)$$

of the power spectrum of the QEF, where τ_F is the coherence time of the QEF.

3. The instantaneous Stark shift $\delta_s(E_0)$, calculated at the amplitude value E_0 of the QEF, the shift being either

$$\delta_s(E_0) = a_1 E_0 \quad (2)$$

for the linear Stark effect or

$$\delta_s(E_0) = a_2 E_0{}^2 \quad (3)$$

for the quadratic Stark effect. In Equations (2) and (3), $a_1(k)$, $a_2(k)$ are the corresponding Stark constants; k denotes the set of quantum numbers of the radiator states.

4. The typical frequency of the variation of the electron microfield

$$\Omega_e(N_e, T_e) = v_{Te}/\min(\rho_{Ne}, \rho_{We}) \quad (4)$$

In Equation (4), $v_{Te} = (T_e/m_e)^{1/2}$ is the thermal velocity of plasma electrons, $\rho_{Ne} \sim 1/N_e{}^{1/3}$ is the mean separation between plasma electrons, and $\rho_{We} \sim n^2\hbar/(m_e v_{Te})$ is the electron Weisskopf radius, with n being the principal quantum number.

5. The typical frequency of the variation of the dynamic part of the ion microfield

$$\Omega_i(N_i, T_i) = v_{Ti}/\min(\rho_{Ni}, \rho_{Wi}) \quad (5)$$

In Equation (5), $v_{Ti} = (T_i/m_i)^{1/2}$ is the thermal velocity of plasma electrons, $\rho_{Ni} \sim 1/N_i{}^{1/3}$ is the mean separation between plasma ions, and $\rho_{We} \sim n^2\hbar/(m_e v_{Ti})$ is the ion Weisskopf radius.

6. The electron plasma frequency

$$\omega_{pe}(N_e) = (4\pi e^2 N_e/m_e)^{1/2} = 1/\tau_{scr} \quad (6)$$

In Equation (6), τ_{scr} is the typical time, after which the screening by plasma electrons becomes effective.

7. The detuning $\Delta\omega$ from the unperturbed position of the radiator spectral line under consideration. This physical quantity influences the typical value of the argument τ of the correlation function, the Fourier transform of which controls the lineshape.

The term "homogeneous", used above, means that the particular quantity is the same for all radiators. The frequencies from the above items 1–3 relate to the subsystem F and its interaction with the subsystem R. The frequencies from the above items 4–6 relate to the subsystem P and its interaction with the subsystem R.

There are four characteristic times that can be derived from the above seven building blocks.

1. The typical time of the formation of quasienergy states:

$$\tau_{QS}(k, E_0, \omega) \sim \min(1/(\omega^2\, \delta_s)^{1/3}, 1/\omega) \quad (7)$$

Under the QEF, the radiator states can exhibit oscillatory behavior at the frequency ω, thus constituting the development of quasienergy states (introduced by Zeldovich [38] and Ritus [39]). Equation (7), derived in paper [35], shows that while for the relatively weak

QEF, the quasienergy states become effective after the typical time $1/\omega$, for the relatively strong QEF, the quasienergy states become effective after the much shorter typical time proportional to $1/E_0^{1/3}$ (for the linear Stark effect) or proportional to $1/E_0^{2/3}$ (for the quadratic Stark effect).

2. The typical time of the formation of the homogeneous Stark broadening by electrons:

$$\tau_e(k, N_e, T_e, \Delta\omega) \sim \min(1/\Omega_e, 1/\omega_{pe}, 1/\Delta\omega) \quad (8)$$

3. The typical time of the formation of the homogeneous Stark broadening by plasma ions:

$$\tau_i(k, N_i, T_i, N_e, \Delta\omega) \sim \min(1/\Omega_i, 1/\omega_{pe}, 1/\Delta\omega) \quad (9)$$

4. The lifetime of the exited state of the radiator

$$\tau_{life}(k, N_e, T_e, N_i, T_i, \gamma, \omega, E_0, \Delta\omega) \sim 1/\Gamma \quad (10)$$

where

$$\Gamma = \gamma_e(k, N_e, T_e, \Delta\omega) + \gamma_i(k, N_i, T_i, N_e, \Delta\omega) + \gamma_F(k, \gamma, \omega, E_0) \quad (11)$$

is the sum of the homogeneous Stark widths due to the electron microfield, to the dynamic part of ion microfield, and to the QEF, respectively. The QEF-caused contribution $\gamma_F(k, \gamma, \omega, E_0)$ was calculated analytically in [40].

It should be underscored that τ_{QS} entangles the parameters of the subsystems QEF and the radiator; τ_e and τ_e entangle the parameters of the subsystems the plasma and the radiator; while τ_{life} entangles the parameters of all three subsystems: the plasma, the QEF, and the radiator.

3. Selected Applications

Below are several illustrative situations, dealing with plasmas containing the QEF, where the interplay of these four characteristic times determines various outcomes of the Stark broadening of spectral lines.

First, let a hydrogenic spectral line be influenced by the QEF $\mathbf{E}(t) = \Sigma_k \mathbf{E_{0k}} \cos(\omega t + \varphi_k)$ in a plasma. (The phase φ_k is uniformly distributed in the interval $(0, 2\pi)$.) Each Stark component of the spectral line can exhibit satellites at the locations $\Delta\omega_p = p\omega$ ($p = 0, \pm1, \pm2, \ldots$). The satellites manifest the occurrence of the quasienergy states of the radiator. Formally, the number of satellites is infinite. However, under the condition

$$\tau_{QS} >> 1/\Delta\omega \quad (12)$$

there is not enough time for the quasienergy states to form because in the correlation function the relevant time will be

$$\tau \sim 1/\Delta\omega << \tau_{QS} \quad (13)$$

In this case, the number of satellites, instead of being infinite, will be limited by the following number

$$p_{max} \sim 1/(\omega\tau_{QS}) \quad (14)$$

Equation (14) stems from the requirement $\tau_{QS} \leq 1/\Delta\omega = 1/(p\omega)$.

As for the satellites intensities, there are two options depending on the interrelation between and the coherence time $\tau_F \sim 1/\gamma$ of the QEF and the lifetime τ_{life} of the excited state of the radiator. Namely, if

$$\tau_{life} << \tau_F \quad (15)$$

the radiator "feels" the QEF as single-mode field $\mathbf{E}(t) = \mathbf{E_0} \cos(\omega t + \varphi)$, so that the intensities of the satellites are described by the Blochinzew result [21]:

$$I(p, \mu) = J_{|p|}{}^2(\mu), \qquad \mu = \delta_s(E_0)/\omega \tag{16}$$

In Equation (16), $J_p(\mu)$ are the Bessel functions.

In the opposite situation, where

$$\tau_{life} >> \tau_F \tag{17}$$

the radiator perceives the multi-mode nature of the QEF, so that the intensities of the satellites are described by Lifshitz result [22]:

$$I(p, \mu) = I_{|p|}(\mu^2/2) \exp(-\mu^2/2) \tag{18}$$

In Equation (18), $I_{|p|}(\mu^2/2)$ are the modified Bessel function.

It should be emphasized that the analytical solutions from papers [21,22] were usually considered to be mutually exclusive. However, the application of the above general principles makes it is clear that the two solutions can follow from the more general view as two limiting cases.

In the multi-satellite case characteristic of the situation where $\mu >> 1$, the shape of the envelope of the satellite's intensity becomes of practical interest. For the Lifshitz's limit, the shape of the envelope is very close to being Gaussian. For the Blochinzew's limit, the shape of the envelope is more complicated: it strongly oscillates. In [35], it was shown that in the vicinity of the most intense satellites, the shape of the envelope is close to the Airy function. This finding facilitated obtaining the analytical results for the widths of the multicomponent spectral lines of hydrogen and hydrogen-like ions [35], bypassing the computationally expensive numerical simulations of the quasienergy states in the multi-satellite case.

Second, let us discuss the relation between the QEF and the homogeneous Stark broadening of spectral lines in plasmas. In the situation where the lifetime of the excited state of the radiator is much greater than the time required for the formation of the quasienergy states, i.e., where

$$\tau_{life}(k, N_e, T_e, N_i, T_i, \gamma, \omega, E_0, \Delta\omega) >> \tau_{QS}(k, E_0, \omega) \tag{19}$$

the homogeneous Stark broadening is applied not to the usual states of the radiator, but to the quasienergy states. Therefore, both the homogeneous Stark width $\gamma_e(k, N_e, T_e, \Delta\omega)$ caused by the electron microfield and the homogeneous Stark width $\gamma_i(k, N_i, T_i, N_e, \Delta\omega)$ caused by the dynamic part of the ion microfield can become "loaded" by the factors $I(p, \mu)$ given by Equations (16) or (18), as presented in paper [35].

Of special interest is the case where $I(p, \mu) = J_{|p|}{}^2(\mu)$. Because of the oscillatory nature of the Bessel functions, at some values of their argument μ, they become zeros. As this happens, the QEF drastically reduces γ_e and/or γ_i, and consequently reduces the full width at half maximum of the spectral line under consideration.

It is worth emphasizing the role of the interrelation between τ_{QS} and τ_e (and/or τ_i). In the conventional theory of the Stark broadening of hydrogen spectral lines by plasma electrons, a lower cutoff is required for the impact parameters. As a result, the factor $\ln(\rho_{max}/\rho_{min}) = \ln(\Omega_{max}/\omega_{pe})$ shows up in γ_e. In the conventional subcase where

$$\tau_{QS} << \tau_e << \tau_{life} \tag{20}$$

the upper cutoff (in the frequency scale) is

$$\Omega_{max} \sim 1/\tau_e \tag{21}$$

However, in the opposite scenario where

$$\tau_e << \tau_{QS} << \tau_{life} \quad (22)$$

the upper cutoff becomes

$$\Omega_{max} \sim 1/\tau_{QS} \quad (23)$$

In this non-conventional situation, the logarithmic factor in the homogeneous Stark width of hydrogen lines becomes $\ln(1/\omega_{pe}\tau_{QS})$, thus demonstrating the additional dependence of the Stark width on the parameters of the QEF.

So, in the situation described by Equation (19), the dependence of the γ_e (as well as of γ_i) on the frequency ω and the amplitude E_0 of the QEF is twofold: (A) through the argument $\mu = \delta_s(E_0)/\omega$ of the Bessel functions; (B) through $\ln(1/\omega_{pe}\tau_{QS})$, the latter being effective if $\tau_e << \tau_{QS}$. Consequently, the dependence of the total homogeneous Stark width

$$\Gamma = \gamma_e + \gamma_i + \gamma_F \quad (24)$$

of the spectral lines becomes quite complicated. For instance, the first two terms in the right side of Equation (24) decrease as E_0 increases, but the third term increases.

In the situation opposite to Equation (19), i.e., where

$$\tau_{life}(k, N_e, T_e, N_i, T_i, \gamma, \omega, E_0, \Delta\omega) << \tau_{QS}(k, E_0, \omega) \quad (25)$$

the lifetime of the excited state of the radiator is too short for the quasienergy states to become effective. Consequently, there the QEF has no effect on the homogeneous Stark broadening. Actually, in this situation, the QEF can be considered quasistatic.

Third, the QEF can affect spectral lines in plasmas indirectly. For instance, interaction V_{FP} can change the distribution of the velocities of plasma electrons, so that spectral lines of the radiator can exhibit an additional shift [35]. Namely, in the situation where $\tau_e << 1/\omega$, as the plasma electron relatively rapidly fly by the radiator, their motion is modulated by the relatively slow oscillations caused by the QEF. Under the additional requirement given by Equation (15), the radiator "feels" the QEF as a single-mode field $\mathbf{E}(t) = \mathbf{E_0} \cos(\omega t + \varphi)$, as explained above. In this situation, the velocity $\mathbf{V}$ of plasma electrons consists of two terms:

$$\mathbf{V} = \mathbf{V}_M + \mathbf{v} \quad (26)$$

In Equation (26), the first term $\mathbf{V}_M$ can be described by the isotropic Maxwell distribution, while the second term $\mathbf{v}$ (where $\mathbf{v} \| \mathbf{E_0}$) has an anisotropic (one-dimensional) distribution as follows:

$$P(v) = \pi^{-1}(2v_0{}^2 - v^2)^{-1/2}, \qquad v_0{}^2 = [eE_0/(m_e\omega)]^2/2 \quad (27)$$

Due to the second term in Equation (26), the distribution of the entire electron velocity $\mathbf{V}$ becomes anisotropic. As a result, in the expansion of the operator of the evolution of the states of the radiator, the first-order term does not vanish (while it vanished in the conventional case). This leads to an additional shift of the radiator energy levels and thus of the spectral lines.

This shift explicitly depends on the parameters of the QEF, namely on the ratio E_0/ω—see the second formula in Equation (27). It was called the electron oscillatory shift in paper [35], where this effect was revealed by analytical calculations. The quadrupole interaction of the plasma electrons with the radiator yields the primary contribution to the electron oscillatory shift. Explicit analytical results for the electron oscillatory shift of hydrogenic spectral lines—the results taking into account all higher multipole interactions—were presented in papers [35,41] and in book [6]. Later, Chichkov et al. [42] also analyzed this effect.

Fourth, studies of the simultaneous interaction of the radiator with the QEF and with the quasistatic part of the plasma electric field led to the discovery of a new sub-area of plasma spectroscopy: intra-Stark spectroscopy. It deals with the Langmuir wave-caused structures (max–min–max structures, also called bump–dip–bump structures) at certain locations of the profiles of hydrogenic spectral lines, the structures being called Langmuir "dips" for brevity—see, e.g., papers [30,31] and books [3,6].

The Langmuir dips have been confirmed and used for plasma diagnostics in numerous laboratory experiments [10,12,14,15,19,20,29,32,33,43–48] and in astrophysical observations [49]. These experiments were performed by a large number of independent experimental groups at various plasma sources.

Fifth, under a high-frequency or strong QEF, dipole matrix elements $\mathbf{r}_{\alpha\alpha'}$ of the hydrogen-like ions become "dressed", acquiring the following factor

$$\mathbf{r}_{\alpha\alpha',\text{dressed}} = \mathbf{r}_{\alpha\alpha'}\, \mathrm{J}_0[(\mathbf{r}_{\alpha\alpha} - \mathbf{r}_{\alpha'\alpha'})\mathrm{E}_0/\omega] \tag{28}$$

as was shown in paper [50] and in chapter 2 of book [6]. In Equation (28), where $\mathrm{J}_0(\mathrm{u})$ is the Bessel function, the atomic units are used. We note in passing that for the quasienergy states manifesting as satellites, the corresponding dressing factor is $\mathrm{J}_\mathrm{p}[(\mathbf{r}_{\alpha\alpha} - \mathbf{r}_{\alpha'\alpha'})\mathrm{E}_0/\omega]$, where p = ±1, ±1, ±2, ...

The Bessel function $\mathrm{J}_0(\mathrm{u})$ is the oscillatory function of its argument. It vanishes at some values of u. Thus, at these values of u, i.e., at the corresponding values of the ratio E_0/ω., the high-frequency or strong QEF (e.g., the laser field) suppresses or even "turns off" the dipole matrix elements responsible for the Stark broadening. The consequences are two-fold and are explained as follows.

The homogeneous Stark width in plasmas, as well as the radiative width (sometimes called the natural width) are proportional to the square of the dipole matrix elements, meaning that they acquire the factor $\{\mathrm{J}_0[(\mathbf{r}_{\alpha\alpha} - \mathbf{r}_{\alpha'\alpha'})\mathrm{E}_0/\omega]\}^2$. Therefore, at the values of $(\mathbf{r}_{\alpha\alpha} - \mathbf{r}_{\alpha'\alpha'})\mathrm{E}_0/\omega$ close to the zeros of this Bessel function (i.e., close to 2.40, 5.52, . . .), both the homogeneous Stark width and the radiative width are drastically suppressed. This narrowing effect can be used for the very significant enhancement of the gain of X-ray lasers. This was first proposed in 1993 [50] and then elaborated in 2000 [51] for dressing by the linearly polarized QEF (studied also in 2001 in paper [52]) and in 2004 in [53] for dressing by the elliptically polarized QEF.

The Stark broadening by the quasistatic part **F** of the plasma electric field is also dramatically affected due to the high-frequency or strong QEF, namely, if

$$\max[\omega, (\mathrm{nE}_0\omega)^{1/2}] >> \mathrm{nF} \tag{29}$$

Equation (29) is given in atomic units. Under this condition, the QES $\mathbf{E}_0 \cos(\omega t)$ suppresses the components of the quasistatic field perpendicular to $\mathbf{E}_0$: according to paper [11], the perpendicular components become

$$\mathbf{F}_{\text{perp,dressed}} = \mathbf{F}_{\text{perp}}\, \mathrm{J}_0[3\mathrm{nE}_0/(2\mathrm{Z}\omega)] \tag{30}$$

In paper [11], this effect was also confirmed experimentally and used for diagnostic purposes. There is also a similar effect of the suppression of the magnetic field **B** by the high-frequency or strong QEF. According to paper by Volodko and Gavrilenko [54], the perpendicular components of the magnetic field become

$$\mathbf{B}_{\text{perp,dressed}} = \mathbf{B}_{\text{perp}}\, \mathrm{J}_0[3\mathrm{nE}_0/(2\mathrm{Z}\omega)] \tag{31}$$

These results for $\mathbf{F}_{\text{perp,dressed}}$ and $\mathbf{B}_{\text{perp,dressed}}$ were also reiterated in chapter 4 of book [6].

Obviously, the above general principles of the spectroscopy of plasmas containing the QEF, presented in 1984 in paper [36], extensively utilize the concept of the quasienergy states, also known as "dressed states". We remind the reader that those are the states of the system "radiator plus QEF".

For completeness, we mention the generalization of the concept presented in paper [55], where the dressing of atomic states by the broadband electron microfield in plasmas was introduced. The most detailed presentations of these broadband-field-dressed states and their diagnostic applications are found in paper [56] and book [57].

All of the above effects in plasmas containing the QEF represent a new class of nonlinear optical phenomena. Indeed, all of these effects are nonlinear with respect to E_0^2 and thus with respect to the energy density of the QEF. Below, we briefly describe some other selected nonlinear phenomena in plasmas containing various types of the QEF.

Blochinzew's satellites under the single-mode one dimensional QEF [21] and Lifshitz's satellites under the multi-mode one-dimensional QEF [22] are two limiting cases, as explained above. For the intermediate case of the two-mode, one-dimensional QEF, the study of the corresponding satellites was presented in section 3.2.1 of book [6] where the following expression was derived for the satellites' intensities (instead of the corresponding results (16) for Blochinzew's satellites and (18) for Lifshitz' satellites):

$$I(p, \mu) = [J_p(\mu)]^2 - J_{p-1}(\mu)J_{p+1}(\mu)] \quad (32)$$

where μ is defined in Equation (16) via $\delta_s(E_0)$ from Equation (2). This is the nonlinear effect. In the multi-satellite regime, corresponding to $\mu >> 1$, the envelope of the satellites' intensities has a bell-shaped form, with the maximum at the center of the hydrogenic spectral line. So, for the envelope of the satellites to have a bell-shaped form, it is not necessary for the one-dimensional QEF to be multi-mode: already for the two-mode case, the envelope is bell-shaped (at $\mu >> 1$), though the shape is not Gaussian, as is the case for the multi-mode QEF.

The effect of the two-dimensional multi-mode QEF on hydrogenic spectral lines was studied analytically in [58]. It was demonstrated that the Stark profile of the main line and of its satellites had a cusp in its center. This unusual shape of the Stark profile was a counterintuitive result.

The effect of the circularly polarized QEF on spectra of hydrogen atoms was studied analytically by Lisitsa [59]. By proceeding to the reference frame rotating with the frequency of the field, he reduced the problem to the hydrogen atom in crossed static electric and magnetic fields. The analytical solution for the latter problem was known since it was presented in paper [60] by Demkov et al. As a result, Lisitsa obtained analytical expressions for the quasienergies and the spectral components of the hydrogen Ly-alpha line. Later, Lisitsa and Sholin [61] used the same approach for obtaining the exact analytical solution for the Stark broadening of hydrogen lines in plasmas in frames of the binary model of the interaction of the radiator with the electron microfield or with the dynamic ion microfield. Then, in paper [62], the authors eliminated the binary assumption and found the exact analytical solution for the most general, multi-particle description of the interaction of the electron or ion microfield with the radiator. Greene et al. [63] and Greene and Cooper [64,65] generalized Lisitsa and Sholin's result to a binary collision of a charged particle with a hydrogen-like ion. Then, they showed that by averaging the binary result over the ensemble of collisions one can obtain an exact multi-particle profile of any hydrogenic spectral line. They performed such averaging explicitly for the case of the HeII L_α line [65].

The effect of the elliptically polarized QEFs on spectra of hydrogenic atoms/ions was studied analytically in [66]. Multiphoton, multifrequency resonances were discovered. The quasienergies were obtained and the dependences of the locations of the corresponding spectral components on the ellipticity degree of the QEF were presented. In the subsequent paper by Gavrilenko [67], the quasienergy states of the hydrogenlike atoms/ions under the high-frequency elliptically polarized QEF were obtained: all the quasienergy states corresponding to the principal quantum numbers n = 1, 2, and 3, as well as 12 quasienergy states corresponding to any n > 3.

In very dense plasmas typical of laser–plasma interactions (especially, for relativistic laser–plasma interactions), intense hydrogenic lines, such as, e.g., the Ly-alpha and Ly-beta lines, can be optically thick. In paper [68], the author proposed a diagnostic method that allows the measurement of both the laser field and the opacity from the experimental spectrum of a hydrogenic line exhibiting satellites. This spectroscopic method is important for developing a better insight into laser–plasma interactions, especially relativistic laser–plasma interactions.

Strong Langmuir waves can become Langmuir solitons. They (or their sequence) can be described as follows [69]:

$$F(x, t) = E(x) \cos \omega t, E(x) = E_0/ch(x/\lambda), \lambda << L \quad (33)$$

In Equation (33), λ is the characteristic dimension of the solitons, L is their separation in the sequence, and

$$\omega = \omega_{pe} - 3T_e/(2m_e\omega_{pe}\lambda^2) \quad (34)$$

To identify solitons and measure their parameters, it is necessary to both experimentally find an electric field oscillating at the frequency ~ ω_{pe} and, most importantly, to ensure that the spatial distribution of the amplitude matches the formfactor E(x) from Equation (33). In paper [70], the authors analytically calculated the shape of satellites of dipole-forbidden lines (such as He, He-like, Li, and Li-like lines) in a spectrum that was spatially integrated through a Langmuir soliton or through its sequence. It was found that in the case of Langmuir solitons, there was a significant enhancement (by more than one order of magnitude) of the peak intensity of the satellites. This property of satellites, caused by the Langmuir solitons, make it possible to distinguish the Langmuir solitons from non-solitonic Langmuir waves. In a subsequent paper [71], a similar study was performed with respect to hydrogenic spectral lines. It was demonstrated that for solitonic cases, some maxima in the profile of the spectral line become less distinct or become shoulders. It was also shown that the increase in the separation between the solitons decreases the number of features in the profile of the hydrogenic spectral line. These findings can be utilized to determine the separation between the solitons and their amplitude.

In paper [27], the authors found the analytical solution for the quasienergies of a three-level quantum system in a strong QEF (for which the perturbation theory was invalid). They applied the result to analytically obtain the spectral positions and intensities of all satellites for the radiative transition in hydrogenlike ions from the following three fine-structure sublevels, $2P_{3/2}$, $2S_{1/2}$, and $2P_{1/2,}$ to the ground level $1S_{1/2}$.

Sauvan and Dalimier [72] combined the Floquet theory, developed to solve time-periodic differential equations, with the time-dependent Liouville equation, used to describe the temporal evolution of quantum systems. The authors called it the Floquet–Liouville formalism.

The work [73] by Ho et al. was the necessary precursor to the Floquet–Liouville formalism. It should be clarified that the so-called Floquet states are the same as quasienergy states.

Several papers provided the groundwork for developing the formalism of generalized quasienergy states in a bichromatic field, i.e., under two QEFs of significantly different frequencies, ω_1 and $\omega_2 << \omega_1$, such as, for example, the laser field at the frequency ω_1 from the visible range and the infra-red or microwave range QEF at the frequency ω_2. The primary application was a tunable amplification of microwaves, driven by laser radiation [74,75]. This novel concept allowed the authors to make inroads into the range of longer wavelengths than the usual concept employing stimulated Raman scattering. It was also applied in paper [76] to map microwave fields in tokamak plasmas. Details on the generalized quasienergy states of hydrogen atoms in the bichromatic field are presented in Appendix D of book [6].

Finally, we list papers that provided an important background for calculating spectral lineshapes in plasmas. Studies of the spectroscopy of plasmas containing the QEF were facilitated by the results from papers by Gavrilenko [77–80] and Peyrusse [81,82].

Additional background information can be found in books by Lisitsa [83], Griem [84], Salzman [85], Bureeva and Lisitsa [86], Fujimoto [87,88], Kunze [89], and Griem [90], as well as in papers [91–116].

4. Conclusions and Future Directions

We reviewed the general principles of the spectroscopy of plasmas containing quasi-monochromatic electric fields (QEFs). We demonstrated that the underlying physics is very rich due to the complicated entanglement of four characteristic times: the typical time required for the formation of the quasienergy states, the lifetime of the excited state of the radiator, the typical time of the formation of the homogeneous Stark broadening by the electron microfield, and the typical time of the formation of the homogeneous Stark broadening by the dynamic part of the ion microfield.

We exemplified how the shape and shift of spectral lines is affected by the mutual interactions of the three subsystems. Specifically, the interaction of the radiator with the plasma can be substantially influenced by the interaction of the radiator with the QEF, and vice versa, as well as by the interaction of the QEF and the plasma with each other. We also provided some applications of these various effects.

Future research can be focused on QEF effects in strongly magnetized plasmas. This can include the combined effects of the QEF and the strong magnetic field **B** on the spectral line profiles of various types of radiators: hydrogenic atoms/ions, He atoms and He-like ions, as well as Li atoms and Li-like ions. In these future directions of research, the diamagnetism (manifested by quadratic terms with respect to B) should be taken into account. This would broaden our horizons in the research area under consideration, both fundamentally and in terms of practical applications.

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