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Branching Ratio and *CP* Violation in $\overline{B_s^0} \to \phi\phi$ Decay in the Framework of QCD Factorization

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Abstract: We present an analytic calculation of Branching Ratio (*BR*) and Charge-Parity (*CP*) violating asymmetries of the $\overline{B_s^0}$ meson decay into the two light vectors $\phi\phi$. In doing this we calculate the helicity amplitude of the present decay in the framework of QCD factorization approach. We find the *BR* of $\overline{B_s^0} \to \phi\phi = (1.56 \pm 0.23) \times 10^{-5}$. We also calculate the direct *CP* violation, *CP* violation in mixing and *CP* violation due to interference which are $\mathcal{A}_{CP}^{\text{dir}} = 0.00355 \pm 0.00152$, $\mathcal{A}_{CP}^{\text{mix}} = -0.00629 \pm 0.03119$ and $\mathcal{A}_{CP}^{\Delta\Gamma} = 0.99997 \pm 0.00019$, respectively. Our results are in agreement with the recent theoretical predictions and experimental measurements.

Keywords: branching ratio; *CP* violation; helicity amplitude; QCD factorization; vector decay

1. Introduction

The Standard Model (SM) of Particle Physics describes the fundamental building blocks of matter and their interactions. One of the deficiencies in the SM is that it does not accommodate the matter-antimatter asymmetry of the universe. One of three proposed conditions to explain this asymmetry is the violation of Charge Conjugation-Parity (*CP*) symmetry [1]. In 1973, Kobayashi and Maskawa (*KM*) proposed an explanation of *CP* violation in the *B* meson and in doing so, predicted the existence of the third generation of quarks [2]. In recent years, sizable *CP* violation has been observed in the decay of B_d meson [3–5]. The charmless two-body non-leptonic B_s decay is another important choice in exploring *CP* violation. At the *B*-factories [6–10] (Belle, BaBar, CDF and LHCb) both B_d-$\bar{B}_d$ and B_s-$\bar{B}_s$ systems are produced. Both systems exhibit the particle-antiparticle mixing phenomenon. Because of the B_s-$\bar{B}_s$ system has higher mass difference ($\Delta m_s = 17.69 \pm 0.08$ ps^{-1}) than that of B_d-$\bar{B}_d$ system ($\Delta m_d = 0.510 \pm 0.004$ ps^{-1}), the B_s meson has faster oscillations than B_d meson [11]. The study of the *CP* violation in the B_s system with the benefit of faster oscillation offers an excellent opportunity to detect the possible deviations from SM predictions and may lead to a new physics beyond the SM. Many authors have studied the decays of $B_s \to PP, PV, VV$ [3,5,12–24], with V being a light vector meson and P being a pseudoscalar meson. The decay of $B_s \to VV$ reveals more dynamics than $B_s \to PV$ or $B_s \to PP$.

In order to get a clear idea of *CP* violation, one needs to know the exact *BR* of the decay modes which motivates us to make an analytic calculation of the $BR(\overline{B_s^0} \to \phi\phi)$. The analytic calculation of the *BR* of $B_s \to VV$ decays is achieved by many approaches as Quantum Chromodynamics Factorization (QCDF) [5,18], the Perturbative QCD (PQCD) [20,21], the Soft-Collinear Effective Theory (SCET) [22,23] and Factorization-Assisted Topological amplitude (FAT) [24].

This paper focuses on the calculation of the *BR* and *CP* violation of the $\overline{B_s^0} \to \phi\phi$ decay in the framework of QCDF approach. In 2003, X. Li et al. [12] studied $\overline{B_s^0} \to \phi\phi$ decay and predicted

$BR(\overline{B_s^0} \to \phi\phi)$ as 1.79×10^{-5} and 3.68×10^{-5} within the naive factorization (NF) and QCDF approaches, respectively. The present decay mode can be used as normalization for the studies of some other channels of charmless B_s^0 meson decays [10]. The first observation of the $BR(\overline{B_s^0} \to \phi\phi) = \left[1.4^{+0.6}_{-0.5}(\text{stat.}) \pm 0.6(\text{syst.})\right] \times 10^{-5}$ was performed by CDF in 2005 [8]. Later on, they updated their result to $[2.32 \pm 0.18(\text{stat.}) \pm 0.82(\text{syst.})] \times 10^{-5}$ in 2011 [9]. Recently, LHCb collaboration reported that $BR(\overline{B_s^0} \to \phi\phi) = [1.84 \pm 0.05(\text{stat.}) \pm 0.07(\text{syst.}) \pm 0.11(f_s/f_d)] \times 10^{-5}$ [10]. A recent theoretical calculation was performed by Yan et al. [20] and they found that $BR(\overline{B_s^0} \to \phi\phi) = \left(1.88^{+0.49}_{-0.38}\right) \times 10^{-5}$ with a $\mathcal{A}_{CP}^{\text{dir}}(\%) = 0.7 \pm 0.2$.

In this paper, we report the calculation of the BR and CP violation of $BR(\overline{B_s^0} \to \phi\phi)$ within QCDF using Mathematica packages.

Since $BR(\overline{B_s^0} \to \phi\phi)$ is a vector decay, we need to find the helicity amplitude to calculate the BR. We formulate the helicity amplitude with neglecting both the annihilation and chiral contributions since they are negligibly small. The calculated $BR(\overline{B_s^0} \to \phi\phi)$ is $(1.56 \pm 0.23) \times 10^{-5}$. Furthermore, we calculated the CP violation in the SM for $\overline{B_s^0} \to \phi\phi$ decay via pure penguin diagram within QCDF. We find that direct CP violation ($\mathcal{A}_{CP}^{\text{dir}}$), CP violation in mixing ($\mathcal{A}_{CP}^{\text{mix}}$) and CP violation due to interference ($\mathcal{A}_{CP}^{\Delta\Gamma}$) are 0.00355 ± 0.00152, -0.00629 ± 0.03119 and 0.99997 ± 0.00019, respectively. We find that the reported results are consistent with other available predictions and experimental observation [5,8–10,18,20,21,23–25].

2. Theoretical Framework

2.1. The Effective Hamiltonian

The effective Hamiltonian ($\mathcal{H}_{\text{eff}}$) describing the transition amplitude of an initial state to a final state follows the Fermi's Golden Rule. In terms of the effective Hamiltonian, the BR of $\overline{B_s^0}$ decays to two vector mesons can be written as [12,16]:

$$BR(\overline{B_s^0} \to V_1 V_2) = \frac{\tau_{B_s^0} p_c s}{8\pi m_{B_s^0}^2} \left| \langle V_1(h_1) V_2(h_2) | \mathcal{H}_{\text{eff}} | \overline{B_s^0} \rangle \right|^2, \tag{1}$$

where h_1,h_2 are the helicities of the final-state vector mesons V_1 and V_2 with four-momentum p_1 and p_2, respectively, the $m_{B_s^0}$ and $\tau_{B_s^0}$ are the mass and lifetime of $\overline{B_s^0}$ meson, the statistical factor $s = \frac{1}{2}, 1$ for two identical and different meson final states, respectively. In the rest frame of $\overline{B_s^0}$ system, since $\overline{B_s^0}$ meson has spin 0, we have $h_1 = h_2 = h$ and $p_c = |\overrightarrow{p_1}| = |\overrightarrow{p_2}|$ is the momentum of either of the two outgoing vector mesons.

Only the penguin operators can contribute to the $\overline{B_s^0} \to \phi\phi$ decay channel which is $b \to s$ transition, so the relevant $\mathcal{H}_{\text{eff}}$ can be written as [12]

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \lambda_t^{(s)} \sum_{i=3}^{10} C_i(\mu) O_i(\mu), \tag{2}$$

where G_F is the Fermi constant and $\lambda_t^{(s)} = V_{tb} V_{ts}^*$ is the CKM factor. The $O_i(\mu)$ are the local four-fermion operators (Wilson operators). The $C_i(\mu)$ are the effective Wilson coefficients which have been reliably evaluated to the next-to-leading logarithmic order (NLL) with $\mu \sim m_b$ being the renormalization scale. The $O_{3,\ldots,6}$ and $O_{7,\ldots,10}$ are the QCD and electroweak penguin operators, respectively and can be expressed as follows [12,26]:

$$\begin{aligned}
O_3 &= (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V-A}, \\
O_4 &= (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V-A}, \\
O_5 &= (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V+A}, \\
O_6 &= (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V+A}, \\
O_7 &= \frac{3}{2}(\bar{s}b)_{V-A}\sum_q e_q(\bar{q}q)_{V+A}, \\
O_8 &= \frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V+A}, \\
O_9 &= \frac{3}{2}(\bar{s}b)_{V-A}\sum_q e_q(\bar{q}q)_{V-A}, \\
O_{10} &= \frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V-A}, \qquad (3)
\end{aligned}$$

where i and j are the color indices, q denotes all the active quarks at the scale $\mu \sim m_b$, i.e., $q \in \{u, d, s, c, b\}$ and e_q are the corresponding quark charges. Also, the $(\bar{q}_1 q_2)_{V\pm A} = \bar{q}_1\gamma_\mu(1 \pm \gamma_5)q_2$ are the right and left-handed vector-axial currents, respectively.

2.2. The Factorizable Amplitude for $\overline{B^0_s} \to V_1 V_2$

The *BR* for $\overline{B^0_s} \to V_1 V_2$ decays can be calculated by inserting Equation (2) into Equation (1). The calculation of the resulting hadronic matrix elements of the local four fermion operators i.e., $\langle V_1(h)V_2(h)|O_i|\overline{B^0_s}\rangle$ represents a theoretical challenge. In order to solve this problem, naive factorization (NF) in which the hadronic matrix elements is replaced by the product of the matrix elements of two currents is carried out as follows [12,16]:

$$\langle V_1 V_2|(\bar{q}_2 q_3)_{V\pm A}(\bar{q}_1 b)_{V-A}|\overline{B^0_s}\rangle \longrightarrow \langle V_2|(\bar{q}_2 q_3)_{V\pm A}|0\rangle\langle V_1|(\bar{q}_1 b)_{V-A}|\overline{B^0_s}\rangle, \qquad (4)$$

with the vector meson V_2 being factored out. The quark content of the two vectors in the above equation are $(q_1\bar{q}_4)_{V_1}$ and $(q_2\bar{q}_3)_{V_2}$. The Equation (4) represents factorizable amplitude, $\chi^{(\overline{B^0_s}V_1,V_2)}$ which is expressed by

$$\chi^{(\overline{B^0_s}V_1,V_2)} = \langle V_2|(\bar{q}_2 q_3)_{V\pm A}|0\rangle\langle V_1|(\bar{q}_1 b)_{V-A}|\overline{B^0_s}\rangle. \qquad (5)$$

The massive vector of spin 1 has three z-components $s_z = \pm 1, 0$ corresponding to three possible helicity states $h = \pm, 0$. So, it can exist in three possible orthogonal polarization states $\varepsilon^{*\mu(\pm)}$ and $\varepsilon^{*\mu(0)}$. These states represent two transverse polarization modes $\varepsilon^{*\mu(\pm)}$ corresponding to $s_z = \pm 1$ and a longitudinal one $\varepsilon^{*\mu(0)}$ corresponding to $s_z = 0$ [27]. For a vector meson V, let p_μ, $\varepsilon^{*\mu}$, f_V and m_V be its momentum, polarization vector, decay constant and mass, respectively. Let $p_{B^0_s}$ and $m_{B^0_s}$ be the momentum and mass of the $\overline{B^0_s}$ meson. If we choose, the coordinate systems in the Jackson convention; that is, in the $\overline{B^0_s}$ rest frame, one of the vector mesons is moving along the $+z$-axis of the coordinate system and the other along the $-z$-axis, while the x-axes of both daughter particles are parallel [17]:

$$\begin{aligned}
\varepsilon_1^{*\mu(0)} &= (p_c, 0, 0, E_1)\,/m_{V_1}, & \varepsilon_2^{*\mu(0)} &= (p_c, 0, 0, E_2)\,/m_{V_2}, \\
\varepsilon_1^{*\mu(\pm)} &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)\,, & \varepsilon_2^{*\mu(\pm)} &= \frac{1}{\sqrt{2}}(0, \mp 1, +i, 0)\,, \\
p_1^\mu &= (E_1, 0, 0, p_c), & p^\mu_{B^0_s} &= (m_{B^0_s}, 0, 0, 0), \qquad (6)
\end{aligned}$$

with

$$
\begin{aligned}
p_c &= \frac{\sqrt{\left[m^2_{B^0_s} - (m_{V_1} - m_{V_2})^2\right]\left[m^2_{B^0_s} - (m_{V_1} + m_{V_2})^2\right]}}{2m_{B^0_s}}, \\
E &= \sqrt{p_c^2 + m^2}.
\end{aligned} \tag{7}
$$

By combining Equations (6) and (7), we obtain

$$
\varepsilon_1^* . \, p_1 = 0, \quad \text{and} \quad \varepsilon^* . \, p_{B^0_s} = \frac{m_{B^0_s} p_c}{m_V}, \tag{8}
$$

$$
\varepsilon_1^{*(0)} . \, \varepsilon_2^{*(0)} = -\frac{m^2_{B^0_s} - m^2_{V_1} - m^2_{V_2}}{2m_{V_1} m_{V_2}} \quad \text{and} \quad \varepsilon_1^{*(\pm)} . \, \varepsilon_2^{*(\pm)} = -1, \tag{9}
$$

$$
(\varepsilon_1^{*(0)} . \, p_{B^0_s})(\varepsilon_2^{*(0)} . \, p_{B^0_s}) = \frac{m^2_{B^0_s} p_c^2}{m_{V_1} m_{V_2}} \quad \text{and} \quad (\varepsilon_1^{*(\pm)} . \, p_{B^0_s})(\varepsilon_2^{*(\pm)} . \, p_{B^0_s}) = 0. \tag{10}
$$

The first part of Equation (5) is given by [12]

$$
\langle V_2(p_2, \varepsilon_2^*) | q_2 \gamma_\mu \bar{q}_3 | 0 \rangle = -i f_{V_2} \varepsilon^*_{2\mu} m_{V_2}, \tag{11}
$$

where $\langle V_2 | \bar{q}_2 \gamma_\mu \gamma_5 q_3 | 0 \rangle = 0$. Whereas, the second part of Equation (5) can be written as [12,17,19]

$$
\begin{aligned}
\langle V_1(p_1, \varepsilon_1^*) \, | (\bar{q}_1 b)_{V-A} | \, \overline{B^0_s}(p_{B^0_s}) \rangle &= -\varepsilon^*_{1\mu} (m_{B^0_s} + m_{V_1}) A_1^{\overline{B^0_s} V_1}(q^2) \\
&+ (p_{B^0_s} + p_1)_\mu (\varepsilon_1^* . \, p_{B^0_s}) \frac{A_2^{\overline{B^0_s} V_1}(q^2)}{m_{B^0_s} + m_{V_1}} \\
&+ q_\mu (\varepsilon_1^* . \, p_{B^0_s}) \frac{2m_{V_1}}{q^2} [A_3^{\overline{B^0_s} V_1}(q^2) - A_0^{\overline{B^0_s} V_1}(q^2)] \\
&- i\epsilon_{\mu\nu\alpha\beta} \, \varepsilon_1^{*\nu} p^\alpha_{B^0_s} \, p_1^\beta \frac{V^{\overline{B^0_s} V_1}(q^2)}{m_{B^0_s} + m_{V_1}},
\end{aligned} \tag{12}
$$

where $A_0^{\overline{B^0_s} V_1}$, $A_1^{\overline{B^0_s} V_1}$, $A_2^{\overline{B^0_s} V_1}$ and $A_3^{\overline{B^0_s} V_1}$ are the transition form factors of the $\overline{B^0_s} \to V_1 V_2$ decay via the axial current while $V^{\overline{B^0_s} V_1}$ is the transition form factor via the vector one. To cancel the poles at $q^2 = 0$ in Equation (12) we have the relation

$$
A_3^{\overline{B^0_s} V_1}(0) = A_0^{\overline{B^0_s} V_1}(0). \tag{13}
$$

Using Equation (13) in Equation (12) we obtain

$$
\begin{aligned}
\langle V_1(p_1, \varepsilon_1^*) \, | (\bar{q}_1 b)_{V-A} | \, \overline{B^0_s}(p_{B^0_s}) \rangle &= -\varepsilon^*_{1\mu} (m_{B^0_s} + m_{V_1}) A_1^{\overline{B^0_s} V_1}(q^2) \\
&+ (p_{B^0_s} + p_1)_\mu (\varepsilon_1^* . \, p_{B^0_s}) \frac{2A_2^{\overline{B^0_s} V_1}(q^2)}{m_{B^0_s} + m_{V_1}} \\
&- i\epsilon_{\mu\nu\alpha\beta} \, \varepsilon_1^{*\nu} p^\alpha_{B^0_s} \, p_1^\beta \frac{2V^{\overline{B^0_s} V_1}(q^2)}{m_{B^0_s} + m_{V_1}}.
\end{aligned} \tag{14}
$$

Thus, the factorizable amplitude $\chi^{(\overline{B^0_s} V_1, V_2)}$ is the product of Equations (11) and (14) which can be written as

$$
\begin{aligned}
\chi_h^{(\overline{B_s^0}V_1,V_2)} &= if_{V_2}m_{V_2}[\varepsilon_1^{*(h)}.\,\varepsilon_2^{*(h)}(m_{B_s^0}+m_{V_1})A_1^{\overline{B_s^0}V_1}(m_{V_2}^2) \\
&- (\varepsilon_1^{*(h)}.\,p_{B_s^0})(\varepsilon_2^{*(h)}.\,p_{B_s^0})\frac{2A_2^{\overline{B_s^0}V_1}(m_{V_2}^2)}{m_{B_s^0}+m_{V_1}} \\
&+ i\epsilon_{\mu\nu\alpha\beta}\,\varepsilon_2^{*\mu(h)}\varepsilon_1^{*\nu(h)}p_{B_s^0}^{\alpha}\,p_1^{\beta}\frac{2V^{\overline{B_s^0}V_1}(m_{V_2}^2)}{m_{B_s^0}+m_{V_1}}],
\end{aligned} \tag{15}
$$

where the momentum transfer is $q = p_{B_s^0} - p_1$ and the totally antisymmetric Levi-Civita tensor is normalized by $\epsilon_{0123} = -1$. By the sum over the non-zero 24 components of the Levi-Civita tensor with the definitions in Equation (6), and for $h = 0$ we find

$$i\epsilon_{\mu\nu\alpha\beta}\,\varepsilon_2^{*\mu(0)}\varepsilon_1^{*\nu(0)}p_B^{\alpha}\,p_1^{\beta} = 0 \tag{16}$$

and for $h = \pm$ the only survived terms are the two terms of $\{\mu,\nu,\alpha,\beta\} = \{1(2),2(1),0,3\}$. Thus we can easily get

$$i\epsilon_{\mu\nu\alpha\beta}\,\varepsilon_2^{*\mu(\pm)}\varepsilon_1^{*\nu(\pm)}p_B^{\alpha}\,p_1^{\beta} = \pm m_{B_s^0}\,p_c. \tag{17}$$

Substituting from Equations (8)–(10), (16) and (17) into Equation (15), one can get

$$
\begin{aligned}
\chi_0^{(\overline{B_s^0}V_1,V_2)} &= -\frac{if_{V_2}\,b}{2m_{V_1}}\left[aA_1^{\overline{B_s^0}V_1}(m_{V_2}^2) - c^2A_2^{\overline{B_s^0}V_1}(m_{V_2}^2)\right], \\
\chi_{\pm}^{(\overline{B_s^0}V_1,V_2)} &= -if_{V_2}m_{V_2}\left[bA_1^{\overline{B_s^0}V_1}(m_{V_2}^2) \pm c\,V^{\overline{B_s^0}V_1}(m_{V_2}^2)\right],
\end{aligned} \tag{18}
$$

with

$$
\begin{aligned}
a &= m_{B_s^0}^2 - m_{V_1}^2 - m_{V_2}^2, \\
b &= m_{B_s^0} + m_{V_1}, \\
c &= \frac{2m_{B_s^0}p_c}{b}.
\end{aligned} \tag{19}
$$

2.3. The Helicity Amplitude of $\overline{B_s^0} \to \phi\phi$

In general, the $\overline{B_s^0} \to V_1V_2$ amplitude can be decomposed into three independent helicity amplitudes A_0, A_+ and A_- corresponding to $h = 0, +$ and $-$, respectively. We use the notation

$$A_h = \langle V_1(h)V_2(h)\,|\mathcal{H}_{\text{eff}}|\,\overline{B_s^0}\rangle. \tag{20}$$

Then, $A_h(\overline{B_s^0} \to \phi\phi)$ can be written as

$$A_h = -\frac{G_F}{\sqrt{2}}\lambda_t^{(s)}\langle\phi\phi|\sum_{i=3}^{10}C_i(\mu)O_i(\mu)|\overline{B_s^0}\rangle. \tag{21}$$

The dynamical details of the decay process are coded in the so-called effective parameters $a_i(\mu)$ which are related to the Wilson coefficients $C_i(\mu)$ through the relation

$$a_i(\mu) = C_i(\mu) + \frac{C_{i\pm1}(\mu)}{N_c}, \tag{22}$$

where $N_c = 3$ is the number of colors and the upper (lower) sign apply when i is odd (even). The helicity amplitude can be written as a linear combination of the $a_i(\mu)$ parameters as follows

$$A_h = -\frac{G_F}{\sqrt{2}}\lambda_t^{(s)}\sum_{i=3}^{10}a_i^h\chi_h^{(\overline{B_s^0}\phi,\,\phi)}. \tag{23}$$

To calculate the exact formula of $A_h(\overline{B_s^0} \to \phi\phi)$, we need to write down the corresponding factorizable amplitude. Moreover, the even and odd terms of a_i that contribute to Equation (23) must be calculated. From Equation (5), the factorizable amplitude of $\overline{B_s^0}(b\bar{s}) \to \phi(s\bar{s})\phi(s\bar{s})$ process with $V_1 = V_2 = \phi$ can be written as:

$$\chi_h^{(\overline{B_s^0}\phi,\,\phi)} = \langle\phi|(\bar{s}s)_{V\pm A}|0\rangle\langle\phi|(\bar{s}b)_{V-A}|\overline{B_s^0}\rangle. \tag{24}$$

Since there are some of the quark flavors of the operators in Equation (3) do not match the quark flavor in Equation (24). So, they must be Fierz-transformed to contribute to the decay using the following transformations [28]

$$\begin{aligned}
(\bar{m}n)_{V-A}(\bar{k}l)_{V-A} &\to (\bar{k}n)_{V-A}(\bar{m}l)_{V-A}, \\
(\bar{m}n)_{V-A}(\bar{k}l)_{V+A} &\to -2(\bar{k}n)_{S-P}(\bar{m}l)_{S+P},
\end{aligned} \tag{25}$$

where $(\bar{m}n)_{S\pm P} = \bar{m}(1 \pm \gamma_5)n$. Also, the color singlet-singlet term of the operators can be obtained by [29]

$$(\bar{m}_i n_j)_{V-A}(\bar{k}_j l_i)_{V\pm A} = \frac{1}{N_c}(\bar{m}n)_{V-A}(\bar{k}l)_{V\pm A}. \tag{26}$$

In the calculation of the $A_h(\overline{B_s^0} \to \phi\phi)$, following up H. Y. Cheng et al. [16,18], the contribution of the odd and even terms of the effective operators $a_i(\mu)$ in Equation (23) can be derived as following:

For the odd terms of a_i, one can directly use the penguin operators from Equation (3) to get

$$\begin{aligned}
\langle O_3\rangle &= \langle O_5\rangle = \chi_h^{(\overline{B_s^0}\phi,\,\phi)}, \\
\langle O_4\rangle &= \langle O_6\rangle = \frac{1}{N_c}\chi_h^{(\overline{B_s^0}\phi,\,\phi)}, \\
\langle O_7\rangle &= \langle O_9\rangle = -\frac{1}{2}\chi_h^{(\overline{B_s^0}\phi,\,\phi)}, \\
\langle O_8\rangle &= \langle O_{10}\rangle = -\frac{1}{2N_c}\chi_h^{(\overline{B_s^0}\phi,\,\phi)},
\end{aligned} \tag{27}$$

where the short-hand notation $\langle O_i\rangle$ stands for $\langle\phi\phi\,|O_i|\,\overline{B_s^0}\rangle$. Then, substitution from Equation (27) into (21) and using Equation (22) yields

$$\left(a_3^h + a_5^h - \frac{1}{2}a_7^h - \frac{1}{2}a_9^h\right)\chi_h^{(\overline{B_s^0}\phi,\,\phi)}. \tag{28}$$

For the even terms of a_i, we can rewrite the operators O_3, O_4, O_9 and O_{10} in Equation (3) in the form

$$\begin{aligned}
O_3 &= \sum_q (\bar{s}_i b_j)_{V-A}(\bar{q}_j q_i)_{V-A}, \\
O_4 &= \sum_q (\bar{q}b)_{V-A}(\bar{s}q)_{V-A}, \\
O_9 &= \sum_q \frac{3}{2}e_q(\bar{s}_i b_j)_{V-A}(\bar{q}_j q_i)_{V-A}, \\
O_{10} &= \sum_q \frac{3}{2}e_q(\bar{q}b)_{V-A}(\bar{s}q)_{V-A}.
\end{aligned} \tag{29}$$

According to Equation (24), only the quark flavors with $q = s$ contribute to the present decay. A straightforward calculation with using Equation (25) and Equation (26) yields

$$\begin{aligned}
\langle O_3\rangle &= -2\langle O_9\rangle = \frac{1}{N_c}\chi_h^{(\overline{B_s^0}\phi,\,\phi)}, \\
\langle O_4\rangle &= -2\langle O_{10}\rangle = \chi_h^{(\overline{B_s^0}\phi,\,\phi)},
\end{aligned} \tag{30}$$

Again, substitution from Equation (30) into (21) and using Equation (22) yields

$$\left(a_4^h - \frac{1}{2}a_{10}^h\right)\ \chi_h^{(\overline{B_s^0}\phi,\,\phi)}. \tag{31}$$

Finally, substituting from Equation (28) and Equation (31) into Equation (23), one can get

$$A_h = -\frac{G_F}{\sqrt{2}}\lambda_t^{(s)}\left\{a_3^h + a_4^h + a_5^h - \frac{1}{2}(a_7^h + a_9^h + a_{10}^h)\right\}\chi_h^{(\overline{B_s^0}\phi,\,\phi)}. \tag{32}$$

Now using Equation (1) and Equation (32) we can write the BR formula of the present decay mode with expanding the helicity amplitude as [12]:

$$BR(\overline{B_s^0} \to \phi\phi) = \frac{\tau_{B_s^0} p_c S}{8\pi m_{B_s^0}^2}\left[|A_0|^2 + |A_+|^2 + |A_-|^2\right]. \tag{33}$$

For the calculation of the $BR(\overline{B_s^0} \to \phi\phi)$ in the framework of QCDF, we define the effective parameters $a_i^h(\mu)$ with $i = 3,4,5,7,9,10$ in the next subsection.

2.4. QCD Factorization for $\overline{B_s^0} \to \phi\phi$ Process

In the framework of QCDF approach the general form of the effective parameters $a_i^h(\mu)$ in the naive dimensional regularization (NDR) scheme at the next-to-leading order (NLO) is given by [12]

$$a_i^h(\mu) = C_i(\mu) + \frac{C_{i\pm1}(\mu)}{N_c} + N_i\frac{C_{i\pm1}(\mu)}{N_c}\frac{\alpha_s C_F}{4\pi}\left(f_1^h + f_2^h\right) + P_{i=4}^h, \tag{34}$$

with

$$N_i = \begin{cases} 1 & \text{for} \quad i = 1,2,3,4,9,10 \\ -1 & \text{for} \quad i = 5,7 \\ 0 & \text{for} \quad i = 6,8 \end{cases}, \tag{35}$$

where the quantities f_1^h account for one loop vertex corrections, f_2^h for hard spectator interactions and $P_{i=4}^h$ for penguin contribution which has been calculated only for $i = 4$. We now write down the explicit expressions for $a_i^h(\mu)$ with $i = 3,4,5,7,9,10$ which are given by [12]

$$\begin{aligned}
a_3^h(\mu) &= C_3 + \frac{C_4}{N_c} + \frac{\alpha_s C_F}{4\pi N_c}C_4(f_1^h + f_2^h), \\
a_4^h(\mu) &= C_4 + \frac{C_3}{N_c} + \frac{\alpha_s C_F}{4\pi N_c}C_3(f_1^h + f_2^h) \\
&+ \frac{\alpha_s C_F}{4\pi N_c}\left(C_3 - \frac{C_9}{2}\right)\left[Q^h(\beta_s) + Q^h(\beta_b) - \begin{pmatrix}\frac{4}{3}\\ \frac{2}{3}\end{pmatrix}\right] \\
&- \frac{\alpha_s C_F}{4\pi N_c}C_1\left[\frac{\lambda_u}{\lambda_t}Q^h(\beta_u) + \frac{\lambda_c}{\lambda_t}Q^h(\beta_c) + \begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\end{pmatrix}\right] \\
&+ \frac{\alpha_s C_F}{4\pi N_c}\left\{\left[(C_4 + C_6) + \frac{3}{2}(C_8 + C_{10})\right]Q^h(\beta_s) + G_{8g}Q_g^h\right\}, \\
a_5^h(\mu) &= C_5 + \frac{C_6}{N_c} - \frac{\alpha_s C_F}{4\pi N_c}C_6(f_1^h + f_2^h), \\
a_7^h(\mu) &= C_7 + \frac{C_8}{N_c} - \frac{\alpha_s C_F}{4\pi N_c}C_8(f_1^h + f_2^h), \\
a_9^h(\mu) &= C_9 + \frac{C_{10}}{N_c} + \frac{\alpha_s C_F}{4\pi N_c}C_{10}(f_1^h + f_2^h), \\
a_{10}^h(\mu) &= C_{10} + \frac{C_9}{N_c} + \frac{\alpha_s C_F}{4\pi N_c}C_9(f_1^h + f_2^h),
\end{aligned} \tag{36}$$

where $C_F = \frac{4}{3}$ and $\beta_q = m_q^2/m_b^2$. In the expression $a_4^h(\mu)$, the upper (lower) value in parenthesis corresponds to $h = 0$ $(\pm)$ state.

In Equation (36), the contribution from the vertex corrections f_1^h are given by

$$\begin{aligned} f_1^0 &= -12\log\left(\frac{\mu}{m_b}\right) - 18 + \int_0^1 \Phi_{\parallel}^{V_2}(u)\varrho(u)du, \\ f_1^{\pm} &= -12\log\left(\frac{\mu}{m_b}\right) - 16 + \int_0^1 du\xi_{V_2}^{\pm}(u)\left[\varrho(u) + 2\int_0^1 \kappa^{\pm}(u,x,y)dxdy\right], \end{aligned} \tag{37}$$

where

$$\varrho(u) = 3\left(\frac{1-2u}{1-u}\log(u) - i\pi\right), \tag{38}$$

$$\xi_V^{\pm}(z) = g_{\perp}^{(v)V}(z) \mp \eta \frac{g_{\perp}^{'(a)V}(z)}{4}, \tag{39}$$

$$\kappa^{\pm}(u,x,y) = \frac{1-x-y}{xy} - \frac{u}{xu+y} \mp \frac{(1-x)u}{y(xu+y)}, \tag{40}$$

with $z \in \{u, v\}$ is the light-cone momentum fraction of the quark in the vector. For $V = V_2$ in Equation (39), the $\eta = +$ or $-$ corresponding to $(V-A)\otimes(V-A)$ or $(V-A)\otimes(V+A)$ current, respectively.

For hard spectator interactions f_2^h are given by [12]

$$\begin{aligned} f_2^0 &= -\frac{4\pi^2}{N_c}\frac{if_{B_s^0}f_{V_1}f_{V_2}}{\chi_0^{(B_s^0V_1,V_2)}}\frac{m_{B_s^0}}{\lambda_{B_s^0}}\int_0^1 dvdu\frac{\Phi_{\parallel}^{V_1}(v)\Phi_{\parallel}^{V_2}(u)}{u\bar{v}}, \\ f_2^{\pm} &= \frac{4\pi^2}{N_c}\frac{if_{B_s^0}f_{V_1}^{\perp}f_{V_2}m_{V_2}}{\lambda_{B_s^0}\chi_{\pm}^{(B_s^0V_1,V_2)}}2(1\pm 1)\int_0^1 dvdu\frac{\Phi_{\perp}^{V_2}(v)\xi_{V_2}^{\pm}(u)}{\bar{v}^2} \\ &- \frac{4\pi^2}{N_c}\frac{if_{B_s^0}f_{V_1}f_{V_2}m_{V_1}m_{V_2}}{m_{B_s^0}\lambda_{B_s^0}\chi_{\pm}^{(B_s^0V_1,V_2)}}\int_0^1 dvdu\xi_{V_1}^{\pm}(v)\xi_{V_2}^{\pm}(u)\frac{u+\bar{v}}{u\bar{v}^2}, \end{aligned} \tag{41}$$

where $\bar{z} = 1 - z$ and the quantity $\lambda_{B_s^0}$ is the parametrization parameter of the distribution amplitude of the B_s^0 meson. Also, the functions $\Phi_{\parallel}^V(z), \Phi_{\perp}^V(z), g_{\perp}^{(v)V}(z)$ and $g_{\perp}^{(a)V}(z)$ are the light-cone distribution amplitudes (LCDAs) of the vector meson and we adopt them in the following asymptotic form [12]

$$\begin{aligned} \Phi_{\parallel}^V(z) &= \Phi_{\perp}^V(z) = g_{\perp}^{(a)V}(z) = 6z\bar{z}, \\ g_{\perp}^{(v)V}(z) &= \frac{3}{4}\left[1 + (2z-1)^2\right]. \end{aligned} \tag{42}$$

Also

$$g_{\perp}^{'(a)V}(z) = \frac{dg_{\perp}^{(a)V}(z)}{dz}. \tag{43}$$

The non-factorizable corrections induced by local four-quark operators O_i can be described by the function $Q^h(\beta_q)$ which is given by [12,19]

$$\begin{aligned} Q^0(\beta_q) &= -\frac{2}{3} + \frac{4}{3}\log\left(\frac{\mu}{m_b}\right) - 4\int_0^1 du\Phi_{\parallel}^{V_2}(u)\mathrm{g}(u,\beta_q), \\ Q^{\pm}(\beta_q) &= -\frac{2}{3} + \frac{2}{3}\log\left(\frac{\mu}{m_b}\right) - 2\int_0^1 du\xi_{V_2}^{\pm}(u)\mathrm{g}(u,\beta_q), \end{aligned} \tag{44}$$

with the function

$$\mathrm{g}(u,\beta_q) = \int_0^1 dx\, x\bar{x} \log[\beta_q - x\bar{x}\bar{u} - i\epsilon]. \tag{45}$$

In Equation (36), we also take into account the contributions of the dipole operator O_{8g} which will give a tree-level contribution described by the function Q_g^h defined as [12,19]

$$Q_g^0 = \int_0^1 du \frac{\Phi_{\parallel}^{V_2}(u)}{\bar{u}}, \quad Q_g^+ = \int_0^1 du \xi_{V_2}^-(u), \quad \text{and} \quad Q_g^- = \int_0^1 du \frac{\xi_{V_2}^-(u)}{\bar{u}}. \tag{46}$$

Finally, one can calculate the effective parameters $a_3^h(\mu), a_4^h(\mu), a_5^h(\mu), a_7^h(\mu), a_9^h(\mu)$ and $a_{10}^h(\mu)$ by substitution the equations from (37) to (46) into Equation (36). Consequently, the the three helicity amplitudes A_0, A_+ and A_- can be calculated from Equation (32) which leads to the calculation of the *BR* by Equation (33).

2.5. CP Violation

We derive the equations for time dependence of the *CP* asymmetry ($\mathcal{A}_{CP}(t)$), direct *CP* violation ($\mathcal{A}_{CP}^{\mathrm{dir}}$), *CP* violation due to mixing ($\mathcal{A}_{CP}^{\mathrm{mix}}$) and *CP* violation due to interference ($\mathcal{A}_{CP}^{\Delta\Gamma}$) as [4]:

$$\mathcal{A}_{CP}(t) = \frac{\mathcal{A}_{CP}^{\mathrm{dir}} \cos(\Delta m_s t) + \mathcal{A}_{CP}^{\mathrm{mix}} \sin(\Delta m_s t)}{\cosh(\Delta\Gamma_s t/2) - \mathcal{A}_{CP}^{\Delta\Gamma} \sinh(\Delta\Gamma_s t/2)}, \tag{47}$$

where $\Delta\Gamma_s = (\Gamma_L^{(s)} - \Gamma_H^{(s)})/2 = \tau_{B_s^0}^{-1}$ is the difference decay width of the B_s system with $\Gamma_H^{(s)}$ and $\Gamma_L^{(s)}$ being the decay widths of the "heavy" and "light" mass eigenstates of the B_s system, respectively. The $\Delta m_s = m_{sL} - m_{sH}$ is the mass difference of B_s system. The three *CP* observables in Equation (47) are defined by [4]

$$\mathcal{A}_{CP}^{\mathrm{dir}}(B_s \to f) \equiv \frac{1 - \left|\zeta_f^{(s)}\right|^2}{1 + \left|\zeta_f^{(s)}\right|^2}, \tag{48}$$

$$\mathcal{A}_{CP}^{\mathrm{mix}}(B_s \to f) \equiv \frac{2\,\mathrm{Im}(\zeta_f^{(s)})}{1 + \left|\zeta_f^{(s)}\right|^2}, \tag{49}$$

$$\mathcal{A}_{CP}^{\Delta\Gamma}(B_s \to f) \equiv \frac{2\,\mathrm{Re}(\zeta_f^{(s)})}{1 + \left|\zeta_f^{(s)}\right|^2}, \tag{50}$$

where

$$\zeta_f^{(s)} = e^{-i\phi_s} \frac{\bar{A}(B_s \to f)}{A(B_s \to f)}. \tag{51}$$

In the SM, for the amplitude of the penguin B_d decays, we can write [4]

$$A(B_d \to f) \propto \left[1 + \lambda^2 b e^{i\theta} e^{i\gamma}\right] \tag{52}$$

Using Equation (52) and in analogy to the penguin B_d system [4], one can rewrite Equation (51) as follows

$$\zeta_f^{(s)} = e^{-i\phi_s} \left[\frac{1 + \lambda^2 b e^{i\theta} e^{-i\gamma}}{1 + \lambda^2 b e^{i\theta} e^{+i\gamma}}\right], \tag{53}$$

where ϕ_s is the mixing phase of B_s system, $be^{i\theta}$ is a penguin parameter, γ is the angle of the unitarity triangle of the *CKM* matrix and λ $(= V_{us})$ is the Wolfenstein parameter with V_{us} is the *CKM* matrix element. The values of ϕ_s, γ and V_{us} are given in Table 1. According to the Ref. [4], one can define the parameter $be^{i\theta}$ by

$$be^{i\theta} \equiv \frac{R_b}{1-\lambda^2}\frac{A_h^u}{A_h^c}. \tag{54}$$

Here $R_b = \left(1-\lambda^2/2\right)|V_{ub}/V_{cb}|/\lambda$, where V_{ub} and V_{cb} are the CKM matrix elements which are given in Table 1. Substitution from Equation (32) into Equation (54) yields

$$be^{i\theta} \equiv \frac{R_b}{1-\lambda^2}\left[\frac{a_3^{u,h}+a_4^{u,h}+a_5^{u,h}-\frac{1}{2}(a_7^{u,h}+a_9^{u,h}+a_{10}^{u,h})}{a_3^{c,h}+a_4^{c,h}+a_5^{c,h}-\frac{1}{2}(a_7^{c,h}+a_9^{c,h}+a_{10}^{c,h})}\right]. \tag{55}$$

From Equation (36), only the definition of the $a_4^{p,h}(\mu)$ parameter can be rewritten as follows

$$\begin{aligned} a_4^{p,h}(\mu) &= C_4+\frac{C_3}{N_c}+\frac{\alpha_s C_F}{4\pi N_c}C_3(f_1^h+f_2^h) \\ &+ \frac{\alpha_s C_F}{4\pi N_c}\left(C_3-\frac{C_9}{2}\right)\left[Q^h(\beta_s)+Q^h(\beta_b)-\begin{pmatrix}\frac{4}{3}\\ \frac{2}{3}\end{pmatrix}\right] \\ &- \frac{\alpha_s C_F}{4\pi N_c}C_1\left[\frac{\lambda_p}{\lambda_t}Q^h(\beta_p)+\begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\end{pmatrix}\right] \\ &+ \frac{\alpha_s C_F}{4\pi N_c}\left\{\left[(C_4+C_6)+\frac{3}{2}(C_8+C_{10})\right]Q^h(\beta_d)+G_{8g}Q_g^h\right\}, \end{aligned} \tag{56}$$

where $p = u, c$. Moreover, we use the same definitions for the other parameters in Equation (36) so that

$$a_i^{p,h}(\mu) = a_i^h(\mu) \quad \text{for} \quad i = 3,5,7,9,10. \tag{57}$$

Hence, the penguin parameter $be^{i\theta}$ can easily be calculated by combining Equations (36) and (55)–(57).

3. Numerical Results and Discussions

3.1. Numerical Results of the Branching Ratio for $\overline{B_s^0} \to \phi\phi$

To calculate the effective parameters a_i^h with $i = 3,4,5,7,9,10$ of Equation (36), we use the Mathematica packages. The used input parameters are given in Tables 1 and 2 where the renormalization scale $\mu = m_b$ is used in the calculations. Since there are a logarithmic infrared divergence integrals in the $f_2^{\pm}$ expression in Equation (41), so we use the following approximations [5,12]

$$\int_0^1 \frac{dz}{z} = \log\left(\frac{m_b}{\Lambda_h}\right) \quad \text{and} \quad \int_0^1 \frac{dz}{z^2} = \frac{m_b}{\Lambda_h}, \tag{58}$$

with $\Lambda_h = 0.5$ GeV. The results of a_i^h with $i = 3,4,5,7,9,10$ corresponding to the three helicities $h = 0, \pm$ are listed in Table 3.

Table 4 shows the calculated values of the helicity amplitudes A_0, A_+ and A_-. One can calculate them by computing the corresponding factorizable amplitudes $\chi_0^{(\overline{B_s^0}\phi,\,\phi)}$, $\chi_+^{(\overline{B_s^0}\phi,\,\phi)}$ and $\chi_-^{(\overline{B_s^0}\phi,\,\phi)}$ from Equation (18) with using the results of Table 3 into Equation (32).

Finally, we get the $BR(\overline{B_s^0} \to \phi\phi)$ by using Equation (33) and Table 4 to be

$$BR(\overline{B_s^0} \to \phi\phi) = (1.56 \pm 0.23) \times 10^{-5}. \tag{59}$$

The main sources of uncertainty of the calculated BR come from the uncertainties of both the decay constants and the form factors.

Table 1. Inputs Parameters.

Parameter	The Value	Ref.
Mass of B_s^0 meson ($m_{B_s^0}$)	5366.88 ± 0.17 MeV	[30]
Mean life time of B_s^0 ($\tau_{B_s^0}$)	1.510 ± 0.004 ps	[30]
Mass of ϕ meson (m_ϕ)	1019.461 ± 0.016 MeV	[30]
Fermi coupling constant (G_F)	1.1663787×10^{-5} GeV^{-2}	[30]
The fine-structure coupling (α)	$1/128$	[30]
The strong coupling constant (α_s)	0.01181 ± 0.0011	[30]
The *CKM* matrix element V_{ub}	0.00394 ± 0.00036	[30]
The *CKM* matrix element V_{us}	0.2243 ± 0.0005	[30]
The *CKM* matrix element V_{cb}	0.0422 ± 0.0008	[30]
The *CKM* matrix element V_{cs}	0.997 ± 0.017	[30]
The *CKM* matrix element V_{tb}	1.019 ± 0.025	[30]
The *CKM* matrix element V_{ts}	0.0394 ± 0.0023	[30]
The mass difference of B_s^0 system (Δm_s)	17.757 ± 0.021 ps^{-1}	[30]
The decay width difference of B_s^0 system ($\Delta\Gamma_s$)	0.090 ± 0.005 ps^{-1}	[30]
Decay constant of B_s^0 meson ($f_{B_s^0}$)	240 ± 30 MeV	[21]
Longitudinal decay constant of ϕ meson (f_ϕ)	215 ± 5 MeV	[21]
Transverse decay constant of ϕ meson ($f_\phi^\perp$)	186 ± 9 MeV	[21]
The form factor $A_1^{B_s \to \phi}$	260 ± 10	[18]
The form factor $A_2^{B_s \to \phi}$	230 ± 10	[18]
The form factor $V^{B_s \to \phi}$	300 ± 10	[18]
The parametrization parameter $\lambda_{B_s^0}$	350 ± 150 MeV	[15]
Mass of b quark (m_b)	4660 MeV	[12]
Mass of c quark (m_c)	1470 MeV	[12]
Mass of u quark (m_u)	0	[12]
Mass of d quark (m_d)	0	[12]
Mass of s quark (m_s)	0	[12]
The phase mixing ϕ_s	-0.021 ± 0.031 rad	[31]
The *CKM* angle γ	$(71.1^{+4.6}_{-5.3})^o$	[31]

Table 2. Wilson coefficients in the NDR scheme at NLO with $\mu = m_b$ [12].

Wilson Coefficient	The Value
C_1	1.078
C_2	-0.176
C_3	0.014
C_4	-0.034
C_5	0.008
C_6	-0.039
C_7/α	-0.011
C_8/α	0.055
C_9/α	-1.341
C_{10}/α	0.264
C_{8g}	-0.146

Table 3. The effective parameters in QCDF approach at NLO.

a_i^h	$h = 0$	$h = +$	$h = -$
a_3^h	$0.00529251 + 0.00133847i$	$-0.354452 + 0.00133847i$	$-0.00587474 + 0.00133847i$
a_4^h	$-0.0536064 - 0.00756553i$	$0.0957453 - 0.00233928i$	$-0.0406903 - 0.0188806i$
a_5^h	$-0.008012 - 0.0015353i$	$11.765 - 0.0015353i$	$0.00654938 - 0.0015353i$
a_7^h	$0.0000904768 + 0.0000169154i$	$-0.12962 + 0.0000169154i$	$-0.0000699551 + 0.0000169154i$
a_9^h	$-0.00994835 - 0.0000811938i$	$0.0118744 - 0.0000811938i$	$-0.00927093 - 0.0000811937i$
a_{10}^h	$-0.000620574 + 0.000412427i$	$-0.11147 + 0.000412427i$	$-0.00406159 + 0.000412427i$

Table 4. Helicity amplitudes in QCDF approach at NLO.

A_h	The Value
A_0	$-0.00521217 + 0.0335507i$
A_+	$-0.000572798 - 2.45607i$
A_-	$-0.000571254 + 0.000988547i$

Table 5 shows a comparison between the present calculation of $BR(\overline{B_s^0} \to \phi\phi)$ in Equation (59) and the available experimental and other theoretical values. This table shows that the present predicted results are very much consistent with the theoretical and experimental ones.

Table 5. Theoretical and experimental $BR(\overline{B_s^0} \to \phi\phi)$ in units of 10^{-5}.

Theory/Experiment	The Value
Present Work	1.56 ± 0.23
QCDF [5]	$2.18^{+3.04}_{-1.71}$
QCDF [18]	$1.67^{+1.19}_{-0.91}$
PQCD [20]	$1.88^{+0.49}_{-0.38}$
PQCD [21]	$1.67^{+0.49}_{-0.38}$
SCET [23]	1.90 ± 0.65
FAT [24]	2.64 ± 0.76
CDF Experiment [8]	$1.4^{+0.6}_{-0.5}(\text{stat.}) \pm 0.6(\text{syst.})$
CDF Experiment [9]	$2.32 \pm 0.18(\text{stat.}) \pm 0.82(\text{syst.})$
LHCb Experiment [10]	$1.84 \pm 0.05(\text{stat.}) \pm 0.07(\text{syst.})$
Particle Data Group [25]	1.87 ± 0.15

3.2. Numerical Results for the CP Violation for $\overline{B_s^0} \to \phi\phi$

To calculate the observables of the CP violation, we have estimated the effective parameters using the Equation (56). For $h = \pm$, we found that the results of these observables are negligibly small, so we report only the calculations for $h = 0$. In this case the estimated effective parameters using Equation (56) are

$$\begin{aligned} a_4^{u,0} &= -0.0373402 - 0.000321103i, \\ a_4^{c,0} &= -0.0537056 - 0.00735794i, \end{aligned} \tag{60}$$

and according to Equation (57), the parameters $a_i^{p,0}$ with $i = 3, 5, 7, 9, 10$ are listed in Table 3. Substitution from Equation (60) into Equation (55), one can get the corresponding value of the penguin parameter to be

$$be^{i\theta} = 0.282421 - 0.0369209i. \tag{61}$$

By imposing the value of Equation (61) into Equation (53) we obtain

$$\zeta_{\phi\phi}^{(s)} = 0.996437 - 0.00626556i. \tag{62}$$

Using Equation (62) with Equations (48)–(50) one can get

$$\mathcal{A}_{CP}^{\text{dir}}(\overline{B_s^0} \to \phi\phi) = 0.00355 \pm 0.00152, \tag{63}$$

$$\mathcal{A}_{CP}^{\text{mix}}(\overline{B_s^0} \to \phi\phi) = -0.00629 \pm 0.03119, \tag{64}$$

$$\mathcal{A}_{CP}^{\Delta\Gamma}(\overline{B_s^0} \to \phi\phi) = 0.99997 \pm 0.00019. \tag{65}$$

The main sources of uncertainty of the CP asymmetries in Equations (63)–(65) come from the uncertainties of the mixing parameters (i.e., $\Delta m_s, \Delta\Gamma_s, \phi_s$, and γ).

Table 6 summarizes the results for $\mathcal{A}_{CP}^{\text{dir}}$ of the present calculation and for comparison we list the available theoretical values. This table shows that the present calculated value is consistent with the other theoretical ones.

The three CP asymmetries $\mathcal{A}_{CP}^{\text{dir}}$, $\mathcal{A}_{CP}^{\text{mix}}$ and $\mathcal{A}_{CP}^{\Delta\Gamma}$ are related observables by the relation [4]

$$\left|A_{CP}^{\text{dir}}\right|^2 + \left|A_{CP}^{\text{mix}}\right|^2 + \left|A_{CP}^{\Delta\Gamma}\right|^2 = 1. \tag{66}$$

Because the $\overline{B_s^0} \to \phi\phi$ mode is induced only by penguin operators, its direct CP asymmetry ($\mathcal{A}_{CP}^{\text{dir}}$) is naturally zero at leading order (LO) contributions [18,20]. After the inclusion of the NLO contributions, its direct CP asymmetry is nonzero but still very small. Also, in the standard model mixing-induced CP asymmetry of pure penguin decays is predicted to be small [18]. Then, the value of $\mathcal{A}_{CP}^{\Delta\Gamma}$ is naturally large as shown in Equation (65).

Table 6. The predicted values of the direct CP asymmetry $\mathcal{A}_{CP}^{\text{dir}}(\%)$.

Approach	$\mathcal{A}_{CP}^{\text{dir}}(\%)$
Present Work	0.35 ± 0.15
QCDF [18]	$0.2^{+0.6}_{-0.3}$
PQCD [20]	0.7 ± 0.2
SCET [23]	-0.39 ± 0.44
FAT [24]	0.83 ± 0.28

The asymmetry as a function of time is shown in Figure 1. To show the asymmetry we use our calculated values in Equations (63)–(65). From this figure one can see a clear CP asymmetry with time for the present decay process.

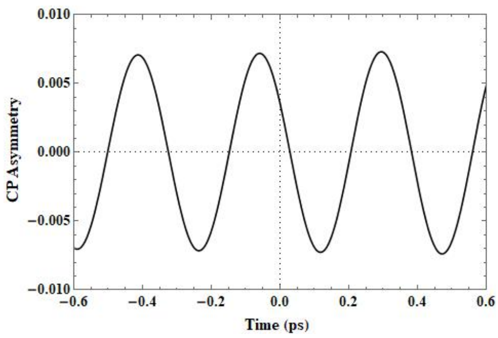

Figure 1. Time dependent CP asymmetry $\mathcal{A}_{CP}$(t).

Even though some previous theorists employed QCDF approach [5,18] but the present work is different in several aspects from them. In the present work, the structure of the effective parameters (a_i^h) is different from the one of Refs. [5,18]. We also have reported the CP violation which has not been done before for the present decay channel. Finally we include the error analysis of the BR and CP violation calculations. Therefore we think that the reported results in this paper will be helpful for future experiment as well as theoretical studies.

4. Conclusions

In this work, we studied the $\overline{B_s^0} \to \phi\phi$ decay mode in the framework of the QCDF approach. In doing this we employed QCDF approach in the NDR scheme at NLO contributions. In this study, we calculated the branching ratio and the CP violation. After numerical evaluation, we found that the BR of $\overline{B_s^0} \to \phi\phi$ decay is $(1.56 \pm 0.23) \times 10^{-5}$. This value for BR is consistent with the experimental values as well as with the other theoretically predicted ones. We also calculated the CP violation asymmetries and we found that the present decay mode is governed by the longitudinal amplitude (i.e., for $h = 0$). The calculated values for $\mathcal{A}_{CP}^{\text{dir}}$, $\mathcal{A}_{CP}^{\text{mix}}$ and $\mathcal{A}_{CP}^{\Delta\Gamma}$ are 0.00355 ± 0.00152, -0.00629 ± 0.03119 and 0.99997 ± 0.00019, respectively. These values are consistent with the available theoretical ones.

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