

Supplementary Materials

Table S1. Execution results for the standard p -median location problem model for varying p .

p	Objective	Solution time (sec)	Iteration	B&B	$\delta_p(\%)$	p	Objective	Solution time (sec)	Iteration	B&B	$\delta_p(\%)$
2	147,016	140.6	149,937	0		17	40,248	122.8	37,302	0	-3.7
3	115,302	99.0	101,461	0	-21.6	18	38,861	118.7	40,776	0	-3.4
4	99,922	120.5	91,934	0	-13.3	19	37,496	95.6	28,464	0	-3.5
5	88,020	120.1	82,280	0	-11.9	20	36,187	89.2	24,548	0	-3.5
6	78,303	117.1	68,083	0	-11.0	21	34,953	898.6	23,784	0	-3.4
7	71,822	123.1	62,915	0	-8.3	22	33,755	84.6	22,544	0	-3.4
8	66,558	157.2	62,420	0	-7.3	23	32,615	79.3	21,230	0	-3.4
9	61,889	191.7	75,405	0	-7.0	24	31,563	79.8	20,437	0	-3.2
10	57,726	147.8	52,329	0	-6.7	25	30,526	77.4	18,710	0	-3.3
11	53,970	120.3	44,970	0	-6.5	26	29,624	76.8	18,646	0	-3.0
12	50,834	96.3	39,699	0	-5.8	27	28,768	81.4	18,086	0	-2.9
13	48,062	120.3	39,109	0	-5.5	28	27,932	80.0	17,939	0	-2.9
14	45,521	125.3	36,183	0	-5.3	29	27,166	80.3	17,657	0	-2.7
15	43,456	113.5	35,462	0	-4.5	30	26,416	82.3	16,911	0	-2.8
16	41,810	153.2	68,678	178	-3.8						

Table S2. Descriptive statistics for the ratio of the maximum and minimum patrol distances for two p -median location problem models, one with and the other without balancing constraints.

p	Standard model					Extended model with balancing constraints						Difference (A-B)
	Min	Mean	Max	Max/Min(A)	Standard Deviation	Min	Mean	Max	Max/Min(B)	Standard Deviation	T	
2	1454.2	1709.5	1964.8	1.35	361.03	1454.2	1709.5	1964.8	1.35	361.03	1965.9	0
3	654.4	974.8	1285.7	1.96	315.74	654.4	974.8	1285.7	1.96	315.74	1286.7	0
4	535.6	638	720.9	1.35	89.26	550.4	638.1	706.5	1.28	79.35	708.2	0.07
5	287.4	448.6	546	1.9	100.84	287.4	448.7	520.7	1.81	94.85	524.8	0.09
6	173.5	340.6	500.9	2.89	124.67	173.5	340.7	496.6	2.86	123.24	497.3	0.03
7	173.8	270.6	483.5	2.78	101.43	173.8	270.7	476.7	2.74	99.48	478.9	0.04
8	123	224	410.2	3.34	95.15	123	224	410.2	3.34	95.15	412.2	0
9	123	183.3	236.4	1.92	44.03	123	183.5	221.6	1.8	40.56	221.8	0.12
10	101.3	157.3	236.4	2.33	46.73	101.3	157.8	204.5	2.02	44.53	204.5	0.31
11	75.9	137	253	3.33	57.57	75.9	137	246.2	3.25	55.85	246.6	0.08
12	67.1	120.7	215.2	3.2	45.80	67.1	120.8	197	2.93	45.07	197.9	0.27
13	62.6	107.4	215.2	3.44	46.87	62.6	107.4	209.1	3.34	46.58	210.4	0.1
14	50.4	96.3	181.7	3.61	42.65	53.2	96.3	179.4	3.37	42.07	180.1	0.24
15	37.7	86.3	181.7	4.82	39.19	37.7	86.3	179.4	4.76	38.59	179.4	0.06
16	38.6	79.4	181.7	4.7	39.63	38.6	79.4	181.7	4.7	39.63	181.9	0
17	38.6	71	121.2	3.14	26.31	38.6	71	121.2	3.14	26.31	121.3	0
18	25	65.2	121.2	4.85	29.05	25	65.2	121.2	4.85	29.05	121.3	0
19	25	60.9	137.6	5.51	31.71	23.2	60.6	110.6	4.78	27.45	110.8	0.73
20	24	55.3	137.6	5.73	30.15	23.2	55	103.7	4.48	25.95	103.9	1.25
21	24	51.5	137.6	5.73	30.05	24	51.5	137.6	5.73	30.05	138.1	0
22	18.7	47.8	137.6	7.37	31.36	18.1	47.5	107.7	5.94	27.89	107.9	1.43
23	18.1	44.7	91.4	5.04	22.62	18.1	44.7	91.4	5.04	22.40	91.5	0
24	17.5	42.5	91.4	5.23	22.36	17.5	42.6	89.9	5.15	22.13	91.0	0.08
25	17.5	40.5	91.4	5.23	21.31	17.5	40.5	91.4	5.23	21.31	91.6	0
26	17.5	38.3	91.4	5.23	19.05	17.5	38.3	91.4	5.23	19.05	91.6	0
27	11.7	35.8	91.4	7.82	18.62	11.7	35.8	91.4	7.82	18.62	91.5	0
28	11.7	33	62.6	5.35	14.29	11.7	33	59.5	5.09	14.01	59.8	0.26
29	9.8	31.3	58.3	5.95	14.51	9.8	31.3	56.4	5.76	14.22	57.3	0.19
30	9.8	29.7	57.8	5.9	12.77	9.8	29.7	57.8	5.9	12.77	57.8	0

Table S3. Execution results for the p -median location problem model implemented with NKDCE.

p	Objective	Solution time (sec)	Iteration	B&B	$\delta_p(\%)$	p	Objective	Solution time (sec)	Iteration	B&B	$\delta_p(\%)$
2	357,590	275.45	154,164	0		17	113,554	1,239.91	377,914	257	-3.6
3	295,477	416.50	142,290	0	-17.4	18	109,548	657.56	178,422	114	-3.5
4	250,096	297.80	110,160	0	-15.4	19	105,906	1,259.58	350,541	148	-3.3
5	223,589	424.86	109,827	0	-10.6	20	102,360	1,151.00	267,201	183	-3.3
6	199,136	377.91	89,008	0	-10.9	21	98,805	659.03	190,627	70	-3.5
7	181,114	300.52	70,967	0	-9.1	22	95,833	379.67	65,159	51	-3.0
8	168,936	354.64	67,443	0	-6.7	23	93,009	346.50	51,154	35	-2.9
9	158,401	266.81	55,682	0	-6.2	24	90,467	351.53	57,495	77	-2.7
10	150,909	370.03	55,543	0	-4.7	25	88,010	295.41	56,972	61	-2.7
11	143,563	422.42	57,350	0	-4.9	26	85,709	292.73	24,056	0	-2.6
12	137,499	465.42	74,853	29	-4.2	27	83,467	249.94	27,079	0	-2.6
13	131,440	637.23	89,298	19	-4.4	28	81,430	245.86	25,764	0	-2.4
14	126,702	571.41	109,531	61	-3.6	29	79,572	238.41	26,569	0	-2.3
15	122,203	1,202.73	288,541	204	-3.6	30	77,831	215.08	18,971	0	-2.2
16	117,785	1,484.28	507,590	312	-3.6						

Table S4. Execution results for the extended p -median location problem model with NKDCE and balancing constraints.

p	Standard model with NKDCE counts						Extended model with NKDCE counts and balancing constraints						Difference (A-B)
	Min	Mean	Max	Max/Min (A)	Standard Deviation		Min	Mean	Max	Max/Min (B)	Standard Deviation	T	
2	1454.2	1709.5	1964.8	1.35	361.0		1454.2	1709.5	1964.8	1.35	361.0	1965.9	0
3	671.5	937.9	1165.5	1.74	249.3		677.0	937.9	1160.1	1.71	243.9	1162.9	0.03
4	459.4	615.8	791.7	1.72	136.7		520.0	607.1	675.6	1.30	66.2	677.3	0.42
5	322.5	438.5	620.6	1.92	128.1		322.5	439.1	582.9	1.81	120.2	583.2	0.11
6	225.9	325.7	456.6	2.02	75.1		229.9	326.0	423.8	1.84	63.5	426.6	0.18
7	181.6	256.5	302.7	1.67	49.4		181.6	256.5	302.7	1.67	49.4	302.7	0
8	160.6	210.9	272.6	1.70	37.6		160.6	211.0	256.1	1.59	32.4	257.3	0.11
9	144.0	172.6	207.0	1.44	21.7		147.1	172.7	194.4	1.32	19.8	195.1	0.12
10	76.1	149.7	183.7	2.41	34.9		76.1	149.9	173.6	2.28	34.0	173.7	0.13
11	90.0	131.3	170.6	1.89	27.2		93.3	131.4	161.5	1.73	25.3	162.8	0.16
12	71.7	114.9	164.2	2.29	27.1		71.7	114.9	154.2	2.15	26.5	155.1	0.14
13	71.7	101.6	138.1	1.93	18.2		81.3	101.8	116.3	1.43	13.6	116.9	0.5
14	53.2	90.8	123.6	2.33	22.7		59.8	91.0	110.6	1.85	17.5	110.8	0.48
15	55.2	82.5	122.8	2.22	20.1		55.2	82.6	107.2	1.94	15.9	107.3	0.28
16	25.7	75.2	121.3	4.73	22.3		25.7	75.2	121.3	4.73	22.3	121.3	0
17	25.7	68.1	121.3	4.73	20.7		25.7	68.1	121.3	4.73	20.7	121.3	0
18	25.7	62.1	113.5	4.42	19.5		25.7	62.1	113.5	4.42	19.5	113.6	0
19	25.7	56.9	113.5	4.42	20.0		25.7	56.9	113.5	4.42	20.0	113.5	0
20	16.2	52.6	89.5	5.52	17.6		16.2	52.6	84.7	5.23	17.5	85.2	0.29
21	16.2	48.8	80.4	4.96	16.6		16.2	48.9	66.7	4.11	14.4	66.8	0.85
22	16.2	45.1	78.3	4.83	14.7		16.2	45.2	59.8	3.69	13.1	60.0	1.14
23	16.2	42.4	72.2	4.45	13.7		16.2	42.4	54.0	3.33	11.8	54.2	1.12
24	16.2	39.0	60.5	3.73	11.6		16.2	39.1	54.0	3.33	11.1	54.2	0.4
25	13.7	36.7	59.5	4.35	11.5		13.7	36.7	53.6	3.92	11.1	53.9	0.43
26	13.7	34.9	62.9	4.59	13.0		13.7	34.9	62.9	4.59	13.0	63.0	0
27	13.7	32.6	59.5	4.35	12.3		13.7	32.6	51.3	3.75	11.8	51.4	0.6
28	13.7	30.9	59.5	4.35	11.8		13.7	30.9	46.9	3.42	10.8	46.9	0.93
29	12.2	29.2	59.5	4.87	12.3		13.7	29.3	46.7	3.41	10.3	46.7	1.46
30	12.2	27.7	47.2	3.86	10.8		12.2	27.7	45.4	3.71	10.8	45.4	0.15