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An Economic Theory with a Formal-Econometric Test of Its Empirical Relevance

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Abstract: The paper contains five parts—a theory about entrepreneurial choice under uncertainty, a formal econometric structure for a test, the test, an appraisal of the test, and a description of the data generating process. Here, an entrepreneur is an individual who manages a firm that produces one commodity with labor, an intermediate good, and capital. He pays dividends to shareholders, invests in bonds and capital, and has an n-period planning horizon. Conditioned on the values of current-period prices, the entrepreneur aims to maximize the expected value of a utility function that varies with the dividends he pays each period and with his firm's balance sheet variables at the end of the planning horizon. The test comprises a family of trials of theorems that I derive from the axioms of the theory part of the formal econometric structure. In the test, the theorems are appraised for their empirical relevance in an empirical context, where each one of a random sample of four hundred entrepreneurs has chosen the first-period part of his optimal n-period expenditure plan. My formal econometric arguments demonstrate that the theorems pass all the trials. At the end, I show that my formal econometric results imply that the theory is empirically relevant.

Keywords: entrepreneur; empirical context; empirical relevance; formal econometric analysis; uncertainty

JEL Classification: C01; C12; C21; C31; C49; C51; D21; D22; D84

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Academic Editor: Julie Le Gallo

Received: 16 August 2024
Revised: 4 November 2024
Accepted: 28 November 2024
Published: 16 January 2025

Citation: Stigum, B. P. (2025). An Economic Theory with a Formal-Econometric Test of Its Empirical Relevance. *Econometrics*, *13*(1), 4. https://doi.org/10.3390/econometrics13010004

1. Introduction

This paper presents a theory of entrepreneurial choice in a world in which the entrepreneur cannot foresee with certainty the behavior of prices during the periods of his planning horizon. I introduced the theory in Stigum (1969a). Here, I develop a formal econometric test of its empirical relevance.

The theory is a natural extension of the neo-classical theory of the firm that David M. Kreps describes in Chapter 7 of his book, Kreps (1990). Since the way entrepreneurs act in the two theories differs, a test of the empirical relevance of my theory is called for. In the last two parts of the paper, I formulate and carry out a formal econometric test that demonstrates that my theory is empirically relevant.

The paper contains six parts in which I present the theory of entrepreneurial choice, delineate a formal econometric structure for the empirical analysis, carry out a test of my theory's empirical relevance, and formulate a description of the data generating process.

1.1. The Theory

In the second part of the paper I present the theory. It is about an entrepreneur who has an n-period planning horizon, and who—subject to the production and financial constraints that he will face—aims to maximize his firm's profit and his own expected utility. For the intended empirical analysis, I show that there exists a function of first-period prices and budget vectors, $U(\cdot)$, with an interesting property. The first-period part of an optimal expenditure plan for n periods can be found by maximizing $U(\cdot)$ subject to the current-period production and financial constraints.

The role $U(\cdot)$ plays in the test is interesting because of the way it highlights the importance of economic theory in empirical analyses. I have observations of the current-period choices of four hundred entrepreneurs and the prices they faced. The function, $U(\cdot)$, enables me to test the empirical relevance of the theory by appraising the empirical relevance of characteristics of the first-period part of an n-period optimal expenditure plan.

1.2. The Formal Econometric Structure

In the third part of the paper, I present a formal econometric structure for an empirical test, and explicate the meaning of its component parts. They comprise a theory universe, a data universe, and a bridge. The theory universe presents the axioms of the theory that is at stake in the empirical analysis. Their empirical relevance is examined in an empirical context that the axioms of the data universe delineate. The two universes are disjoint and the bridge describes how their variables are related to one another.

Two of the axioms in the theory universe indicate the way my theory is an extension of the neo-classical theory of the firm. One describes the characteristics of an entrepreneur's choice of production variables. These characteristics are, also, choice characteristics of entrepreneurs in the neo-classical theory of the firm. Consequently, the theorems that I derive from my axiom and try for their empirical relevance are theorems in the neo-classical theory as well.

The other describes the characteristics of an entrepreneur's choice of dividends and balance sheet variables. It has no counterpart in the neo-classical theory of the firm. My axiom delineates the necessary conditions for an optimal first-period choice of the given variables. The theorems that I derive from them and try for their empirical relevance are not theorems in the neo-classical theory.

I have used Stata's number generators to generate the data in the data universe. The data comprise a random sample of four hundred instances of the components of a twenty-nine dimensional random vector. The given vector satisfies four axioms which insist that its components are real valued, satisfy six linear equations, and have finite means and finite positive variances.

The data generation process was not bound by a specific joint probability distribution of the components of a twenty-nine dimensional random vector. Consequently, a random sample of any twenty-nine dimensional random vector that satisfies the four data axioms constitutes an empirical context in which my theory can be tested.

If the researcher in charge has observations of the components of a twenty-nine dimensional random vector that do not satisfy the axioms of the data universe, he can use David Hendry's Autometrics to estimate the parameters of the mechanism that generated his data. With the estimates, he can formulate new axioms for the data universe and use my formal econometric structure to test the empirical relevance of my theory in the new empirical context. A good reference is Hendry and Doornik (2015).

The bridge principles play two roles in the data analysis. In the first role, they relate equilibrium values of theory variables and functions of theory variables to observed values of pertinent data variables and functions of data variables. Whether the observed values are the true values of the respective variables and functions the researcher in charge must decide.

In the other role, the bridge principles and the probability distribution of the theory variables are used to form a probability distribution of the theory-related values of the data variables. This distribution may be very different from the probability distribution which the researcher in charge has taken to be the true probability distribution of the data generating process.

1.3. The Test

The test comprises two parts. One is described in the fourth part of the paper and the other in the fifth part. In the fourth part, I present the formal econometric test of my theory. The test adds up to a test of the empirical relevance of each one of a family of theorems that I derive from the axioms of the theory universe. If my theory is empirically relevant, these theorems describe characteristics of entrepreneurial choice that the choices of the entrepreneurs in my sample share.

In the fifth part of the paper, I establish the empirical relevance of the theory that I present in the second part. For the empirical analysis, the theory is a family of models of a probability distribution, $Q(dP)$, and four equations, (8)–(11). I demonstrate that it is empirically relevant by showing that all its theorems have empirically relevant analogues in the theory universe. In the process of looking for analogues, I discovered new theorems of the family of models of A1–A6 that I show are empirically relevant.

The test appraises the empirical relevance of a theory of entrepreneurial choice. It differs in many ways from the dominant trials of the theory of consumer choice; e.g., Theil and Barten's (Theil, 1965; Barten, 1969) Rotterdam Model, Cristensen, Jorgenson, and Lau's (Christensen et al., 1975) Translog Model, and Deaton and Muellbauer's (1980) An Almost Ideal Demand System. The three most important differences are listed below.

Firstly, the theory at stake in my test is a theory about the characteristics of variables that live and function in an imaginary model world. The dominant trials of consumer choice are tests of a theory about the characteristics of variables that live and function in the real world.

Secondly, the present trial tests the empirical relevance of a family of necessary conditions for an entrepreneur's optimal choice of production and investment variables. The dominant trials of consumer choice are tests of the empirical relevance of complicated non-linear approximations to a solution of the necessary conditions for an optimal choice of a consumer's decision variables.

Thirdly, in my test, the data variables are distributed in accordance with a probability distribution that is induced by the probability distribution of the theory variables and the bridge principles. In the dominant tests of consumer choice, the data variables are taken to be distributed in accordance with the true probability distribution of the data generating process. This is so even if the researcher in charge is an econometrician working in the tradition of Trygve Haavelmo (Haavelmo, 1944, pp. 7–8). He identifies the values of theory variables with the true values of pertinent data variables, formulates his economic theory with the true values of data variables in place of the original theory variables, and assumes that the data variables are distributed in accordance with the true probability distribution of the data generating process.

The dominant trials of consumer choice have rejected the theory. There is a good reason for that. In Stigum (2022), I developed the comparative-statics properties of a theory of consumer choice under uncertainty that I presented in Stigum (1969b). My theory is a natural extension of the certainty theory. Still, there are models of the theory that do not satisfy the restrictions which the dominant trials imposed on their Marshallian and Hicksian demand functions. That suggests that the certainty theory of consumer choice is unfit to describe characteristics of consumer behavior in an uncertain world.

The comparative-statics properties of my theory of entrepreneurial choice are like the comparative-statics properties of my uncertainty theory of consumer choice. It is, therefore, interesting that I in Stigum (2022) with Haavelmo's ideas used formal econometric arguments to test an uncertainty version of Stone's Linear Expenditure System (Stone, 1954). Stone's system passed the test (see Spanos (1989, 2015) and Qin (2015) for a discussion of the legacy of Haavelmo).

1.4. Appendix

In the Appendix A at the end of the paper, I describe the functions that I have used to generate my data.

2. A Theory of Entrepreneurial Choice Under Uncertainty

In this paper, the entrepreneur is an individual who operates a firm that is owned by many investors, each one of which possesses a portion of the firm's outstanding shares. I assume that the entrepreneur owns one share himself, and that they under no circumstances will sell it. The shares and their price I denote by the letters M and p_M.

The firm produces one output, y, with three inputs, L, x, and K, in accordance with the prescriptions of a production function, $g(\cdot)$, as follows:

$$y = g(L, x, K), \text{ with } (y, L, x, K) \in \mathbb{R}^3_+ \times \mathbb{R}_{++} \quad (1)$$

Here, L is short for labor, x for an intermediate good, and K for capital. The function, $g(\cdot)$, is an instantaneous point-input–point-output variety production function. I assume that $g(\cdot)$ is increasing, strictly concave, and twice differentiable with $\frac{\partial^2 g(L,x,K)}{\partial L \partial x} > 0$. The prices of y, L, x, and K I denote by the letters p_y, w, p_x, and p_K.

The entrepreneur is a price taker in all markets. He uses the firm's profit, $p_y y - wL - p_x x$, to pay the shareholders dividends, d, to invest in capital and in bonds that mature in one period, μ, and to adjust the number of outstanding shares. In a given period, i, the budget constraint for this activity is

$$p_{yi}y_i - w_i L_i - p_{xi}x_i - d_i - (p_{\mu i}\mu_i - \mu_{i-1}) - p_{Ki}(K_i - K_{i-1}) + p_{Mi}(M_i - M_{i-1}) \geq 0 \quad (2)$$

where μ_{i-1}, K_{i-1}, and M_{i-1} record, respectively, the bonds and capital that the firm owns and the number of outstanding shares at the beginning of period i. I take bonds and shares to be continuous variables. Moreover, I take capital to be a fixed factor of production. Hence, the entrepreneur's investment in new capital in one period cannot be used in the production of y before the next period. Finally, I assume that there is no market for K_{i-1} in period i, and that there is no storage facility for commodities and intermediate goods.

A period is a week or a month. I assume that the entrepreneur has an n-period planning horizon, a utility function, V, and a subjective probability distribution, $Q(dP)$, of the values which the respective prices assume in each period. The utility function is a function of the dividends that the entrepreneur pays the shareholders in each period and of the firm's balance sheet variables at the end of his planning horizon. Thus,

$$V = V(d_1, \ldots, d_n, \mu_n, K_n, M_n) \quad (3)$$

where the function $V(\cdot) : \mathbb{R}^n_+ \times [-N_\mu, N_\mu] \times \mathbb{R}_+ \times [1, N_M) \rightarrow R_+$ is taken to be twice differentiable, strictly concave, increasing in the d_i's, μ_n, and K_n, and decreasing in M_n. Moreover, a positive value of μ_n is an investment. A negative value of μ_n is a one-period loan. The interest rate in period n on such loans, r_n, equals $((1/p_{\mu n}) - 1)$. Finally, N_μ and N_M are finite positive constants with $N_M > 1$.

Let a circumstance be a vector of positive prices. I assume that the entrepreneur in the first period of his planning horizon chooses an optimal expenditure plan—that is, a family of vectors

$$(y_1, L_1, x_1, d_1, \mu_1, K_1, M_1, \ldots, y_n, L_n, x_n, d_n, \mu_n, K_n, M_n),$$

that, for $i = 1, \ldots, n$, and for each and every circumstance that may occur, satisfies the conditions,

$$(y_i, L_i, x_i, d_i, K_i) \geq 0, \quad N_\mu \geq \mu_i \geq -N_\mu, \quad N_M \geq M_i \geq 1 \tag{4}$$

$$y_i = g(L_i, x_i, K_{i-1}), \tag{5}$$

$$K_i \geq K_{i-1}, \text{ with } K_0 \text{ equal to a positive constant} \tag{6}$$

$$p_{yi}y_i - w_iL_i - p_{xi}x_i - d_i - (p_{\mu i}\mu_i - \mu_{i-1}) - p_{Ki}(K_i - K_{i-1}) + p_{Mi}(M_i - M_{i-1}) \geq 0 \tag{7}$$

and maximizes the expected value of $V(\cdot)$ with respect to $Q(dP)$ conditioned upon the observed values of p_{y1}, w_1, p_{x1}, $p_{\mu 1}$, p_{K1}, and p_{M1}.

Formulating an optimal expenditure plan is a cumbersome way to determine what the entrepreneur's optimal first-period choice of variables is. However, under reasonable conditions on $Q(dP)$, one can show—cf., Theorem T 30.5, p. 813 in (Stigum, 1990)—that there exists a function, $U(\cdot)$, such that the first-period part of an optimal expenditure plan, $(y_1, L_1, x_1, d_1, \mu_1, K_1, M_1)$, is a vector that maximizes the value of $U(\cdot)$ subject to the first-period production and budget constraints. Specifically, there is a function,

$$U(\cdot) : \mathbb{R}^6_{++} \times \mathbb{R}_+ \times [-N_\mu, N_\mu] \times \mathbb{R}_+ \times [1, N_M) \to \mathbb{R}_+, \tag{8}$$

of $((p_{y1}, w_1, p_{x1}, p_{\mu 1}, p_{K1}, p_{M1}), d_1, \mu_1, K_1, M_1)$, such that the entrepreneur in the first period of his planning horizon chooses a vector, $(y_1, L_1, x_1, d_1, \mu_1, K_1, M_1)$, that maximizes the value of $U(\cdot)$ subject to the conditions

$$(y_1, L_1, x_1, d_1, K_1 - K_0) \geq 0, \quad N_\mu \geq \mu_1 \geq -N_\mu, \quad N_M \geq M_1 \geq 1 \tag{9}$$

$$y_1 = g(L_1, x_1, K_0), \text{ and} \tag{10}$$

$$p_{y1}y_1 - w_1L_1 - p_{x1}x_1 - d_1 - (p_{\mu 1}\mu_1 - \mu_0) - p_{K1}(K_1 - K_0) + p_{M1}(M_1 - M_0) \geq 0 \tag{11}$$

where K_0, μ_0 , M_0, N_μ, and N_M are suitable positive constants. In this paper, I assume that $U(\cdot)$ is twice differentiable, strictly concave in (d_1, μ_1, K_1, M_1), increasing in (d_1, μ_1, K_1), and decreasing in M_1.

Here, an example may be useful. Example 1 describes a two-period version of the theory I presented above.

Example 1. *In this example,* $n = 2$, $\mu_0 = A$, $K_0 = 5$, $M_0 = 25$, *and for* $i = 1, 2$, $(y_i, L_i, x_i) \in \mathbb{R}^3_+$, $(d_i, \mu_i, K_i) \in \mathbb{R}^3_+$, *and* $M_i \in [1, 49]$. *The corresponding prices are*

$$P1 = (p_{y1}, w_1, p_{x1}, p_{\mu 1}, p_{K1}, p_{M1}),$$
$$P2 = (p_{y2}, w_2, p_{x2}, p_{\mu 2}, p_{K2}, p_{M2}),$$

with $Pi \in \mathbb{R}^6_{++}$, $i = 1, 2$, $(p_{\mu 1}, p_{K1}) < 1$, *and* $(p_{\mu 2}, p_{K2}) < 1$. *For* $i = 1, 2$, *the production and budget constraints are, respectively:*

$$y_i = g(L_i, x_i, K_{i-1}) = L_i^{(1/4)} x_i^{(1/4)} + \gamma \log K_{i-1},$$
$$K_i \geq K_{i-1}, \textit{ and}$$
$$p_{yi}y_i - w_iL_i - p_{xi}x_i - d_i - (p_{\mu i}\mu_i - \mu_{i-1}) - p_{Ki}(K_i - K_{i-1}) + p_{Mi}(M_i - M_{i-1}) \geq 0.$$

Finally, the two-period utility function, $V(\cdot)$, is

$$V(d_1, d_2, \mu_2, K_2, M_2) = d_1^{(1/3)} \cdot (d_2 \cdot \mu_2 \cdot K_2 \cdot (50 - M_2))^{(1/6)}.$$

In this two-period theory, the first-period utility function is

$$U(P_1, d_1, \mu_1, K_1, M_1) = (1/4)^{(2/3)} d_1^{(1/3)}$$
$$E\Big\{(p_{\mu 2} p_{K2} p_{M2})^{-(1/6)} \left[\pi(p_{y2}, w_2, p_{x2}, K_1) + p_{K2}K_1 + \mu_1 + p_{M2}(50 - M_1)\right]^{(2/3)} \Big| P1\Big\},$$

where $E\{(\cdot)|P1\}$ denotes the expected value of $(\cdot)$ conditioned on the value of $P1$, and the value of $\pi(p_{y2}, w_2, p_{x2}, K_1)$ equals the second-period profit of the firm. The latter depends on the value of K_1.

3. A Formal Econometric Structure for an Empirical Test

The theory of entrepreneurial choice under uncertainty that I outlined in the second part of the paper is a family of models of $Q(dP)$ and the equations in (1)–(3). The theory is not meant to describe entrepreneurial behavior under uncertainty. Instead, it is a family of models that describe characteristic features of entrepreneurial choice in a world in which the entrepreneur cannot foresee with certainty the behavior of prices during his planning horizon.

Different families of models of $Q(dP)$ and the equations in (1)–(3) constitute different theories of entrepreneurial choice under uncertainty. I assume that the functions, $g(\cdot)$ and $V(\cdot)$, and the model of $Q(dP)$ vary among theories, and may vary with the models of a given family of models of $Q(dP)$ and the equations in (1)–(3). Thus, members of a given family of models may be very different even though they describe characteristics of entrepreneurial choice in one and the same theory.

The way entrepreneurial choice varies with the models is interesting and of fundamental importance to the way theory is used in the empirical analysis of entrepreneurial choice under uncertainty. For example, even though the members of a given family describe choice characteristics of many different entrepreneurs, the entrepreneurs share many characteristics. Their choice of y, L, and x satisfies Hotelling's Lemma, ensures that marginal cost equals the price of y, and maximizes the firm's profit. Similarly, their choice of d, μ, K, and M ensures that the marginal efficiency of the entrepreneur's investments in μ and K equal, respectively, the interest rate on one-period loans and the firm's conditionally expected rate of return from an additional unit of capital in period one.

A theory of entrepreneurial choice under uncertainty; i.e., a particular family of models of $Q(dP)$ and the equations in (1)–(3), is empirically relevant if it contains a model that is empirically relevant. Looking for an empirically relevant model is not meaningful. To test the empirical relevance of the theory, one must look for choice characteristics which the models of the given family of models share. The theory is empirically relevant only if the data do not reject the validity of one of them.

My data comprise observations of a sample of entrepreneurs' choices of first-period budget vectors and of the prices they faced. In the following applied formal econometric analysis, I will use these data to see if a family of models of $Q(dP)$ and the equations in (8)–(11) are empirically relevant. If they are, I may claim that the corresponding family of models of $Q(dP)$ and the equations in (1)–(3) are empirically relevant.

3.1. The Theory Universe

I imagine that the variables in the family of models of $Q(dP)$ and the equations in (8)–(11) belong to a theory universe. This theory universe is a triple, $(\Omega_T, \Gamma_T, (\Omega_T, \aleph_T, P_T(\cdot)))$, where Ω_T is a subset of a vector space, Γ_T is a finite set of assertions concerning properties of vectors in Ω_T, and $(\Omega_T, \aleph_T, P_T(\cdot))$ is a probability space. The latter comprises Ω_T, a σfield of subsets of Ω_T, $\aleph_T$, and a probability measure, $P_T(\cdot) : \aleph_T \to [0, 1]$.

The assertions in Γ_T consist of six axioms, A1–A6.

A1 $\Omega_T \subset \mathbb{R}^3 \times \mathbb{R}^4 \times \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R} \times \mathbb{R}^7 \times \mathbb{R}^2$. Thus, $\omega_T \in \Omega_T$ only if $\omega_T = (y, L, x, d, \mu, K, M, p_y, w, p_x, p_\mu, p_K, p_M, \chi, u, z)$ for some $(y, L, x) \in \mathbb{R}^3$, $(d, \mu, K, M) \in \mathbb{R}^4$, $(p_y, w, p_x) \in \mathbb{R}^3$, $(p_\mu, p_K, p_M) \in \mathbb{R}^3$, $\chi \in \mathbb{R}$, $u \in \mathbb{R}^7$, $z \in \mathbb{R}^2$, and $(y, L, x, d, \mu, K, M, p_y, w, p_x, p_\mu, p_K, p_M, \chi, u, z) \in \mathbb{R}^{23}$.

A2 For all $\omega_T \in \Omega_T$, $(y, L, x) \in \mathbb{R}^3_+$, and $(d, \mu, K, M) \in \mathbb{R}_+ \times [-N_\mu, N_\mu] \times \mathbb{R}_+ \times [1, N_M)$. Moreover, $(p_y, w, p_x, p_M) \in (0, 50)^4$, and $(p_\mu, p_K) \in (0, 1)^2$.

In the intended interpretation of y, L, x, d, μ, K, M, p_y, w, p_x, p_μ, p_K, and p_M, y denotes the firm's output, and (L, x) denotes a pair of inputs. Moreover, d denotes dividends, a positive μ denotes a bond that matures in one period, and a negative μ denotes a one-period loan, K denotes the capital that is used in the production of y, and M denotes the firm's outstanding shares. Finally, the components of (p_y, w, p_x) denote the respective first-period prices of y, L, and x; and the components of (p_μ, p_K, p_M) denote the respective first-period prices of μ, K, and M. The χ and the components of u and z are error terms. The u and z are to be used to describe the relationship between theoretical variables and data variables.

The given theory variables also satisfy the conditions in axioms A3 and A4. In them, K_0 in A3 and μ_0, K_0, and M_0 in A4 denote initial quantities of μ, K, and M.

A3 There is a function, $g(\cdot) : \mathbb{R}^3_+ \to \mathbb{R}_+$, which is increasing, strictly concave, and twice continuously differentiable with $\frac{\partial^2 g(L,x,K)}{\partial L \partial x} > 0$ such that, for all $\omega_T \in \Omega_T$,

$$y = g(L, x, K_0); \qquad p_y y - wL - p_x x = 0;$$
$$\frac{p_y \partial g(L, x, K_0)}{\partial L} = w; \qquad \frac{p_y \partial g(L, x, K_0)}{\partial x} = p_x.$$

A4 Let $\pi = p_y y - wL - p_x x$, and let $\pi^* = \pi + \mu_0 + p_K K_0 - p_M M_0$. In addition, let P and D, respectively, be short for $(p_y, w, p_x, p_\mu, p_K, p_M)$ and (d, μ, K, M). There exists a twice continuously differentiable function,

$$U(\cdot) : \mathbb{R}^6_{++} \times \mathbb{R}_+ \times [-N_\mu, N_\mu] \times \mathbb{R}_+ \times [1, N_M) \to \mathbb{R}_+,$$

of $(p_y, w, p_x, p_\mu, p_K, p_M)$, d, μ, K, and M that is strictly concave in D, increasing in (d, μ, K), and decreasing in M. Moreover, for all $\omega_T \in \Omega_T$,

$$\frac{\partial U(P, D)}{\partial d} = A + \chi; \qquad \frac{\partial U(P, D)}{\partial \mu} = p_\mu \frac{\partial U(P, D)}{\partial d};$$
$$\frac{\partial U(P, D)}{\partial K} = p_K \frac{\partial U(P, D)}{\partial d}; \qquad \frac{\partial U(P, D)}{\partial M} = -p_M \frac{\partial U(P, D)}{\partial d};$$
$$\pi^* - d - p_\mu \mu - p_K K + p_M M = 0.$$

In the intended interpretation of A3 and A4, the equations in A3 record the necessary conditions on the entrepreneur's choice of y, L, and x that ensure that his choice maximizes the firm's profit. The equations in A4 record the necessary conditions on the entrepreneur's choice of D that ensure that his choice maximizes his utility. The equations in both axioms concern the equilibrium values of $g(\cdot)$ and the partial derivatives of $g(\cdot)$ and $U(\cdot)$ and not properties of the functions themselves.

A5 Let $(y, L, x)(\cdot) : \Omega_T \to \mathbb{R}^3_+$, $(p_y, w, p_x)(\cdot) : \Omega_T \to \mathbb{R}^3_{++}$, $(d, \mu, K, M)(\cdot) : \Omega_T \to \mathbb{R}_+ \times [-N_\mu, N_\mu] \times \mathbb{R}_+ \times [1, N_M)$, $(p_\mu, p_K, p_M)(\cdot) : \Omega_T \to \mathbb{R}^3_{++}$, and $(\chi, u, z)(\cdot) : \Omega_T \to \mathbb{R}^{10}$ be defined by the equations,

$$\big[(y, L, x)(\omega_T), (d, \mu, K, M)(\omega_T), (p_y, w, p_x)(\omega_T),$$
$$(p_\mu, p_K, p_M)(\omega_T), (\chi, u, z)(\omega_T)\big] = \omega_T, \text{ and } \omega_T \in \Omega_T.$$

The vector-valued functions,

$$(y, L, x)(\cdot),\ (d, \mu, K, M)(\cdot),\ (p_y, w, p_x)(\cdot),\ (p_\mu, p_K, p_M)(\cdot),\ (\chi, u, z)(\cdot),$$
$$(p_\mu^{-1}, p_K^{-1})(\cdot),\ \left(\frac{\partial g(L, x, K_0)}{\partial L}, \frac{\partial g(L, x, K_0)}{\partial x}\right)(\cdot), \text{ and}$$
$$\left(\frac{\partial U(P, D)}{\partial d}, \frac{\partial U(P, D)}{\partial \mu}, \frac{\partial U(P, D)}{\partial K}, \frac{\partial U(P, D)}{\partial M}\right)(\cdot)$$

are measurable with respect to $\aleph_T$. They have, subject to the conditions on which Γ_T insists, a well-defined joint probability distribution relative to $P_T(\cdot)$, the RPD, where R is short for researcher, P for probability, and D for distribution.

A6 Relative to $P_T(\cdot)$, the components of

$$(y, L, x, d, \mu, K, M, p_y, w, p_x, p_\mu, p_K, p_M, \chi, u, z)(\cdot),\ (p_\mu^{-1}, p_K^{-1})(\cdot),$$
$$\left(\frac{\partial g(L, x, K_0)}{\partial L}, \frac{\partial g(L, x, K_0)}{\partial x}\right)(\cdot), \text{ and}$$
$$\left(\frac{\partial U(P, D)}{\partial d}, \frac{\partial U(P, D)}{\partial \mu}, \frac{\partial U(P, D)}{\partial K}, \frac{\partial U(P, D)}{\partial M}\right)(\cdot)$$

have finite means and finite positive variances. Moreover, the $\chi(\cdot)$ and the components of $u(\cdot)$ and $z(\cdot)$ have means zero and are independently distributed of each other, of the components of P and D, and of the partial derivatives of $g(\cdot)$ and $U(\cdot)$.

In the intended interpretation of A5 and A6, the RPD delineates statistical properties of the theoretical variables. Thus, the RPD of $(p_y, w, p_x, p_\mu, p_K, p_M)(\cdot)$ is not a model of the entrepreneur's subjective probability distribution of current-period prices. I assume that the $P_T(\cdot)$ and the ranges of the variables in A1 and A2 may vary with the families of models of A1–A6. However, they do not vary with the models in a given family.

3.2. The Data Universe

I imagine that the data I will use to test the empirical relevance of my theory axioms belong in a data universe. This data universe is a triple, $(\Omega_P, \Gamma_P, (\Omega_P, \aleph_P, P_P(\cdot)))$, where Ω_P is a subset of a vector space, Γ_P is a finite set of assertions concerning properties of vectors in Ω_P, and $(\Omega_P, \aleph_P, P_P(\cdot))$ is a probability space. The latter comprises Ω_P, a σ field of subsets of Ω_P, $\aleph_P$, and a probability measure, $P_P(\cdot) : \aleph_P \to [0, 1]$.

The assertions in Γ_P consist of four axioms, D1–D4.

D1 $\Omega_P \subset \mathbb{R}^7 \times \mathbb{R}^6 \times \mathbb{R}^2 \times \mathbb{R}^4 \times \mathbb{R}^4 \times \mathbb{R}^6$. Thus, $\omega_P \in \Omega_P$, only if $\omega_P = (Y, V, \text{mg}, \text{mu}, \eta, \delta)$ for some $Y \in \mathbb{R}^7$, $V \in \mathbb{R}^6$, $\text{mg} \in \mathbb{R}^2$, $\text{mu} \in \mathbb{R}^4$, $\eta \in \mathbb{R}^4$, $\delta \in \mathbb{R}^6$, and $(Y, V, \text{mg}, \text{mu}, \eta, \delta) \in \mathbb{R}^{29}$.

D2 Suppose that $\omega_P \in \Omega_P$ and that $\omega_P = (Y, V, \text{mg}, \text{mu}, \eta, \delta)$ for some $(Y, V, \text{mg}, \text{mu}, \eta, \delta) \in \mathbb{R}^{29}$. There exist constants, $a_i, i = 1, \ldots, 6$, such that

$$V_1 \text{mg}_1 = a_1 V_2 + \delta_1, \qquad V_1 \text{mg}_2 = a_2 V_3 + \delta_2; \tag{12}$$

$$\text{mu}_1 = a_3 + \delta_3, \qquad \text{mu}_2 = a_4 \cdot V_4 + \delta_4,$$
$$\text{mu}_3 = a_5 \cdot V_5 + \delta_5, \qquad \text{mu}_4 = a_6 \cdot V_6 + \delta_6. \tag{13}$$

In the intended interpretation of these axioms, the denotation of the components of Y are observations of the respective components of (y, L, x, d, μ, K, M), and the denotation of the components of V are observations of the respective components of $(p_y, w, p_x, p_\mu, p_K, p_M)$. Moreover, the components of mg are observations of the respective values of the partial derivatives, $\frac{\partial g(L,x,K_0)}{\partial L}$ and $\frac{\partial g(L,x,K_0)}{\partial x}$; the components of mu are observations of the respective values of the partial derivatives, $\frac{\partial U(P,D)}{\partial d}$, $\frac{\partial U(P,D)}{\partial \mu}$, $\frac{\partial U(P,D)}{\partial K}$, and $\frac{\partial U(P,D)}{\partial M}$; and the components of η and δ are error terms.

D3 Let $Y(\cdot) : \Omega_P \to \mathbb{R}^7$, $V(\cdot) : \Omega_P \to \mathbb{R}^6$, $\mathrm{mg}(\cdot) : \Omega_P \to \mathbb{R}^2$, $\mathrm{mu}(\cdot) : \Omega_P \to \mathbb{R}^4$, $\eta(\cdot) : \Omega_P \to \mathbb{R}^4$, and $\delta(\cdot) : \Omega_P \to \mathbb{R}^6$ be defined by the equations, $(Y(\omega_P), V(\omega_P), \mathrm{mg}(\omega_P), mu(\omega_P), \eta(\omega_P), \delta(\omega_P)) = \omega_P$ and $\omega_P \in \Omega_P$.
The vector-valued functions, $Y(\cdot)$, $V(\cdot)$, $\mathrm{mg}(\cdot)$, $\mathrm{mu}(\cdot)$, $\eta(\cdot)$, $\delta(\cdot)$, and $(V4^{-1}, V5^{-1})(\cdot)$ are measurable with respect to $\aleph_P$ and have, subject to the conditions on which Γ_P insists, a well-defined joint probability distribution, the TPD, where T is short for true, P for probability, and D for distribution.

D4 Relative to $P_P(\cdot)$, $Y(\cdot)$, $V(\cdot)$, $\mathrm{mg}(\cdot)$, $\mathrm{mu}(\cdot)$, $\eta(\cdot)$, $\delta(\cdot)$, and $(V_4^{-1}, V_5^{-1})(\cdot)$ have finite means and finite positive variances. Moreover, the components of δ are orthogonal to the components of V, and the components of η and δ have zero means and are independently distributed of each other.

In the intended interpretation of D1–D4, the TPD plays the role of the data generating process. Specifically, I assume that TPD has one model, and that this model is a true rendition of the data generating process. According to D4, the variables in TPD have finite means and finite positive variances. Moreover, D1–D4 implies that the equations in (12) and (13) have a TPD model. The researcher does not know the model of TPD.

For the empirical analysis, I have a random sample of 400 observations of the components of Y, V, mg, and mu. If my assumptions about the TPD are valid, I can obtain good estimates of the variables' TPD means and variances and of the TPD values of the parameters in Equations (12) and (13).

I begin with the six production variables, Y_1, Y_2, Y_3, V_1, V_2, and V_3. They must have finite means. Table 1 attests to that. Table 2 records estimates of the TPD values of the parameters in (12)—with mv1 and mv2 short for $V_1\mathrm{mg}_1$ and $V_1\mathrm{mg}_2$. In the table, $RMSE$ is short for the square root of the mean square error of the residual, $R - sq$ is short for R square, F designates the F-statistic, and P is short for $Prob. > F$.

Table 1. TPD means of production variables.

	Mean	**Std. Err.**	**95% Conf. Interval**
Y_1	444.3416	1.7283	[440.9438, 447.7393]
Y_2	125.3647	0.2608	[124.8521, 125.8774]
Y_3	223.5203	2.3923	[218.8171, 228.2234]
V_1	3.7201	0.0812	[3.5605, 3.8798]
V_2	5.1477	0.1034	[4.9445, 5.3509]
V_3	4.5191	0.0704	[4.3808, 4.6575]

So much for the production variables. Next, I must consider Y_4, Y_5, Y_6, Y_7, V_4, V_5, and V_6. All of them except Y_5 must have positive means. In addition, the means of V_4 and V_5 ought to be less than one. Table 3 attests to that. Table 4 records an estimate of the TPD values of the parameters in (13).

Table 2. Estimates of the TPD values of the parameters in (12).

Equation	**Obs.**	**Parms**	$RMSE$	$R-sq$	F	$P>F$
mv1	400	1	0.4182	0.9944	70,383.36	0.0000
mv2	400	1	0.5233	0.9881	33,137.81	0.0000
Variable	Coefficient	Std. err.	t	$P>\|t\|$	95% conf. interval	
mv1 on V_2	1.0001	0.0038	265.30	0.000	[0.9927, 1.0075]	
mv2 on V_3	1.0065	0.0055	182.04	0.000	[0.9956, 1.0174]	

Table 3. TPD means of dividends and balance sheet variables.

Variable	**Mean**	**Std. Err.**	**95% Conf. Interval**
Y_4	16.1481	0.2066	[15.7419, 16.5543]
Y_5	21.8662	0.4076	[21.0648, 22.6676]
Y_6	70.8180	0.4958	[69.8433, 71.7927]
Y_7	59.8945	0.3401	[59.2259, 60.5632]
V_4	0.9089	0.0015	[0.9060, 0.9119]
V_5	0.9017	0.0013	[0.8993, 0.9042]
V_6	3.9878	0.0136	[3.9610, 4.0145]

It is important to observe that I have formulated D1–D4 without using the theory axioms. Hence, in the TPD, there are no theory-based true values of the parameters in (12) and (13). I introduce the theory into the empirical analysis with the bridge principles in B1–B6. In reading them, note that I relate the entrepreneur's decision variables, y, L, x, d, μ, K, M, and the partial derivatives of $g(\cdot)$ and $U(\cdot)$, to the observed values of the corresponding components of Y, mg, and mu. In contrast and in the tradition of Trygve Haavelmo (cf. (Haavelmo, 1944, pp. 7–8)), I relate the variables over which the entrepreneur has no control, p_y, w, p_x, p_μ, p_K, and p_M, to the true values in the data universe of the corresponding components of V.

Table 4. Estimates of TPD values of the parameters in (13).

Equation	**Obs.**	**Parms**	$RMSE$	$R-sq$	F	$P>F$
mu_2	400	1	0.0581	0.9984	252,750.7	0.0000
mu_3	400	1	0.1463	0.9898	38,813.54	0.0000
mu_4	400	1	0.0099	1.0000	1.66×10^8	0.0000
Variable	Coefficient	Std. err.	t	$P>\|t\|$	95% conf. interval	
mean of mu_1	1.5998	0.0064	-	-	[1.5872, 1.6124]	
mu_2 on V_4	1.6065	0.0032	502.74	0.000	[1.6003, 1.6128]	
mu_3 on V_5	1.5980	0.0081	197.01	0.000	[1.5821, 1.6140]	
mu_4 on V_6	−1.6001	0.0001	-1.3×10^4	0.000	[−1.6003, −1.5998]	

3.3. The Bridge

The bridge is a pair, (Ω, Γ_{TP}), where Ω is a subset of $\Omega_T \times \Omega_P$, and Γ_{TP} is a set of six assertions about the vectors in Ω. It is understood that a researcher's observations consist of pairs, (ω_T, ω_P), where $\omega_T \in \Omega_T$, $\omega_P \in \Omega_P$, and $(\omega_T, \omega_P) \in \Omega$.

The components of ω_T are unobservable, while the components of ω_P that are not error terms are observable. For example, in the present bridge, one of the components of ω_T may record the entrepreneur's intended payment of dividends to shareholders, while the corresponding component of ω_P will record a sample entrepreneur's actual payment of dividends to his shareholders.

B1 $\Omega \subset \Omega_T \times \Omega_P$. Thus, $\omega \in \Omega$ only if $\omega = (\omega_T, \omega_P)$ for some $\omega_T \in \Omega_T, \omega_P \in \Omega_P$, and $(\omega_T, \omega_P) \in \Omega_T \times \Omega_P$; i.e., $\omega \in \Omega$ only if

$\omega = ((y, L, x, d, \mu, K, M, p_y, w, p_x, p_\mu, p_K, p_M, \chi, u, z),$
$(Y, V, mg, mu, \eta, \delta))$ for some $(y, L, x, d, \mu, K, M, p_y, w, p_x, p_\mu, p_K, p_M, \chi, u, z) \in \Omega_T$, $(Y, V, mg, mu, \eta, \delta) \in \Omega_P$, and $((y, L, x, d, \mu, K, M, p_y, w, p_x, p_\mu, p_K, p_M, \chi, u, z),$
$(Y, V, mg, mu, \eta, \delta)) \in \Omega_T \times \Omega_P$.

B2 Ω_T and Ω_P are disjoint, and $\aleph_T$ and $\aleph_P$ are stochastically independent.

B3 In the probability space, $(\Omega_T \times \Omega_P, \aleph, P(\cdot))$, which the probability spaces in the theory universe and the data universe generate, $\Omega \in \aleph$, and $P(\Omega) > 0$.

B4 $\Omega_T \subset \{(y, L, x, d, \mu, K, M, p_y, w, p_x, p_\mu, p_K, p_M, \chi, u, z) \in \Omega_T$ for which there is a $(Y, V, mg, mu, \eta, \delta) \in \Omega_P$ such that $((y, L, x, d, \mu, K, M, p_y, w, p_x, p_\mu, p_K, p_M, \chi, u, z),$
$(Y, V, mg, mu, \eta, \delta)) \in \Omega\}$.

B5 For all $(\omega_T, \omega_P) \in \Omega$,
$(y, L, x)(\omega_T) + (u_1, u_2, u_3)(\omega_T) = (Y_1, Y_2, Y_3)(\omega_P)$
$(d, \mu, K, M)(\omega_T) + (u_4, u_5, u_6, u_7)(\omega_T) = (Y_4, Y_5, Y_6, Y_7)(\omega_P)$
$(p_y, w, p_x)(\omega_T) = (V_1, V_2, V_3)(\omega_P) - (\eta_1, \eta_2, \eta_3)(\omega_P)$
$(p_\mu, p_K, p_M)(\omega_T) = (V_4, V_5, V_6)(\omega_P) - (\eta_4, \eta_5, \eta_6)(\omega_P)$
$\left(\frac{\partial g(L,x,K_0)}{\partial L}, \frac{\partial g(L,x,K_0)}{\partial x}\right)(\omega_T) + (z_1, z_2)(\omega_T) = (mg_1, mg_2)(\omega_P)$; and
$\left(\frac{\partial U(P,D)}{\partial d}, \frac{\partial U(P,D)}{\partial \mu}, \frac{\partial U(P,D)}{\partial K}, \frac{\partial U(P,D)}{\partial M}\right)(\omega_T) + (z_3, z_4, z_5, z_6)(\omega_T) = (mu_1, mu_2, mu_3, mu_4)(\omega_P)$.

In the intended interpretation of these axioms, Axiom B5 is not meant to establish just an ordinary errors-in-variables relationship between theoretical variables and data variables. Instead, the first two equations and the last two equations delineate how the RPD of the left-hand variables is to be assigned to the corresponding data variables. This distribution, the MPD, may be very different from their TPD. The third, fourth, and fifth equation describe how the RPD of p_y, w, p_x, p_μ, p_K, and p_M is to be assigned to the true values of the corresponding components of V. This is the MPD of the true values of the components of V.

To obtain the MPD of the observed values of V, it is necessary to establish a theorem, and to add an assumption, B6, about $\aleph_T$, the σ field of subsets of Ω_T. The theorem is an easy consequence of axioms *A*, *D*, and *B*. I will sketch a proof of it.

Theorem 1. *Suppose that the A, D, and B axioms are valid. For all $(\omega_T, \omega_P) \in \Omega$, let*

$$u_{7+j}(\omega_T) = \eta_j(\omega_P), \quad j = 1, \ldots, 4.$$

The four $u_{7+j}(\cdot)$'s are well defined on Ω, and the third, fourth, and fifth equation in B5 can be rewritten as follows:

$$(p_y, w, p_x)(\omega_T) + (u_8, u_9, u_{10})(\omega_T) = (V_1, V_2, V_3)(\omega_P),$$
$$(\mu, p_K)(\omega_T) = (V_4, V_5)(\omega_P),$$
$$p_M(\omega_T) + u_{11}(\omega_T) = V_6(\omega_P).$$

It suffices to consider one case in the proof of Theorem 1. Let $j = 2$ and consider the equation, $u_9(\omega_T) = \eta_2(\omega_P)$. Suppose that there are two pairs in Ω, (ω_T^0, ω_P^0) and (ω_T^1, ω_P^0), at which the two values of $u_9(\cdot)$ differ: i.e., where $u_9(\omega_T^0) \neq u_9(\omega_T^1)$. The two equations,

$$V_2(\omega_P^0) - \eta_2(\omega_P^0) = p_K(\omega_T^0),$$
$$V_2(\omega_P^0) - \eta_2(\omega_P^0) = p_K(\omega_T^1),$$

imply that $p_K(\omega_T^0) = p_K(\omega_T^1)$. But if that is so, the two equations,

$$V_2(\omega_P^0) = u_9(\omega_T^0) + p_K(\omega_T^0),$$
$$V_2(\omega_P^0) = u_9(\omega_T^1) + p_K(\omega_T^1),$$

imply that $u_9(\omega_T^1) = u_9(\omega_T^0)$.

Then the final assumption about the bridge.

B6 The vector valued function, $(u_8, \ldots, u_{11})(\cdot)$ is measurable with respect to $\aleph_T$. Relative to $P_T(\cdot)$, its components have zero means, finite positive variances, and are independently distributed of each other and of $\chi(\cdot)$, $z(\cdot)$, $(u_1, \ldots, u_7)(\cdot)$, the P and D in $A4 - A6$, and the partial derivatives of $g(\cdot)$ and $U(\cdot)$ in $A6$.

3.4. B4 and the MPD

It remains to say a few words about the role of B4 in the construction of the MPD. To show how B4 helps to determine the meaning of the MPD, I let $\Omega(T, P) = \{\omega_T \in \Omega_T$ for which there is an $\omega_P \in \Omega_P$ with $(\omega_T, \omega_P) \in \Omega\}$, and observe that according to B4, $\Omega_T \subset \Omega(T, P)$. Next, I let G and H, respectively, be sets in the ranges of $Y(\cdot)$ and $V(\cdot)$, and observe that, with $y = (y, L, x, d, \mu, K, M)$, $u = (u_1, \ldots, u_7)$, $x = (p_y, w, p_x, p_\mu, p_K, p_M)$, and $v = (u_8, \ldots, u_{10}, 0, 0, u_{11})$,

$$MPD(\{(Y, V) \in G \times H\}) =$$
$$\frac{P_T\{\omega_T \in \Omega_T : y(\omega_T) + u(\omega_T) \in G, x(\omega_T) + v(\omega_T) \in H\} \cap \Omega(T, P)}{P_T(\Omega(T, P))}$$
$$= P_T(\{\omega_T \in \Omega_T : y(\omega_T) + u(\omega_T) \in G, x(\omega_T) + v(\omega_T) \in H\}).$$

Thus, the MPD of (Y, V) equals the RPD distribution of $(y + u, x + v)$.

4. The Empirical Analysis

In this section, the fourth part of the paper, the theory at stake in the empirical analysis is a family of models of the axioms, A1–A6. Consequently, the words "theory" and "my theory", must be taken to be short for, "the given family of models of A1–A6".

My sample of 400 observations of the components of $(Y, V, \text{mg}, \text{mu})$ is a random sample. According to A6 and B2-B5, the components of $(Y, \text{mg}, \text{mu})$ have finite means and finite positive variances in the MPD. According to A6, B2–B6, and Theorem 1, the components of V have, also, finite means and finite positive variances in the MPD.

From this, it follows that Tables 1–4 TPD estimates of the means of Y and V and of the parameters in Equations (12) and (13) are, also, estimates of the values of the same means and parameters in the MPD. In the MPD there are theory-based true values of a_1, a_2, and a_3. As I shall show, they are, respectively, 1, 1, and A. $A4$ does not insist on a true value of A, but the MPD estimate of A in Table 4 suggests that the true value of A with 95% certainty lies in the interval, $(1.5872, 1.6124)$.

4.1. The Empirical Relevance of A3

In the intended interpretation of Axiom A3, the axiom describes characteristics of an entrepreneur's choice of production variables that maximize his firm's profit. With that interpretation in mind, I can deduce from A3, B2–B6, and Theorem 1 all the characteristics of such a choice that the entrepreneurs in my sample must share if my theory is empirically relevant.

To see if my sample entrepreneurs' choices have the required characteristics, I begin by recording in Table 5 the correlation matrix of the production variables. According to A3 and the theorems that I can deduce from it, $\partial y/\partial p_y > 0$; $\partial L/\partial w < 0$; and $\partial x/\partial p_x < 0$. Hence, my theory is empirically relevant only if the table shows that an entrepreneur's supply of y varies positively with its price, and that his demand for an input varies negatively with its price.

Table 5. MPD correlation matrix of production variables.

	Y_1	Y_2	Y_3	V_1	V_2	V_3
Y_1	1.0000					
Y_2	−0.0129	1.0000				
Y_3	−0.1837	−0.2179	1.0000			
V_1	0.1158	0.1330	−0.3485	1.0000		
V_2	0.0331	−0.0319	−0.0462	−0.0173	1.0000	
V_3	0.0150	−0.0603	−0.1308	0.0547	0.1287	1.0000

The table gives me no reason to reject the theory. To see why, let a, b, and c denote, respectively, the MPD means of the current-period values of y, L, and x; and let α, β, and γ denote, respectively, the mean values of the current-period prices of y, L, and x. Then, observe that

$$(y - a + u_1)(p_y - \alpha + u_8) = (Y_1 - a)(V_1 - \alpha),$$
$$(L - b + u_2)(w - \beta + u_9) = (Y_2 - b)(V_2 - \beta),$$
$$(x - c + u_3)(p_x - \gamma + u_{10}) = (Y_3 - c)(V_3 - \gamma).$$

From these equations and A6, B2–B6, Theorem 1, and the table, it follows that, in the MPD,

$$E(Y_1 - a)(V_1 - \alpha) = E(y - a)(p_y - \alpha) > 0;$$
$$E(Y_2 - b)(V_2 - \beta) = E(L - b)(w - \beta) < 0; \text{ and}$$
$$E(Y_3 - c)(V_3 - \gamma) = E(x - c)(p_x - \gamma) < 0,$$

in accordance with the predictions of my theory.

Next, I will obtain estimates of the data version of the relations which the last two equations in A3 depict. I will carry this out by regressing $V_1 \cdot mg_1$ on V_2 and $V_1 \cdot mg_2$ on V_3. The rationale that underlies my arguments is as follows: there is an MPD model of the equations in (12) in which

$$V_1 mg_1 = \alpha_1 V_2 + \xi_1,$$
$$V_1 mg_2 = \alpha_2 V_3 + \xi_2,$$

and ξ_1 and ξ_2 have mean zero and finite positive variances. To see why, observe that by A3, B5, B6, and Theorem 1, the first equation has an MPD model with $\alpha_1 = 1$.

$$V_1 mg_1 = (p_y + u_8)\left(\frac{\partial g}{\partial L} + z_1\right) = w + p_y z_1 + \frac{\partial g}{\partial L} \cdot u_8 + u_8 z_1 = V_2 + \left(p_y z_1 + \frac{\partial g}{\partial L} \cdot u_8 + u_8 z_1 - u_9\right).$$

Now, by A6, D4, B2, $EV_1 mg_1 = EV_2$. Consequently, the true value of α_1 must equal 1. By a similar argument, I find that $\alpha_2 = 1$. But if that is so, I can conclude that my theory is empirically relevant only if the confidence intervals of the MPD estimates of the coefficients in Table 2 contain the number one, which they do.

It will be interesting to see if my observations, also, are in accordance with Hotelling's Lemma. For that purpose, let

$$rm\pi = V_1 Y_1 - V_2 Y_2 - V_3 Y_3,$$

and observe first that my assumptions about TPD and MPD imply that there is an MPD model of the equation,

$$rm\pi = \alpha + aV_1 + bV_2 + cV_3 + \xi, \tag{14}$$

in which ξ has mean zero and finite positive variance. Then, let

$$\begin{aligned} \pi &= p_y \cdot y - wL - p_x \cdot x, \text{ and} \\ m\pi &= (p_y + u_8)(y + u_1) - (w + u_9)(L + u_2) - (p_x + u_{10})(x + u_3). \end{aligned}$$

A3 implies that Hotelling's Lemma is valid in the theory-to wit:

$$\frac{\partial \pi}{\partial p_y} = y + p_y \frac{\partial y}{\partial p_y} - p_y \frac{\partial g}{\partial L} \frac{\partial L}{\partial p_y} - p_y \frac{\partial g}{\partial x} \frac{\partial x}{\partial p_y} = y,$$

and by a similar argument, $\frac{\partial \pi}{\partial w} = -L$, and $\frac{\partial \pi}{\partial p_x} = -x$. In addition, by A3, B5, and Theorem 1,

$$\begin{aligned} m\pi &= rm\pi, \\ \frac{\partial m\pi}{\partial p_y} &= y + u_1 = Y_1 = \frac{\partial rm\pi}{\partial V_1}, \\ \frac{\partial m\pi}{\partial w} &= -(L + u_2) = -Y_2 = \frac{\partial rm\pi}{\partial V_2}, \\ \frac{\partial m\pi}{\partial p_x} &= -(x + u_3) = -Y_3 = \frac{\partial rm\pi}{\partial V_3}. \end{aligned}$$

When regressing $rm\pi$ on V_1, V_2, and V_3, it follows from the observations above that the constant in (14) equals zero and that Hotelling's Lemma and my theory are empirically relevant only if the confidence intervals of the estimated coefficients of V_1, V_2, and V_3 contain the mean values of Y_1, $-Y_2$, and $-Y_3$. Tables 1 and 6 show that they do.

Table 6. An MPD test of Hotelling's Lemma.

Equation	**Obs**	**Parms**	***RMSE***	R^2	***F***	$P > F$
rmπ1	400	3	259.7794	0.9140	1406.773	0.000
Variable	Coefficient	Std.err.	t	$P > \|t\|$	95% conf. interval	
V_1	452.2555	7.0351	64.29	0.000	[438.4248, 466.086]	
V_2	−130.2235	5.5835	−23.32	0.000	[−141.2006, −119.2465]	
V_3	−222.8214	7.3486	−30.32	0.000	[−237.2684, −208.3744]	

It remains to be seen if the entrepreneurs in my sample allocate their resources so that the marginal cost of producing y equals its price. Let

$$c(y) = wL + p_x x, \quad \text{and} \quad rmc(Y_1) = V_2Y_2 + V_3Y_3$$

be the cost of producing y in the theory and data universe, and let

$$mc(y + u_1) = (w + u_9)(L + u_2) + (p_x + u_{10})(x + u_3).$$

According to A3, B5, and Theorem 1, $mc(y + u_1) = rmc(Y_1)$. Moreover,

$$mc(y + u_1) = (p_y + u_8)(y + u_1) - m\pi(y + u_1); \quad \frac{\partial m\pi}{\partial y} = 0; \quad \text{and} \quad \frac{\partial mc}{\partial y} = (p_y + u_8).$$

Likewise,

$$rmc(Y_1) = V_1Y_1 - rm\pi(Y_1); \quad \frac{\partial rm\pi}{\partial Y_1} = 0; \quad \text{and} \quad \frac{\partial rmc(Y_1)}{\partial Y_1} = V_1.$$

Finally, observe that my assumptions about the TPD and MPD imply that there are constants, a and b, and an error term, ξ, with mean zero and finite positive variance, such that

$$rmc = aY_1 + brm\pi_1 + \xi.$$

Hence, it is the case that

$$(1 + b)rmc(Y_1) = (a + bV_1)Y_1 + \xi,$$

and that

$$(1 + b)\frac{\partial rmc(Y_1)}{\partial Y_1} = (a + bV_1).$$

But if that is so, then

$$\frac{\partial rmc(Y_1)}{\partial Y_1} = V_1$$

if and only if $b \neq 0$, and $a = V_1$. Thus, I can test whether the marginal cost of producing Y_1 equals its price by checking if the estimate of b is significantly different from zero and if the confidence interval of the estimate of a contains V_1. Tables 1 and 7 show that the two conditions are satisfied.

Table 7. An MPD estimate of the marginal cost of Y_1.

Equation	**Obs**	**Parms**	***RMSE***	R^2	***F***	$P > F$
rmc	400	2	409.5867	0.9440	3354.542	0.0000
Variable	Coefficient	Std. err.	t	$P > \|t\|$	95% conf. interval	
Y_1	3.7216	0.0460	80.97	0.000	[3.6312, 3.8119]	
rmπ	−0.3259	0.0232	−14.04	0.000	[−0.3715, −0.2795]	

4.2. The Empirical Relevance of A4

So much for the production variables. Next, I must consider the interpretation of Y_4, Y_5, Y_6, Y_7, V_4, V_5, and V_6. In the intended interpretation of Axiom A4, the axiom describes characteristics of an entrepreneur's choice of dividends and balance sheet variables that maximizes the value of his utility in (8) subject to the conditions in (9)–(11). With that interpretation in mind, I can deduce from A4, B2–B6, and Theorem 1 all the characteristics of such choices that depict characteristics that the entrepreneurs in my sample must share if my theory is empirically relevant.

I begin with the first four equations in A4. It follows from A4, A6, B2–B6, and Theorem 1 that there exist four random variables, ξ_1, ξ_2, ξ_3, and ξ_4 with MPD means zero and finite positive variances such that

$$\begin{aligned}
\text{mu}_1 &= A + \chi = A + \xi_1 \\
\text{mu}_2 &= V_4(A + \chi) = AV_4 + \xi_2 \\
\text{mu}_3 &= V_5(A + \chi) = AV_5 + \xi_3 \\
\text{mu}_4 &= -(V_6 - u_{11})(A + \chi) = -AV_6 + \xi_4
\end{aligned}$$

MPD estimates of the mean of mu_1 and the coefficients in the last three equations are recorded in Table 4. My theory is empirically relevant only if the three estimates of A lie in the confidence interval of the mean of mu_1. All three do.

Next, I must check the marginal efficiency condition for investments in bonds. Before I display my results, a few words about the meaning of marginal efficiency of capital are called for. In the neo-classical theory, the marginal efficiency of capital is the rate of discount that will equate the price of fixed capital with the present value of the entrepreneur's income from the firm's fixed capital during his planning horizon (cf. Keynes, 1936, p. 135). My idea of the marginal efficiency of capital under conditions of uncertainty differs. It is like Irving Fisher's idea of a consumer's rate of time preference (Fisher, 1961, p. 62). I describe it below for investments in μ and K.

Let $r = (1/p_\mu) - 1$ be the rate of interest on one-period loans; let m_K be the entrepreneur's expected return during the planning horizon from a first-period additional unit of capital conditioned on the observed values of first period prices; and let r_K be defined by the equation, $m_K/(1 + r_K) = p_K$. It follows from A4 that the entrepreneur invests in μ and K up to the point, where

$$\frac{\partial U/\partial d - \partial U/\partial \mu}{\partial U/\partial \mu} = r \tag{15}$$

$$\frac{m_K \cdot \partial U/\partial d - \partial U/\partial K}{\partial U/\partial K} = r_K \tag{16}$$

In (15) and (16), the term, $\partial U/\partial d$, records the expected value of the marginal utility of an extra unit of dividends in period one. In the same period, $\partial U/\partial \mu$ equals the expected value of the marginal utility to the entrepreneur of the income that would be forgone if one unit less is invested in μ. The two concepts combine to form what I in Stigum (1969a) called the marginal efficiency of an extra unit of investment in μ. Similarly, $\mathrm{m}_K \cdot \partial U/\partial d$ and $\partial U/\partial K$ combine to form a relation that I will call the marginal efficiency of capital.

With these concepts in mind, (15) and (16) insist that in equilibrium the entrepreneur invests in μ and K up to the point, where the marginal efficiency of investments in μ and K equal, respectively, the interest rate on one-period loans and the conditionally expected rate of return from an additional unit of capital in period one.

There are six variables involved in the analysis of the entrepreneur's investment in bonds, dividends —Y_4, bonds—Y_5, price of bonds—V_4, two of the marginal-utility variables in the equations in (13)—mu_1 and mu_2, the interest rate on one-period loans—ccr1, and the marginal efficiency of the investment in Y_5—mefmu1. The definition of the last two variables are as follows: $\mathrm{mefmu1} = (\mathrm{mu}_1 - \mathrm{mu}_2)/\mathrm{mu}_2$ and $\mathrm{ccr1} = 1/V_4 - 1$. The mean values of the two mus, mefmu1, and ccr1 are listed in Table 8. According to A4–A6, B5, and (15), my theory is empirically relevant in the present empirical context only if the mean value of ccr1 lies in the confidence interval of the mean value of mefmu1. It does.

Table 8. MPD means of variables involved in bond Investment.

Variables	Mean	Std. Err.	95% Conf. Interval
V_4	0.9089	0.0015	[0.9060, 0.9119]
ccr1	0.1014	0.0018	[0.0979, 0.1049]
mu_1	1.5998	0.0064	[1.5872, 1.6124]
mu_2	1.4520	0.0037	[1.4447, 1.4593]
mefmu1	0.1042	0.0050	[0.0944, 0.1141]

Next, the marginal efficiency condition on investment in capital. There are six variables involved in the empirical analysis of the entrepreneur's investment in capital, capital—Y_6, price of capital—V_5, two of the marginal-utility variables in the equations in (13)—mu_1 and mu_3, the rate of return to capital—ccr3, and the marginal efficiency of the investment in Y_6—mefmu3. With the $\mathrm{m}_K = 1$ in (16), the definitions of the last two variables are as follows:

$$\text{mefmu3} = ((\text{mu}_1 - \text{mu}_3)/\text{mu}_3) \text{ and ccr3} = (1/V_5) - 1.$$

The mean values of the two mus and mefmu3 and ccr3 are listed in Table 9.

Table 9. MPD means of variables involved in capital investment.

Variables	Mean	Std. Err.	95% Conf. Interval
V_5	0.9017	0.0013	[0.8993, 0.9042]
ccr3	0.1098	0.0015	[0.1068, 0.1129]
mu_1	1.5998	0.0064	[1.5872, 1.6124]
mu_3	1.4410	0.0076	[1.4261, 1.4559]
mefmu3	0.1222	0.0074	[0.1078, 0.1367]

According to A4–A6, B5, and (16), my theory is empirically relevant in the present empirical context only if the mean value of ccr3 lies in the confidence interval of the mean of mefmu3. It does.

For the present test the value of m_K is irrelevant since $(\mathrm{m}_K \cdot \text{mu}_1/\text{mu}_3) - 1 = (\mathrm{m}_K/V_5) - 1$, and the 1 and the m_K cancel.

That ends my discussion of the empirical relevance of A4. Without specific assumptions about the way $U(\cdot, D)$ varies with P, there is no analogue of Theorem 3 in the fifth part of the paper for A4.

4.3. Concluding Remarks

I have, now, checked the empirical relevance of all the characteristics that my sample entrepreneurs must share if the theory is empirically relevant. The checks were carried out with MPD distributed data variables. They did not give me reasons to reject the empirical relevance of the theory in an empirical context in which the data are MPD distributed.

It remains to show that the theory is, also, empirically relevant in an empirical context in which the TPD is the data generating process—i.e., in the present empirical context. To carry this out, I must demonstrate that the bridge principles, B1–B6, are valid in the present empirical context. They are valid—according to the Status of bridge principles in applied econometrics—only if all the data admissible models of the MPD are congruent models of the TPD (cf. p. 7 in Stigum (2016)).

A model of the MPD is data admissible only if its parameters lie in the 95% confidence band of the parameters of a meaningful estimate of the MPD. It is a congruent model of the TPD only if it encompasses the TPD and is coherent with the a priori theory in D1 and D2 by containing a model of the equations in (12) and (13) (cf. Definition 2 on p. 6 in Stigum (2016)).

To demonstrate that a data admissible model of the MPD is a congruent model of the TPD, I show, first, that an MPD model in some sense encompasses the TPD. Let M_T and M_P be econometric models whose variables are listed in D1 and satisfy the conditions imposed on them in D1 and D2. Assume that the M_T variables are MPD distributed, that the M_P variable are TPD distributed, and let Δ be a vector whose components are the parameters whose estimated values are listed in Tables 1–9. Moreover, let s_n denote a sample of n observations of the data variables, and let $\mathrm{m}_P^0(\cdot)$ and $\mathrm{m}_T^0(\cdot)$ be, respectively, the Stata 17 estimators of the components of Δ in the TPD and the MPD distributions. Finally, let $\mathrm{TP}(\cdot) : \aleph_P \to [0,1]$ be the probability measure on $(\Omega_P, \aleph_P)$ corresponding to TPD, and let $\mathrm{MP}(\cdot) : \aleph_P \to [0,1]$ be the probability measure on $(\Omega_P, \aleph_P)$ which—in accordance with Kolmogorov's Consistency Theorem (cf. Theorem T 15.23 on p. 347 in Stigum (1990))—is induced by a given MPD. This measure varies with the MPD in question.

Since the two estimators are identical, it is the case, both in $\mathrm{TP}(\cdot)$ measure and in $\mathrm{MP}(\cdot)$ measure, that $\mathrm{m}_T^0(\mathrm{s}_n) = \mathrm{m}_P^0(\mathrm{s}_n)$, a.e. The estimates in Tables 1–4 are MPD estimates as well as TPD estimates. Similarly, the estimates in Tables 5–9 are TPD estimates as well as MPD estimates. Consequently, the two pairs, $(\mathrm{M}_P, \mathrm{m}_P^0(\mathrm{s}_n))$ and $(\mathrm{M}_T, \mathrm{m}_T^0(\mathrm{s}_n))$, in fact, mutually encompass each other (cf. in this context, Bontemps & Mizon, 2008, pp. 727–728).

Since a data admissible model of the MPD contains a model of the equations in (12) and (13), the preceding observations imply that a data admissible model of the MPD is a congruent model of the TPD. From this and the status of bridge principles in applied econometrics, it follows that the bridge principles, B1–B6, are empirically valid in an empirical context in which the data are TPD distributed.

In the present case, the validity of B1–B6 and the fact that my theory is empirically relevant in an empirical context with MPD distributed data imply that the theory is, also, empirically relevant in an empirical context in which the data are TPD distributed.

5. The Empirical Relevance of a Family of Models of $Q(dP)$ and Equations (8)–(11)

In the fourth part of the paper, I established the empirical relevance of a family of models of the axioms, A1–A6. In this section, I will establish the empirical relevance of a family of models of $Q(dP)$ and the equations in (8)–(11). To carry this out, it suffices to show that any one of its theorems has an empirically relevant analogue in the theory universe. In this section, the fifth part of the paper, "my theory" will be taken to be short for "the family of models of $Q(dP)$ and the equations in (8)–(11)".

5.1. The Theorems

In my theory, the entrepreneur's choice of production variables maximizes the firm's profit only if it satisfies the following conditions:

$$\frac{\partial \pi}{\partial y_1} = 0, \quad \frac{\partial c}{\partial y_1} = p_{y1}, \quad \frac{\partial \pi}{\partial p_{y1}} = y_1, \quad \frac{\partial \pi}{\partial w_1} = -L_1 \text{ and } \frac{\partial \pi}{\partial p_{x1}} = -x_1$$

where $\pi = p_{y1}y_1 - w_1L_1 - p_{x1}x_1$ and $c = w_1L_1 + p_{x1}x_1$. Theorem 2 shows that analogous relations are theorems in the theory universe.

Theorem 2. *Suppose that the axioms of the theory universe with my interpretation of them are valid. Then, in* Ω_T*,*

$$\frac{\partial \pi}{\partial y} = 0, \quad \frac{\partial c}{\partial y} = p_y, \quad \frac{\partial \pi}{\partial p_y} = y, \quad \frac{\partial \pi}{\partial w} = -L \textit{ and } \frac{\partial \pi}{\partial p_x} = -x$$

where $\pi = p_y y - wL - p_x x$ *and* $c = wL + p_x x$.

To see why Theorem 2 is valid, observe that in Ω_T

$$\begin{aligned} 1 &= \left(\frac{\partial g(L,x,K_0)}{\partial L}\right)\frac{\partial L}{\partial y} + \left(\frac{\partial g(L,x,K_0)}{\partial x}\right)\frac{\partial x}{\partial y}, \\ \frac{\partial \pi}{\partial y} &= p_y - p_y\left(\frac{\partial g(L,x,K_0)}{\partial L}\right)\frac{\partial L}{\partial y} - p_y\left(\frac{\partial g(L,x,K_0)}{\partial x}\right)\frac{\partial x}{\partial y}, \\ \text{and } \frac{\partial c}{\partial y} &= p_y\left(\frac{\partial g(L,x,K_0)}{\partial L}\right)\frac{\partial L}{\partial y} + p_y\left(\frac{\partial g(L,x,K_0)}{\partial x}\right)\frac{\partial x}{\partial y} \end{aligned}$$

Consequently in Ω_T, $\frac{\partial \pi}{\partial y} = 0$, and $\frac{\partial c}{\partial y} = p_y$.

It is also the case in Ω_T that

$$\frac{\partial \pi}{\partial p_y} = y + p_y \frac{\partial y}{\partial p_y} - w \frac{\partial L}{\partial p_y} - p_x \frac{\partial x}{\partial p_y} =$$
$$y + p_y \left(\frac{\partial g(L, x, K_0)}{\partial L} \right) \frac{\partial L}{\partial p_y} + p_y \left(\frac{\partial g(L, x, K_0)}{\partial x} \right) \frac{\partial x}{\partial p_y}$$
$$- p_y \frac{\partial g(L, x, K_0)}{\partial L} \frac{\partial L}{\partial p_y} - p_y \frac{\partial g(L, x, K_0)}{\partial x} \frac{\partial x}{\partial p_y}.$$

Consequently, $\frac{\partial \pi}{\partial p_y} = y$

By a similar argument I find that $\frac{\partial \pi}{\partial w} = -L$ and $\frac{\partial \pi}{\partial p_x} = -x$.

Next, I totally differentiate the necessary conditions for the optimal choice of production variables in my theory. My aim is to find out how the partial derivatives of y_1, L_1, and x_1, with respect to p_{y1}, w_1, and p_{x1}, are related to one another. The results of my differentiation are summarized in the system of equations in (17). From this system, I deduce that in my theory the entrepreneur's choice of production variables maximizes the firm's profit only if it satisfies the following conditions:

$$\frac{\partial y_1}{\partial p_{y_1}} > 0, \quad \frac{\partial L_1}{\partial w_1} < 0, \quad \frac{\partial x_1}{\partial p_{x_1}} < 0 \text{ and}$$
$$\frac{\partial y_1}{\partial w_1} = -\frac{\partial L_1}{\partial p_{y_1}}, \quad \frac{\partial y_1}{\partial p_{x_1}} = -\frac{\partial x_1}{\partial p_{y_1}} \text{ and } \frac{\partial L_1}{\partial p_{x_1}} = \frac{\partial x_1}{\partial w_1}$$

Theorem 3 shows that analogous relations are theorems in the theory universe.

$$\begin{pmatrix} 0 & p_{y1} \frac{\partial^2 g}{\partial L_1 \partial L_1} & p_{y1} \frac{\partial^2 g}{\partial L_1 \partial x_1} \\ 0 & p_{y1} \frac{\partial^2 g}{\partial x_1 \partial L_1} & p_{y1} \frac{\partial^2 g}{\partial x_1 \partial x_1} \\ 1 & -\frac{\partial g}{\partial L_1} & -\frac{\partial g}{\partial x_1} \end{pmatrix} \begin{pmatrix} \frac{\partial y_1}{\partial p_{y1}} & \frac{\partial y_1}{\partial w_1} & \frac{\partial y_1}{\partial p_{x1}} \\ \frac{\partial L_1}{\partial p_{y1}} & \frac{\partial L_1}{\partial w_1} & \frac{\partial L_1}{\partial p_{x1}} \\ \frac{\partial x_1}{\partial p_{y1}} & \frac{\partial x_1}{\partial w_1} & \frac{\partial x_1}{\partial p_{x1}} \end{pmatrix} \begin{pmatrix} -\frac{\partial g}{\partial L_1} & 1 & 0 \\ -\frac{\partial g}{\partial x_1} & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{17}$$

Theorem 3. *Suppose that the axioms in the theory universe with my interpretation of them are valid. By totally differentiating the first, third, and fourth equation in A3 and summarizing the results in a system of equations, I find that in Ω_T the partial derivatives of y, L, and x, with respect to p_y, w, and p_x, satisfy the following relations:*

$$\frac{\partial y}{\partial p_y} > 0, \quad \frac{\partial L}{\partial w} < 0, \quad \frac{\partial x}{\partial p_x} < 0 \textit{ and}$$
$$\frac{\partial y}{\partial w} = -\frac{\partial L}{\partial p_y}, \quad \frac{\partial y}{\partial p_x} = -\frac{\partial x}{\partial p_y} \textit{ and } \frac{\partial L}{\partial p_x} = \frac{\partial x}{\partial w}$$

To see that Theorem 3 is valid, let $-D$ be the determinant of the matrix

$$\begin{pmatrix} 0 & p_y \frac{\partial^2 g}{\partial L \partial L} & p_y \frac{\partial^2 g}{\partial L \partial x} \\ 0 & p_y \frac{\partial^2 g}{\partial x \partial L} & p_y \frac{\partial^2 g}{\partial x \partial x} \\ 1 & -\frac{\partial g}{\partial L} & -\frac{\partial g}{\partial x} \end{pmatrix}$$

and observe that $D = p_y^2 \left(\frac{\partial^2 g}{\partial L \partial L} \right) \left(\frac{\partial^2 g}{\partial x \partial x} \right) - p_y^2 \left(\frac{\partial^2 g}{\partial L \partial x} \right)^2$. I assume that $D > 0$.

Next note that

$$\frac{\partial x}{\partial p_x} = D^{-1} p_y \left(\frac{\partial^2 g}{\partial L \partial L} \right) < 0, \quad \frac{\partial L}{\partial W} = D^{-1} p_y \left(\frac{\partial^2 g}{\partial x \partial x} \right) < 0 \text{ and}$$
$$\frac{\partial y}{\partial p_y} = \frac{\partial g}{\partial L} \frac{\partial L}{\partial p_y} + \frac{\partial g}{\partial x} \frac{\partial x}{\partial p_y} > 0$$

Hence the inequalities in Theorem 3 are valid.

To see that the equalities are valid too, observe first that

$$\frac{\partial L}{\partial p_x} = -D^{-1} p_y \frac{\partial^2 g}{\partial L \partial x} < 0 \text{ and } \frac{\partial x}{\partial w} = -D^{-1} p_y \frac{\partial^2 g}{\partial x \partial L} < 0$$

They are equal and negative. Next, note that

$$\begin{aligned}
\frac{\partial y}{\partial w} &= D^{-1} p_y \frac{\partial^2 g}{\partial x \partial x} \frac{\partial g}{\partial L} - D^{-1} p_y \frac{\partial^2 g}{\partial x \partial L} \frac{\partial g}{\partial x} < 0, \\
\frac{\partial L}{\partial p_y} &= -D^{-1} p_y \frac{\partial^2 g}{\partial x \partial x} \frac{\partial g}{\partial L} + D^{-1} p_y \frac{\partial^2 g}{\partial L \partial x} \frac{\partial g}{\partial x} > 0, \\
\frac{\partial y}{\partial p_x} &= D^{-1} p_y \frac{\partial^2 g}{\partial L \partial x} \frac{\partial g}{\partial L} + D^{-1} p_y \frac{\partial^2 g}{\partial L \partial L} \frac{\partial g}{\partial x} < 0 \text{ and} \\
\frac{\partial x}{\partial p_y} &= D^{-1} p_y \frac{\partial^2 g}{\partial x \partial L} \frac{\partial g}{\partial L} - D^{-1} p_y \frac{\partial^2 g}{\partial L \partial L} \frac{\partial g}{\partial x} > 0.
\end{aligned}$$

Hence, the given equalities are valid.

5.2. The New Theorems' Empirical Relevance

The preceding observations go to show that Theorem 3 is valid. In the fourth part of the paper, I established the empirical relevance of Theorem 2 and the three inequalities in Theorem 3. It remains to show that the three equalities of Theorem 3 have empirical relevance. For that purpose, several remarks are called for. Note, first, the properties of the data variables in the MPD imply that there exist constants and error terms with zero means and finite positive variances such that

$$Y_2 = a + bV_2 + cV_3 + \xi \text{ and } Y_3 = d + eV_2 + fV_3 + \eta$$
$$Y_1 = \alpha + \beta V_1 + \gamma V_3 + \lambda V_6 + \chi \text{ and } Y_3 = g + hV_1 + kV_2 + mV_3 + \pi$$

A look at the equation

$$\frac{\partial L}{\partial p_y} = -\, D^{-1} p_y \frac{\partial^2 g}{\partial x \partial x} \frac{\partial g}{\partial L} + D^{-1} p_y \frac{\partial^2 g}{\partial x \partial L} \frac{\partial g}{\partial x} =$$
$$w \cdot \left(-D^{-1} \frac{\partial^2 g}{\partial x \partial x} \right) + p_x \cdot \left(D^{-1} \frac{\partial^2 g}{\partial x \partial L} \right)$$

suggests that the constants in the preceding equations can be taken to equal zero.

Below (Table 10) are the regressions I use to test whether the three equality relations are empirically relevant. With one exception, the variables are as described in the Appendix A. The exception is cddv1. It equals –ddv1. I take an equality relation to be empirically relevant if it cannot be contradicted. All the equalities pass the test.

Table 10. Estimates for the equality relations in Theorem 3.

Equation	**Obs**	**Parms**	$RMSE$	$R-sq$	F	P
ddy1	400	3	61.94738	0.9808	6767.304	0.0000
ddy1	Coefficient	Std. err.	t	$P>t$	[95% conf. interval]	
ddv1	11.69772	1.835127	6.37	0.000	8.089934, 15.3055	
cddv2	8.21989	1.525233	5.39	0.000	5.221347, 11.21843	
wy6	5.013005	0.1437563	34.87	0.000	4.730386, 5.295624	
Equation	Obs	Parms	$RMSE$	$R-sq$	F	$P>F$
ddy2	400	3	24.06712	0.9635	3492.587	0.0000
ddy2	Coefficient	Std. err.	t	$P>t$	[95% conf. interval]	
cddv1	−6.110936	0.6813045	−8.97	0.000	−7.450352, −4.77152	
cddv2	7.808191	0.4967596	15.72	0.000	6.831583, 8.784799	
ddv3	12.87144	0.6289732	20.46	0.000	11.63491, 14.10798	
Equation	Obs	Parms	$RMSE$	$R-sq$	F	$P>F$
ddy1	400	3	59.3506	0.9824	7384.28	0.0000
ddy1	Coefficient	Std. err.	t	$P>t$	[95% conf. interval]	
ddv1	10.4313	1.770994	5.89	0.000	6.949602, 13.913	
ddv3	16.51212	2.015146	8.19	0.000	12.55043, 20.47382	
wy6	4.629229	0.1500069	30.86	0.000	4.334322, 4.924136	
Equation	Obs	Parms	$RMSE$	$R-sq$	F	$P>F$
cdcy3	400	3	64.71511	0.9208	1538.081	0.0000
cdcy3	Coefficient	Std. err.	t	$P>t$	[95% conf. interval]	
cddv1	−10.78102	1.831989	−5.88	0.000	−14.38263, −7.179	
cddv2	14.521	1.335758	10.87	0.000	11.89496, 17.14705	
ddv3	22.39877	1.691273	13.24	0.000	19.0738, 25.72374	
Equation	Obs	Parms	$RMSE$	$R-sq$	F	$P>F$
ddy2	400	2	26.3601	0.9561	4333.561	0.0000
cdcy3	400	2	67.3939	0.9139	2111.389	0.0000
	Coefficient	Std. err.	t	$P>t$	[95% conf. interval]	
ddy2						
cddv2	9.470035	0.5048279	18.76	0.000	8.477572, 10.4625	
ddv3	15.80457	0.5884696	26.86	0.000	14.64767, 16.96147	
cdcy3						
cddv2	17.45286	1.290676	13.52	0.000	14.91546, 19.99025	
ddv3	27.57344	1.504519	18.33	0.000	24.61564, 30.53123	

5.3. Concluding Remarks

This paper presents a formal econometric empirical test of an economic theory. To me, the test is an example of an empirical test in applied econometrics in the tradition of Ragnar Frisch. So a few words about applied econometrics in the tradition of Frisch are called for.

At the beginning, applied econometrics in the tradition of Frisch comprised two worlds, a model world and an observational world. In the model world, Frisch developed theories about things and events that he had seen or experienced in the observational world. He considered the rational laws that he discovered in the model world to be totally different from the empirical laws that he observed in the observational world. Hence, there was no bridge between the two worlds in Frisch's applied econometrics (Bjerkholt & Qin, 2011, p. 34).

The missing bridge notwithstanding, Frisch found no difficulties in using his model-world theories to give economic meaning to the variables and parameters of pertinent empirical laws. A good example is Frisch's 1935 interpretation of observed additions of cocoa fat and changes in the moulding-casting work at Freya Chocolate Factory (Frisch, 1935). He used his model-world theory of production to show that they were observations of changes in the values of two factors of production in the production of ordinary nut chocolate. Frisch's theory-based interpretation gave economic meaning to the variables and parameters of the empirical law of nut-chocolate production at Freya.

In this paper, I have added a theory universe, a data universe, a bridge between the two universes, and statistical arguments to Frisch's original applied econometrics. The additions constitute building blocks for a bridge between Frrisch's two worlds. My arguments in the last four parts of the paper describe how the blocks fit together and demonstrate that the resulting bridge is up to its intended task.

Funding: This research received no external funding.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

Acknowledgments: In writing the paper, I have benefited from interesting and constructive criticisms of my work by Edwin Leuven and *Econometrics*' three referees. With K. Waagan's help, I used Stata for my statistical calculations. Konrad Kristofer Reksjø Almås used LaTeX to prepare the manuscript for the submission.

Conflicts of Interest: The author declares no conflicts of interest.

Appendix A

In this, Appendix A, I use Stata's number generator (StataCorp, 2021) to construct the functions that I use to generate my data.

Appendix A.1. Auxiliary Variables

z1 = wz1 = runiform(0, 1) + 0.01393
z2 = wz2 = rbeta(0.75, 0.75) + 0.012386
u1 = du1 = rgamma(7.5, 3) − 22 = 4·cxu1
u2 = du2 = rweibull(5, 25) − 22
u3 = du3 = rchi2(100) − 100
u4 = du4 = rnormal(2, 2) − 2
u5 = du5 = rlaplace(2, 1) − 1
u6 = du6 = rt(100)
z7 = rchi2(100)
wv423 = the end of the following sequence of calculations:

- generate wv41 = 0.083 * rgamma(7.5, 1) + 0.01 * rlaplace(2, 1)
- wv411 = ((wv41 + 0.4)/2)
- wv412 = ((wv411 + 0.3)/1.4)
- wv413 = ((wv412 + 0.3)/1.2)
- wv414 = ((wv413 + 0.3)/1.2)
- wv415 = ((wv414 − 0.05)/0.99)
- wv416 = ((wv415 + 0.1)/1.1)
- wv417 = ((wv416 + 0.1)/1.1)
- wv418 = ((wv417 + 0.1)/1.1)
- wv419 = ((wv418 + 0.1)/1.1
- wv420 = ((wv419 + 0.1)/1.1)
- wv421 = ((wv420 + 0.1)/1.1)
- wv422 = ((wv421 + 0.1)/1.1)
- wv423 = ((wv422 + 0.1)/1.1)

xwv58 = the end of the following sequence of calculations:

- generate xwv5 = 0.02 * rhypergeometric(500, 70, 300) + 0.01 * rnormal(0, 1)
- xwv51 = ((xwv5 + 0.3)/1.5)
- xwv52 = ((xwv51 + 0.15)/1.1)
- xwv53 = ((xwv52 + 0.1)/1.1)
- xwv54 = ((xwv53 + 0.1)/1.1)
- xwv55 = ((xwv54 + 0.1)/1.1)
- xwv56 = ((xwv55 + 0.1)/1.1)
- xwv57 = ((xwv56 + 0.1)/1.1)
- xwv58 = ((xwv57 + 0.1)/1.1)

Appendix A.2. The Variables in Table 1

Y_1 = ddy1 = 4 * (107.686 + 5 * wz1 + cxu1)
Y_2 = ddy2 = 121.6389 + 5 * wz1 + du2
Y_3 = cdcy3 = 18.911 + 9 * rweibull(5, 25) + 2 * runiform (0, 1) + 0.5056
V_1 = ddv1 = 1.7686 + 2 * wz1 + du5
V_2 = cddv2 = (4 − 2*wz2 + du4) − 0.006
V_3 = ddv3 = (6 − 3 * wz2 + du6) + 0.105

Appendix A.3. The Variables in Table 2

mg1 = (cddv2/ddv1) + rnormal(1, 1) − 1
mg2 = (ddv3/ddv1) + rnormal(1, 1) − 1

Appendix A.4. The Variables in Table 3

Y_4 = wy4 = 1.5 * rgamma(7.5, 1) + 0.1 * rbinomial(100, 0.5)
Y_5 = wy5 = 3 * rgamma(7.5, 1) + 0.01 * (rbinomial(100, 0.5) − 50)
Y_6 = wy6 = 10 + 3 * rbinomial(40, 0.5) + 0.1 * (rnormal(2, 1) − 1)
Y_7 = wy7 = 10 + 0.5*z7 + 0.1 * rlaplace(2, 1)
V_4 = wv423
V_5 = xwv58
V_6 = wxv6 = 2 + 0.02 * z7 + 0.001 * rt(100)

Appendix A.5. The Variables in Table 4

mu1 = wmu1 = 1.6 + 0.025 * (rbinomial(100, 0.5) − 50)
mu2 = xwmu2 = 1.6 * wv423 + 0.1 * runiform(1, 3) − 0.2
mu3 = wxwmu3 = 1.6 * xwv58 + 0.01 *(rchi2(100) − 100) + 0.005
mu4 = xwmu4 = 1.6 * wxv6 + 0.01 * rt(100)

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