

# Covariance Prediction in Large Portfolio Allocation: Supplementary Material\*

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In this supplementary material we discuss the results corresponding to the case without short-selling constraints of the empirical application in the paper of Trucíos et al. (2019). These results are given in Tables 9-16.

[Tables 9 to 16 around here]

The analysis of the tables reveals that no method is the best in all scenarios and the performance depends on the criterion, period and rebalancing strategy. The analysis is first done according to the standard deviation (SD) criterion, because portfolios are chosen in order to have the minimum variance. For portfolios rebalanced daily (Tables 9-12), the largest SD is reported by the SNL-OGARCH

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(full and pre-crisis periods) and RM1994 (crisis and post-crisis periods) procedures, and the best performance is achieved by the DCC with composite likelihood. For portfolios rebalanced monthly (Tables 13-16) the worst performance is obtained by the RM1994 method and the smallest SD is obtained by the DCC procedure with composite likelihood in all cases.

The best performance in terms of the average portfolio return (AV) criterion differs depending on the period and rebalancing strategy. For daily rebalancing, RM1994-LS exhibits the best performance in the pre-crisis, crisis and post-crisis periods, whereas in the full period the best performance is achieved by the RM1994 method. However, for monthly rebalancing, the best performance in the full, pre-crisis and crisis periods is achieved by the RM1994 while the best performance in the post-crisis period is achieved by the RM1994 with linear shrinkage applied to the one-step-ahead conditional covariance matrix.

An interesting finding regarding minimum variance portfolios under no short-selling constraints is described next. Since no limits are imposed on the weights of the portfolio, large turnover values can be obtained, and consequently, the quantity  $(1 - c \times turnover_t)$ , where  $c$  is the basis point, can be large in negative value, changing the sign of the  $(1 + r_{p,t})$  values. As a consequence, there can be large loss (average return) but huge net gain (average net portfolio return taking into account transaction costs). This occurs, for example, in Table 10 (pre-crisis period) using the SNL-OGARCH procedure, where the turnover is 3.951, the average portfolio return is near  $-116$ , but the average portfolio return net of transaction costs is near 621 and 1726 considering 20 bp and 50 bp, respectively. Therefore, the turnover and average net returns in the case without short-selling constraints must be carefully analysed.

Next we focus the analysis only on the full period (Tables 9 and 13) in order to assess the performance of the methods in a period with different volatility levels.

In terms of the best five performances according to each criterion, for daily rebalancing, DCC methods are the best regarding SD and are among the best in terms of information ratio (IR) and Sortino's ratio (SR). RM1994-LS and RM2006-LS are the best according to AV, annualised average portfolio return net of transaction costs considering 20bp and 50bp ( $AV_{20bp}^{net}$  and  $AV_{50bp}^{net}$ , respectively), IR and SR. In

addition, some RPVC methods are among the best in terms of AV. For monthly rebalancing, DCC-LS and LS-DCC-LS are among the best in terms of SD, while RM2006-NLS is the best in terms of SD and is among the best regarding IR and SR. RM1994 and RM1994-LS are the first and second in terms of AV,  $AV_{20bp}^{net}$  and  $AV_{50bp}^{net}$ , but are among the worst in terms of SD.

Now we discuss whether shrinkage improves the performance of the methods in terms of SD. The analysis is again done in the full period (Tables 9 and 13). For daily and monthly rebalancing, shrinkage always improves the performance of CCC, RM2006, RM1994, GPVC, RPVC and OGARCH (except NLS-OGARCH for daily rebalancing and LS-OGARCH-LS for monthly rebalancing). In contrast, shrinkage does not improve the performance of DCC and DECO methods for daily rebalancing. However, shrinkage always improves the performance of DCC for monthly rebalancing (except LS-DCC). In addition, LS-DECO and NLS-DECO improve the performance of the DECO method for monthly rebalancing. This improvement produced by the shrinkage can be important. This occurs, for example, in daily rebalancing applying SNL-CCC, and in monthly rebalancing applying NLS-DCC, DCC-LS, RM2006-NLS, RM1994-LS and RM1994-NLS.

In summary, we can say that in the full period, DCC and Riskmetrics based methods are the best and shrinkage improves the performance of the methods in terms of SD. Additionally, for minimum variance portfolios under no short-selling constraints, the results must be analysed with care.

## References

Trucíos, C., Zevallos, M., Hotta, L. K., and Santos, A. A. P. (2019). Covariance prediction in large portfolio allocation. *Submitted to Econometrics*.

Table 9: Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns.  $AV_{20bp}^{net}$  and  $AV_{50bp}^{net}$  stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2004 - November 2017. The shaded cells denote the top five for each criterion. Weights are rebalanced on a daily basis considering no short-selling constraints

	AV	SD	IR	SR	TO	$AV_{20bp}^{net}$	$AV_{50bp}^{net}$
1/N	8.302	20.058 (44)	0.414	0.570	-	-	-
CCC	8.052	11.033 (21)	0.730	1.017	0.838	7.575	6.924
CCC LS	6.546	10.899 (18)	0.601	0.838	0.631	6.194	5.704
CCC NLS	7.939	10.844 (15)	0.732	1.023	0.666	7.554	7.033
LS CCC	7.940	10.853 (17)	0.732	1.019	0.758	7.506	6.917
NLS CCC	7.852	10.686 (12)	0.735	1.023	0.649	7.474	6.969
LS CCC LS	6.242	10.916 (19)	0.572	0.796	0.579	5.916	5.466
NLS CCC NLS	7.939	10.844 (15)	0.732	1.023	0.666	7.554	7.033
DCC	8.942	9.264 (1)	0.965	1.357	0.859	8.454	7.784
DCC LS	7.479	9.632 (4)	0.777	1.093	0.685	7.099	6.567
DCC NLS	9.192	9.658 (6)	0.952	1.327	0.645	8.806	8.302
LS DCC	8.950	9.279 (2)	0.965	1.356	0.862	8.460	7.787
NLS DCC	9.083	9.300 (3)	0.977	1.377	0.872	8.592	7.911
LS DCC LS	7.493	9.640 (5)	0.777	1.094	0.686	7.111	6.578
NLS DCC NLS	9.247	9.677 (7)	0.956	1.334	0.649	8.862	8.355
RM2006	8.456	11.923 (26)	0.709	0.976	0.546	8.141	7.720
RM2006 LS	12.098	10.772 (13)	1.123	1.582	0.565	11.771	11.327
RM2006 NLS	9.445	10.013 (8)	0.943	1.297	0.330	9.228	8.970
RM1994	7.364	26.518 (46)	0.278	0.382	2.933	5.727	3.444
RM1994 LS	12.213	11.390 (24)	1.072	1.527	0.590	11.869	11.403
RM1994 NLS	9.773	10.452 (9)	0.935	1.292	0.361	9.526	9.243
DECO	6.312	12.132 (27)	0.520	0.730	0.368	6.096	5.810
DECO NLS	6.150	12.385 (34)	0.497	0.700	0.436	5.899	5.558
LS DECO	6.463	12.141 (28)	0.532	0.748	0.368	6.248	5.961
NLS DECO	6.462	12.141 (29)	0.532	0.748	0.368	6.247	5.960
NLS DECO NLS	6.210	12.401 (35)	0.501	0.707	0.437	5.963	5.621
OGARCH	-1.751	26.101 (45)	-0.067	-0.071	1.971	3.377	11.124
OGARCH LS	7.545	11.135 (22)	0.678	0.954	0.322	7.372	7.121
OGARCH NLS	8.834	10.574 (11)	0.835	1.167	0.373	8.610	8.321
LS OGARCH	8.799	17.372 (43)	0.507	0.666	1.519	10.426	12.923
NLS OGARCH	-32.209	116.156 (47)	-0.277	-0.278	3.951	181.760	502.767
LS OGARCH LS	8.250	11.174 (23)	0.738	1.038	0.305	8.082	7.842
NLS OGARCH NLS	8.826	10.573 (10)	0.835	1.166	0.372	8.603	8.314
GPVC	8.573	12.993 (38)	0.660	0.931	0.971	7.996	7.218
GPVC LS	8.398	10.799 (14)	0.778	1.083	0.290	8.210	7.985
GPVC NLS	7.634	12.173 (30)	0.627	0.877	0.560	7.289	6.859
LS GPVC	9.764	12.323 (32)	0.792	1.121	0.712	9.320	8.749
NLS GPVC	7.872	12.586 (36)	0.626	0.875	0.818	7.382	6.736
LS GPVC LS	9.257	10.997 (20)	0.842	1.176	0.242	9.092	8.905
NLS GPVC NLS	7.478	12.331 (33)	0.606	0.848	0.556	7.134	6.707
RPVC	8.183	15.938 (42)	0.513	0.704	2.243	7.189	5.805
RPCV LS	7.499	11.453 (25)	0.655	0.917	0.572	7.165	6.712
RPVC NLS	9.905	13.300 (39)	0.745	1.063	1.043	9.279	8.444
LS RPVC	8.595	13.774 (40)	0.624	0.902	1.366	7.797	6.682
NLS RPVC	10.533	14.281 (41)	0.738	1.076	1.771	9.394	7.750
LS RPCV LS	6.519	12.201 (31)	0.534	0.754	0.489	6.240	5.852
NLS RPVC NLS	10.317	12.606 (37)	0.818	1.182	0.968	9.765	9.000

Table 10: Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns.  $AV_{20bp}^{net}$  and  $AV_{50bp}^{net}$  stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2004 - December 2007. The shaded cells denote the top five for each criterion. Weights are rebalanced on a daily basis considering no short-selling constraints

	AV	SD	IR	SR	TO	$AV_{20bp}^{net}$	$AV_{50bp}^{net}$
1/N	12.732	12.755 (42)	0.998	1.418	-	-	
CCC	8.069	8.066 (16)	1.000	1.416	0.698	7.562	7.023
CCC LS	5.542	7.888 (13)	0.703	0.985	0.527	5.183	4.777
CCC NLS	7.567	7.809 (11)	0.969	1.371	0.581	7.133	6.684
LS CCC	7.706	7.906 (14)	0.975	1.377	0.639	7.238	6.745
NLS CCC	7.675	7.736 (10)	0.992	1.402	0.535	7.260	6.847
LS CCC LS	4.464	8.130 (17)	0.549	0.763	0.483	4.128	3.757
NLS CCC NLS	7.567	7.809 (11)	0.969	1.371	0.581	7.133	6.684
DCC	8.249	6.833 (1)	1.207	1.716	0.708	7.738	7.189
DCC LS	5.738	7.258 (6)	0.791	1.109	0.537	5.373	4.959
DCC NLS	8.319	6.934 (4)	1.200	1.702	0.608	7.831	7.359
LS DCC	8.297	6.846 (2)	1.212	1.724	0.711	7.785	7.234
NLS DCC	8.737	6.894 (3)	1.267	1.815	0.732	8.232	7.663
LS DCC LS	5.752	7.262 (7)	0.792	1.111	0.538	5.386	4.972
NLS DCC NLS	8.553	6.967 (5)	1.228	1.751	0.625	8.068	7.582
RM2006	11.477	8.578 (24)	1.338	1.904	0.411	11.139	10.813
RM2006 LS	9.038	7.725 (9)	1.170	1.684	0.439	8.702	8.361
RM2006 NLS	9.335	7.482 (8)	1.248	1.748	0.280	9.031	8.810
RM1994	15.516	18.682 (45)	0.831	1.230	2.305	13.811	11.852
RM1994 LS	9.073	8.393 (22)	1.081	1.570	0.483	8.703	8.324
RM1994 NLS	8.887	8.030 (15)	1.107	1.554	0.332	8.510	8.249
DECO	3.499	9.350 (27)	0.374	0.527	0.354	3.232	2.960
DECO NLS	3.138	9.479 (32)	0.331	0.467	0.424	2.838	2.512
LS DECO	3.756	9.437 (29)	0.398	0.563	0.354	3.494	3.221
NLS DECO	3.756	9.438 (30)	0.398	0.563	0.354	3.494	3.221
NLS DECO NLS	3.315	9.569 (33)	0.346	0.491	0.427	3.027	2.698
OGARCH	-9.878	40.590 (46)	-0.243	-0.249	3.690	0.870	17.183
OGARCH LS	9.036	9.177 (26)	0.985	1.369	0.224	8.898	8.723
OGARCH NLS	10.538	8.310 (19)	1.268	1.809	0.328	10.258	9.998
LS OGARCH	14.665	13.437 (44)	1.091	2.155	0.925	12.351	9.083
NLS OGARCH	-115.725	214.848 (47)	-0.539	-0.539	10.570	620.764	1725.689
LS OGARCH LS	9.733	9.470 (31)	1.028	1.440	0.205	9.598	9.435
NLS OGARCH NLS	10.490	8.307 (18)	1.263	1.801	0.326	10.211	9.953
GPVC	9.169	10.284 (39)	0.892	1.300	0.710	8.594	8.038
GPVC LS	9.234	8.421 (23)	1.097	1.550	0.186	9.007	8.862
GPVC NLS	8.647	9.681 (34)	0.893	1.284	0.412	8.230	7.908
LS GPVC	10.544	9.409 (28)	1.121	1.614	0.505	10.055	9.654
NLS GPVC	7.160	9.930 (37)	0.721	1.033	0.631	6.633	6.156
LS GPVC LS	9.859	8.370 (21)	1.178	1.677	0.168	9.628	9.497
NLS GPVC NLS	8.475	9.794 (36)	0.865	1.247	0.401	8.060	7.747
RPVC	6.495	13.255 (43)	0.490	0.671	2.567	4.998	3.121
RPCV LS	7.862	8.322 (20)	0.945	1.334	0.419	7.527	7.194
RPVC NLS	9.168	10.129 (38)	0.905	1.298	1.066	8.376	7.543
LS RPVC	8.201	10.627 (40)	0.772	1.130	1.451	7.239	6.079
NLS RPVC	9.091	12.002 (41)	0.758	1.162	2.093	7.397	5.080
LS RPCV LS	5.947	8.845 (25)	0.672	0.928	0.383	5.679	5.378
NLS RPVC NLS	8.018	9.751 (35)	0.822	1.181	0.973	7.364	6.604

Table 11: Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns.  $AV_{20bp}^{net}$  and  $AV_{50bp}^{net}$  stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2008 - June 2009. The shaded cells denote the top five for each criterion. Weights are rebalanced on a daily basis considering no short-selling constraints

	AV	SD	IR	SR	TO	$AV_{20bp}^{net}$	$AV_{50bp}^{net}$
1/N	-30.668	43.046 (46)	-0.713	-0.960	-	-	
CCC	-20.400	18.640 (22)	-1.094	-1.415	0.626	-20.692	-21.130
CCC LS	-20.078	18.605 (21)	-1.079	-1.395	0.609	-20.363	-20.789
CCC NLS	-18.955	19.082 (25)	-0.993	-1.292	0.430	-19.163	-19.475
LS CCC	-20.230	18.470 (18)	-1.095	-1.413	0.585	-20.503	-20.913
NLS CCC	-19.128	18.475 (19)	-1.035	-1.336	0.516	-19.371	-19.734
LS CCC LS	-19.795	18.460 (17)	-1.072	-1.384	0.572	-20.063	-20.464
NLS CCC NLS	-18.955	19.082 (25)	-0.993	-1.292	0.430	-19.163	-19.475
DCC	-15.158	15.823 (1)	-0.958	-1.236	0.704	-15.484	-15.972
DCC LS	-15.414	15.935 (4)	-0.967	-1.246	0.684	-15.731	-16.206
DCC NLS	-15.999	17.389 (6)	-0.920	-1.186	0.465	-16.217	-16.543
LS DCC	-15.042	15.863 (2)	-0.948	-1.224	0.709	-15.370	-15.863
NLS DCC	-14.016	15.877 (3)	-0.883	-1.148	0.715	-14.351	-14.852
LS DCC LS	-15.304	15.975 (5)	-0.958	-1.234	0.689	-15.623	-16.102
NLS DCC NLS	-15.692	17.417 (7)	-0.901	-1.165	0.467	-15.912	-16.243
RM2006	-23.481	20.689 (40)	-1.135	-1.432	0.594	-23.726	-24.094
RM2006 LS	-6.744	19.072 (24)	-0.354	-0.459	0.567	-7.020	-7.433
RM2006 NLS	-15.222	17.827 (14)	-0.854	-1.090	0.302	-15.355	-15.555
RM1994	-50.810	44.772 (47)	-1.135	-1.430	2.718	-51.440	-52.387
RM1994 LS	-3.433	20.355 (38)	-0.169	-0.227	0.648	-3.757	-4.244
RM1994 NLS	-13.731	18.562 (20)	-0.740	-0.959	0.340	-13.882	-14.109
DECO	-16.632	19.927 (32)	-0.835	-1.092	0.382	-16.809	-17.076
DECO NLS	-15.162	20.246 (35)	-0.749	-0.999	0.360	-15.333	-15.590
LS DECO	-16.049	19.878 (29)	-0.807	-1.059	0.376	-16.224	-16.487
NLS DECO	-16.053	19.880 (30)	-0.808	-1.059	0.376	-16.228	-16.491
NLS DECO NLS	-14.939	20.221 (34)	-0.739	-0.988	0.357	-15.108	-15.362
OGARCH	-11.771	20.285 (36)	-0.580	-0.785	0.670	-12.074	-12.530
OGARCH LS	-8.367	19.246 (27)	-0.435	-0.589	0.572	-8.638	-9.044
OGARCH NLS	-12.362	17.519 (12)	-0.706	-0.911	0.392	-12.538	-12.801
LS OGARCH	10.914	27.407 (45)	0.398	0.759	3.161	-5.631	-30.447
NLS OGARCH	-11.771	20.285 (37)	-0.580	-0.785	0.670	-12.075	-12.530
LS OGARCH LS	-4.829	19.032 (23)	-0.254	-0.341	0.575	-5.101	-5.510
NLS OGARCH NLS	-12.362	17.519 (11)	-0.706	-0.911	0.392	-12.537	-12.800
GPVC	-13.708	17.897 (15)	-0.766	-0.999	0.174	-13.773	-13.871
GPVC LS	-10.723	17.680 (13)	-0.607	-0.777	0.188	-10.797	-10.909
GPVC NLS	-14.336	17.466 (8)	-0.821	-1.053	0.142	-14.393	-14.479
LS GPVC	-8.419	17.959 (16)	-0.469	-0.618	0.155	-8.484	-8.581
NLS GPVC	-12.460	17.484 (9)	-0.713	-0.915	0.150	-12.519	-12.608
LS GPVC LS	-4.383	19.411 (28)	-0.226	-0.294	0.154	-4.442	-4.531
NLS GPVC NLS	-14.441	17.494 (10)	-0.826	-1.059	0.142	-14.498	-14.583
RPVC	-12.858	21.823 (42)	-0.589	-0.800	0.699	-13.188	-13.681
RPCV LS	-11.876	19.917 (31)	-0.596	-0.792	0.515	-12.118	-12.481
RPVC NLS	-17.311	19.989 (33)	-0.866	-1.163	0.373	-17.478	-17.729
LS RPVC	-21.358	23.955 (44)	-0.892	-1.231	0.783	-21.767	-22.380
NLS RPVC	-13.316	20.778 (41)	-0.641	-0.890	0.484	-13.564	-13.936
LS RPCV LS	-18.578	23.219 (43)	-0.800	-1.098	0.598	-18.899	-19.382
NLS RPVC NLS	-15.170	20.458 (39)	-0.742	-1.028	0.384	-15.355	-15.631

Table 12: Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns.  $AV_{20bp}^{net}$  and  $AV_{50bp}^{net}$  stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period July 2009 - November 2017. The shaded cells denote the top five for each criterion. Weights are rebalanced on a daily basis considering no short-selling constraints

	AV	SD	IR	SR	TO	$AV_{20bp}^{net}$	$AV_{50bp}^{net}$
1/N	13.130	16.057 (43)	0.818	1.148	-	-	-
CCC	13.108	10.393 (24)	1.261	1.832	0.942	12.613	11.870
CCC LS	11.763	10.238 (22)	1.149	1.681	0.684	11.402	10.861
CCC NLS	12.904	10.011 (17)	1.289	1.887	0.748	12.510	11.918
LS CCC	13.065	10.192 (20)	1.282	1.868	0.846	12.620	11.952
NLS CCC	12.738	9.961 (14)	1.279	1.866	0.728	12.355	11.780
LS CCC LS	11.722	10.225 (21)	1.146	1.677	0.627	11.391	10.895
NLS CCC NLS	12.904	10.011 (17)	1.289	1.887	0.748	12.510	11.918
DCC	13.561	8.649 (1)	1.568	2.326	0.958	13.055	12.295
DCC LS	12.383	9.102 (8)	1.361	2.025	0.755	11.984	11.386
DCC NLS	14.092	8.785 (4)	1.604	2.360	0.694	13.724	13.173
LS DCC	13.531	8.657 (2)	1.563	2.318	0.961	13.023	12.260
NLS DCC	13.359	8.674 (3)	1.540	2.278	0.967	12.848	12.081
LS DCC LS	12.379	9.101 (7)	1.360	2.024	0.757	11.979	11.379
NLS DCC NLS	14.017	8.798 (5)	1.593	2.341	0.692	13.651	13.101
RM2006	12.703	11.106 (27)	1.144	1.655	0.602	12.386	11.912
RM2006 LS	16.909	9.930 (12)	1.703	2.530	0.624	16.576	16.078
RM2006 NLS	13.887	9.071 (6)	1.531	2.210	0.359	13.697	13.411
RM1994	13.838	25.270 (47)	0.548	0.781	3.270	12.055	9.381
RM1994 LS	16.492	10.356 (23)	1.593	2.351	0.630	16.156	15.653
RM1994 NLS	14.378	9.401 (9)	1.529	2.202	0.379	14.176	13.872
DECO	11.735	11.440 (30)	1.026	1.497	0.372	11.537	11.239
DECO NLS	11.376	11.736 (31)	0.969	1.410	0.455	11.135	10.773
LS DECO	11.758	11.437 (28)	1.028	1.500	0.373	11.559	11.260
NLS DECO	11.757	11.437 (28)	1.028	1.500	0.373	11.558	11.260
NLS DECO NLS	11.353	11.739 (32)	0.967	1.407	0.457	11.110	10.747
OGARCH	3.901	16.429 (44)	0.238	0.264	1.384	7.321	12.450
OGARCH LS	9.668	9.952 (13)	0.972	1.412	0.325	9.495	9.235
OGARCH NLS	11.796	9.861 (10)	1.196	1.741	0.390	11.590	11.282
LS OGARCH	5.632	16.728 (46)	0.337	0.371	1.509	12.367	22.471
NLS OGARCH	3.901	16.450 (45)	0.237	0.263	1.385	7.331	12.475
LS OGARCH LS	9.872	9.970 (15)	0.990	1.441	0.305	9.708	9.461
NLS OGARCH NLS	11.806	9.862 (11)	1.197	1.742	0.390	11.600	11.292
GPVC	12.255	13.101 (39)	0.935	1.345	1.236	11.586	10.582
GPVC LS	11.403	10.166 (19)	1.122	1.636	0.358	11.214	10.931
GPVC NLS	11.062	12.082 (34)	0.916	1.316	0.704	10.701	10.158
LS GPVC	12.629	12.311 (36)	1.026	1.486	0.909	12.139	11.404
NLS GPVC	11.830	12.669 (38)	0.934	1.339	1.026	11.280	10.456
LS GPVC LS	11.399	9.983 (16)	1.142	1.674	0.292	11.245	11.015
NLS GPVC NLS	10.905	12.294 (35)	0.887	1.271	0.703	10.544	10.003
RPVC	12.731	15.864 (42)	0.803	1.103	2.364	11.859	10.551
RPCV LS	10.775	10.643 (26)	1.012	1.466	0.654	10.425	9.900
RPVC NLS	15.099	13.122 (40)	1.151	1.683	1.151	14.472	13.532
LS RPVC	14.113	12.547 (37)	1.125	1.667	1.428	13.325	12.142
NLS RPVC	15.465	13.844 (41)	1.117	1.634	1.847	14.431	12.880
LS RPCV LS	11.258	10.619 (25)	1.060	1.545	0.520	10.982	10.569
NLS RPVC NLS	15.948	11.946 (33)	1.335	1.972	1.069	15.378	14.524

Table 13: Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns.  $AV_{20bp}^{net}$  and  $AV_{50bp}^{net}$  stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2004 - November 2017. The shaded cells denote the top five for each criterion. Weights are rebalanced on a monthly basis considering no short-selling constraints

	AV	SD	IR	SR	TO	$AV_{20bp}^{net}$	$AV_{50bp}^{net}$
1/N	8.302	20.058 (46)	0.414	0.570	-	-	-
CCC	7.729	11.407 (24)	0.678	0.951	0.881	7.704	7.668
CCC LS	6.248	11.146 (9)	0.561	0.788	0.671	6.230	6.203
CCC NLS	7.605	11.274 (18)	0.675	0.946	0.701	7.586	7.557
LS CCC	7.733	11.252 (16)	0.687	0.964	0.799	7.712	7.679
NLS CCC	7.588	11.106 (7)	0.683	0.957	0.686	7.569	7.541
LS CCC LS	6.025	11.176 (10)	0.539	0.757	0.619	6.008	5.984
NLS CCC NLS	7.605	11.274 (18)	0.675	0.946	0.701	7.586	7.557
DCC	8.257	11.250 (15)	0.734	1.031	0.910	8.232	8.195
DCC LS	6.848	11.024 (2)	0.621	0.871	0.728	6.828	6.799
DCC NLS	8.440	11.131 (8)	0.758	1.063	0.673	8.422	8.394
LS DCC	8.265	11.255 (17)	0.734	1.031	0.912	8.240	8.202
NLS DCC	8.318	11.194 (12)	0.743	1.047	0.921	8.293	8.255
LS DCC LS	6.867	11.025 (3)	0.623	0.874	0.730	6.848	6.818
NLS DCC NLS	8.461	11.093 (6)	0.763	1.071	0.675	8.443	8.415
RM2006	9.381	12.396 (29)	0.757	1.057	0.606	9.366	9.344
RM2006 LS	10.542	11.219 (13)	0.940	1.328	0.598	10.527	10.504
RM2006 NLS	9.359	10.715 (1)	0.874	1.229	0.357	9.350	9.335
RM1994	13.708	27.177 (47)	0.504	0.705	3.014	13.647	13.555
RM1994 LS	10.822	11.643 (27)	0.930	1.312	0.627	10.805	10.780
RM1994 NLS	10.289	11.178 (11)	0.920	1.301	0.373	10.278	10.263
DECO	5.270	12.607 (33)	0.418	0.585	0.388	5.260	5.245
DECO NLS	5.013	12.908 (35)	0.388	0.546	0.461	5.001	4.984
LS DECO	5.480	12.606 (31)	0.435	0.610	0.388	5.470	5.455
NLS DECO	5.479	12.606 (32)	0.435	0.610	0.388	5.469	5.454
NLS DECO NLS	5.123	12.912 (36)	0.397	0.559	0.461	5.111	5.094
OGARCH	9.695	11.380 (22)	0.852	1.204	0.405	9.684	9.668
OGARCH LS	8.272	11.341 (20)	0.729	1.028	0.340	8.263	8.250
OGARCH NLS	9.085	11.056 (5)	0.822	1.154	0.382	9.075	9.061
LS OGARCH	9.780	11.236 (14)	0.870	1.230	0.405	9.769	9.753
NLS OGARCH	9.698	11.380 (21)	0.852	1.204	0.405	9.688	9.672
LS OGARCH LS	8.506	11.438 (25)	0.744	1.047	0.317	8.498	8.486
NLS OGARCH NLS	9.091	11.055 (4)	0.822	1.155	0.382	9.082	9.067
GPVC	9.390	14.185 (42)	0.662	0.936	1.143	9.350	9.289
GPVC LS	8.907	11.389 (23)	0.782	1.101	0.306	8.899	8.887
GPVC NLS	8.204	12.901 (34)	0.636	0.891	0.600	8.189	8.166
LS GPVC	9.730	13.074 (38)	0.744	1.053	0.875	9.704	9.663
NLS GPVC	9.003	13.674 (40)	0.658	0.930	0.891	8.975	8.934
LS GPVC LS	9.120	11.480 (26)	0.794	1.117	0.254	9.114	9.104
NLS GPVC NLS	8.351	13.050 (37)	0.640	0.898	0.602	8.335	8.311
RPVC	6.701	17.016 (45)	0.394	0.548	2.343	6.629	6.521
RPCV LS	6.113	12.126 (28)	0.504	0.687	0.619	6.098	6.075
RPVC NLS	6.720	13.824 (41)	0.486	0.671	1.141	6.691	6.648
LS RPVC	6.025	14.267 (43)	0.422	0.584	1.590	5.974	5.899
NLS RPVC	8.390	15.095 (44)	0.556	0.778	2.077	8.359	8.314
LS RPCV LS	5.365	12.557 (30)	0.427	0.596	0.585	5.348	5.321
NLS RPVC NLS	8.782	13.116 (39)	0.670	0.945	0.969	8.757	8.719



Table 14: Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns.  $AV_{20bp}^{net}$  and  $AV_{50bp}^{net}$  stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2004 - December 2007. The shaded cells denote the top five for each criterion. Weights are rebalanced on a monthly basis considering no short-selling constraints

	AV	SD	IR	SR	TO	$AV_{20bp}^{net}$	$AV_{50bp}^{net}$
1/N	12.732	12.755 (44)	0.998	1.418	-	-	-
CCC	7.456	8.310 (13)	0.897	1.255	0.693	7.437	7.410
CCC LS	4.815	8.248 (11)	0.584	0.806	0.538	4.801	4.779
CCC NLS	6.327	8.105 (5)	0.781	1.080	0.588	6.311	6.287
LS CCC	7.057	8.178 (8)	0.863	1.203	0.636	7.040	7.014
NLS CCC	6.770	8.017 (2)	0.844	1.172	0.533	6.756	6.734
LS CCC LS	4.145	8.501 (22)	0.488	0.672	0.500	4.132	4.112
NLS CCC NLS	6.327	8.105 (5)	0.781	1.080	0.588	6.311	6.287
DCC	7.898	8.319 (14)	0.950	1.328	0.710	7.879	7.851
DCC LS	4.882	8.229 (9)	0.593	0.817	0.557	4.868	4.846
DCC NLS	7.714	8.144 (7)	0.947	1.321	0.607	7.698	7.674
LS DCC	7.885	8.322 (15)	0.948	1.325	0.712	7.866	7.838
NLS DCC	7.807	8.270 (12)	0.944	1.326	0.725	7.788	7.760
LS DCC LS	4.855	8.230 (10)	0.590	0.812	0.557	4.841	4.819
NLS DCC NLS	7.532	8.092 (4)	0.931	1.302	0.617	7.516	7.491
RM2006	11.344	8.814 (27)	1.287	1.856	0.342	11.334	11.318
RM2006 LS	7.710	8.024 (3)	0.961	1.369	0.410	7.699	7.681
RM2006 NLS	8.949	7.878 (1)	1.136	1.593	0.298	8.940	8.927
RM1994	14.778	18.815 (47)	0.786	1.137	2.155	14.726	14.647
RM1994 LS	7.445	8.725 (26)	0.853	1.218	0.449	7.432	7.412
RM1994 NLS	8.704	8.357 (16)	1.042	1.469	0.334	8.694	8.679
DECO	2.809	9.805 (32)	0.287	0.401	0.351	2.800	2.786
DECO NLS	2.679	9.970 (37)	0.269	0.376	0.431	2.668	2.651
LS DECO	2.790	9.857 (34)	0.283	0.397	0.347	2.780	2.766
NLS DECO	2.787	9.858 (35)	0.283	0.397	0.349	2.777	2.763
NLS DECO NLS	2.581	10.013 (38)	0.258	0.362	0.426	2.570	2.553
OGARCH	11.656	8.429 (17)	1.383	1.988	0.298	11.648	11.636
OGARCH LS	9.445	9.353 (29)	1.010	1.419	0.225	9.439	9.430
OGARCH NLS	10.876	8.456 (20)	1.286	1.841	0.305	10.868	10.855
LS OGARCH	10.483	8.509 (23)	1.232	1.758	0.296	10.475	10.463
NLS OGARCH	11.656	8.429 (17)	1.383	1.988	0.298	11.648	11.636
LS OGARCH LS	9.537	9.633 (30)	0.990	1.392	0.202	9.531	9.523
NLS OGARCH NLS	10.876	8.456 (20)	1.286	1.841	0.305	10.868	10.855
GPVC	8.499	11.088 (42)	0.767	1.089	1.042	8.446	8.366
GPVC LS	9.312	8.604 (24)	1.082	1.525	0.218	9.306	9.296
GPVC NLS	7.126	9.940 (36)	0.717	1.007	0.475	7.112	7.091
LS GPVC	10.212	9.637 (31)	1.060	1.517	0.743	10.188	10.152
NLS GPVC	7.733	10.320 (41)	0.749	1.061	0.939	7.700	7.651
LS GPVC LS	10.241	8.609 (25)	1.190	1.693	0.189	10.235	10.227
NLS GPVC NLS	7.244	10.047 (39)	0.721	1.015	0.464	7.230	7.209
RPVC	2.919	14.295 (46)	0.204	0.282	3.150	2.804	2.630
RPCV LS	7.305	8.449 (19)	0.865	1.216	0.483	7.291	7.270
RPVC NLS	6.695	10.147 (40)	0.660	0.937	1.250	6.659	6.605
LS RPVC	6.624	11.475 (43)	0.577	0.771	1.751	6.568	6.483
NLS RPVC	3.934	14.113 (45)	0.279	0.375	2.174	3.877	3.791
LS RPCV LS	6.994	9.090 (28)	0.769	1.062	0.460	6.979	6.956
NLS RPVC NLS	6.838	9.806 (33)	0.697	0.984	0.985	6.810	6.769

Table 15: Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns.  $AV_{20bp}^{net}$  and  $AV_{50bp}^{net}$  stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2008 - June 2009. The shaded cells denote the top five for each criterion. Weights are rebalanced on a monthly basis considering no short-selling constraints

	AV	SD	IR	SR	TO	$AV_{20bp}^{net}$	$AV_{50bp}^{net}$
1/N	-30.668	43.046 (46)	-0.713	-0.960	-	-	-
CCC	-19.417	19.608 (14)	-0.990	-1.310	0.704	-19.437	-19.467
CCC LS	-19.353	19.584 (13)	-0.988	-1.306	0.687	-19.373	-19.402
CCC NLS	-19.407	20.329 (22)	-0.955	-1.267	0.488	-19.422	-19.444
LS CCC	-19.380	19.505 (10)	-0.994	-1.313	0.658	-19.398	-19.426
NLS CCC	-19.602	19.582 (12)	-1.001	-1.322	0.580	-19.618	-19.643
LS CCC LS	-19.290	19.505 (11)	-0.989	-1.307	0.645	-19.308	-19.335
NLS CCC NLS	-19.407	20.329 (22)	-0.955	-1.267	0.488	-19.422	-19.444
DCC	-15.559	19.673 (16)	-0.791	-1.052	0.750	-15.582	-15.616
DCC LS	-15.812	19.672 (15)	-0.804	-1.068	0.729	-15.834	-15.867
DCC NLS	-17.319	20.625 (27)	-0.840	-1.119	0.503	-17.335	-17.359
LS DCC	-15.327	19.682 (18)	-0.779	-1.037	0.754	-15.350	-15.384
NLS DCC	-13.128	19.361 (6)	-0.678	-0.910	0.754	-13.150	-13.184
LS DCC LS	-15.601	19.682 (17)	-0.793	-1.054	0.735	-15.624	-15.657
NLS DCC NLS	-15.795	20.507 (25)	-0.770	-1.030	0.507	-15.811	-15.836
RM2006	-12.434	22.419 (40)	-0.555	-0.733	0.641	-12.440	-12.450
RM2006 LS	-7.281	20.495 (24)	-0.355	-0.474	0.584	-7.293	-7.312
RM2006 NLS	-11.703	20.320 (21)	-0.576	-0.781	0.300	-11.709	-11.719
RM1994	15.202	46.693 (47)	0.326	0.436	3.228	15.164	15.108
RM1994 LS	-5.687	21.025 (34)	-0.271	-0.366	0.704	-5.704	-5.728
RM1994 NLS	-8.392	21.197 (36)	-0.396	-0.545	0.360	-8.401	-8.415
DECO	-16.709	20.834 (31)	-0.802	-1.069	0.423	-16.719	-16.733
DECO NLS	-16.353	21.212 (37)	-0.771	-1.041	0.404	-16.362	-16.377
LS DECO	-15.192	20.757 (29)	-0.732	-0.979	0.427	-15.202	-15.216
NLS DECO	-15.196	20.759 (30)	-0.732	-0.980	0.427	-15.206	-15.221
NLS DECO NLS	-15.481	21.182 (35)	-0.731	-0.989	0.408	-15.490	-15.505
OGARCH	-4.790	21.283 (39)	-0.225	-0.307	0.727	-4.807	-4.833
OGARCH LS	-4.561	20.065 (19)	-0.227	-0.307	0.635	-4.576	-4.599
OGARCH NLS	-12.047	19.405 (8)	-0.621	-0.822	0.390	-12.056	-12.070
LS OGARCH	-0.780	20.706 (28)	-0.038	-0.051	0.674	-0.795	-0.819
NLS OGARCH	-4.790	21.283 (38)	-0.225	-0.307	0.727	-4.807	-4.832
LS OGARCH LS	-2.276	20.191 (20)	-0.113	-0.152	0.603	-2.290	-2.311
NLS OGARCH NLS	-12.046	19.405 (7)	-0.621	-0.822	0.390	-12.055	-12.069
GPVC	-5.286	19.325 (5)	-0.274	-0.370	0.157	-5.290	-5.296
GPVC LS	-3.173	19.204 (4)	-0.165	-0.223	0.226	-3.179	-3.187
GPVC NLS	-8.691	18.815 (1)	-0.462	-0.617	0.134	-8.695	-8.700
LS GPVC	-2.206	19.423 (9)	-0.114	-0.155	0.142	-2.210	-2.215
NLS GPVC	-5.957	18.959 (3)	-0.314	-0.420	0.140	-5.961	-5.966
LS GPVC LS	-1.798	20.834 (32)	-0.086	-0.116	0.184	-1.800	-1.804
NLS GPVC NLS	-8.761	18.841 (2)	-0.465	-0.621	0.134	-8.765	-8.770
RPVC	-27.965	26.268 (45)	-1.065	-1.324	0.819	-27.972	-27.981
RPCV LS	-20.952	22.964 (42)	-0.912	-1.158	0.647	-20.959	-20.970
RPVC NLS	-24.506	22.441 (41)	-1.092	-1.407	0.490	-24.513	-24.522
LS RPVC	-31.971	25.057 (44)	-1.276	-1.708	1.121	-32.000	-32.043
NLS RPVC	-12.300	20.894 (33)	-0.589	-0.816	0.800	-12.340	-12.400
LS RPCV LS	-28.707	23.988 (43)	-1.197	-1.616	0.848	-28.728	-28.759
NLS RPVC NLS	-14.866	20.620 (26)	-0.721	-0.991	0.538	-14.885	-14.913

Table 16: Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns.  $AV_{20bp}^{net}$  and  $AV_{50bp}^{net}$  stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period July 2009 - November 2017. The shaded cells denote the top five for each criterion. Weights are rebalanced on a monthly basis considering no short-selling constraints

	AV	SD	IR	SR	TO	$AV_{20bp}^{net}$	$AV_{50bp}^{net}$
1/N	13.130	16.057 (45)	0.818	1.148	-	-	-
CCC	12.690	10.654 (26)	1.191	1.737	1.004	12.663	12.621
CCC LS	11.486	10.220 (16)	1.124	1.659	0.732	11.467	11.438
CCC NLS	13.022	10.245 (18)	1.271	1.864	0.791	13.000	12.968
LS CCC	12.882	10.461 (25)	1.231	1.799	0.902	12.857	12.820
NLS CCC	12.816	10.234 (17)	1.252	1.833	0.778	12.795	12.763
LS CCC LS	11.425	10.203 (15)	1.120	1.652	0.669	11.408	11.382
NLS CCC NLS	13.022	10.245 (18)	1.271	1.864	0.791	13.000	12.968
DCC	12.668	10.356 (21)	1.223	1.784	1.034	12.639	12.597
DCC LS	11.818	9.981 (8)	1.184	1.740	0.812	11.796	11.763
DCC NLS	13.371	9.863 (4)	1.356	1.985	0.736	13.351	13.321
LS DCC	12.645	10.360 (22)	1.221	1.780	1.038	12.616	12.573
NLS DCC	12.378	10.382 (23)	1.192	1.735	1.044	12.350	12.306
LS DCC LS	11.824	9.980 (7)	1.185	1.741	0.814	11.802	11.769
NLS DCC NLS	13.221	9.861 (3)	1.341	1.960	0.734	13.201	13.171
RM2006	12.330	11.295 (30)	1.092	1.575	0.726	12.311	12.284
RM2006 LS	15.062	10.132 (14)	1.487	2.194	0.690	15.044	15.018
RM2006 NLS	13.304	9.311 (1)	1.429	2.079	0.394	13.293	13.277
RM1994	12.933	25.810 (47)	0.501	0.713	3.385	12.863	12.759
RM1994 LS	15.368	10.450 (24)	1.471	2.136	0.696	15.349	15.322
RM1994 NLS	14.368	9.662 (2)	1.487	2.153	0.396	14.357	14.341
DECO	10.353	11.818 (31)	0.876	1.263	0.398	10.343	10.327
DECO NLS	9.927	12.165 (34)	0.816	1.177	0.484	9.915	9.896
LS DECO	10.441	11.820 (32)	0.883	1.274	0.401	10.430	10.415
NLS DECO	10.440	11.820 (32)	0.883	1.274	0.401	10.430	10.415
NLS DECO NLS	10.000	12.166 (35)	0.822	1.186	0.489	9.988	9.969
OGARCH	11.340	9.992 (11)	1.135	1.643	0.398	11.329	11.313
OGARCH LS	9.998	9.975 (6)	1.002	1.462	0.344	9.989	9.976
OGARCH NLS	11.993	10.047 (13)	1.194	1.736	0.417	11.983	11.968
LS OGARCH	11.324	9.909 (5)	1.143	1.661	0.407	11.313	11.297
NLS OGARCH	11.345	9.991 (10)	1.136	1.644	0.398	11.335	11.319
LS OGARCH LS	9.934	9.989 (9)	0.995	1.449	0.321	9.926	9.914
NLS OGARCH NLS	12.005	10.047 (12)	1.195	1.738	0.417	11.994	11.979
GPVC	12.427	14.417 (44)	0.862	1.235	1.367	12.386	12.325
GPVC LS	10.865	10.664 (27)	1.019	1.473	0.363	10.856	10.842
GPVC NLS	11.725	12.853 (38)	0.912	1.303	0.742	11.707	11.679
LS GPVC	11.626	13.094 (40)	0.888	1.272	1.065	11.593	11.545
NLS GPVC	12.270	13.950 (42)	0.880	1.267	1.000	12.240	12.196
LS GPVC LS	10.531	10.275 (20)	1.025	1.489	0.298	10.523	10.511
NLS GPVC NLS	11.923	13.054 (39)	0.913	1.305	0.753	11.904	11.876
RPVC	14.672	16.065 (46)	0.913	1.357	2.231	14.609	14.514
RPCV LS	10.363	10.724 (28)	0.966	1.392	0.677	10.346	10.320
RPVC NLS	12.290	13.300 (41)	0.924	1.313	1.206	12.261	12.218
LS RPVC	12.503	12.702 (36)	0.984	1.411	1.598	12.451	12.374
NLS RPVC	14.193	14.288 (43)	0.993	1.423	2.256	14.177	14.153
LS RPCV LS	10.655	10.881 (29)	0.979	1.411	0.600	10.637	10.610
NLS RPVC NLS	13.916	12.760 (37)	1.091	1.559	1.040	13.891	13.854