"Pitfalls of Two Step Testing for Changes in the Error Variance and Coefficients of a Linear Regression Model"
Supplementary Material (for online publication only)

Supplement I: Experiment with a dynamic regression model

We repeated the full experiments using regression models that include a lagged dependent variable as a regressor. The data are generated by

$$y_t = \mu + \delta_2 I(t > T^c) + \alpha y_{t-1} + e_t \tag{1}$$

where $e_t \sim i.i.d.N(0, 1 + \delta_1 I(t > T^v))$. Throughout, the coefficient α is assumed to be constant, hence it is a correctly specified partial structural change model.

We consider two cases in which the true values are $\alpha = 0$ and $\alpha = 0.5$ but y_{t-1} is included as a regressor in both cases. The results for $\alpha = 0$ are presented in Figures 1A to 6A and those for $\alpha = 0.5$ are presented in Figures 1B to 6B, respectively. The power functions are qualitatively the same as for the benchmark case reported in the text. There are some cases for which the size distortions are larger (see Figure 1B) and some cases with smaller size distortions (see Figures 3A and 3B). However, the qualitative results remain unchanged.

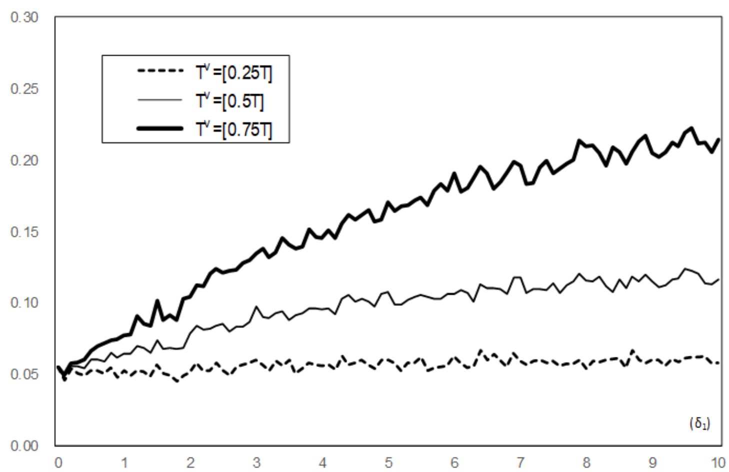

Figure 1A: Size of the Sup-LR test for a coefficient change ignoring a variance change (dynamic model: $\alpha = 0$ in DGP)

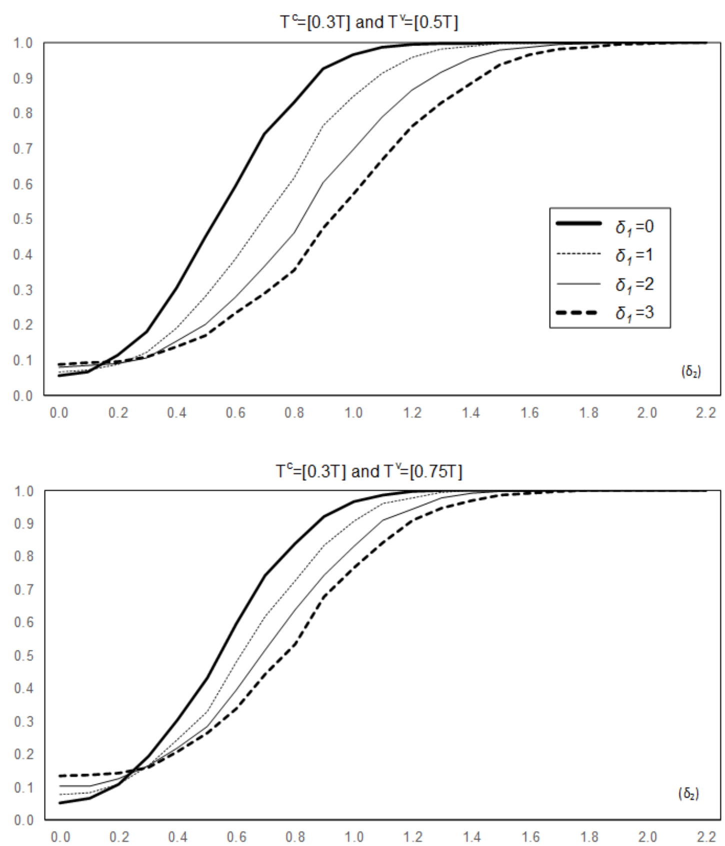

Figure 2A: Power of the Sup-LR test for a coefficient change ignoring a variance change (dynamic model: $\alpha = 0$ in DGP)

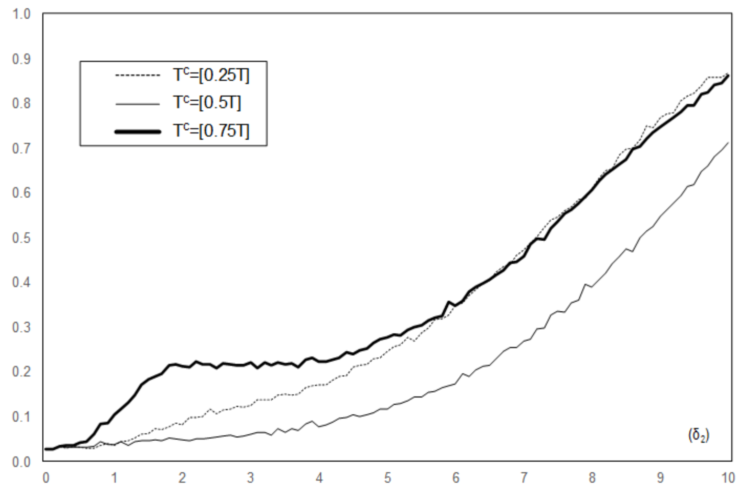

Figure 3A: Size of the CUSQ test for a variance change ignoring a coefficient change (dynamic model: $\alpha = 0$ in DGP)

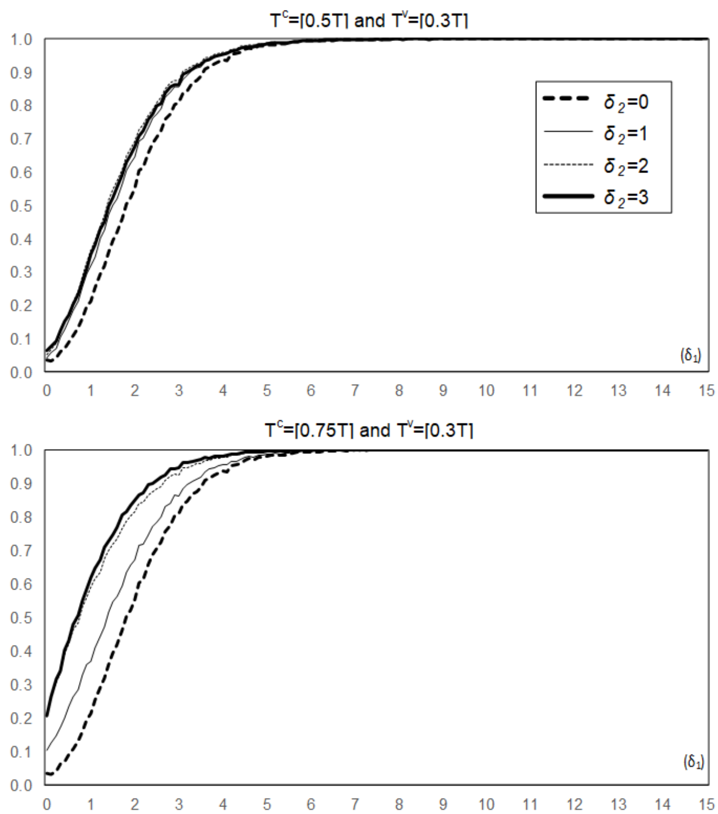

Figure 4A: Power of the CUSQ test for a variance change ignoring a coefficient change (dynamic model: $\alpha = 0$ in DGP)

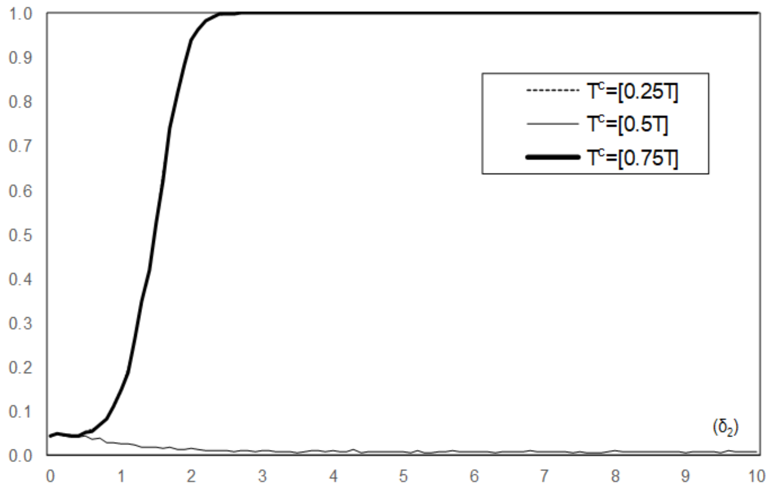

Figure 5A: Size of the two step test for a variance change ignoring a coefficient change (dynamic model: $\alpha = 0$ in DGP)

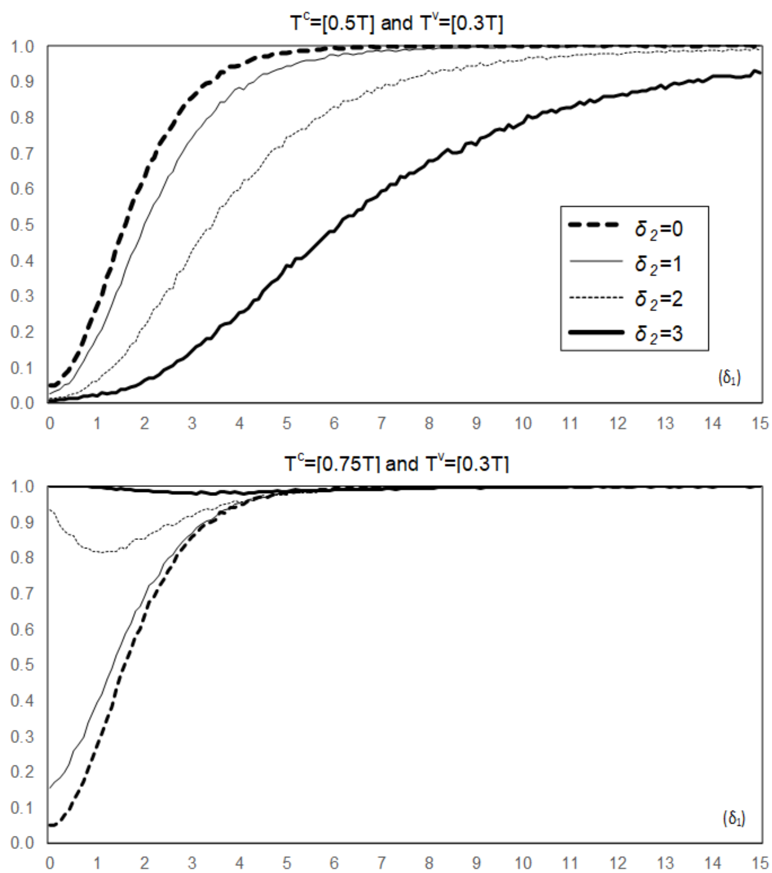

Figure 6A: Power of the two step test for a variance change ignoring a coefficient change (dynamic model: $\alpha = 0$ in DGP)

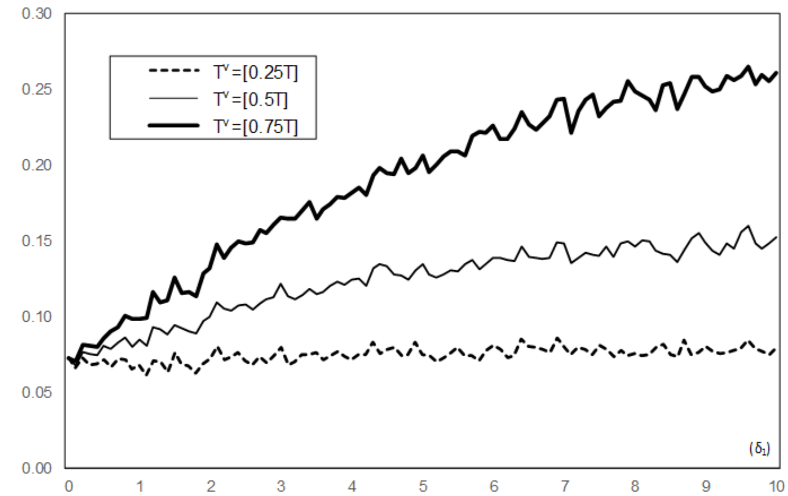

Figure 1B: Size of the Sup-LR test for a coefficient change ignoring a variance change (dynamic model: $\alpha = 0.5$ in DGP)

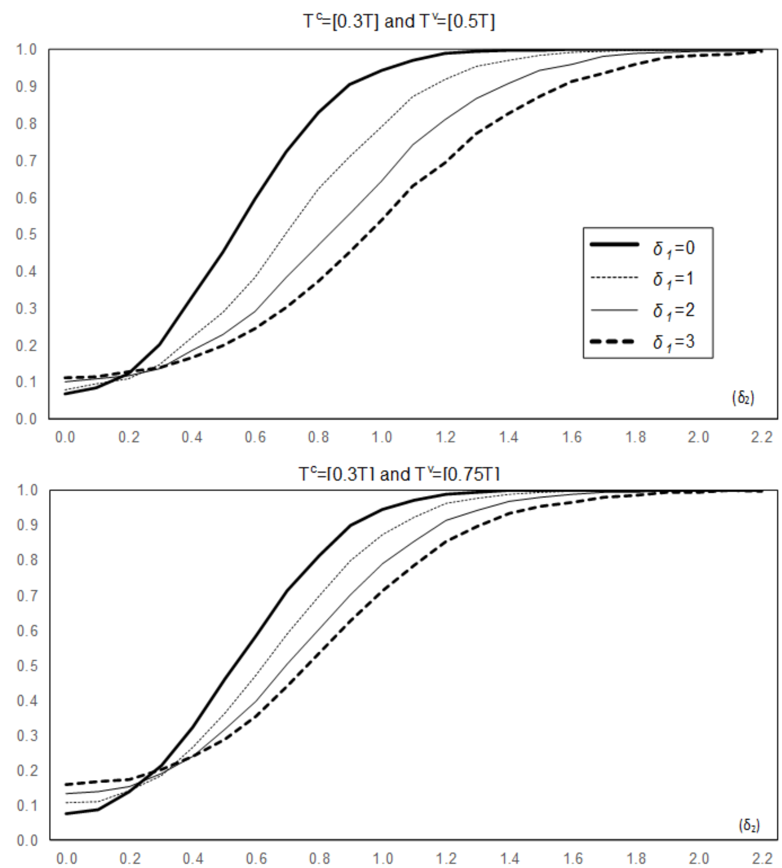

Figure 2B: Power of the Sup-LR test for a coefficient change ignoring a variance change (dynamic model: $\alpha = 0.5$ in DGP)

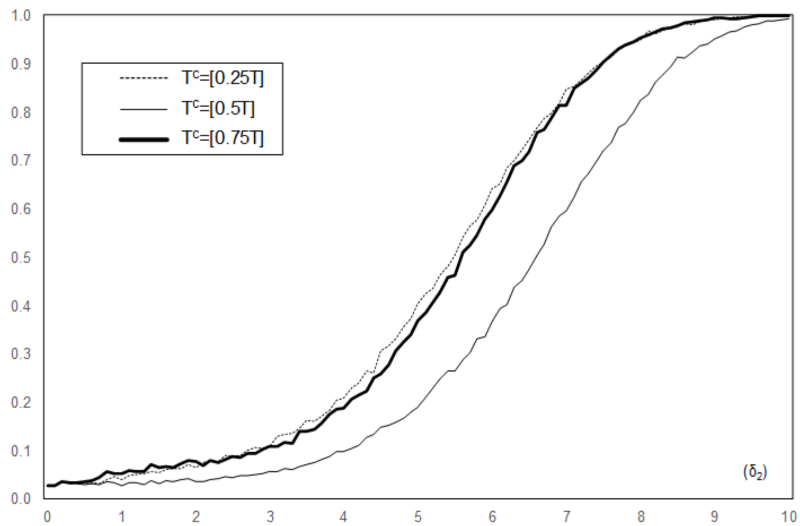

Figure 3B: Size of the CUSQ test for a variance change ignoring a coefficient change (dynamic model: $\alpha = 0.5$ in DGP)

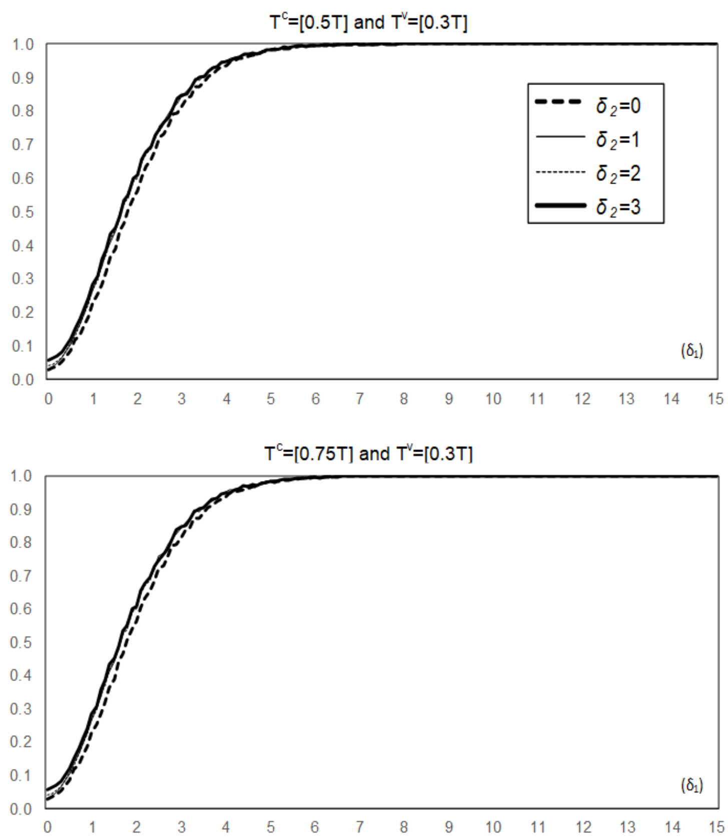

Figure 4B: Power of the CUSQ test for a variance change ignoring a coefficient change (dynamic model: $\alpha = 0.5$ in DGP)

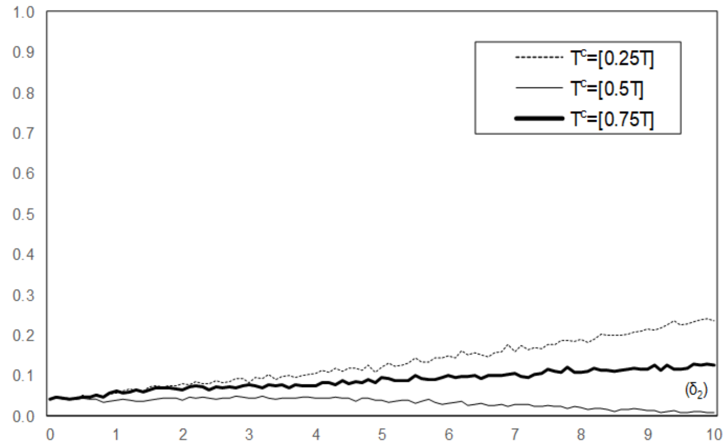

Figure 5B: Size of the two step test for a variance change ignoring a coefficient change (dynamic model: $\alpha = 0.5$ in DGP)

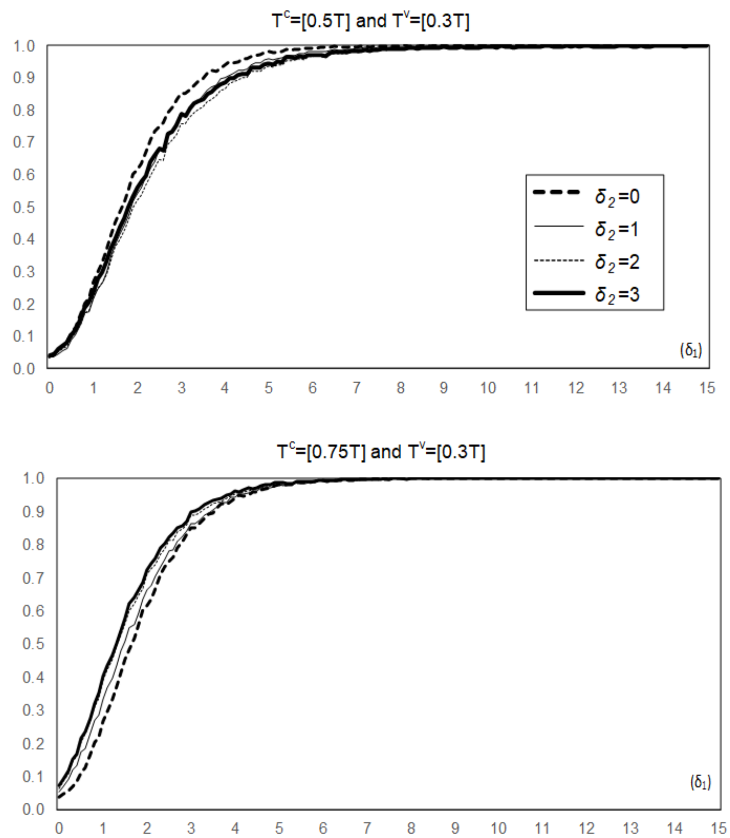

Figure 6B: Power of the two step test for a variance change ignoring a coefficient change (dynamic model: $\alpha = 0.5$ in DGP)

Supplement II: Experiments with multiple structural changes

We repeated the full experiments using DGPs which include m structural changes in the component not accounted for; that is, a model with m changes in the error variance when coefficients changes are tested and a model with m changes in the conditional mean when changes in the error variance are tested. We set $m = 2$ and the two breaks have exactly the same magnitude in the opposite directions, getting back to the original level after the second break. The results presented in Figures 1C-6C show that the benchmark results reported in the text remain qualitatively the same, though Figures 1C and 6C show smaller distortions. Although these results can be DGP-specific and can become quantitatively different when different magnitudes and directions are considered, the same qualitative results should hold.

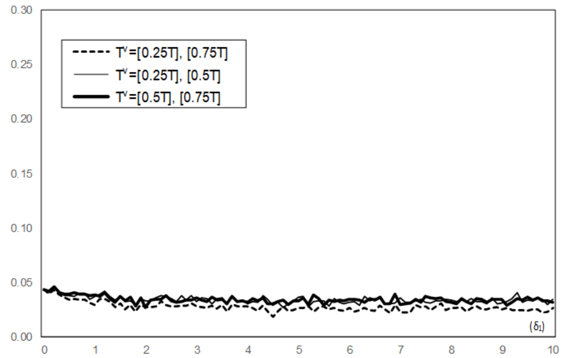

Figure 1C: Size of the Sup-LR test for a coefficient change ignoring variance changes (two variance changes in the opposite directions)

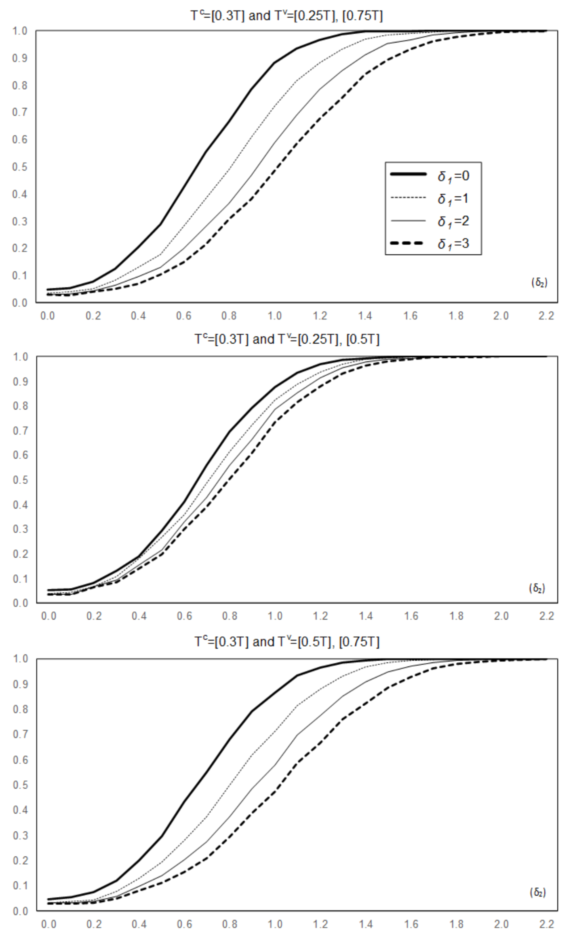

Figure 2C: Power of the Sup-LR test for a coefficient change ignoring variance changes (two variance changes in the opposite directions)

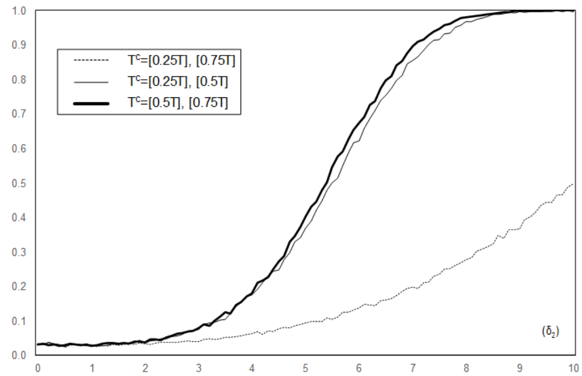

Figure 3C: Size of the CUSQ test for a variance change ignoring coefficient changes (two coefficient changes in the opposite directions)

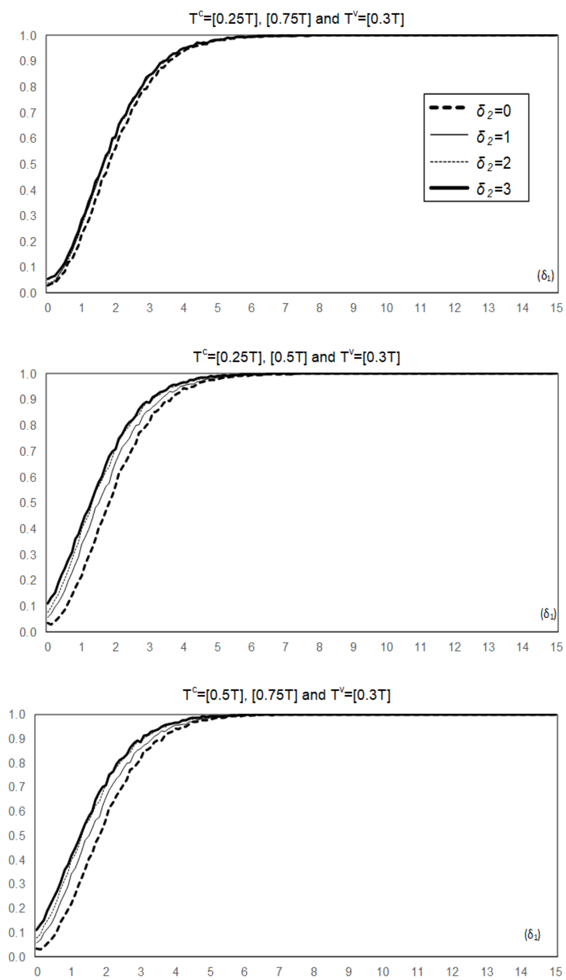

Figure 4C: Power of the CUSQ test for a variance change ignoring coefficient changes (two coefficient changes in the opposite directions)

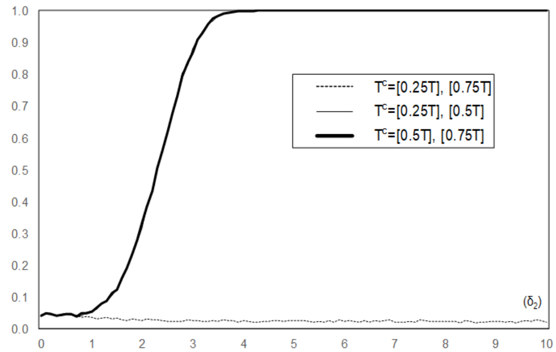

Figure 5C: Size of the two step test for a variance change ignoring coefficient changes (two coefficient changes in the opposite directions)

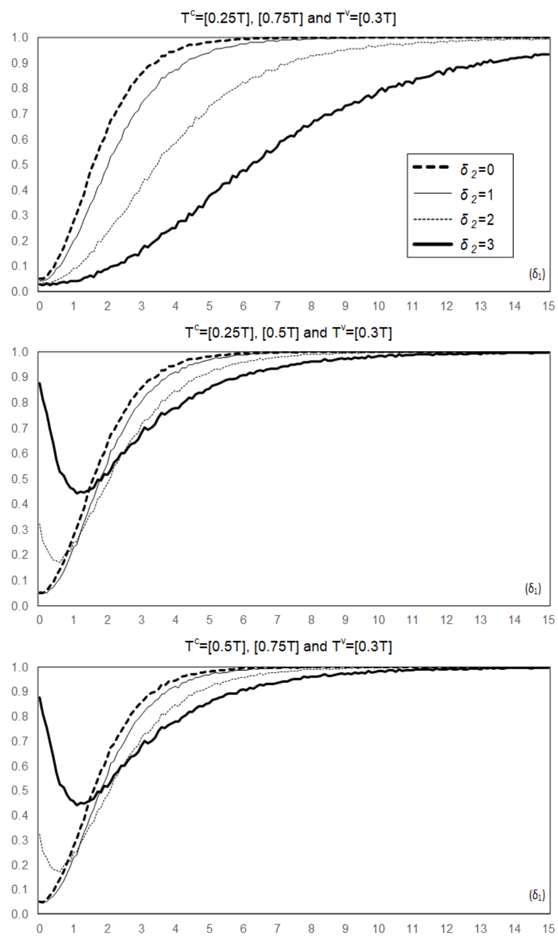

Figure 6C: Power of the two step test for a variance change ignoring coefficient changes (two changes in the opposite directions)

Supplement III: Accounting for conditional heteroskedasticity

We repeated the experiment of testing for a change in variance ignoring the change in the conditional mean presented in Figure 4 when conditional heteroskedasticity is suspected. This can be performed using the extended version of the CUSQ test analyzed in Deng and Perron (2008a) given by

$$CUSQ^* = \frac{\sup_{\lambda\in[0,1]} \left| T^{-1/2} \left[\sum_{t=1}^{[T\lambda]} \widetilde{v}_t^2 - \frac{[T\lambda]}{T} \sum_{t=1}^{T} \widetilde{v}_t^2 \right] \right|}{\hat{\varphi}_a^{1/2}}$$

with

$$\hat{\varphi}_a = \frac{1}{T} \sum_{j=-(T-1)}^{(T-1)} \omega\left(j, b_T\right) \sum_{t=|j|+1}^{T} \hat{\eta}_t \hat{\eta}_{t-j}$$

where $\hat{\eta}_t = \widetilde{v}_t^2 - \hat{\sigma}^2$, with $\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \widetilde{v}_t^2$, with $\widetilde{v}_t$ denoting the recursive residuals. Here $\omega\left(j, b_T\right)$ is the Quadratic Spectral kernel and the bandwidth b_T is selected using Andrews' (1991) method with an AR(1) approximation. In the DGP, we consider Normal errors but this particular choice is inconsequential. In Figure 7D, we present the power of this version under the same DGP as for Figure 4 with the case $T^c = [0.25T]$ added. For illustrative purposes, we use a larger set of values for the change in the conditional mean $\delta_2 = 5, 10, 15, 20$. The results provide evidence that the power of the CUSQ test decreases as the change in the conditional mean becomes large. The power can be near zero in extreme cases such as $\delta_2 = 15, 20$ with $T^c = [0.25T]$.

We can also expect similar results for the two step procedure in which the serial correlations in the second step is corrected via a long-run variance estimate constructed with the residuals under the null hypothesis. The same can be said when a lagged dependent variable is included in the second step and the SupLM test is used.

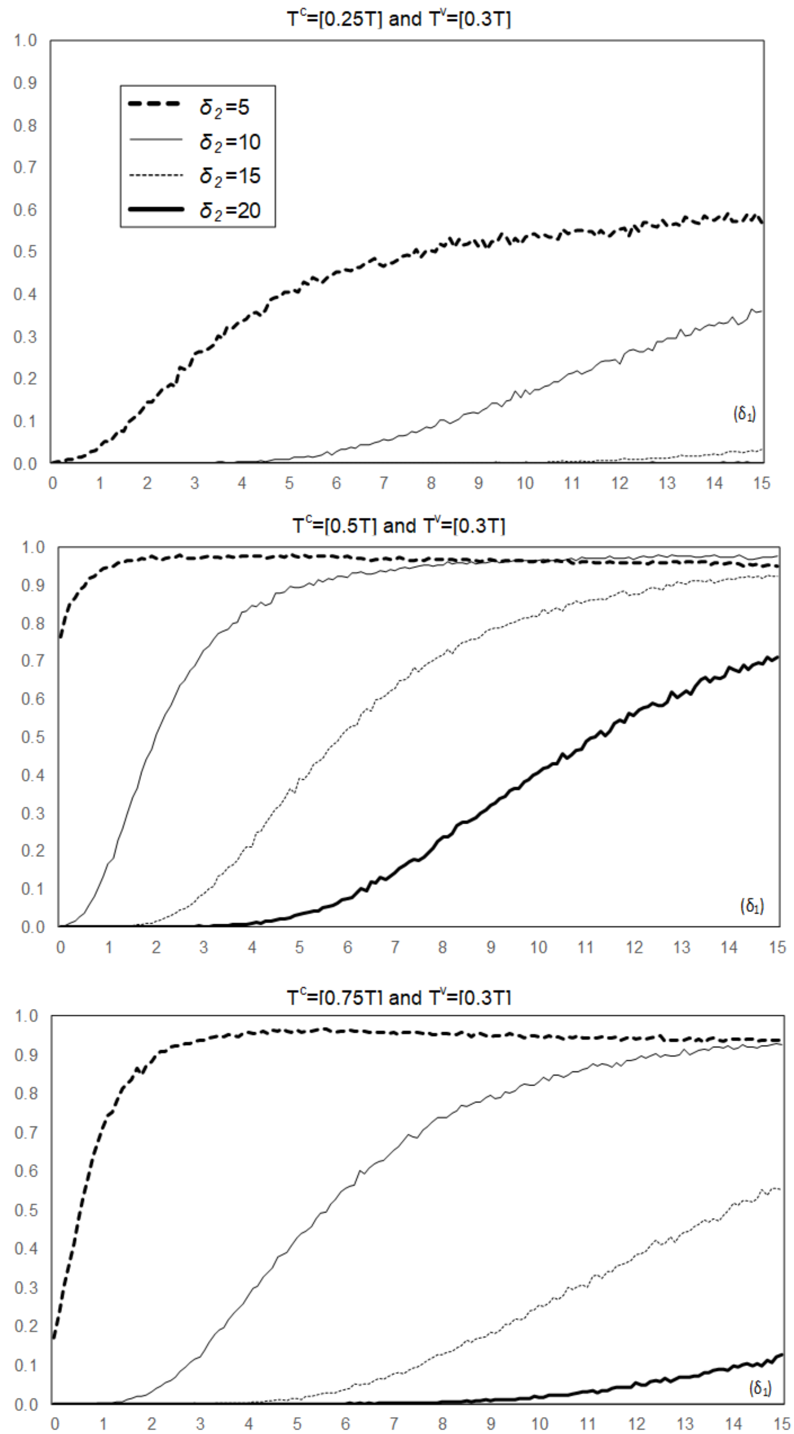

Figure 7D: Power of the CUSQ test for a variance change ignoring a coefficient change (accounting for conditional heteroskedasticity)