Article

Robust Optimization Model of Airport Group Coordinated Timetable with Uncertain Flight Time

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Abstract: This study develops a robust 0–1 linear optimization programming model for airport group timetable coordination, aiming at assigning each flight at an airport to a unique time slot to avoid conflicts between multiple flights from different airports at the same shared waypoint in an uncertain environment. Flight times between airports and shared waypoints are assumed to have an arbitrary distribution in the interval. Furthermore, some practical constraints, such as the time-varying capacity of each airport, waypoints affected by factors such as weather and traffic control, and maximum delay times for each flight, are considered in this study. The objective is to minimize the total delay time for all flights. The solution is obtained using the RSOME solver. Finally, a real-world case of the Beijing–Tianjin–Hebei airport group, China, is used to optimize the schedules of four airports to prove the accuracy and effectiveness of the method developed in this study. The influence of the budget of uncertainty parameters on model performance is also analyzed.

Keywords: airport group coordinated timetable; robust optimization model; uncertain travel time

1. Introduction

Flight timetables are the core element of the daily operation of airlines, airports, and air traffic management bureaus. They aim to determine the order in which aircraft depart and arrive at the airport. They are determined by the flight performance of aircraft, affecting the level of the safety interval in the process of aircraft taking off and landing on the runway and of aircraft passing through the waypoint. Obviously, flight timetables determine the maximum number of aircraft in the air. In an airport group, there are various airports and shared waypoints, where each airport directly connects to different waypoints according to the distribution of air traffic flow. If several airports are connected to the same waypoint, the lack of coordinated flight arrival and departure times in these airports may cause flights to reach waypoints at the same time, resulting in flight conflicts and delays. Compared with single-airport timetables (SATs), which depend on their own spatial and temporal distribution of travel demands and ignore how the flight schedules of nearby airports affect them, airport group coordinated timetables (AGCTs) aim at assigning all flights of each airport to time slots by coordinating their arrival and departure times in multiple airports to avoid conflicts at shared waypoints. Therefore, it is necessary for authorities to design reasonable AGCTs to find the optimal relationship between network layouts and the capacity of each airport and waypoint, the space–time distribution of traffic flow, and the management rules for the airlines and air traffic controllers to improve operational efficiency and reduce flight delays and carbon emissions.

Flight times between airports and shared waypoints, related to factors such as the flight performance of aircraft, bad weather, and pilot behavior, are one of the important inputs for AGCT design. Most studies have focused on AGCTs with certain flight times [1–4] and neglected the real-life conditions of uncertain flight times [5–7]. Obviously, AGCT schedules without consideration of uncertain times are difficult to apply in practice. Various

stochastic models, such as the expected value model [8] and chance constraints [4], have been used to deal with such uncertainties. However, the drawbacks of these stochastic models lie in (1) the distribution functions of uncertain parameters, which are difficult to obtain but are needed; and (2) the fact that the solution of equivalent deterministic forms does not satisfy all parameter values. Robust optimization theory is used to avoid the above defects by tackling the worst-case decision variables in the interval of uncertain parameters to strictly satisfy all constraints [9]. Hence, it is necessary to study robust optimization models for AGCTs with interval flight times [4,10].

The main goal of this study was to develop a robust optimization model for coordinating airport group timetables with uncertain flight times. Two tasks guide our study: (1) the determination of an optimal airport group timetable by simultaneously coordinating the arrival and departure times of flights in different airports to avoid conflicts between them at the same shared waypoint and considering some practical constraints such as the capacity of each airport and waypoint at different time slots as well as maximum delay times; (2) the creation of a robust optimization model that would assume interval flight times to analyze the influence of the budget of uncertainty parameters on model performance. Finally, a real-world case of the Beijing–Tianjin–Hebei airport group, China, is used to illustrate the accuracy and effectiveness of our methodology.

The structure of this study is as follows: Section 2 summarizes the relevant literature. Section 3 describes the methodology of the proposed model and its mathematical model. Section 4 compares the difference in the timetables of four airports in the Beijing–Tianjin–Hebei airport group before and after optimization and discusses the sensitivity analysis of different parameters on optimal schedules. Finally, concluding remarks and future directions are given in Section 5.

2. Related Works

The time slot allocation problem is often viewed as a resource-constrained project scheduling challenge, as scrutinized by Brucker et al. [11]. In the aviation domain, the specific aim [12] is to distribute a fixed number of resources (i.e., time slots) over time for relevant activities (i.e., arrivals/departures) while adhering to various hard and soft constraints [13,14] such as resource availability, timing, capacity, and continuity. Time slot allocation can be categorized into two types based on the geographical scope of the scheduling problem: single-airport models and airport network models. Currently, the majority of research endeavors are predominantly concentrated on single-airport time slot allocation. A widely used criterion for assessing the feasibility of flight schedule requests is the total schedule displacement at a single airport, which reflects the efficiency of schedule allocation. For example, Zografos et al. [15] proposed an integer linear model aimed at minimizing total displacement to comply with existing EU/IATA regulations, operational constraints, and coordination procedures. Subsequent studies have introduced additional objectives, such as minimizing rejected requests, maximizing airline profitability, enhancing passenger welfare, and expanding flight slots beyond acceptable time windows [2,16–19]. It is noteworthy that much of the research has focused on single-day operations or entire seasons at busy airports. Ribeiro et al. [20] extended this to encompass the flight schedule allocation problem for entire seasons at medium-sized airports. However, solutions that merely minimize total displacement may not ensure equitable allocation among competing airline flight schedule requests. Multi-objective models for time slot allocation have been devised [1,2,21], and recent studies have integrated fairness [22–24] and acceptability considerations [18,19,25,26] into existing scheduling models. Experimental findings suggest that sacrificing a small number of flight schedule coordination efficiencies can significantly enhance other performance metrics while boosting the actual utilization of airport flight slots. Nevertheless, the existing literature predominantly addresses constraints from the supply side without explicitly considering airline preferences regarding which requests to replace and by how much. Katsigiannis et al. [2] introduced the Time Flexibility Index (TFI) from the airline perspective to express preferences for flight scheduling flexibility,

thereby facilitating the dynamic allocation of airport apron and terminal resources. This approach reduces previously considered scheduling quality metrics (improvement ranging from 5.5% to 24%), minimizes maximum and total displacement, and notably improves flight schedule request fulfillment and passenger satisfaction. Subsequently, Fairbrother et al. [27] proposed a two-stage mechanism model for congested airport flight schedule scheduling that not only addresses efficiency and fairness objectives but also accounts for airline preferences when allocating total displacement associated with each airline's flights. This model enhances airport capacity utilization, refines airport landing and take-off schedules, and exhibits considerable flexibility.

While the benefits of single-airport time slot allocation are apparent, there are also several challenges. Specifically, allocating flight slots at a single airport demands meeting airline requirements while navigating various constraints, presenting significant challenges for slot coordinators. Additionally, the practical management of time remains subject to numerous random factors. In addressing the single-airport flight scheduling problem, a notable feature of airport network collaborative scheduling is the sharing of terminal area airspace resources, focusing on departure and arrival convergence points. Scholars have extensively studied flight schedule optimization within airport networks. Castelli et al. [28] introduced a network-level slot allocation model that considers interdependencies among different airport schedules, aiming to minimize airline-requested slot displacements caused by inconsistencies in slot allocations between departure and destination airports. Building upon this, Pellegrini et al. [29] expanded the model using metaheuristic algorithms like Iterated Local Search (ILS) and Variable Neighborhood Search (VNS) for resolution. However, both models oversimplify matters by failing to consider the allocation of requests across a series of flight slots. Corolli et al. [5] developed stochastic programming models allocating requests to a series of slots, serving as tools for slot coordinators to balance schedule/request discrepancies and future delays. It is worth noting that optimization models for flight scheduling within airport networks struggle with handling large traffic samples due to capacity constraints and scalability issues. Although Pellegrini et al. [29] proposed model algorithms, they could not prove the optimality of the returned solutions. Consequently, a wealth of literature has emerged employing various algorithms to solve large-scale, intricate instances, including stochastic programming [4] and heuristic algorithms [30]. Experimental findings indicate that optimizing flight schedules within airport groups effectively alleviates operational safety and efficiency declines caused by large-scale and overly busy airports. Through unified management and cooperative division of labor among member airports, each airport's flight schedule resource utility is maximized, achieving functional complementarity and reducing the number of flight conflicts between adjacent airports.

Most studies addressing flight schedule optimization have been developed under deterministic conditions, with relatively few focusing on uncertainties in air traffic flow management. Those that do primarily address the assignment of ground delays to aircraft amidst uncertain airport capacities, known as stochastic ground holding problems, spanning both single-airport and airport network scenarios [31–33]. Corolli et al. [5] proposed a scenario-based model for flight schedule allocation within airport networks under uncertain capacities, utilizing the Sample Average Approximation (SAA) method to manage uncertainties. Various approaches exist for dealing with uncertainty, including the use of Markov decision processes [34], chance constraints [4,8], and heuristic algorithms [35]. Subsequently, Gupta and Bertsimas [36] addressed stochastic air traffic flow management problems, simultaneously considering adaptability and robustness in uncertain airport arrival and departure capacities due to weather. They noted that robust problems inherit attractive features from deterministic problems, often at a minimal cost of robustness. Recently, some scholars have explored robust runway scheduling problems under uncertain conditions [37] and robust optimization models for uncertain airport capacities [9]. However, few studies have employed robust programming to address flight slot allocation optimization problems for network airports with uncertain flight times [38].

3. Methodology

3.1. Research Framework

An airport group network comprises some airports, shared waypoints, and links between them. There are sets of arrival and departure flights with their planned take-off or landing times in an airport, where arrival flights arrive at the airport at their planned landing time after a short flight from the waypoint, and departure flights fly through the waypoint for some time and fly for a certain amount of time to the shared waypoint after departing from the airport at their planned take-off times. The whole day is divided into 288 time slots based on 5-min intervals, i.e., the first and last time slots are 00:00–00:05 and 23:55–24:00. Each airport flight is assigned a unique time slot by considering the planned time and maximum delay time. Obviously, the lack of coordinated arrival and departure times of the related flights in these airports must cause them to reach a waypoint at the same time, resulting in flight conflicts and delays. Furthermore, if the number of flights in an airport assigned to a time slot is less than the airport capacity, the number of flights through a waypoint assigned to a time slot is less than the waypoint capacity. In this study, an uncertain flight time between airports and shared waypoints, affected by bad weather, etc., is assumed to be an interval variable. The main purpose of this study is to reveal the optimal coupling relationship between the space–time distribution of flights, the budget of uncertainty during flight time, the AGCT, and operational efficiency.

Figure 1 illustrates the principles and scope of the proposed methodology. The study area contains three airports (A1–A3) and two shared waypoints (P1 and P2). The number on the line between the airport and the waypoint denotes the flight time. In this case, there are three arrival flights (IF001–IF003) and six departure flights (OF001–OF006) with their assigned time slots at three airports. For example, IF001 arrives at waypoint P1 at time slot 4 and will land at airport A1 after flying during the 4 time slots. For A3, two flights, IF003 and OF005, are assigned to slot 6. If the airport capacity for this time slot is less than 1, the schedule is not feasible. The two flights IF002 and OF001 of P1 at time slot 6 experience the same phenomenon.
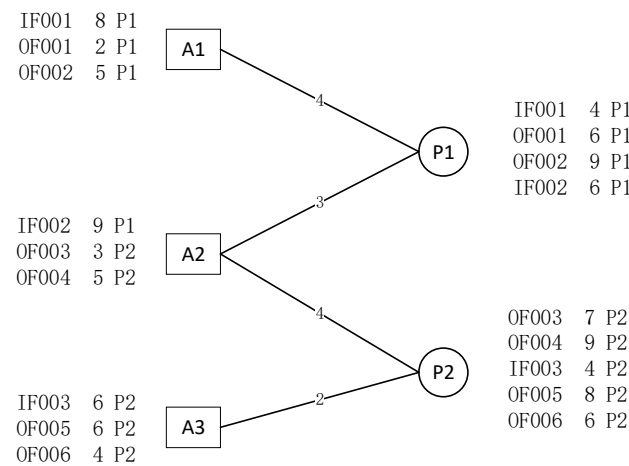

Figure 1. Diagram of the proposed model.

The purpose of a robust 0–1 linear optimization programming model is to assign all flights of each airport to time slots by coordinating their arrival and departure times in multiple airports to avoid conflicts between them at the same shared waypoint and minimize the total delay time of all flights. To ensure that this study could fit well with real-world situations, we made the following assumptions:

(1) Full flight information for each airport is available in advance.

(2) Time-varying airport capacity is mainly determined by the safety interval between the aircraft on a runway as well as the number of runways available for aircraft to take

off or land at different time slots, which is affected by some factors such as weather and traffic control.

(3) Time-varying waypoint capacity is mainly determined by the horizontal and vertical separation between flight levels in a waypoint, as well as the number of flight levels, which is also affected by some factors such as weather and traffic control.

(4) Upper and lower bounds of flight times between airports and shared waypoints could be obtained from big data.

3.2. Notation

All definitions and notations used hereafter are summarized in Table A1.

3.3. Formulation

In this section, a novel 0–1 linear integer programming model of the AGCT with a certain flight time is first given as follows:

$$\text{Min } f = \sum_{\forall a \in A} \sum_{\forall i \in F_a^{Out} \cup F_a^{In}} \sum_{\forall k \in K} h_{ik}(k - T_i) \tag{1}$$

such that

$$\sum_{\forall k \in K} h_{ik} = 1, \forall i \in F_a^{Out} \cup F_a^{In}, \forall a \in A \tag{2}$$

$$0 \le h_{ik}(k - T_i) \le d_i, \forall i \in F_a^{Out} \cup F_a^{In}, \forall a \in A, \forall k \in K \tag{3}$$

$$\sum_{\forall i \in F_a^{Out} \cup F_a^{In}} h_{ik} \le C_a^k, \forall k \in K, \forall a \in A \tag{4}$$

$$\sum_{\forall i \in F_a^{Out} \cup F_a^{In}} \sum_{j=0}^{2} h_{ik} \le C_a^{15}, \forall k \in K, \forall a \in A \tag{5}$$

$$\sum_{\forall i \in F_a^{Out} \cup F_a^{In}} \sum_{j=0}^{5} h_{ik} \le C_a^{30}, \forall k \in K, \forall a \in A \tag{6}$$

$$\sum_{\forall i \in F_a^{Out} \cup F_a^{In}} \sum_{j=0}^{11} h_{ik} \le C_a^{60}, \forall k \in K, \forall a \in A \tag{7}$$

$$\sum_{\forall a \in A} \left[\sum_{\forall i \in F_{a,w}^{Out}} h_{ik - T_{aw}} + \sum_{\forall i \in F_{a,w}^{In}} h_{ik + T_{aw}} \right] \le Q_w^k, \forall w \in N, \forall k \in K \tag{8}$$

$$\sum_{\forall a \in A} \left[\sum_{\forall i \in F_{a,w}^{Out}} \sum_{j=0}^{2} h_{ik + j - T_{aw}} + \sum_{\forall i \in F_{a,w}^{In}} \sum_{j=0}^{2} h_{ik + T_{aw}} \right] \le Q_w^{15}, \forall w \in N, \forall k \in K \tag{9}$$

$$\sum_{\forall a \in A} \left[\sum_{\forall i \in F_{a,w}^{Out}} \sum_{j=0}^{5} h_{ik + j - T_{aw}} + \sum_{\forall i \in F_{a,w}^{In}} \sum_{j=0}^{5} h_{ik + T_{aw}} \right] \le Q_w^{30}, \forall w \in N, \forall k \in K \tag{10}$$

$$\sum_{\forall a \in A} \left[\sum_{\forall i \in F_{a,w}^{Out}} \sum_{j=0}^{11} h_{ik + j - T_{aw}} + \sum_{\forall i \in F_{a,w}^{In}} \sum_{j=0}^{11} h_{ik + T_{aw}} \right] \le Q_w^{60}, \forall w \in N, \forall k \in K \tag{11}$$

The objective function, represented by Equation (1), is to minimize the total delay time of all arrival and departure flights for all airports. Equations (2)–(10) are constraints, where Equation (2) ensures that each flight must be assigned to a slot time; Equation (3) ensures that the deviation between the planned time slot and the actual one of each flight is within a certain range; Equation (4) guarantees that the number of arrival and departure flights at an airport at each time slot cannot exceed its maximum capacity; Equations (5)–(7) ensure that the number of arrival and departure flights at an airport in any 15 min/30 min/60 min cannot exceed its maximum capacity; Equation (8) guarantees that the number of arrival and departure flights at a waypoint in each time slot cannot exceed its maximum capacity; and Equations (9)–(11) ensure that the number of arrival and departure flights at an airport in any 15 min/30 min/60 min cannot exceed its maximum capacity.

To deal with an AGCT with uncertain flight time, the affine term $\ddot{T}_{aw} + \hat{T}_{aw}z$ depicts the random flight time between airport a and waypoint w, and the random variable z is in the range $[-1, 1]$, so flight time has an arbitrary distribution in the interval $[\ddot{T}_{aw} - \hat{T}_{aw}, \ddot{T}_{aw} + \hat{T}_{aw}]$. The robust model of the AGCT is given below.

$$\underset{z \in \mathbb{Z}}{\text{Min}} f = \sum_{\forall a \in A} \sum_{\forall i \in F_a^{Out} \cup F_a^{In}} \sum_{\forall k \in K} h_{ik}(k - T_i) \tag{12}$$

such that

$$\sum_{\forall a \in A} \left[\sum_{\forall i \in F_{a,w}^{Out}} h_{ik - [\ddot{T}_{aw} + \hat{T}_{aw} z]} + \sum_{\forall i \in F_{a,w}^{In}} h_{ik + [\ddot{T}_{aw} + \hat{T}_{aw} z]} \right] \le Q_w^k, \forall w \in N, \forall k \in K \quad (13)$$

$$\sum_{\forall a \in A} \left[\sum_{\forall i \in F_{a,w}^{Out}} \sum_{j=0}^{2} h_{ik + j - [\ddot{T}_{aw} + \hat{T}_{aw} z]} + \sum_{\forall i \in F_{a,w}^{In}} \sum_{j=0}^{2} h_{ik + j + [\ddot{T}_{aw} + \hat{T}_{aw} z]} \right] \le Q_w^{15}, \forall w \in N, \forall k \in K \quad (14)$$

$$\sum_{\forall a \in A} \left[\sum_{\forall i \in F_{a,w}^{Out}} \sum_{j=0}^{5} h_{ik + j - [\ddot{T}_{aw} + \hat{T}_{aw} z]} + \sum_{\forall i \in F_{a,w}^{In}} \sum_{j=0}^{5} h_{ik + j + [\ddot{T}_{aw} + \hat{T}_{aw} z]} \right] \le Q_w^{30}, \forall w \in N, \forall k \in K \quad (15)$$

$$\sum_{\forall a \in A} \left[\sum_{\forall i \in F_{a,w}^{Out}} \sum_{j=0}^{11} h_{ik + j - [\ddot{T}_{aw} + \hat{T}_{aw} z]} + \sum_{\forall i \in F_{a,w}^{In}} \sum_{j=0}^{11} h_{ik + j + [\ddot{T}_{aw} + \hat{T}_{aw} z]} \right] \le Q_w^{60}, \forall w \in N, \forall k \in K \quad (16)$$

The other constraints are the same as above. The uncertainty set $\mathbb{Z}$ of the flight time is described as follows: $\mathbb{Z} = \{z : \| z \|_\infty \le 1, \| z \|_1 \le \Gamma\}$, where Γ denotes the budget of the uncertainty parameter.

4. Solution Method

The framework for a data-driven solution for the AGCT is shown in Figure 2. The interval of the time slot is related to the safety of aircraft operation, which depends on the relevant standards of the ICAO. The whole day is divided into time slots based on this interval. According to the trajectories of all arriving and departing aircraft at different airports in the airport group, the busy waypoints can be identified. In this case, we can determine the busy waypoints as well as the planned departure/arrival time slot of each flight. We can also calculate the upper and lower bounds of flight time slots between airports and busy waypoints. These are the two inputs for the robust model. After we manually set the budget of the uncertainty parameter, the model can be solved using the RSOME solver.
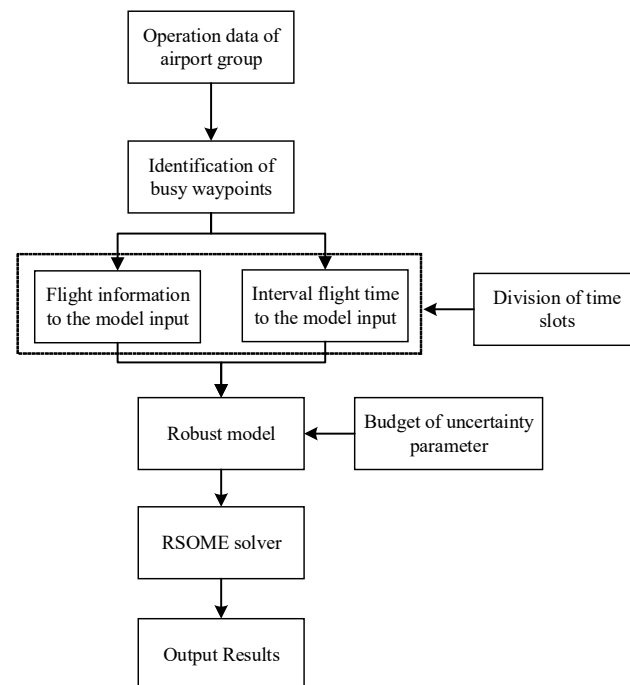

Figure 2. Flowchart of the RSOME solver for the robust model.

5. Case Study

5.1. Example Description

A realistic airport group network consisting of four airports and four waypoints in the Beijing–Tianjin–Hebei region, China, shown in Figure 3, is used to verify the feasibility and accuracy of the proposed model. As shown in Table 1, there are a total of 1291 arrival and

1240 departure flights at all airports, and 52.6% of these flights pass through these busy waypoints. The other parameters are set as follows: $d_i = 120$ min, $C_a^{15} = 3C_a^k$, $C_a^{30} = 6C_a^k$, $C_a^{60} = 12C_a^k$, $Q_w^k = 4$, $Q_w^{15} = 3Q_w^k$, $Q_w^{30} = 6Q_w^k$, $Q_w^{60} = 12Q_w^k$.
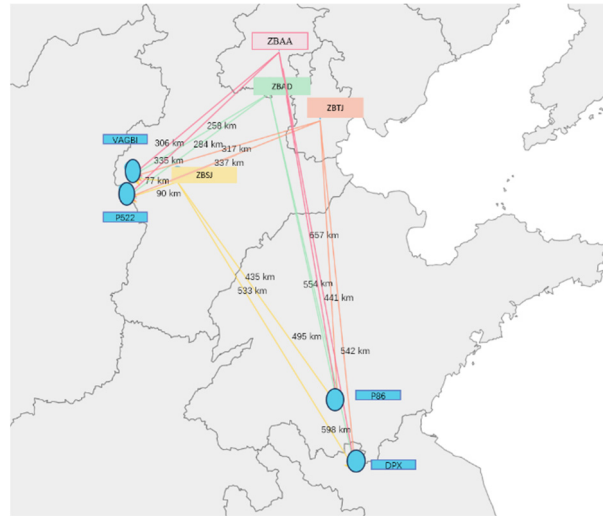

Figure 3. Spatial distribution of the four airports and four waypoints in the Beijing–Tianjin–Hebei airport group network, China.

Table 1. Flight information.

Airport			Waypoint		
Name	Number of Arrival/Departure Flights	C_a^k	Name	Flight Times (slots)	Number of Arrival/Departure Flights
ZBAA	799 (444/355)	8	P522	4	118 (95/23)
			P86	8	95 (31/64)
			VAGBI	7	129 (29/100)
			DPX	3	89 (65/24)
ZBAD	1033 (515/518)	6	P522	5	104 (65/39)
			P86	8	154 (60/94)
			VAGBI	8	120 (53/67)
			DPX	4	142 (109/33)
ZBTJ	455 (216/239)	4	P522	2	64 (54/10)
			P86	7	48 (13/35)
			VAGBI	6	68 (14/54)
			DPX	1	47 (37/10)
ZBSJ	244 (116/128)	2	P522	5	54 (45/9)
			P86	7	31 (10/21)
			VAGBI	6	48 (7/41)
			DPX	4	22 (16/6)

5.2. Results

The optimal solution for 2531 flights assigned to their actual time slots with a total delay of 1305 slots was derived using the RSOME solver. Figure 4 depicts the offset between planned and actual times for all flights at several airports. Most flights were delayed for less than half an hour (i.e., six slots), rarely more than an hour (i.e., twelve slots). As shown in Table 2, we found that (1) the total delay time, average delay time, and number of delayed

flights of the four airports are ranked as follows: ZBAD is the largest, ZBAA is second, ZBTJ is third, and ZBSJ is the smallest; (2) except for at ZBAD, there were no flights delayed for more than half an hour at the airports.
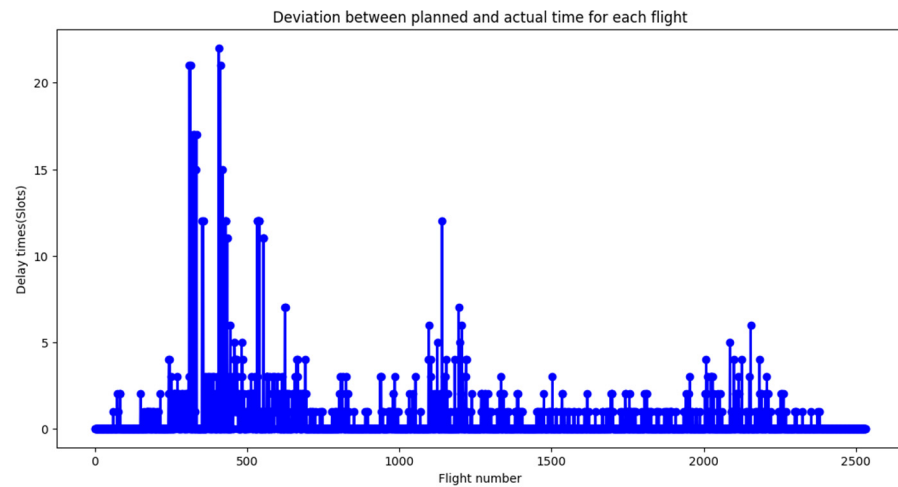

Figure 4. Deviation between planned and actual times for all flights.

Table 2. Basic information for the optimal solution.

Airport	Total Delay Time (slots)	Average Delay Time (slots)	Number of Flights Not Delayed	Number of Delayed Flights		
				More than 30 min	More than 60 min	More than 120 min
ZBAA	**192**	**0.24**	668	0	0	0
ZBAD	960	0.93	670	33	12	0
ZBTJ	78	0.17	398	0	0	0
ZBSJ	75	0.31	189	0	0	0

Figures 5 and 6 depict the number of flights at different time periods at each waypoint and airport before and after optimization. The blue and red lines indicate the results before and after optimization, respectively. Based on these results, we can conclude that (1) according to the planned arrival or departure times of all flights before optimization, each airport/waypoint's flight flow is greater than its capacity in some periods (i.e., the timetable is not feasible), while the former is less than the latter in other periods (i.e., time slot resources are wasted); (2) after optimization, the number of flights always at the airport is smaller than its capacity at different times, while that of the waypoint is also smaller than its capacity at different times, so the timetable is feasible; (3) for each airport or waypoint, flight delays during peak hours are longer than those during low peak hours.

5.3. Sensitivity Analysis

Figure 7 shows how the degree of uncertainty for the flight time influences model performance. When Γ is set as 0, 0.5, 1, and 1.5, the total flight delay times for assigning all flights to their unique time slots are 1305 slots, 6608 slots, 6608 slots, and 6608 slots, respectively. The reason for this phenomenon is that the increase in the degree of uncertainty may lead to a wider feasible time window of flight time and result in a reduction in the solution space satisfying all relaxation variables, which in turn increases the total flight delay time. If an increase in Γ, such as from $\Gamma = 0$ to 0.5, leads to a better solution in a reduced solution space, the objective function will increase. If an increase in Γ, such as from $\Gamma = 0.5$ to 1, does not lead to a better solution in the reduced solution space, the objective function remains unchanged. Although the increase in Γ generates a schedule with more flight delays, it is robust and more easily applied in practice.
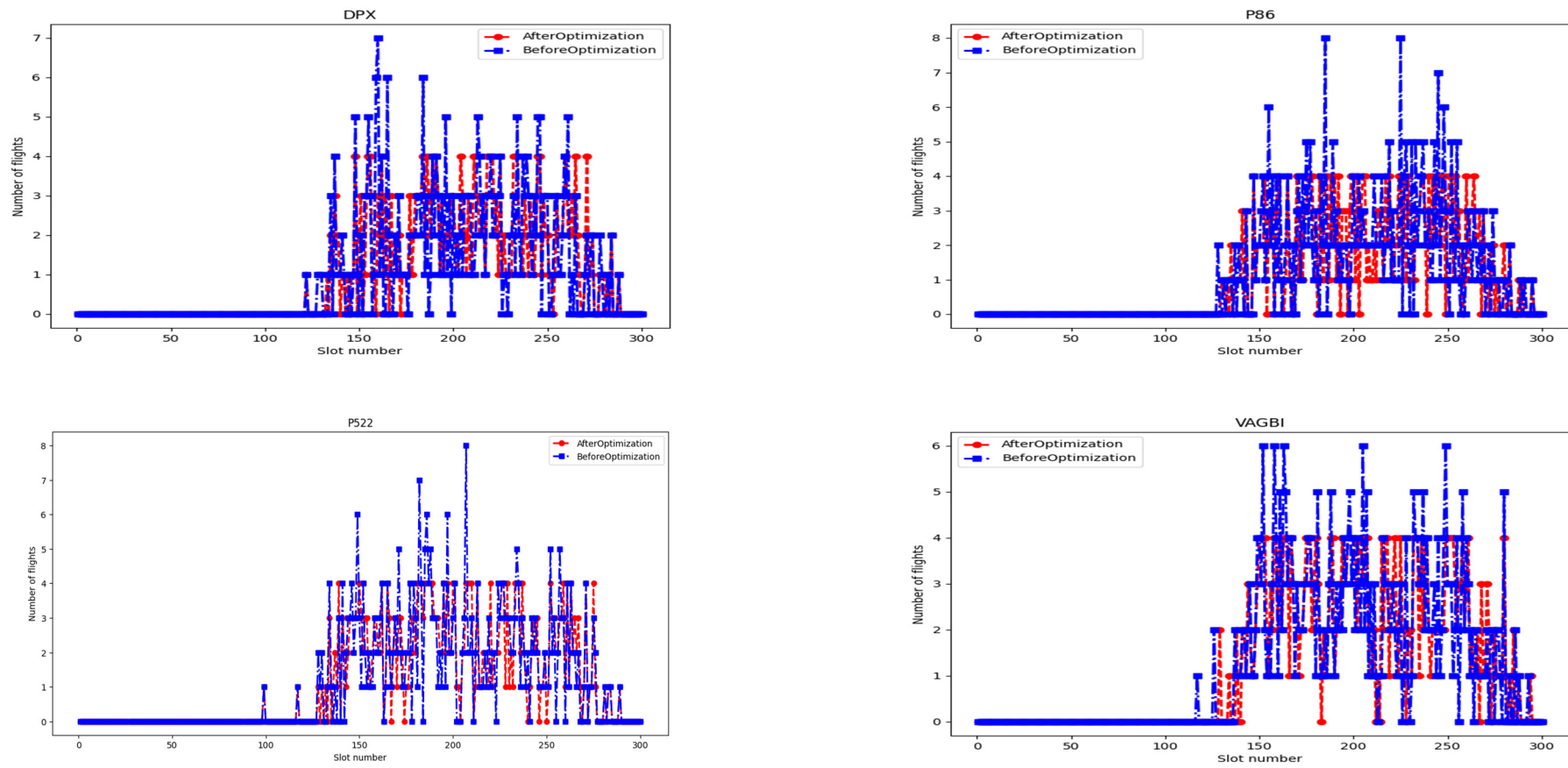
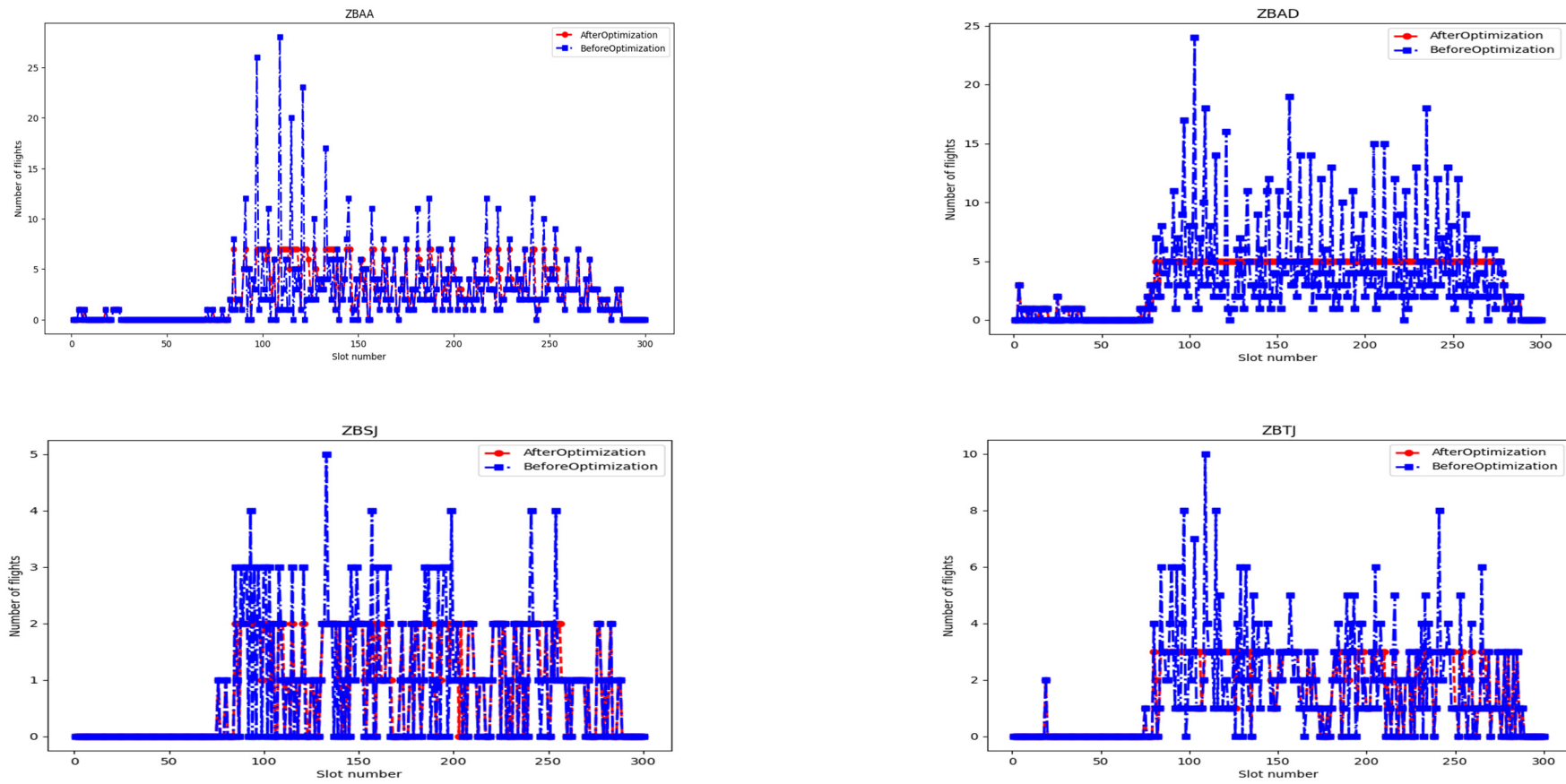**Figure 5.** Flight flow of the four waypoints at different time periods before and after optimization.

Figure 6. Flight flow of the four airports at different time periods before and after optimization.
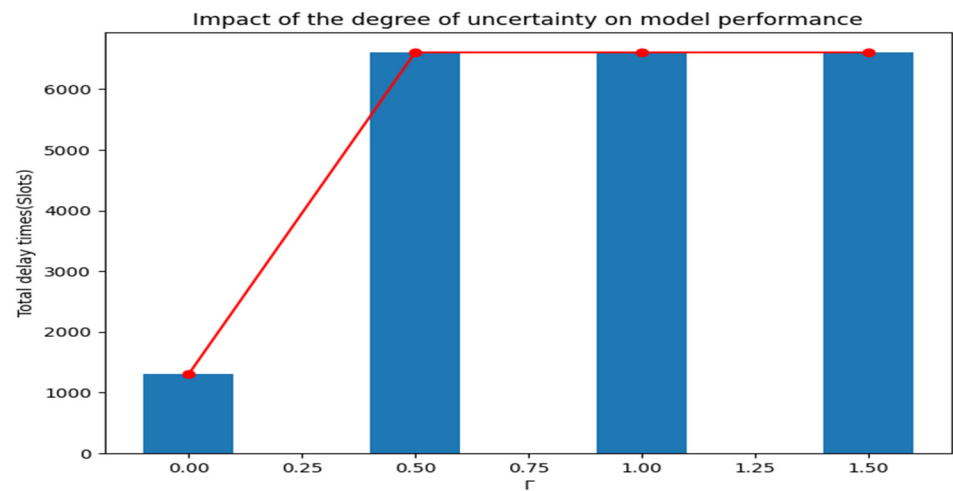
Figure 7. Comparison of optimal solutions based on the degree of uncertainty.

Figure 8 shows how different capacities of airports affect model performance. Airport capacity is mainly determined by a safety interval between the aircraft as well as the number of runways available for aircraft take-off or landing. When it is gradually expanded by 1, 1.1, 1.2, and 1.3 times, the total flight delay times for assigning all flights to their unique time slots are 1305 slots, 1305 slots, 702 slots, and 632 slots, respectively. The reason for this phenomenon is that the increase in airport capacity will gradually ease the contradiction between runway supply and demand, since the number of flights has not changed. If it reaches a critical value, the total flight delay times will decrease; otherwise, the value will stay the same.
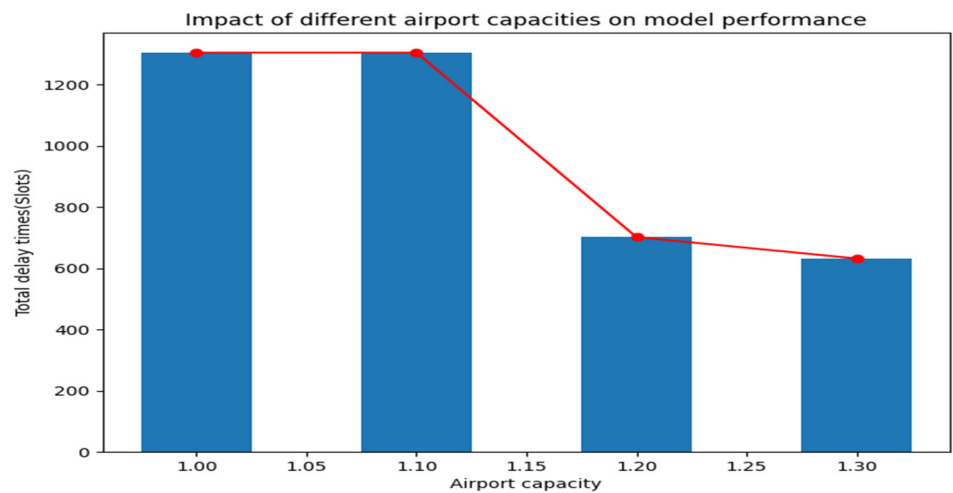

Figure 8. Comparison of optimal solutions according to different airport capacities.

Figure 9 indicates how different capacities of waypoints influence the model's performance. The results presented in Figure 9 are similar to those in Figure 8. When it is gradually expanded by 1, 1.1, 1.2, and 1.3 times, the total flight delay times for assigning all flights to their unique time slots are 1305 slots, 1305 slots, 1305 slots, and 1280 slots, respectively. The increase in waypoint capacity allows more flights to pass through the location at the same time, resulting in a reduction in the total flight delay time. However, when it is increased to a certain extent, the flight delay may not depend on the limited waypoint capacity because its capacity is greater than the number of aircraft passing through the waypoint.
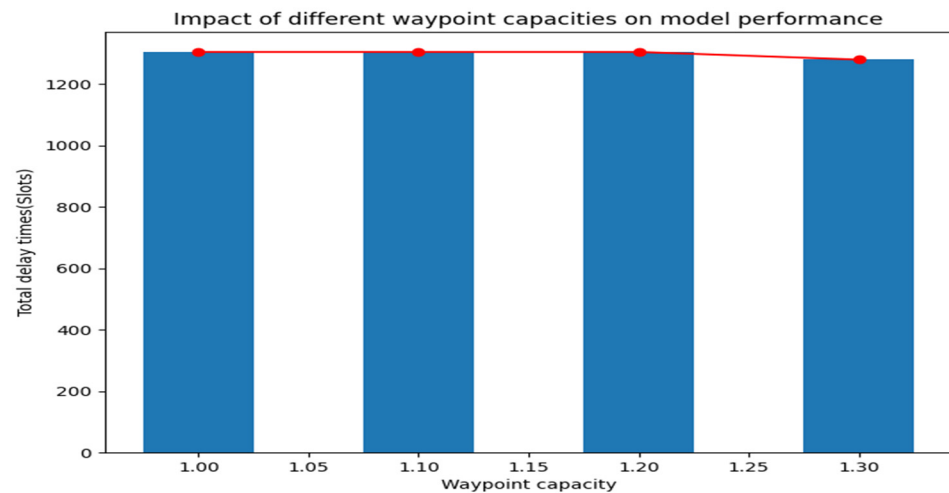
Figure 9. Comparison of optimal solutions according to different waypoint capacities.

6. Conclusions

This study introduced a robust optimization model for airport group coordinated timetables with interval flight times to coordinate the arrival and departure times of flights at different airports and avoid conflicts between them at the same shared waypoint. This was carried out in order to determine the optimal relationship between the spatial and temporal distributions of flight travel demand, the budget of the uncertainty parameter, and the total delay time of all flights. Practical constraints such as the capacity of each airport and waypoint at different time slots and the maximum delay time for each flight were also considered in the proposed model. The RSOME solver was used to solve our model. Finally, a practical test in a real-world case of the Beijing–Tianjin–Hebei airport group, China, was conducted to prove the accuracy and effectiveness of this study's model.

The main findings include the following points:

(1) In the original flight planning data, the number of flights at each airport or waypoint for some of the time slots is greater than its capacity, making the current timetable unworkable. Once the current timetable has been optimized, it will satisfy some practical constraints, such as the capacity of each airport and waypoint at different time slots and the maximum delay time for each flight. Hence, this study can be used as a tool for authorities to yield valid arrival and departure times of flights at different airports in an airport group to improve operational efficiency.

(2) When flight flow is greater than the airport or waypoint capacity, the reduced capacity of the airport or waypoint at different time slots increases the total delay time of all flights by requiring each flight to relax its maximum delay time limit to obtain a feasible timetable.

(3) As the budget of uncertainty in flight time gradually increases, each flight with a wider feasible time window of flight time may lead to a reduction in the solution space for assigning all flights to their unique time slots to satisfy all relaxation variables, resulting in increased total flight delay times. Although a schedule with a larger budget of uncertainty in flight time leads to more flight delays, it is robust and easily applied in practice.

However, the shortcomings of our model lie in two aspects: (1) it is necessary to balance multiple competing objectives, such as airline and airport fairness and flight delays, to find the best schedule; and (2) some realistic constraints, such as the priority of flights, transfers between flights, and workload of air traffic controllers, are neglected. Extending this study to a multi-objective one with more realistic constraints is the aim of our future research.

Appendix A

Table A1. Mathematical symbols used in the AGCT model.

Sets:	
A	Set of airports
K	Set of time slots
N	Set of waypoints
F_a^{Out}	Set of departure flights at airport a; $\forall a \in A$
F_a^{In}	Set of arrival flights at airport a; $\forall a \in A$
$F_{a,w}^{Out}$	Set of departure flights at airport a passing though waypoint w; $\forall a \in A, \forall w \in N$
$F_{a,w}^{In}$	Set of arrival flights at airport a passing though waypoint w; $\forall a \in A, \forall w \in N$
Indices:	
k	Time slot
i	Flight
a	Airport
w	Waypoint
Parameters:	
T_i	Planned time slot of flight i; $\forall i \in F_a^{Out} \cup F_a^{In}, \forall a \in A$
d_i	Maximization deviation in time slot of flight i; $\forall i \in F_a^{Out} \cup F_a^{In}, \forall a \in A$
C_a^k	Capacity of airport a at time slot k; $\forall a \in A, \forall k \in K$
C_a^{15}	Capacity of airport a during 15 min; $\forall a \in A$
C_a^{30}	Capacity of airport a during 30 min; $\forall a \in A$
C_a^{60}	Capacity of airport a during 60 min; $\forall a \in A$
Q_w^k	Capacity of waypoint w at time slot k; $\forall w \in N, \forall k \in K$
Q_w^{15}	Capacity of waypoint w during 15 min; $\forall w \in N$
Q_w^{30}	Capacity of waypoint w during 30 min; $\forall w \in N$
Q_w^{60}	Capacity of waypoint w during 60 min; $\forall w \in N$
T_{aw}	Flight time between airport a and waypoint w; $\forall w \in N, \forall a \in A$
$\ddot{T}_{aw}$	Average flight time between airport a and waypoint w; $\forall w \in N, \forall a \in A$
$\hat{T}_{aw}$	Deviation of flight time between airport a and waypoint w; $\forall w \in N, \forall a \in A$
z	Random variable
$\mathbb{Z}$	Uncertainty set
Γ	Budget of uncertainty in flight time
Decision variables:	
h_{ik}	Whether the flight i is assigned to time slot k; $\forall i \in F_a^{Out} \cup F_a^{In}, \forall a \in A, \forall k \in K$

References

1. Feng, H.; Hu, R.; Wang, D.; Zhang, J.; Wu, C. Bi-objective airport slot scheduling considering scheduling efficiency and noise abatement. *Transp. Res. Part D Transp. Environ.* **2023**, *115*, 103591. [CrossRef]
2. Katsigiannis, F.A.; Zografos, K.G.; Fairbrother, J. Modelling and solving the airport slot-scheduling problem with multi-objective, multi-level considerations. *Transp. Res. Part C Emerg. Technol.* **2021**, *124*, 102914. [CrossRef]
3. Katsigiannis, F.A.; Zografos, K.G. Optimising airport slot allocation considering flight-scheduling flexibility and total airport capacity constraints. *Transp. Res. Part B Methodol.* **2021**, *146*, 50–87. [CrossRef]
4. Wang, Y.; Liu, C.; Wang, H.; Duong, V. Slot allocation for a multiple-airport system considering airspace capacity and flying time uncertainty. *Transp. Res. Part C Emerg. Technol.* **2023**, *153*, 104185. [CrossRef]

5. Corolli, L.; Lulli, G.; Ntaimo, L. The time slot allocation problem under uncertain capacity. *Transp. Res. Part C Emerg. Technol.* **2014**, *46*, 16–29. [CrossRef]

6. Delahaye, D.; Wang, Y. Slot allocation in a multi-airport system under flying time uncertainty. In Proceedings of the International Workshop on ATM/CNS (IWAC) 2022, Tokyo, Japan, 25–27 October 2022.

7. Liu, W.; Zhao, Q.; Delahaye, D. Research on slot allocation for airport network in the presence of uncertainty. *J. Air Transp. Manag.* **2022**, *104*, 102269. [CrossRef]

8. Clare, G.; Richards, A. Air traffic flow management under uncertainty: Application of chance constraints. In Proceedings of the 2nd International Conference on Application and Theory of Automation in Command and Control Systems, London, UK, 29–31 May 2012; pp. 20–26.

9. Wang, D.; Zhao, Q. A simultaneous optimization model for airport network slot allocation under uncertain capacity. *Sustainability* **2020**, *12*, 5512. [CrossRef]

10. Agogino, A.; Rios, J. Robustness of two air traffic scheduling approaches to departure uncertainty. In Proceedings of the 2011 IEEE/AIAA 30th Digital Avionics Systems Conference, Seattle, WA, USA, 16–20 October 2011; pp. 2C6-1–2C6-8.

11. Brucker, P.; Drexl, A.; Möhring, R.; Neumann, K.; Pesch, E. Resource-constrained project scheduling: Notation, classification, models, and methods. *Eur. J. Oper. Res.* **1999**, *112*, 3–41. [CrossRef]

12. Zografos, K.G.; Madas, M.A.; Androutsopoulos, K.N. Increasing airport capacity utilisation through optimum slot scheduling: Review of current developments and identification of future needs. *J. Sched.* **2017**, *20*, 3–24. [CrossRef]

13. Burke, E.; Jackson, K.; Kingston, J.H.; Weare, R. Automated university timetabling: The state of the art. *Comput. J.* **1997**, *40*, 565–571. [CrossRef]

14. Burke, E.K.; Petrovic, S.; Qu, R. Case-based heuristic selection for timetabling problems. *J. Sched.* **2006**, *9*, 115–132. [CrossRef]

15. Zografos, K.G.; Salouras, Y.; Madas, M.A. Dealing with the efficient allocation of scarce resources at congested airports. *Transp. Res. Part C Emerg. Technol.* **2012**, *21*, 244–256. [CrossRef]

16. Androutsopoulos, K.N.; Manousakis, E.G.; Madas, M.A. Modeling and solving a bi-objective airport slot scheduling problem. *Eur. J. Oper. Res.* **2020**, *284*, 135–151. [CrossRef]

17. Jorge, D.; Ribeiro, N.A.; Antunes, A.P. Towards a decision-support tool for airport slot allocation: Application to Guarulhos (Sao Paulo, Brazil). *J. Air Transp. Manag.* **2021**, *93*, 102048. [CrossRef]

18. Swaroop, P.; Zou, B.; Ball, M.O.; Hansen, M. Do more US airports need slot controls? A welfare based approach to determine slot levels. *Transp. Res. Part B Methodol.* **2012**, *46*, 1239–1259. [CrossRef]

19. Vaze, V.; Barnhart, C. Modeling airline frequency competition for airport congestion mitigation. *Transp. Sci.* **2012**, *46*, 512–535. [CrossRef]

20. Ribeiro, N.A.; Jacquillat, A.; Antunes, A.P.; Odoni, A.R.; Pita, J.P. An optimization approach for airport slot allocation under IATA guidelines. *Transp. Res. Part B Methodol.* **2018**, *112*, 132–156. [CrossRef]

21. Zografos, K.; Jiang, Y. Modelling and solving the airport slot scheduling problem with efficiency, fairness, and accessibility considerations. In Proceedings of the 9th Triennial Symposium on Transportation Analysis (TRISTAN 2016), Oranjestad, Aruba, 12–17 June 2016; pp. 13–17.

22. Androutsopoulos, K.N.; Madas, M.A. Being fair or efficient? A fairness-driven modeling extension to the strategic airport slot scheduling problem. *Transp. Res. Part E Logist. Transp. Rev.* **2019**, *130*, 37–60. [CrossRef]

23. Jiang, Y.; Zografos, K.G. A decision making framework for incorporating fairness in allocating slots at capacity-constrained airports. *Transp. Res. Part C Emerg. Technol.* **2021**, *126*, 103039. [CrossRef]

24. Zografos, K.G.; Jiang, Y. A bi-objective efficiency-fairness model for scheduling slots at congested airports. *Transp. Res. Part C Emerg. Technol.* **2019**, *102*, 336–350. [CrossRef]

25. Pyrgiotis, N.; Odoni, A. On the impact of scheduling limits: A case study at newark liberty international airport. *Transp. Sci.* **2016**, *50*, 150–165. [CrossRef]

26. Zografos, K.G.; Androutsopoulos, K.N.; Madas, M.A. Minding the gap: Optimizing airport schedule displacement and acceptability. *Transp. Res. Part A Policy Pract.* **2018**, *114*, 203–221. [CrossRef]

27. Fairbrother, J.; Zografos, K.G.; Glazebrook, K.D. A slot-scheduling mechanism at congested airports that incorporates efficiency, fairness, and airline preferences. *Transp. Sci.* **2020**, *54*, 115–138. [CrossRef]

28. Castelli, L.; Pellegrini, P.; Pesenti, R. Airport slot allocation in Europe: Economic efficiency and fairness. *Int. J. Revenue Manag.* **2012**, *6*, 28–44. [CrossRef]

29. Pellegrini, P.; Castelli, L.; Pesenti, R. Metaheuristic algorithms for the simultaneous slot allocation problem. *IET Intell. Transp. Syst.* **2012**, *6*, 453–462. [CrossRef]

30. Benlic, U. Heuristic search for allocation of slots at network level. *Transp. Res. Part C Emerg. Technol.* **2018**, *86*, 488–509. [CrossRef]

31. Andreatta, G.; Dell'Olmo, P.; Lulli, G. An aggregate stochastic programming model for air traffic flow management. *Eur. J. Oper. Res.* **2011**, *215*, 697–704. [CrossRef]

32. Mukherjee, A.; Hansen, M. A dynamic stochastic model for the single airport ground holding problem. *Transp. Sci.* **2007**, *41*, 444–456. [CrossRef]

33. Richetta, O.; Odoni, A.R. Dynamic solution to the ground-holding problem in air traffic control. *Transp. Res. Part A Policy Pract.* **1994**, *28*, 167–185. [CrossRef]

34. Nilim, A.; El Ghaoui, L. Algorithms for air traffic flow management under stochastic environments. In Proceedings of the 2004 American Control Conference, Boston, MA, USA, 30 June–2 July 2004; Volume 4, pp. 3429–3434.
35. Corolli, L.; Lulli, G.; Ntaimo, L.; Venkatachalam, S. A two-stage stochastic integer programming model for air traffic flow management. *IMA J. Manag. Math.* **2017**, *28*, 19–40. [CrossRef]
36. Gupta, S.; Bertsimas, D.J. Multistage air traffic flow management under capacity uncertainty: A robust and adaptive optimization approach. In Proceedings of the 51st AGIFORS Annual Symposium and Study Group Meeting 2011, Antalya, Turkey, 10–14 October 2011; pp. 692–721.
37. Sidiropoulos, S.; Han, K.; Majumdar, A.; Ochieng, W.Y. Robust identification of air traffic flow patterns in Metroplex terminal areas under demand uncertainty. *Transp. Res. Part C Emerg. Technol.* **2017**, *75*, 212–227. [CrossRef]
38. Wei, M.; Zhang, S.P.; Liu, T.; Sun, B. The adjusted passenger transportation efficiency of nine airports in China with consideration of the impact of high-speed rail network development: A two-step DEA-OLS method. *J. Air Transp. Manag.* **2023**, *109*, 102395. [CrossRef]