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Number of Blades' Influence on the Performance of Rotor with Equal Solidity in Open and Shrouded Configurations: Experimental Analysis

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Abstract: This study explores the implications of the number of blades on the performance of both open and shrouded rotors. By conducting a thorough experimental analysis at a fixed solidity ratio, this research seeks to enhance our understanding of rotor dynamics and efficiency. Two-, three-, four-, and five-bladed rotors were designed and manufactured to have the same solidity ratio. This leads to smaller chord distribution values for higher blade numbers. The experimental analysis aims to quantify the effects of the number of blades and provides a comparative analysis of performance differences between the two rotor configurations (shrouded and open). For the open rotor, results indicate that increasing the number of blades has a minimal impact on overall performance. This is due to the decrease in the tip loss factor being counterbalanced by a decline in efficiency caused by the two-dimensional airfoil performance, which results from a smaller chord and a lower Reynolds number. In contrast, the shrouded rotor exhibits a noticeable performance decay with an increased blade count. Since tip loss is inherently absent in shrouded designs, the decline is primarily attributed to the two-dimensional airfoil performance. This decay occurs while maintaining a constant solidity ratio, highlighting the significant effect of blade count on shrouded rotor efficiency, thereby contributing to the optimization of rotor design in various engineering applications.

Keywords: number of blades; shrouded rotor; open rotors; experimental aerodynamics

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1. Introduction

The performance and efficiency of rotors, crucial components in numerous engineering applications, have been thoroughly researched and refined over time. In the fields of propulsion and energy production, rotor efficiency and performance are essential for achieving desired outcomes across various applications. Whether in wind turbines, industrial fans, or aviation propulsion, the design and optimization of rotor systems are vital for maximizing efficiency and minimizing energy consumption [1,2].

Ever since the first aircraft propellers were introduced, a multitude of designs have emerged to keep up with the rapid advancement of aerospace technologies. These developments are driven by the need to enhance performance, efficiency, and adaptability to various operational requirements. Rotor design approaches are tailored to meet the specific mission characteristics of different types of aircraft, from small unmanned aerial vehicles to large commercial airliners and military aircraft [3,4].

Modifications to rotor design involve a range of adjustments. Changing the number of blades can affect the thrust and efficiency, with more blades (along with increasing the solidity ratio) often being used to increase the thrust for heavier aircraft. The type of airfoil used in the blades influences aerodynamic performance, with different shapes and profiles selected to maximize lift and minimize drag. Adjusting the propeller diameter can also impact performance, as a larger diameter can produce more thrust but may require more power to turn. Additionally, the angle and twist of the blades are critical factors in propeller design. The blade angle, or pitch, determines how much air is displaced by the propeller, directly affecting the thrust and efficiency [5,6].

Overall, these modifications and design considerations are essential for achieving highly efficient rotor performance, contributing to the effectiveness and sustainability of modern aviation. The continuous evolution of rotor technology plays a crucial role in the advancement of aerospace engineering, enabling aircraft to meet increasingly demanding performance and efficiency standards [7–9].

Current research and development efforts have focused on the two types of rotor configurations: open and shrouded. Open rotors, which do not have an outer casing, allow the flow field to circulate freely around the blades. Open rotors, commonly used in helicopters and certain types of drones, have a simple design that results in significant tip losses and increased noise, which can be problematic in some applications. Shrouded rotors, on the other hand, are encased within a duct, which alters flow dynamics and significantly impacts performance parameters. This design is particularly effective in reducing tip losses, which can enhance the overall efficiency of the rotor. The shroud also helps accelerate airflow, enhancing thrust while reducing noise [10,11].

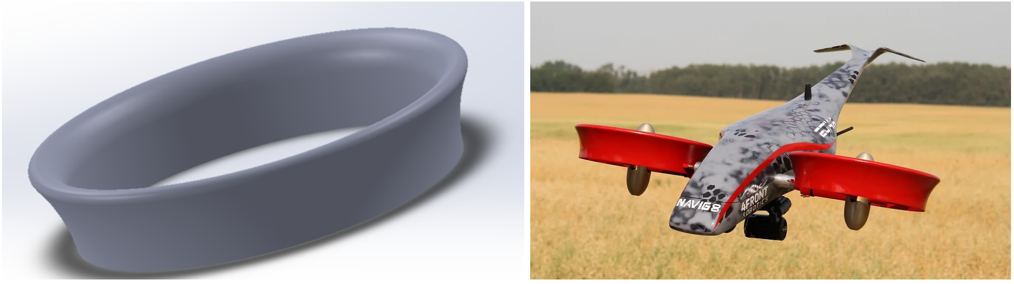
Understanding the performance differences between shrouded and open rotors is critical for optimizing rotor designs for particular applications, such as improved aerodynamic efficiency [12], noise production [13], and structural integrity [14], requiring additional research and testing. Shrouded rotors can be a better choice for applications that require high efficiency and low noise levels, such as certain types of industrial fans, wind turbines, and ducted fan propulsion systems in aircraft.

The proposed work makes a significant contribution to the field by addressing a critical gap in the current research on shrouded rotors. Specifically, Hrishikeshavan et al. [15] investigated the impact of varying the blade number from two to six on shrouded rotor performance. The researchers concluded that, in both shrouded and unshrouded rotor configurations, the performance did not change significantly with the number of blades. However, the solidity ratio as a variable affecting performance was not isolated, which could have influenced their findings. The present research uniquely focuses on this aspect, providing insights into how varying the number of blades influences performance while keeping other parameters constant. The findings from this study will contribute insights into optimizing rotor designs for various shrouded rotor applications, enhancing our understanding of the trade-offs involved in rotor performance.

In this research, two-, three-, four-, and five-bladed rotors were designed and manufactured to maintain the same solidity ratio, resulting in smaller chord distribution values for rotors with a higher number of blades. This study employs experimental analysis to investigate the performance of these rotors. It aims to quantify the effects of varying numbers of blades on performance differences between open and shrouded rotor configurations. By ensuring the same solidity ratio across different blade counts, this research isolates the impact of blade number on rotor efficiency and performance.

2. Case Study

The Navig8 UAV, shown in Figure 1b, is a highly maneuverable Vertical Take-Off and Landing (VTOL) aircraft designed for high payloads via small power to perform complex flight maneuvers in places inaccessible to traditional VTOL aircraft, such as the interior of collapsed structures or beneath the forest canopy [16]. It uses shrouded rotors designed specifically to improve the aerodynamic performance and efficiency of the propeller system by optimizing airflow dynamics toward the blades. Its design guarantees that the propeller functions within the optimal parameters for thrust generation and maneuverability, which are critical for the UAV's navigation and operational requirements.



(**a**) The used shroud. (**b**) Navig8 UAV in flight.

Figure 1. Navig8 UAV.

The shroud referenced in Figure 1a is custom-designed to accommodate a 9-inch propeller used in the thruster configuration of the Navig8 UAV. It was designed by Trek Aerospace, a company that developed a proprietary CFD software program, TASPA (Trek Aerospace Shrouded Propeller Analysis), specifically for designing shrouded propellers. The shroud features a 6.25% lip radius, a 28% duct length, and a throat positioned at 44% of the shroud chord (0.06 m). By placing the throat farther into the shroud (0.025 m from the shroud inlet), a gentler velocity profile into the rotor was achieved, allowing the shroud supports to be placed aft of the rotor plane. The exterior shape is designed to resist flow separation at moderate angles of attack and side flow angles.

The three-bladed rotor used in this research was designed by optimizing the chord and twist distributions using a third-degree variation for the chord ($c(r)$) and twist ($\theta(r)$) using the Navig8 UAV as a reference frame to define the feasible regions for the selected function coefficients. The rotor was optimized using Blade Element Momentum Theory (BEMT) to surpass the Navig8 original rotor's performance in terms of thrust-to-power ratio. It is capable of providing the necessary thrust so that it can lift the UAV at around 8500 RPM, while the original shrouded rotor could do it at around 10,500 [17]. The equations representing chord and twist variations are shown in Equations (1) and (2), respectively:

$$c(r) = 1.66e^{-6}\, r^3 - 2.87e^{-2}\, r^2 + 0.022\, r + 0.033 \tag{1}$$

$$\theta(r) = -5.06\, r^3 + 60.55\, r^2 - 93\, r + 51.23 \tag{2}$$

where r is the non-dimensional radial position. The design maintained a constant solidity ratio, the rotors were 3D-printed with 100% filament density. The surface was then coated with epoxy to enhance smoothness and increase strength.

The airfoils were selected based on Reynolds number performance as the design incorporates the S7055 airfoil for the inboard sections, ranging from 0% to 35% of the rotor radius, chosen for its favorable performance at low Reynolds numbers typical of slower flow velocities nearer the hub. In contrast, the S1223 airfoil is employed for the outer sections spanning from 45% of the rotor radius to the tip, renowned for its efficiency at higher Reynolds numbers associated with faster tangential velocities at the blade tips. The transitional region between 35% and 45%, depicted in Figure 2, facilitates a smooth aerodynamic shift between these airfoils, ensuring optimal performance and efficiency across the entire span of the rotor.

The two-, four-, and five-bladed rotors were designed with the same rotor airfoil and twist distributions as that of the base rotor (three blades), and the chord length was modified to keep the solidity ratio constant ($\sigma = 0.297$), as shown in Figure 3.

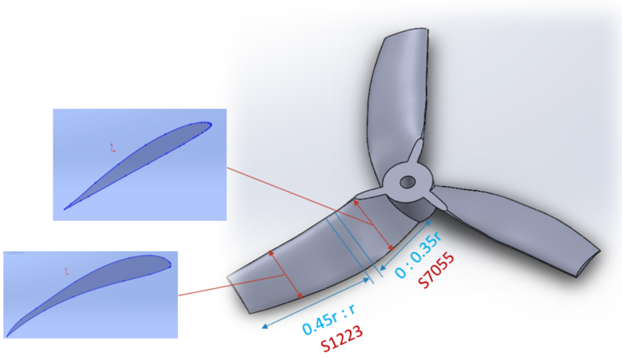


Figure 2. Airfoil distribution of the selected rotor.

(**a**) 2-bladed rotor. (**b**) 3-bladed rotor. (**c**) 4-bladed rotor. (**d**) 5-bladed rotor.

Figure 3. Two-, three-, four-, and five-bladed rotors.

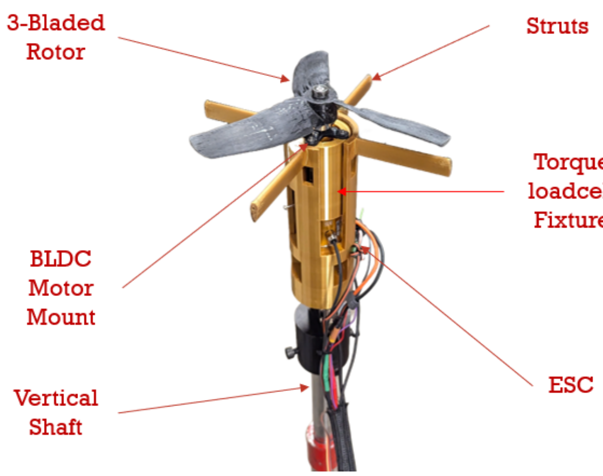
3. Experimental Setup

3.1. Test Rig Design

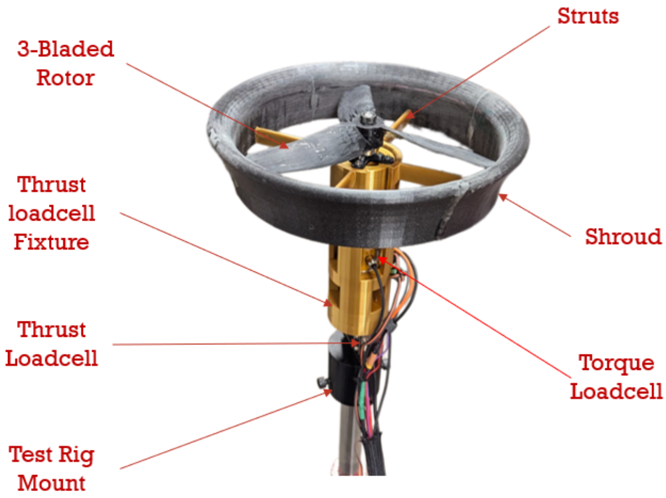
The experimental objective is to determine the effect of different blade numbers in both open and shrouded rotor cases. Test rig fixtures were designed and manufactured to hold the rotor/shrouded rotor in position, as shown in Figure 4. The struts that hold the shroud in position are left intact even when testing the open rotor configuration, as shown in Figure 4a. This ensures that the impact of the number of blades is isolated, as the struts exert a consistent influence in both open and shrouded rotor scenarios, maintaining the same structural elements in both configurations. A torque load cell measures the torque at various rotational speeds. It is connected directly to the rotor to eliminate any influence from the struts on the measured torque value. This direct connection ensures that the torque readings accurately reflect the rotor's performance, unaffected by external structural elements.

The thrust load cell supports the entire assembly to measure the total thrust produced in both open and shrouded configurations. This setup ensures accurate measurement of the overall thrust generated by the rotor systems, providing reliable data for performance analysis. The RPM is measured by capturing the voltage frequency of two phases from the Brushless Direct Current (BLDC) Motor, enabling the calculation of mechanical power. A Pololu Maestro Controller is used to send the pulse width modulation to the electronic speed controller (ESC) to control the BLDC motor RPM. This ensures precise monitoring and control of the rotational speed, allowing for accurate assessment of the rotor's performance.

National Instruments Data Acquisition (DAQ) Systems are used for automated data collection. During the experiment, the rotor's RPM was varied from 4000 to 8000, and the thrust, torque, and RPM were measured accordingly. The RPM, total thrust, and torque were measured simultaneously and recorded for each run lasting at least 15 s. Each second recording represents the average of ten readings during that second. Since all measurements were taken in a steady hovering regime, the average value of each measurand was considered.



(a) Open rotor test rig.



(b) Shrouded rotor test rig.

Figure 4. Shrouded and open rotor fixtures.

The initial design specified a tip clearance for the shrouded rotor of 0.02 times the rotor radius. However, this specification was not compatible with the test rig design. To ensure accurate torque measurements unaffected by the shroud and struts, the torque load cell was directly connected to the BLDC motor. As a result, the rotor and BLDC motor could not be connected to the external shroud as previously described. Any vibrations in the rotor shaft could potentially cause interference between the rotor and the shroud.

To mitigate this risk, a larger tip clearance of approximately 0.03 times the rotor radius was implemented in this research. Consequently, the expected thrust increase in the shrouded rotor compared with the open rotor for a given power was not fully achieved, resulting in only an 11% increase in thrust. However, previous testing for a tip clearance of 0.02 of the rotor radius had shown that the shrouded rotor achieved a thrust increase of 23% to 28% over the open rotor at different RPMs for a three-bladed rotor. By maintaining a consistent tip clearance for all tested rotors, the effects of tip clearance were isolated. Additionally, all experimental tests were performed at the same time to ensure consistent operating conditions, as presented in Table 1, for all cases.

Table 1. Experimental operating conditions.

Atmospheric Pressure [Pa]	Temperature [K]	Density [kg/m^3]
89,100	293.5	1.0574

3.2. Uncertainty Analysis

Uncertainty analysis is used to qualify the results of the experimental test to be conducted in the present research. The purpose of the uncertainty analysis is to determine numerical errors for the random (precision) errors and systematic or fixed (bias) errors encountered in rotor performance analysis.

The random error estimate is performed based on a statistical evaluation of the results for numerous experiment repeats, in some respects a measure of the repeatability of the experimental measurement and inherent unsteadiness seen in rotor performance. On the other hand, a systematic error will be accounted for to be "certain about the uncertainty to be presented in the measurement process".

3.2.1. Random (Precision) Error

During the experimental work, it is anticipated that random errors may arise due to inaccuracies in measuring instruments and human factors. These errors will be addressed by conducting repeated experiments to estimate and manage them effectively. The estimation of random errors will be based on principles of probability and statistics. This approach allows for a qualitative assessment of the measurement precision and provides insights into potential variations that may occur [18].

The averaged thrust and torque represent the mean values obtained from recorded measurements. The uncertainty analysis at the 95% level of confidence is selected, and the obtained uncertainties are to be illustrated. The sample size of each test point is three, and the arithmetic average, $\bar{x}$, is calculated as per the following Equation (3). The standard deviation of each sample, S_x, is calculated according to the following Equation (4):

$$\overline{x} = \frac{\Sigma_{i=1}^{N} x_i}{N} \tag{3}$$

where x_i is the values of the measured sample and N is the sample size.

$$S_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{4}$$

The distribution function, $t_{2,95} = 4.303$, is selected based on the required level of confidence of 95% and a sample size of three [19]; the random uncertainty is calculated as per Equation (5).

$$U_{ran} = t_{2,95}\frac{S_x}{\sqrt{N}} \tag{5}$$

As an example of the obtained random uncertainty, calculations are applied to the shrouded three-bladed rotor's thrust and torque calculated at different RPMs; the maximum value of uncertainties is 0.47% for the RPM, 1.3% for the total thrust, and 1.4% for the mechanical power, which are deemed acceptable for the specified confidence level. Increasing the sample size would decrease the uncertainty, but it would also incur higher costs and require more time.

3.2.2. Systematic (Bias) Error

The systematic error estimates are conducted at the level of individual parameter measurements. The systematic uncertainty is typically specified in the measuring instrument datasheet, which, in the current case, is 0.03% for the thrust sensor and 0.3% for the torque sensor of the reading value. The RPM measurement has an accuracy of 0.3% of the reading value.

There are mainly two measured parameters during the experiment to calculate the mechanical power (P [W]) (the rotational speed (Ω [rad/s]) and the torque (Q [N.m])). The overall systematic uncertainty of the mechanical power is calculated according to Equation (6), which is based upon a first-order Taylor series expansion [20].

$$U_{sys} = \sqrt{\left[\frac{\partial P}{\partial \Omega}\delta(\Omega)\right]^2 + \left[\frac{\partial P}{\partial Q}\delta(Q)\right]^2} \tag{6}$$

The first partial derivatives of each term in Equation (6) ($\frac{\partial P}{\partial \Omega}$ and $\frac{\partial P}{\partial Q}$) are sensitivities that define the relative change in the calculated parameter with respect to an individual measured variable. The second part of each term defines the systematic error in the measuring instrument for each parameter, as defined by the instrument manufacturer in its manual. Calculating the sensitivities can be performed using two approaches depending on how they are obtained. If the equation of the calculated parameter is straightforward and differentiable, sensitivities are obtained using the parameter's first partial derivatives. Conversely, if differentiation is not feasible (as in the case of complex models), sensitivities can be determined using measured data. Given that mechanical power can be calculated according to Equation (7), the uncertainty of mechanical power can be determined as per Equation (8).

$$P = Q\,\Omega \tag{7}$$

$$U_{sys} = \sqrt{[Q\ \delta(\Omega)]^2 + [\Omega\ \delta(Q)]^2} \tag{8}$$

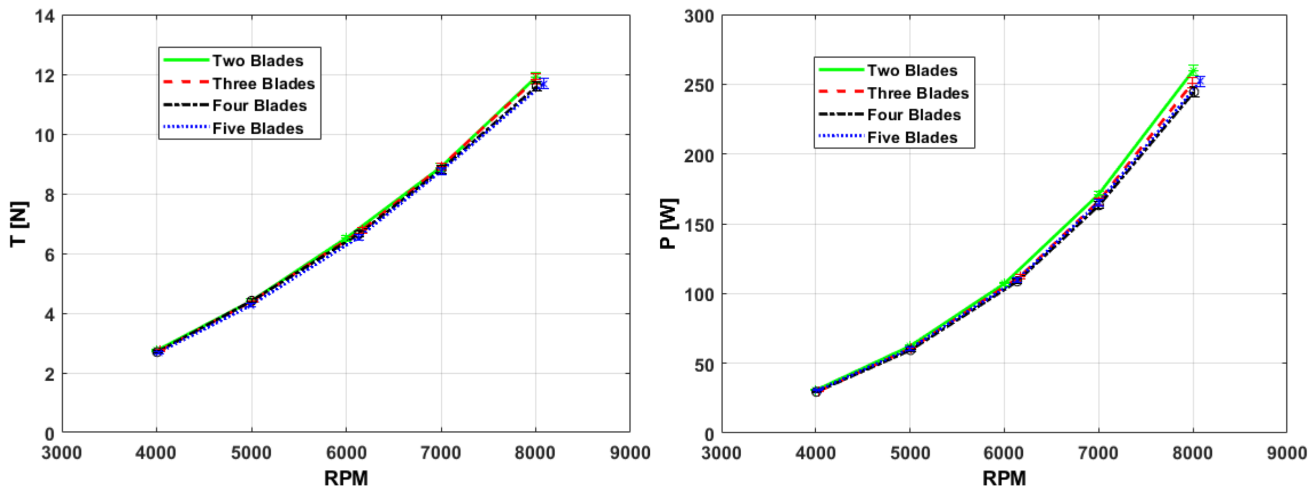
Therefore, the combined uncertainty based on error propagation for the mechanical power is found to be ±1.42%.

The total uncertainty in measurement is the result of combining both random and systematic uncertainties. By identifying and quantifying both types of uncertainties, the total measurement uncertainty provides a comprehensive understanding of the potential inaccuracies in the measurement process, enabling more reliable and accurate data interpretation.

4. Results

4.1. Open Rotors

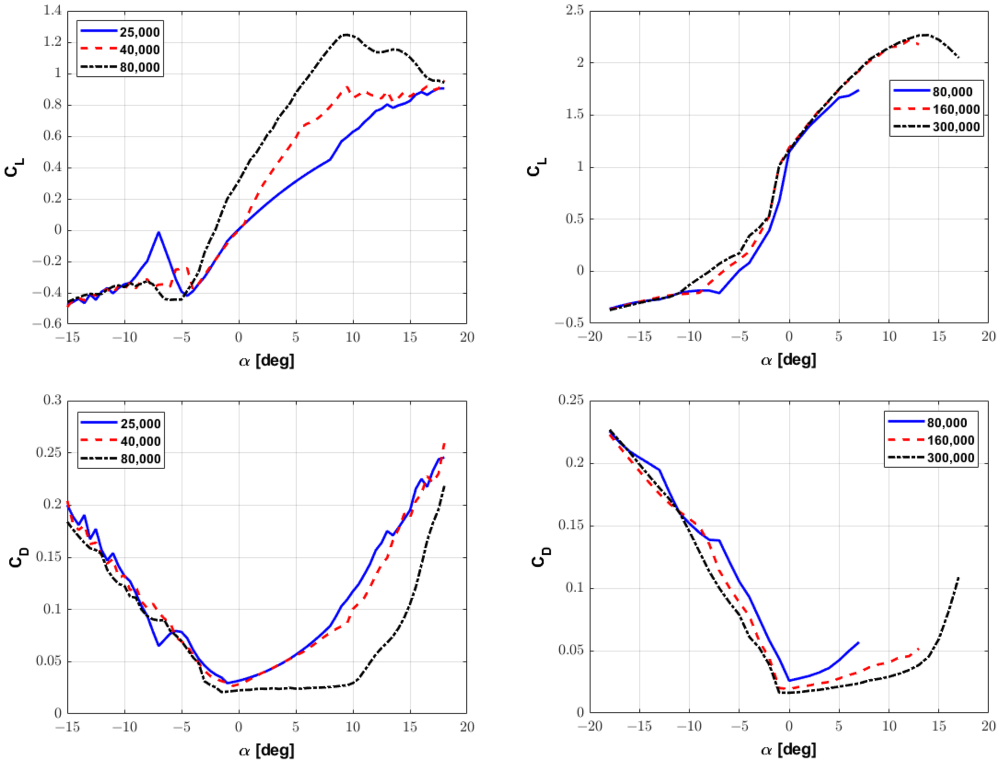
The measured thrust and mechanical power for open rotors are plotted against rotational speeds in Figure 5. From the data, it is evident that, at higher rotational speeds, both the generated thrust and power of the two- and three-bladed rotors are slightly higher than those of the four-bladed and five-bladed rotors. Despite variations in the number of blades, the overall conclusion indicates that all four open rotor configurations performed similarly.



(**a**) Total thrust vs. RPM. (**b**) Mechanical power vs. RPM.

Figure 5. Thrust and mechanical power for open rotors.

Generally, increasing the number of blades on a rotor reduces the tip loss effect, thereby potentially enhancing rotor efficiency. However, increasing the blade number also introduces a counterbalancing factor that diminishes efficiency. This reduction is primarily attributed to the two-dimensional airfoil performance, which is adversely affected by a smaller chord and lower Reynolds number. The impact of two-dimensional performance, particularly on local lift and drag coefficients, becomes more pronounced with the five-bladed rotor due to its smaller chord profile. This results in reduced thrust and power compared with rotors with fewer blades operating at similar rotational speeds. The influence of two-dimensional performance is derived from XFOIL simulations (an interactive program for the design and analysis of subsonic isolated airfoils), illustrated in Figure 6. The results from these simulations demonstrate that, at lower Reynolds numbers, there is a decrease in the lift coefficient and an increase in the drag coefficient.



(**a**) S7055 airfoil lift and drag coefficients. (**b**) S1223 airfoil lift and drag coefficients.

Figure 6. Lift and drag coefficients for S7055 and S1223 airfoils.

The measured thrust versus mechanical power for open rotors is plotted in Figure 7. The results indicate that increasing the number of blades has a minimal impact on overall performance. Specifically, although both thrust and power decrease for the five-bladed rotor, the overall performance (thrust versus power) remains nearly identical to that of rotors with fewer blades. This suggests that the overall performance is not significantly influenced by the number of blades on the rotor.

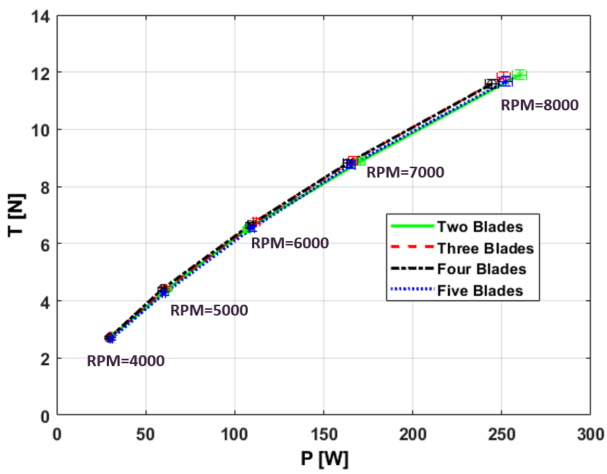
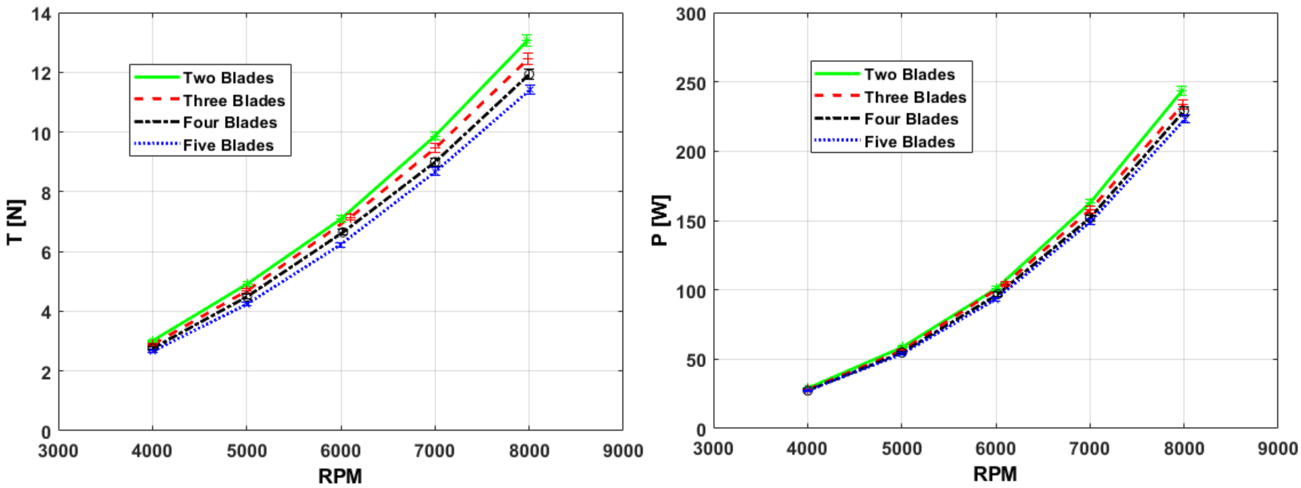


Figure 7. Thrust vs. power for open rotors.

4.2. Shrouded Rotors

The measured thrust and mechanical power for shrouded rotors are plotted against rotational speeds in Figure 8. Additionally, Figure 9 illustrates the relationship between measured thrust and mechanical power specifically for shrouded rotors. The results highlight a noticeable deterioration in the performance of shrouded rotors as the number of blades increases. This degradation suggests that increasing the number of blades adversely affects the overall efficiency and effectiveness of the shrouded rotor configuration (unlike the open rotor case).



(**a**) Thrust vs. RPM. (**b**) Power vs. RPM.

Figure 8. Thrust and power for shrouded rotors.

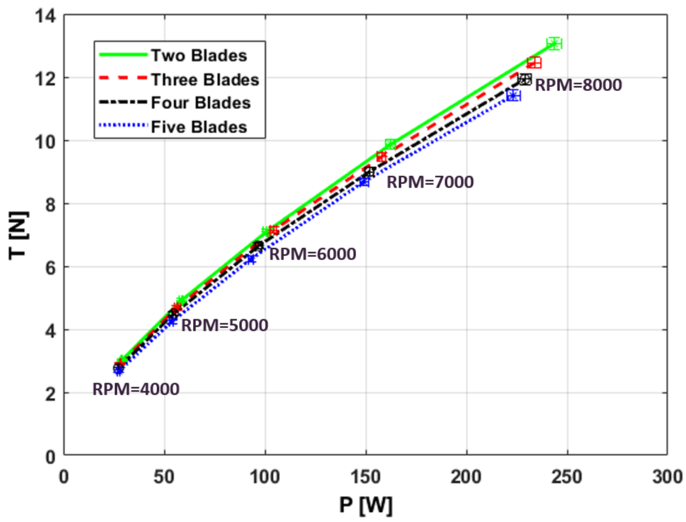


Figure 9. Thrust vs. power for shrouded rotors.

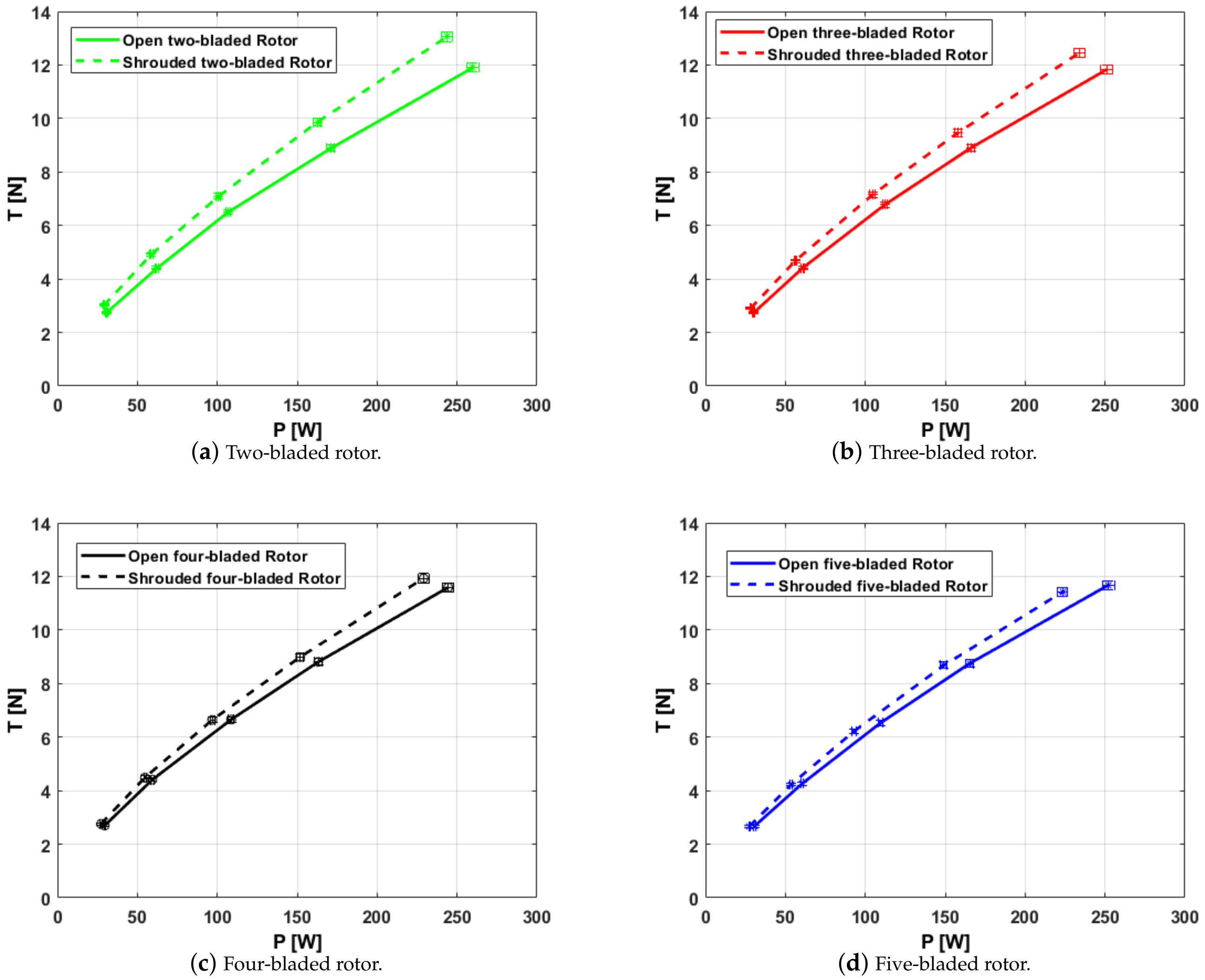
In shrouded rotor designs, the absence of tip loss, a common occurrence in open rotors, shifts the focus to the aerodynamic characteristics of the blade profiles. Specifically, the performance decline in shrouded rotors due to an increasing blade number can largely be attributed to the aerodynamic behavior of the two-dimensional airfoils used at a lower Reynolds number. Additionally, as the number of blades increases, aerodynamic efficiency decreases due to another factor, which is heightened blade-to-blade interaction. This factor collectively leads to lower lift-to-drag ratios and overall efficiency, underscoring the critical role of blade design and spacing in optimizing shrouded rotor systems.

Furthermore, increasing the number of blades can contribute to reducing the pressure difference across each blade because each blade bears a proportionately lesser load to generate thrust. This reduction in pressure difference is particularly pronounced in shrouded rotors compared with open rotors, where the shroud helps to mitigate tip losses and maintain more uniform airflow around the blades.

4.3. Open vs. Shrouded Rotors

Figure 10 provides a comparison between open and shrouded rotors with configurations of two, three, four, and five blades in terms of thrust versus mechanical power. The four sub-figures clearly demonstrate the superior performance of the shrouded rotor compared with the open rotor.

For each blade configuration, the shrouded rotor consistently generates higher thrust for a given mechanical power input. This improved performance can be attributed to the shroud's ability to enhance airflow through the rotor, thereby increasing its efficiency. Even though there is a reduction in performance as the number of blades increases due to maintaining the same solidity ratio, the shrouded rotor continues to outperform the open rotor. Overall, the data underscore the effectiveness of the shrouded design in improving rotor performance across different blade configurations.



(**a**) Two-bladed rotor. (**b**) Three-bladed rotor.

(**c**) Four-bladed rotor. (**d**) Five-bladed rotor.

Figure 10. Thrust-to-power ratio for open vs. shrouded rotors.

5. Conclusions

This research investigates the influence of blade count on the performance of open and shrouded rotors through experimental analysis, maintaining a constant solidity ratio. To ensure consistency, two-, three-, four-, and five-bladed rotors were designed and manufactured with an identical solidity ratio. This approach resulted in smaller chord distributions for rotors with higher blade numbers. Experimental analyses were conducted to quantify the impact of the blade number on the performance of both open and shrouded rotors.

For the open rotor, the findings indicate that increasing the number of blades has a minimal impact on overall performance. This phenomenon can be explained by the balance between the decrease in the tip loss factor and the decline in efficiency caused by the two-dimensional airfoil performance. The smaller chord and lower Reynolds number associated with higher blade counts lead to reduced aerodynamic efficiency, thereby offsetting the benefits gained from reduced tip losses. Consequently, the overall performance remains largely unchanged.

In contrast, the shrouded rotor exhibits a noticeable decline in performance with an increased blade count. In shrouded designs, the absence of tip loss shifts the primary focus to the performance of two-dimensional airfoils. As the number of blades increases, aerodynamic efficiency deteriorates due to a lower effective Reynolds number. This results in a reduction in lift-to-drag ratios and overall efficiency. This decay occurs while maintaining a constant solidity ratio, highlighting the significant impact of blade count on the efficiency of shrouded rotors. Moreover, increasing the number of blades can help reduce the pressure difference across each blade since each blade needs to generate less lift. Overall, the increase in the number of blades in shrouded rotors significantly degrades performance due to decreased aerodynamic efficiency, whereas open rotors exhibit a minimal impact from changes in blade count.

Understanding the impact of blade count on the performance of both open and shrouded rotors is crucial for optimizing rotor design across various engineering applications. The findings are essential for advancing rotor technology, enhancing efficiency, and meeting stringent performance criteria in applications ranging from unmanned aerial vehicles to industrial fans and wind turbines.

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Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

BEMT	Blade Element Momentum Theory
BLDC	Brushless Direct Current
DAQ	Data Acquisition
ESC	electronic speed controller
TASPA	Trek Aerospace Shrouded Propeller Analysis
UAV	unmanned aerial vehicle
VTOL	Vertical Take-Off and Landing

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