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Multi-Objective Portfolio Optimization: An Application of the Non-Dominated Sorting Genetic Algorithm III

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Abstract: This study evaluates the effectiveness of the Non-dominated Sorting Genetic Algorithm III (NSGA-III) in comparison to the traditional Mean–Variance optimization method for financial portfolio management. Leveraging a dataset of global financial assets, we applied both approaches to optimize portfolios across multiple objectives, including risk, return, skewness, and kurtosis. The findings reveal that NSGA-III significantly outperforms the Mean–Variance method by generating a more diverse set of Pareto-optimal portfolios. Portfolios optimized with NSGA-III exhibited superior performance, achieving higher Sharpe ratios, more favorable skewness, and reduced kurtosis, indicating a better balance between risk and return. Moreover, NSGA-III's capability to handle conflicting objectives underscores its utility in navigating complex financial environments and enhancing portfolio resilience. In contrast, while the Mean–Variance method effectively balances risk and return, it demonstrates limitations in addressing higher-order moments of the return distribution. These results emphasize the potential of NSGA-III as a robust and comprehensive tool for portfolio optimization in modern financial markets characterized by multifaceted objectives.

Keywords: multi-objective optimization; NSGA-III algorithm; portfolio management; higher-order moments; risk–return trade-off

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1. Introduction

In the realm of portfolio management, achieving an optimal balance between risk and return remains a paramount objective, both in academic discourse and in practical investment strategies. The seminal work by Markowitz (1952) introduced the Mean–Variance (MV) optimization framework, establishing a foundational paradigm in modern portfolio theory. This framework revolutionized portfolio selection by offering a methodical approach to constructing portfolios that maximize expected returns for a given level of risk, thereby laying the groundwork for subsequent developments in financial economics. The MV optimization model, which quantifies the trade-off between risk and return, has since become a cornerstone of modern financial practices and continues to influence contemporary portfolio management strategies. Additionally, the MV framework is inherently limited by its focus on a single-objective function optimization problem—the trade-off between risk and return. This narrow focus does not account for the multitude of other factors that influence investor decision-making in modern financial markets. For instance, considerations such as liquidity constraints, transaction costs, and regulatory requirements add layers of complexity that the traditional Mean–Variance (MV) model cannot adequately handle (Harvey et al., 2016).

As financial markets have evolved, there has been increasing recognition that more than just the risk–return trade-off influence investors' decisions.

The dynamic nature of financial markets and the diverse objectives of investors necessitate more sophisticated optimization methodologies that can accommodate the multifaceted nature of portfolio management. In response to these challenges, multi-objective optimization frameworks have gained popularity in the field of portfolio management. These frameworks provide a more comprehensive approach to portfolio optimization by allowing for the simultaneous consideration of multiple, often conflicting, goals.

Genetic Algorithms (GAs) are a cornerstone in the field of evolutionary computation, widely employed for addressing complex optimization challenges in financial portfolio management. As a subset of evolutionary algorithms, GAs emulate natural selection processes observed in biological evolution, where fitness, variation, and survival are pivotal (Mujahid, 2014). According to Vijay Kanade (2023), GAs are computational optimization techniques inspired by principles of genetics and natural selection, tailored to navigate multi-dimensional solution spaces effectively. The GA workflow is characterized by a robust iterative process involving selection, crossover, and mutation. Initially, the algorithm defines a chromosome structure and fitness function, generating a diverse, randomly initialized population of potential solutions, each representing distinct portfolio configurations. The fitness function evaluates these portfolios against optimization objectives, such as maximizing return and skewness while minimizing risk and kurtosis, ensuring that high-performing solutions are retained for subsequent iterations.

Following this evaluation, the algorithm iteratively refines the population through selection, crossover, and mutation. Selection identifies high-performing portfolios based on their fitness scores, ensuring that superior solutions progress to the next generation while maintaining genetic diversity. Crossover combines these solutions, blending advantageous traits to produce new "offspring" portfolios, while mutation introduces random variations, expanding the search space and preventing premature convergence to suboptimal solutions. This evolutionary cycle continues until the termination criteria—such as a defined number of generations or minimal improvement—are met, culminating in a refined set of optimal portfolios. Unlike traditional single-solution methods, GAs excel as population-based metaheuristics, maintaining a diverse portfolio of solutions and offering superior adaptability in navigating complex financial optimization landscapes (Abo-Alsabeh & Salhi, 2022; Abid et al., 2023).

Building upon these foundations, the Non-dominated Sorting Genetic Algorithm III (NSGA-III) marks a significant advancement in evolutionary algorithms, tailored to address high-dimensional multi-objective optimization problems (Deb & Jain, 2014). NSGA-III introduces a reference-point-based selection mechanism that ensures a well-distributed set of Pareto-optimal solutions, making it particularly effective in scenarios with competing objectives, such as balancing risk, return, skewness, and kurtosis. Unlike its predecessor, NSGA-II, which is constrained to bi-objective or tri-objective problems, NSGA-III's scalability enables optimization across numerous objectives, ensuring a comprehensive exploration of the solution space (Jain & Deb, 2013). This capability is indispensable in modern finance, where portfolio managers require tools that address the intricate dynamics of financial markets, incorporating metrics like Sharpe ratios, Value at Risk (VaR), and drawdowns to create resilient investment strategies (Cheng et al., 2018; Dorokhov, 2023).

Empirical evidence further underscores the superiority of NSGA-III in financial portfolio management. By generating diverse Pareto-optimal portfolios, NSGA-III facilitates better trade-offs among multiple objectives, outperforming algorithms like NSGA-II and MOEA/D[1] in terms of convergence speed, solution diversity, and robustness under varying market conditions (Li et al., 2018). This adaptability makes NSGA-III particularly effective in handling extreme market scenarios and long-term portfolio rebalancing, demonstrating its potential as a transformative tool in modern financial optimization. Recent studies confirm that NSGA-III not only aligns with theoretical advancements but also addresses practical challenges in high-stakes financial decision-making, offering a sophisticated alternative to traditional approaches like the Mean–Variance model (Deb et al., 2014; Silva et al., 2024).

This paper extends the application of NSGA-III to global financial markets, optimizing portfolios across multi-dimensional objectives using a diverse dataset of financial assets. The findings reveal that NSGA-III generates portfolios with superior risk-adjusted returns, showcasing better risk–return trade-offs and enhanced skewness and kurtosis compared to the Mean–Variance framework. By incorporating real-time data and simulating market extremes, the analysis validates NSGA-III's robustness in volatile environments, affirming its capability to meet the demands of dynamic portfolio management. This research significantly contributes to the literature, demonstrating NSGA-III's applicability beyond theoretical constructs and offering actionable insights for investors and policymakers navigating the complexities of modern financial markets.

Thomas and Douglas (2024) argue that incorporating asymmetric risk measures provides a superior framework for assessing portfolio performance, demonstrating that unsquared deviation-based risk metrics outperform their squared-deviation counterparts. Their study highlights the effectiveness of measures such as semi-absolute deviation, mean absolute deviation, and downside semi-deviation in achieving higher returns, emphasizing the importance of asymmetric risk considerations in modern optimization models (Thomas & Douglas, 2024). These findings align with the increasing focus on tailored risk measures that better capture portfolio behavior under real-world market conditions.

Building on this premise, this study conducts an empirical evaluation of NSGA-III's capabilities to manage downside risks and sustain long-term portfolio returns, positioning the algorithm as a more nuanced and adaptable tool for portfolio management. By employing multiple performance metrics, including skewness, kurtosis, and Sharpe ratios, this paper advances the understanding of NSGA-III's effectiveness in addressing the complexities of real-world financial environments where traditional methods often fall short. Additionally, the results underscore NSGA-III's ability to mitigate risks in volatile market conditions, making it especially valuable for risk-averse investors seeking robust strategies.

The structure of this paper is as follows. Section 2 provides a comprehensive review of the existing literature on evolutionary algorithms and portfolio optimization, situating this research within the broader academic context. Section 3 outlines the methodology, detailing the models, computational strategies, and implementation of NSGA-III alongside the performance metrics employed. Section 4 presents an empirical analysis of the data and findings, offering a comparative evaluation of NSGA-III against traditional approaches. Finally, Section 5 synthesizes this study's key insights, discusses implications for financial portfolio management, and proposes directions for future research.

2. Literature Review

The literature on portfolio optimization has evolved significantly with the advent of multi-objective optimization techniques, addressing the growing complexities of modern financial markets. Markowitz's (1952) Modern Portfolio Theory (MPT) established the foundational framework for portfolio optimization, emphasizing the trade-off between risk and return through the efficient frontier. Building on this, Ali et al. (2022) applied a four-moment modified Value at Risk (VaR) model to explore diversification benefits across Global Business and Industry Classifications (GBICs), revealing superior outcomes compared to traditional Mean–Variance results. Similarly, Peeters (2023) extended the relevance of the Mean–Variance framework to non-normal return distributions, while

Fernando et al. (2020) introduced a fuzzy multi-objective variant incorporating liquidity, outperforming the S&P100 index in terms of return and risk.

Empirical studies further support the efficacy of Mean–Variance optimization, showing its ability to outperform equal-weighted or market-based portfolios, particularly in markets like the Chinese A-share (Tong, 2024). Recent advancements have integrated collaborative filtering techniques into the Mean–Variance framework, refining stock recommendations by balancing risk–return preferences with investor-specific requirements (Chung, 2023; Wang et al., 2023). However, the single-objective nature of the Mean–Variance approach has led to the development of advanced methodologies like the Non-dominated Sorting Genetic Algorithm II (NSGA-II) (Deb et al., 2002), which sorts solutions into Pareto fronts while maintaining diversity. Despite its success, NSGA-II struggles with higher-dimensional objectives, spurring the introduction of NSGA-III, which employs reference-point-based selection for more refined optimization in complex financial scenarios (Deb & Jain, 2014).

NSGA-III represents a significant advancement, addressing the limitations of its predecessor by offering well-distributed Pareto-optimal solutions in high-dimensional objective spaces. Studies show NSGA-III's superiority in optimizing portfolios with metrics such as Sharpe ratios, drawdowns, and kurtosis, making it particularly effective in balancing conflicting objectives like risk, return, and liquidity (Deb & Jain, 2014; Ouyang et al., 2018). Moreira and Muir (2017) highlighted the utility of multi-objective frameworks in evaluating risk–return trade-offs, principles that NSGA-III extends by navigating multi-dimensional optimization spaces. Gupta et al. (2021) further confirmed NSGA-III's dominance over other algorithms like NSGA-II and SPEA2, particularly in maintaining diverse and robust Pareto-optimal solutions.

The importance of incorporating asymmetric and holistic risk measures has also gained traction. Rudolph et al. (2016) and Deb et al. (2017) underscored the value of metrics like drawdowns and Conditional Value at Risk (CVaR) for capturing asymmetric risks and enhancing portfolio resilience in volatile markets. These advancements have shifted the focus of the optimization literature from traditional volatility measures toward more comprehensive approaches. Recent applications of NSGA-III in credit risk assessment and portfolio optimization further emphasize its flexibility and utility in financial engineering (Ouyang et al., 2018).

This study builds on foundational work by Naqvi et al. (2017) and aligns with the latest trends in the literature, incorporating performance metrics like standard deviation, Sharpe ratio, skewness, and kurtosis. By comparing NSGA-III to the Mean–Variance framework under diverse market conditions, this study offers a nuanced evaluation of risk-adjusted returns and portfolio resilience. The comprehensive multi-objective approach adopted here responds to calls in the literature for robust, adaptable methodologies that address the complexities of modern financial markets (Gupta et al., 2021; He & Zhou, 2020).

Building on the foundational work of Gupta et al. (2021), this study advances the application of NSGA-III in portfolio optimization by moving beyond traditional performance metrics such as return and risk. While Gupta et al. (2021) demonstrated NSGA-III's effectiveness in generating diverse Pareto-optimal solutions, our research extends their methodology by integrating additional metrics, including skewness, kurtosis, Value at Risk (VaR), and drawdowns. These enhancements align with the refinements introduced by Deb and Jain (2014), particularly their development of a reference direction framework in NSGA-III, which bolsters the algorithm's robustness in navigating dynamic and high-dimensional market conditions. By broadening the range of performance metrics, this study provides a more holistic evaluation of NSGA-III's capabilities, making significant contributions to both theoretical understanding and practical applications in multi-dimensional portfolio optimization.

This research also contrasts with the findings of Ali et al. (2022), who explored diversification benefits using a four-moment modified VaR framework. Their study highlighted the superior performance of DJI equity pairings under this model, results that traditional Mean–Variance approaches failed to corroborate. By incorporating an expanded analytical scope and leveraging NSGA-III's advanced optimization capabilities, this study offers deeper insights into addressing complex portfolio objectives. These contributions underscore the algorithm's potential to adapt to the evolving demands of financial markets, enhancing its utility for both academic inquiry and practical portfolio management.

3. Methodology of the Study

This paper enhances portfolio management practices by incorporating Sharpe ratio, VaR, skewness, and kurtosis using the NSGA-III and the traditional Mean–Variance framework. The Mean–Variance portfolio selection problem involves finding the optimal weights of a portfolio composed of a set of assets. Let $W^T = (w_1, w_2, \ldots, w_n)$ represent the vector of portfolio weights, where (w_i) is the weight of the ith asset in the portfolio. The portfolio weights satisfy the following constraints:

$$\sum_{\{i=1\}_i^{\{n\}w}} w_i = w^T e = 1 \tag{1}$$

and

$$w_i \geq 0 \;\; for\; all \;\; i, \tag{2}$$

where e is a $(n \times 1)$ vector of ones. The return of a portfolio is the weighted sum of the mean returns of each asset, given by:

$$R_P(w, r) = w^T r \tag{3}$$

The variance of the return of a portfolio is expressed as:

$$\Sigma_P(w, r) = w^T \Sigma_r w \tag{4}$$

In the Mean–Variance optimization framework, the main objective is to maximize a single-objective function that maximizes the expected return while simultaneously minimizing the portfolio risk as measured by its variance. This framework suffers from limitations that include the loss of information, the subjectivity in weighting, and the risk of oversimplification. To address these limitations, especially in the case of non-normal return distributions, a multi-objective approach that includes higher moments of the return distribution such as skewness and kurtosis becomes crucial and is adopted in this study. The incorporation of higher-order moments, such as skewness and kurtosis, necessitates a carefully designed fitness function that effectively balances these objectives alongside risk and return. Equally critical is the determination of an appropriate stopping criterion for the algorithm, which ensures a balance between computational efficiency and the accurate approximation of the true Pareto front. Optimizing these parameters often involves experimentation or leveraging hyperparameter optimization techniques, enabling the configuration to be tailored precisely to the specific dataset and the unique characteristics of the optimization problem.

This paper assumes that the return distribution follows a skew–normal distribution, which allows for non-zero skewness. The probability density function of a skew–normal distribution is given by:

$$f(y, \alpha) = 2\varphi(y)\Phi(\alpha y) \tag{5}$$

where $\varphi(y)$ and $\Phi(y)$ are the standard normal density and cumulative distribution functions, respectively, and α regulates the skewness. We therefore formulate the following objective functions:

$$f_1(w) = \left(R_P(w,\, r) = w^T r\right) \tag{6}$$

$$f_2(w) = \left(\Sigma_P(w,\, r) = w^T \Sigma_r w\right) \tag{7}$$

$$f_3(w) = \left(S_P(w,\, r) = w^T S(r)(w \otimes w)\right) \tag{8}$$

$$f_4(w) = \left(K_P(w,\, r) = w^T K(r)(w \otimes w \otimes w)\right) \tag{9}$$

where $S(r)$ and $K(r)$ are the skewness and kurtosis matrices, respectively.

3.1. NSGA-III for Multi-Objective Optimization

The NSGA-III is a sophisticated evolutionary algorithm developed to address many-objective optimization problems, particularly those involving more than two or three conflicting objectives. Unlike its predecessor, NSGA-II, which was primarily effective for bi-objective or tri-objective problems, NSGA-III employs a reference-point-based selection mechanism, enabling it to effectively navigate high-dimensional objective spaces (Deb & Jain, 2014). This mechanism uses a set of evenly distributed reference points to guide the selection process, ensuring a diverse array of solutions along the Pareto front. In the context of portfolio optimization, where objectives often include maximizing returns and skewness while minimizing risk and kurtosis, NSGA-III stands out as a robust method for managing complex, multi-dimensional trade-offs. By leveraging its non-dominated sorting technique and reference-point-based selection operator, NSGA-III explores the solution space comprehensively while preserving diversity across multiple objectives.

NSGA-III's ability to converge toward a well-distributed Pareto-optimal front, even in the presence of conflicting objectives, provides significant advantages for portfolio optimization. This feature is particularly useful in financial contexts where balancing competing objectives is essential to address varying investor preferences. Traditional methods, such as the Mean–Variance model or NSGA-II, often oversimplify these complexities, whereas NSGA-III offers a more nuanced exploration of trade-offs, resulting in a diverse range of optimal solutions. Studies, such as those by Simo et al. (2024), highlight NSGA-III's ability to compute the complete Pareto front when applied to problems with three or more objectives, outperforming NSGA-II in scalability and precision. Similarly, Awad et al. (2022) demonstrate that while NSGA-II is effective for problems involving two objectives, NSGA-III is more suited for scenarios with three or more objectives, making it highly relevant for modern portfolio management challenges.

Recent advancements in the NSGA-III framework underscore its versatility. Vesikar et al. (2018) proposed a reference-point-based evolutionary many-objective optimization procedure, further refined by Wietheger and Doerr (2024), who replaced crowding distance with reference points in the survival process. This adjustment ensures that non-dominated solutions are preserved across iterations, maintaining the algorithm's robustness. Empirical studies also showcase NSGA-III's efficacy in solving complex multi-objective problems. For instance, Jafari et al. (2022) demonstrated its effectiveness in addressing a five-objective sensor placement problem, outperforming benchmark methods. Similarly, Jafari et al. (2022) illustrated its utility in solving complex multi-dimensional challenges, reinforcing its applicability across domains.

The NSGA-III algorithm begins by generating an initial population of potential portfolio configurations. These portfolios evolve iteratively through selection, crossover, and mutation processes. The selection phase prioritizes portfolios that excel across multiple objectives, ensuring the survival of high-performing solutions. Crossover combines features of selected portfolios to produce new configurations, while mutation introduces slight random variations to maintain diversity and prevent premature convergence. NSGA-III organizes solutions into Pareto-optimal fronts, where the first front contains the most balanced trade-offs across all objectives. Each subsequent front comprises solutions dominated only by those in preceding fronts. This structured ranking provides investors with a broad spectrum of portfolio options, each tailored to balance competing financial objectives effectively.

For a multi-objective optimization problem involving k objective functions, NSGA-III aims to identify a diverse set of solutions that optimize the following:

$$Maximize\ \ F(w) = \left(f_{1(w)},\ f_{2(w)},\ \ldots,\ f_{k(w)}\right) \tag{10}$$

where each $f_{j(w)}$ represents an objective function. A solution w_1 is said to dominate another solution w_2 if $f_{j(w_1)} \geq f_{j(w_2)}$ for all $(j = 1, 2, \ldots, k)$, and at least one strict inequality holds.

The NSGA-III is employed to optimize multiple conflicting objectives such as maximizing returns, minimizing risk (variance), maximizing skewness, and minimizing kurtosis. The portfolio optimization problem used in this paper can be formulated as:

$$Maximize\ \ R_{P(w)} = w^T r \tag{11}$$

$$Minimize\ \ \Sigma_{P(w)} = w^T \Sigma_r w \tag{12}$$

$$Maximize\ \ S_{P(w)} = w^T S(r)(w\ \otimes w) \tag{13}$$

$$Minimize\ \ K_{P(w)} = w^T K(r)(w\ \otimes w\ \otimes w) \tag{14}$$

To understand how the NSGA-III is employed in solving this multi-objective portfolio optimization problem, we discuss each of the main steps involved in this process.

3.1.1. Initialization

NSGA-III starts with an initial population of potential solutions (i.e., portfolios), each representing a different allocation of weights w across assets. These weights are generated randomly, ensuring that they satisfy the constraints:

$$\sum_{i=1}^{n} w_i = 1 \text{and } w_i \geq 0 \text{ for all } i \tag{15}$$

each solution in the population is a vector of portfolio weights w.

3.1.2. Reference Point Generation

The NSGA-III uses reference points to guide the search process. In multi-objective optimization, these reference points represent ideal solutions that maximize or minimize objectives to the extreme. For the four-objective portfolio optimization problem, reference points are generated for each of the following objectives:

- Maximize $R_P(w)$.
- Minimize $\Sigma_P(w)$.
- Maximize $S_P(w)$.
- Minimize $K_P(w)$.

The reference points form a well-distributed set in the objective space, and NSGA-III ensures that solutions are spread uniformly across these points to maintain diversity in the population. For example, one reference point might represent the portfolio with the highest possible return and lowest possible risk, while another point might emphasize high skewness and low kurtosis.

3.1.3. Selection

Selection is the process of choosing the fittest individuals (portfolios) from the population to be used for crossover and mutation. In the NSGA-III, selection is based on the concept of Pareto dominance. A portfolio w_1 dominates another portfolio w_2 if

$$f_i(w_1) \geq f_i(w_2) \text{ for all objectives } i \text{ and } f_j(w_1) > f_j(w_2) \text{ for at least one objective} j,$$

where $f_1(w) = R_P(w)$, $f_2(w) = \Sigma_P(w)$, $f_3(w) = S_P(w)$, and $f_4(w) = K_P(w)$.

The NSGA-III ranks the solutions into different fronts based on their dominance. The first front consists of solutions that are not dominated by any other solution, followed by the second front, and so on.

3.1.4. Reference Point Association

Each solution in the population is associated with a reference point based on its proximity to that reference point in the objective space. The distance between a solution and a reference point is calculated, and each solution is mapped to its closest reference point. This ensures that the population is diverse, with solutions spread out across the different objectives.

For example, a portfolio with high returns and high skewness may be associated with a reference point that emphasizes maximizing $R_P(w)$ and $S_P(w)$, while another portfolio with low risk and kurtosis may be associated with a reference point that focuses on minimizing $\Sigma_P(w)$ and $K_P(w)$.

3.1.5. Niching and Survival Selection

The NSGA-III uses a niching strategy to maintain diversity among the solutions. After the solutions are associated with reference points, the algorithm calculates how many solutions are associated with each reference point. Reference points that have fewer solutions associated with them are given priority to ensure that underrepresented areas of the objective space are explored.

The survival selection step then selects solutions to form the next generation. Solutions that are closest to underrepresented reference points are selected first to maintain diversity. The idea is to ensure that the population does not converge prematurely to a small region in the objective space but rather covers the entire range of possible trade-offs between maximizing return (and skewness) and minimizing risk (and kurtosis).

3.1.6. Crossover and Mutation

Once the best solutions are selected, they are combined through crossover to produce offspring solutions. Crossover involves taking two selected portfolios and creating new ones by combining their weights. For instance, the weights of two portfolios w_1 and w_2 may be combined to create a new portfolio w_3 with weights that are a mix of w_1 and w_2. Mutation introduces small random changes to the portfolio weights to ensure exploration of the solution space. For example, one or more weights in the portfolio might be slightly altered to explore new potential portfolios that were not present in the initial population.

3.1.7. Termination

The algorithm repeats the process of selection, crossover, mutation, and survival selection for a fixed number of generations or until the solutions converge. Convergence occurs when the population shows little improvement in subsequent generations, indicating that the algorithm has found a good approximation of the Pareto-optimal front. At termination, the NSGA-III outputs a set of Pareto-optimal solutions. These solutions represent a diverse set of portfolios that balance the trade-offs between maximizing returns and skewness while minimizing risk and kurtosis.

4. Empirical Analysis

4.1. Data

The empirical analysis begins with an examination of the dataset, which comprises the daily returns of major global financial indices, including the FTSE100, SP500, NASDAQ, DAX, ALSI, MOEX, BOVESPA, Shanghai SE Composite, Sensex, and Hang Seng, along with the exchange rate of the South African rand against the U.S. dollar. A total of 4795 daily observations were utilized in this analysis, spanning from 30 March 2005 to 18 March 2024.

Such a substantial dataset enhances population diversity, crucial for a thorough exploration of the solution space in complex optimization tasks. By incorporating developed markets, such as the S&P 500—a benchmark with substantial liquidity and global economic representation—alongside emerging markets like Bovespa, Hang Seng, Sensex, Moex, Shanghai Composite, and ALSI, we aim to achieve a more comprehensive understanding of the algorithm's performance across diverse economic environments. These indices reflect a range of geographic regions and economic sectors, thereby increasing the robustness of our findings. Recognizing that regional markets react uniquely to macroeconomic influences, the inclusion of diverse markets provides broader insights into the algorithm's adaptability. A simple random sampling method was applied to ensure an unbiased selection of markets, thus supporting a balanced examination of algorithmic performance across varied economic landscapes. The data span from 30 March 2005 to 18 March 2024 and were collected from the Thomson Reuters database.

The descriptive statistics of the log-returns are presented in Table 1, providing an initial overview of the return's key characteristics and variability across the selected financial instruments.

Table 1. Descriptive statistics.

	Mean	Min	25%	50%	75%	Max	Std
FTSE100	0.0002	−0.1087	−0.0048	0.0005	0.0056	0.0984	0.0113
SP500	0.0004	−0.0911	−0.0039	0.0008	0.0055	0.1067	0.0112
NASDAQ	0.0006	−0.1219	−0.0053	0.0011	0.0074	0.1258	0.0140
DAX	0.0004	−0.1224	−0.0055	0.0009	0.0068	0.1140	0.0133
ALSI	0.0004	−0.0972	−0.0058	0.0006	0.0070	0.0947	0.0122
MOEX	0.0005	−0.3328	−0.0065	0.0007	0.0083	0.2869	0.0190
BOVESPA	0.0005	−0.1478	−0.0082	0.0007	0.0095	0.1466	0.0168
Shanghai	0.0003	−0.0884	−0.0061	0.0006	0.0074	0.0946	0.0150
Sensex	0.0006	−0.1315	−0.0053	0.0009	0.0070	0.1734	0.0135
HangSeng	0.0002	−0.1270	−0.0071	0.0004	0.0075	0.1435	0.0150
ZAR/USD	−0.0002	−0.1482	−0.0063	0.0001	0.0064	0.0729	0.0106

Note: "mean", "min", "25%", "50%", "75%", "max", and "std" represent the mean, minimum, 25th percentile, median (50th percentile), 75th percentile, maximum, and standard deviation, respectively, of the return distributions for each asset. These assets include the FTSE100, SP500, NASDAQ, DAX, ALSI, MOEX, BOVESPA, Shanghai SE, Sensex, Hang Seng, and the ZAR/USD exchange rate. This table provides a detailed overview of key descriptive statistics for each asset's returns, serving as a foundation for understanding their performance characteristics.

Table 1 reveals that the mean returns across the selected financial indices and the ZAR/USD exchange rate are generally positive, with NASDAQ having the highest mean return (0.00062), indicating strong average performance. In contrast, the ZAR/USD exchange rate has a slightly negative mean return (−0.0002), signaling a slight depreciation of the South African rand against the U.S. dollar. The standard deviation, representing volatility, varies significantly, with the MOEX index showing the highest volatility (0.019) and the ZAR/USD exchange rate being the lowest (0.0106), reflecting differing risk levels across assets.

However, the data suggest that while most indices have experienced growth, the risk associated with them varies, underscoring the importance of considering both return and volatility in portfolio optimization.

Figure 1 represents the trade-off between risk, as measured by standard deviation, and reward, as indicated by the mean return, for each asset in the portfolio. Assets such as the index and the BOVESPA index are positioned in the top right quadrant of the plot, reflecting both high risk and high mean returns, suggesting that these assets offer the potential for higher returns but come with greater volatility, making them riskier investments. In contrast, the FTSE100 index and the ZAR/USD exchange rate are found in the bottom left quadrant, displaying low risk and low mean returns, which may appeal to investors seeking safer investments, albeit with modest expected returns.

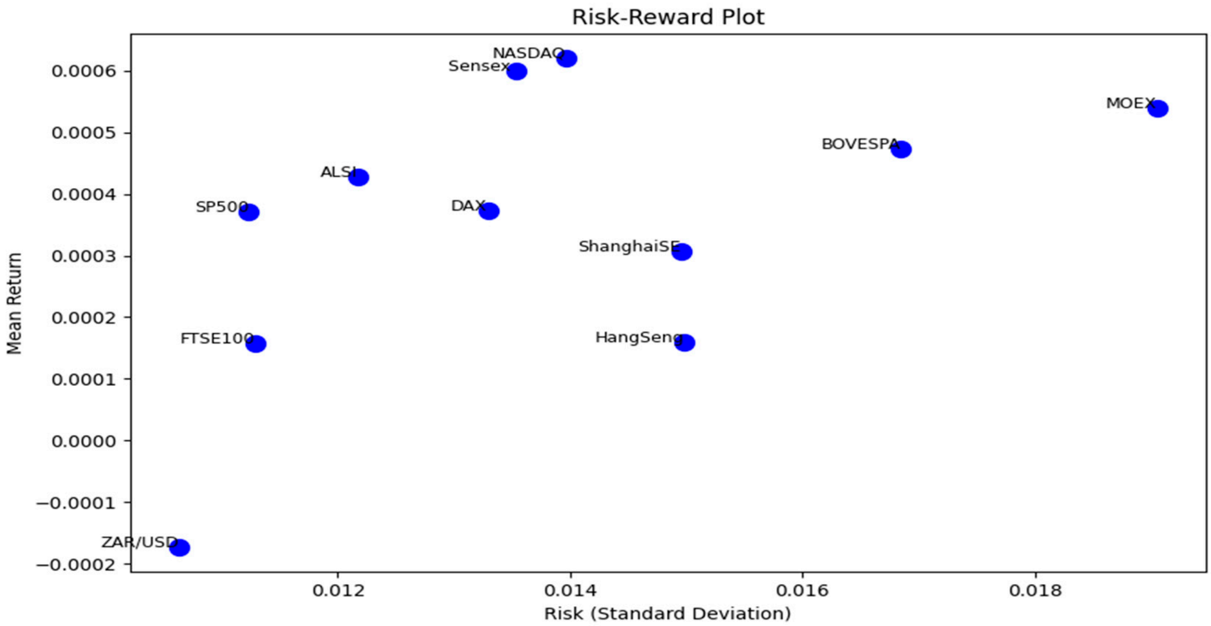

Figure 1. Risk–reward plot. This figure presents the assets based on their respective mean and standard deviation values. Assets positioned most favorably exhibit lower risk and higher mean returns. Among the observed extremes, MOEX displays a high mean return accompanied by elevated risk, while the SP500 shows both a relatively low mean return and a lower risk. Positioned between these two extremes, NASDAQ and Sensex represent optimal balance points with comparatively favorable mean returns and manageable risk levels.

Indices like SP500, DAX, and ALSI occupy the middle of the plot, indicating a balance between risk and return, making them suitable for those seeking a moderate level of risk with corresponding returns. Notably, the ZAR/USD exchange rate stands out with a slightly negative mean return and low risk, suggesting that holding the South African rand relative to the U.S. dollar might have resulted in a slight loss over the sample period, albeit with low volatility. Additionally, the clustering of assets such as NASDAQ, Sensex, and BOVESPA suggests similar returns with varying levels of risk, providing insight for investors based on their individual risk tolerance.

4.2. Optimization Results

The NSGA-III algorithm was applied to the dataset with the goal of generating portfolios that simultaneously optimize four key objective functions: maximizing the portfolio expected return function (f_1), minimizing the portfolio risk (measured by variance) function (f_2), maximizing the portfolio skewness function (f_3), and minimizing the portfolio Kurtosis function (f_4). We can understand that when utilizing performance measures like the Sharpe ratio, a higher Sharpe ratio for one model over another indicates that the first model has achieved superior performance. In this study, we compare two models—the NSGA-III and the Mean–Variance model. The findings reveal that the NSGA-III model consistently exhibits a higher Sharpe ratio than the Mean–Variance model, indicating its relative outperformance in optimizing risk-adjusted returns.

The efficient frontier with the NSGA-III is displayed below.

Figure 2 is a pairwise scatter plot matrix that illustrates the relationships between four key objective functions in the optimization process: portfolio risk (f_2), return (f_1), skewness (f_3), and kurtosis (f_4). The plots provide insights into the trade-offs between these objectives. For instance, the f_1 vs. f_2 plot shows the classic risk–return trade-off, where higher risk typically corresponds to higher returns. The f_1 vs. f_3 and f_1 vs. f_4 plots help assess how lower-risk portfolios align with skewness (which indicates the potential for higher positive returns) and kurtosis (which measures the likelihood of extreme outcomes). These visualizations are crucial for understanding how the portfolio's risk level correlates with these higher moments.

These scatter plots offer valuable insights into constructing portfolios that optimize multiple objective functions simultaneously. By examining the relationships between risk, return, skewness, and kurtosis, they can design investment strategies that balance these factors effectively. This approach helps in crafting portfolios that not only match the risk tolerance of investors but also maximize the potential for positive returns while minimizing the risks associated with extreme market events. The comprehensive understanding gained from this analysis is essential for making informed decisions in complex financial environments, ensuring more resilient and sustainable portfolio performance.

We, thereafter, compare the optimal portfolios obtained with the NSGA-III and the traditional Mean–Variance model. The results are as follows.

The identical weights in the Mean–Variance column of Table 2 reflect the model's tendency to distribute weights equally across assets when balancing multiple objectives without a specific asset preference. This outcome aligns with findings in portfolio theory, where an equal-weighted strategy can often emerge as a straightforward solution in scenarios without a dominant risk–return tradeoff focus (DeMiguel et al., 2009). Such a uniform allocation serves as a baseline for comparison, highlighting the more tailored and adaptive weight distribution achieved through NSGA-III's multi-objective approach (Markowitz, 1952).

Table 2. Optimal weights—NSGA-III vs. Mean–Variance.

Asset	NSGA-III Weights	Mean–Variance Weights
FTSE100	0.01622029	0.09090909
SP500	0.07597128	0.09090909
NASDAQ	0.10497395	0.09090909
DAX	0.25618561	0.09090909
ALSI	0.02443839	0.09090909
MOEX	0.12726688	0.09090909
BOVESPA	0.02645138	0.09090909
ShanghaiSE	0.07819937	0.09090909
Sensex	0.24180201	0.09090909
HangSeng	0.02431681	0.09090809
ZAR/USD	0.02417403	0.09090709

Note: Table 2 presents the portfolio weights allocated to various global financial assets by the NSGA-III algorithm and the traditional Mean–Variance approach. Each asset, ranging from major stock indices like the FTSE100 and NASDAQ to currency pairs like ZAR/USD, has been assigned a weight representing its proportion in the portfolio.

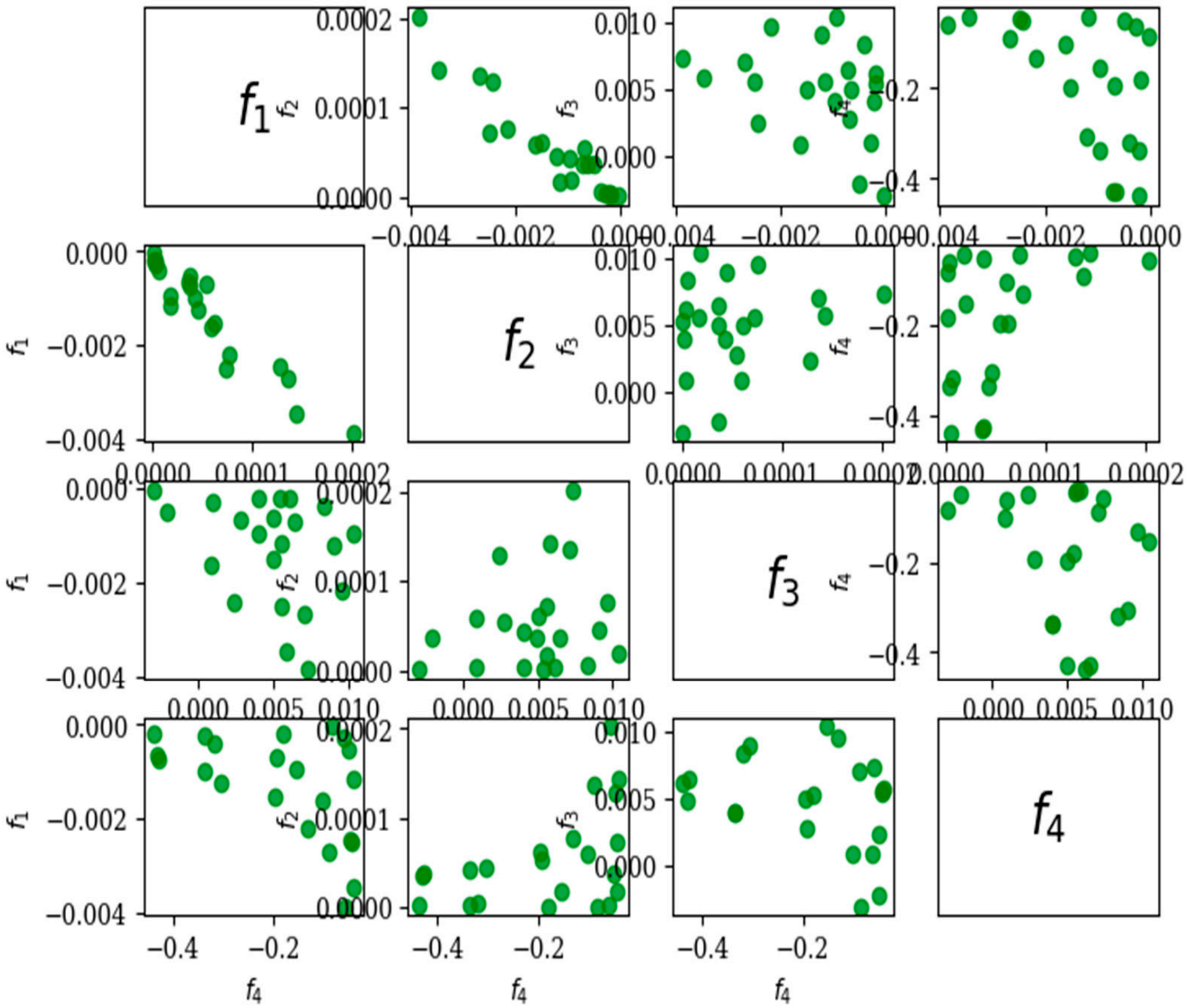

Figure 2. Efficient frontier for multi-objective optimization using NSGA-III. In this figure, each component or moment of our objective function (f_1, f_2, f_3, f_4) is systematically paired with each other, creating a matrix of 16 subplots. In each subplot, the x-axis represents one moment, (f_i), while the y-axis represents another moment, $\left(f_j\right)$, allowing us to observe the interactions and trade-offs between all possible pairs of moments.

In contrast, the results indicate that NSGA-III successfully generated a diverse set of Pareto-optimal portfolios, as demonstrated by the 13 non-dominated solutions identified after 20,000 evaluations. The NSGA-III algorithm shows a more dynamic and varied allocation, with significant differences in weights across assets, as shown in Figure 3.

DAX and Sensex receive higher allocations (0.2453 and 0.2309, respectively) under NSGA-III, suggesting that these assets might offer a favorable balance of return and risk when higher-order moments are considered. Conversely, assets like FTSE100 and Hang Seng are given much lower weights, indicating that they may contribute less favorably to the portfolio's overall performance under a multi-objective optimization approach. The advantage of NSGA-III lies in its ability to adaptively allocate capital based on a more comprehensive risk–return profile, potentially leading to a more resilient and optimized portfolio compared to the more uniform and potentially less efficient allocation by the Mean–Variance method.

We then plot the portfolio performance measure corresponding to each method used in this study, as follows.

Figure 4 visualizes the heatmap for the comparison between the NSGA-III model and the Mean–Variance model across several portfolio metrics, including weights, mean return, risk, kurtosis, skewness, and Sharpe ratio. Each row represents an asset, and the columns show the corresponding values under both the NSGA-III and MV models. The color scheme plays a pivotal role in facilitating rapid and intuitive comparisons of portfolio performance across various metrics and optimization algorithms. By visually representing dimensions such as risk, return, and higher-order moments, these heatmaps enable investors and researchers to clearly discern the trade-offs inherent in different portfolio choices. This approach allows for direct visual analysis of how each optimization method, specifically NSGA-III versus Mean–Variance, performs across multiple critical dimensions.

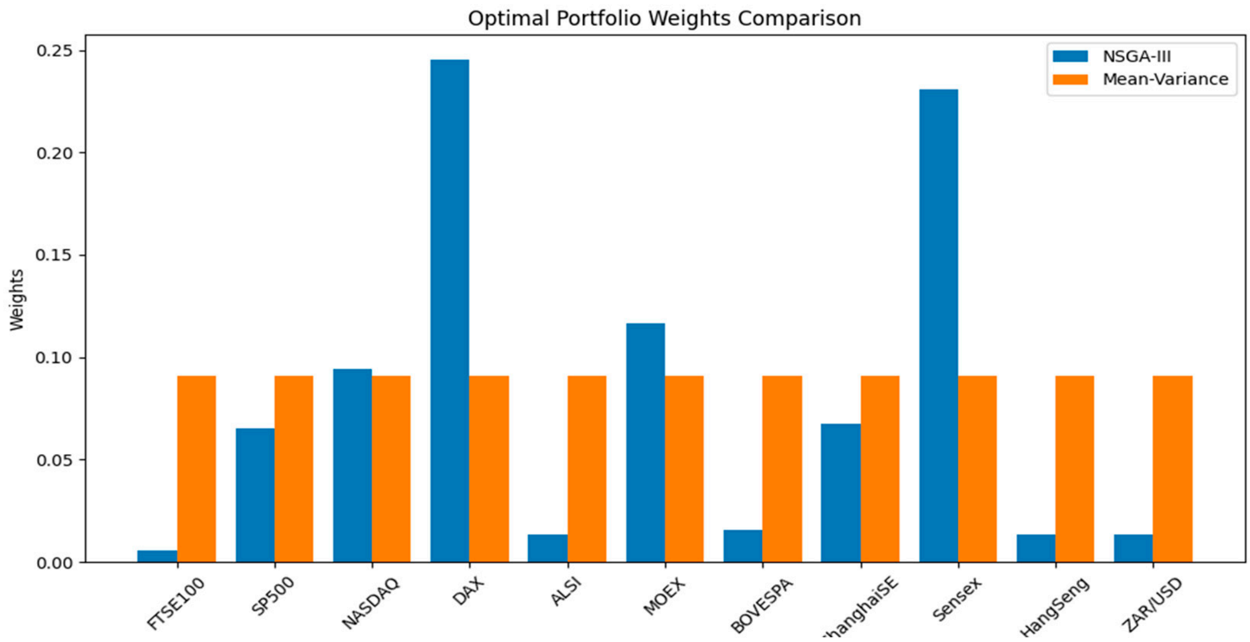

Figure 3. Portfolio optimal weights—NSGA-III vs. Mean–Variance. The chart illustrates the allocation proportions of each asset within the portfolio, comparing results from the NSGA-III and Mean–Variance optimization methods. Each color represents the weight assigned to assets across various categories, including developed and emerging market stocks, as well as currency exchange rates. This visual comparison underscores the distinct allocation strategies generated by each method, reflecting their differing approaches to balancing risk and return.

The NSGA-III model displays distinct advantages in portfolio diversification, as evident in the weights assigned to various assets. Unlike the Mean–Variance model, which assigns almost equal weights to all assets, the NSGA-III provides a more nuanced distribution of weights. This indicates that NSGA-III is better equipped to tailor the portfolio to capture more specific risks and returns associated with individual assets, resulting in a potentially more optimized portfolio.

Moreover, the NSGA-III model demonstrates slightly better risk management capabilities, as seen in the slightly lower kurtosis and skewness values. These lower values suggest that the NSGA-III model may produce a portfolio that is less prone to extreme fluctuations and tail risks, offering a more stable return profile. Additionally, the Sharpe ratios for both models are similar, but the diversified weights in NSGA-III could lead to a more resilient portfolio over time, which is particularly advantageous in volatile market conditions.

Finally, we visualize the relationship between the Sharpe ratio and the portfolio risk in Figure 5.

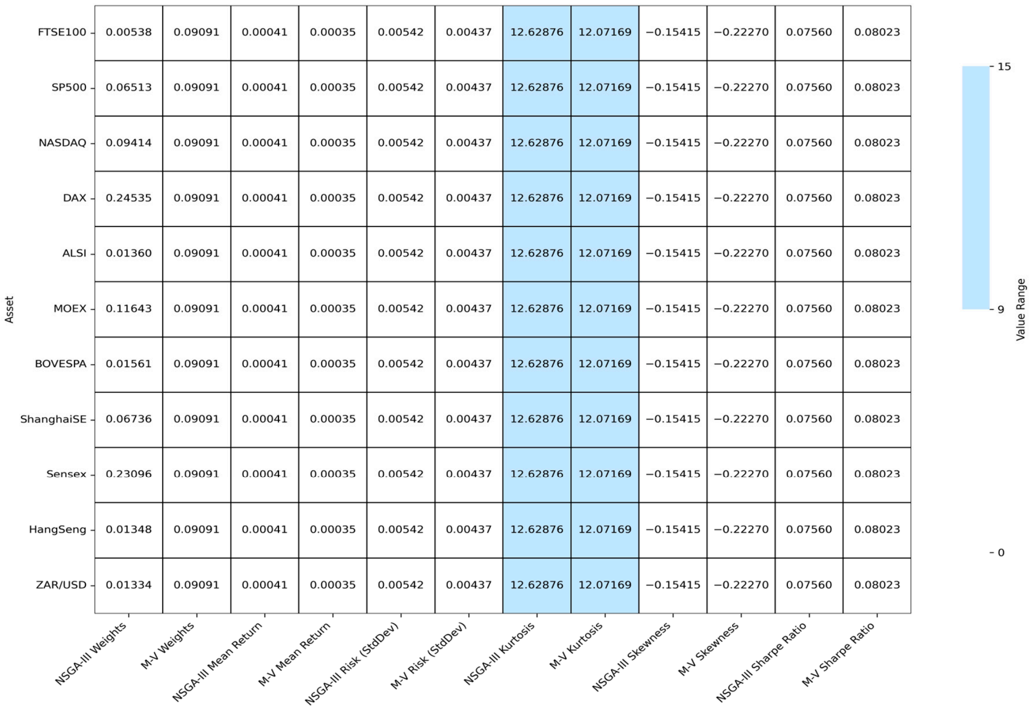

Figure 4. Heatmap of NSGA-III and M–V portfolio metrics. Darker shades, such as deep blue, typically signify poorer performance—indicating metrics like higher risk or lower returns. Conversely, lighter hues, such as yellow, represent improved performance, corresponding to lower risk or higher returns. The gradient spectrum effectively aids in comparing portfolios or performance metrics across varying solutions, offering an intuitive understanding of each portfolio's relative standing.

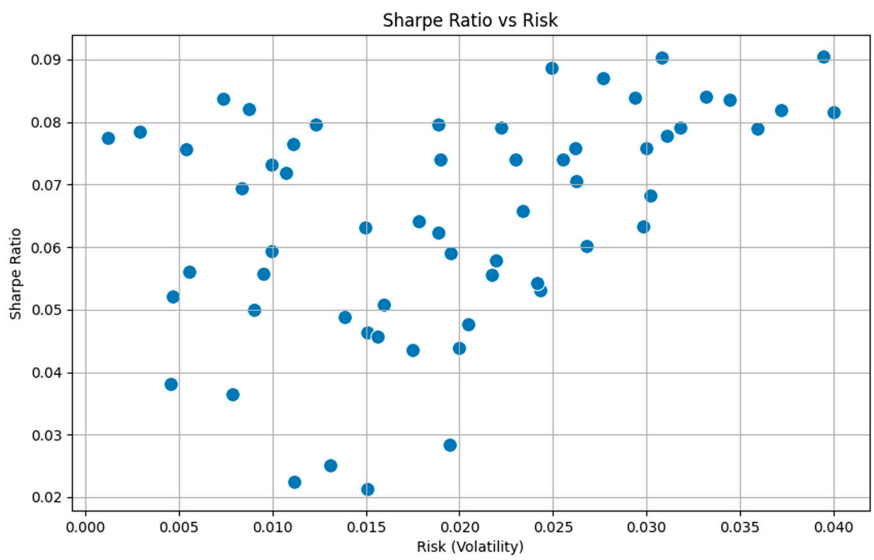

Figure 5. Sharpe ratio vs. portfolio risk.

Figure 5 presents a scatter plot illustrating the relationship between the Sharpe ratio and portfolio risk across a range of portfolio configurations. Each point in the plot represents a distinct portfolio, with its x-axis position indicating risk (measured as standard deviation) and its y-axis position showing the Sharpe ratio, which reflects risk-adjusted return. This visual representation allows us to assess how various portfolios perform in balancing risk and return.

In this plot, each portfolio's unique configuration is marked by its respective risk and Sharpe ratio values. The optimal portfolio, as indicated in this context, is one that minimizes risk while maximizing the Sharpe ratio. Here, the Sharpe ratio is derived as the ratio of a portfolio's return to its standard deviation, offering a quantifiable measure of performance relative to risk exposure.

The results suggest a general trend where higher-risk portfolios often exhibit elevated Sharpe ratios, implying improved risk-adjusted returns. Nevertheless, the spread of points reveals that not all high-risk portfolios achieve higher Sharpe ratios, underscoring the variability in outcomes influenced by differing portfolio configurations. This dispersion highlights the importance of selecting portfolio structures that align with both risk tolerance and return objectives.

Figure 5 underscores the importance of considering both risk and return simultaneously. While higher risk can potentially lead to higher returns, it is crucial to evaluate portfolios on a risk-adjusted basis, as not all high-risk portfolios offer superior returns. This insight supports the use of multi-objective optimization techniques like NSGA-III, which can help in identifying portfolios that not only optimally balance risk and return but also the skewness and kurtosis of the entire portfolio.

For investors, the NSGA-III results highlight a new frontier in portfolio management. Traditional methods, while foundational, often fail to account for the complexities and uncertainties inherent in modern financial markets. The NSGA-III's multi-objective optimization provides investors with a more nuanced tool for balancing risk and return, particularly in the context of managing extreme risks and asymmetric return distributions.

This study corroborates findings from Deb and Jain (2014), who demonstrated the superiority of evolutionary algorithms in handling multi-objective problems. Investors can leverage these findings to construct portfolios that are better aligned with their individual risk tolerances and financial goals, especially in unpredictable market conditions.

A significant insight from this paper's findings is the enhanced stability of returns in portfolios optimized using NSGA-III. This stability is reflected in a less negative Sharpe ratio—a widely regarded measure of risk-adjusted returns—when compared to portfolios constructed with conventional optimization techniques (Sharpe, 1966). This outcome holds substantial importance, particularly within volatile financial markets, where sustaining consistent performance is a persistent challenge. DeMiguel et al. (2009) similarly addressed this limitation, observing that traditional Mean–Variance models often struggle to maintain performance consistency under varying market conditions. Thus, the empirical evidence presented here supports the view that NSGA-III offers a more resilient framework for achieving steady and reliable returns over time, benefiting both institutional and individual investors.

Our findings further underscore the increasing acknowledgment of evolutionary algorithms as both theoretically robust and practically applicable in real-world financial contexts. By incorporating performance metrics such as the Sharpe ratio, Value at Risk (VaR), skewness, kurtosis, and drawdowns, this study illustrates NSGA-III's practical advantages in driving superior portfolio performance across multiple dimensions. This multidimensional optimization approach represents a marked advancement over traditional methodologies, equipping portfolio managers with a comprehensive toolkit to construct portfolios that not only balance risk and return but also fulfill a broader array of performance objectives.

5. Conclusions

This paper applied the NSGA-III to a dataset of global financial assets to investigate its effectiveness in dealing with conflicting objective functions in financial portfolio management. Using a dataset comprising global financial assets, this study demonstrated the algorithm's robust ability to generate portfolios that effectively balance the trade-offs between risk and return. The results revealed that NSGA-III can achieve a diverse set of Pareto-optimal portfolios, offering superior risk-adjusted returns, as evidenced by metrics such as the Sharpe ratio, Value at Risk (VaR), and drawdowns. These findings underscore the potential of NSGA-III as a powerful tool for portfolio managers, providing a sophisticated alternative to conventional single-objective optimization techniques and addressing the complexities of modern financial markets, where multiple conflicting objectives must be navigated.

The implications for financial portfolio management are significant. The NSGA-III's ability to optimize across multiple dimensions simultaneously suggests that it can enhance decision-making processes, particularly in volatile and uncertain market conditions. This capability is especially valuable in the current financial landscape, where traditional optimization methods may not fully capture the multifaceted nature of portfolio performance.

In light of these findings, several recommendations emerge for both practitioners and researchers in the field of financial portfolio management. For practitioners, it is recommended that NSGA-III be considered as a viable tool for portfolio optimization, especially in scenarios where multiple objectives, such as risk, return, skewness, kurtosis, and liquidity, need to be balanced. Portfolio managers should explore incorporating NSGA-III into their decision-making frameworks, potentially in conjunction with other optimization techniques, to enhance the robustness of their portfolios.

For researchers, this study opens several avenues for future exploration. There is a need to extend the application of NSGA-III to a broader range of asset classes and market environments, which would test the algorithm's versatility and effectiveness in different financial contexts. Additionally, further research could investigate the integration of the NSGA-III with other advanced optimization techniques or machine learning models, which could enhance predictive capabilities and improve portfolio performance. Exploring NSGA-III's potential in stress-testing scenarios, where portfolios are subjected to extreme market conditions, would also be valuable in assessing the algorithm's resilience.

Overall, this study contributes to the ongoing development of sophisticated portfolio optimization strategies that meet the challenges of an increasingly complex financial world. By leveraging the advanced capabilities of NSGA-III, both academics and practitioners can gain deeper insights into multi-objective optimization, ultimately leading to more effective and sustainable financial management practices.

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Conflicts of Interest: The authors declare no conflicts of interest.

Note

1 MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition

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