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Exciting of Strong Electrostatic Fields and Electromagnetic Resonators at the Plasma Boundary by a Power Electromagnetic Beam

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Abstract: The interaction of an electromagnetic beam with a sharp boundary of a dense cold semi-limited plasma was considered in the case of a normal wave incidence on the plasma surface. The possibility of the appearance of an electrostatic field outside the plasma was revealed, the intensity of which decreased according to the power law with a distance from the plasma and the center of the beam. It was possible to form cavities with a reduced electron density, being each electromagnetic resonators, which probed deeply into the dense plasma and couldexist in a stable state for a long period.

Keywords: nonlinear properties; electrostatic field; resonator; electromagnetic beam; irradiation; surface charge

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1. Introduction

The origination of many phenomena taking place during the interaction of electromagnetic radiation with a dense plasma occurs on the interface of media where the possibility of the appearance of certain effects is determined depending on the conditions and the ratio of parameters. Therefore, identifying such conditions and characterizing interactions in simple modeling cases appearsto be an important primary step toward detecting and predicting many interesting phenomena. It was within the framework of the simplest models of cold plasma with a sharp boundary that the effects of nonlinear transparency [1–3], complete absorption [4,5], and anomalous radiation [6] of electromagnetic radiation were investigated. The construction of such a model impliedan accurate representation of the physical essence of the phenomenon under study and those basic features of the interaction of radiation with plasma that ensured its existence.

In this work, the possibility of the formation of globe-shaped resonators, being cavities with a rarefied electron density created at the plasma boundary under the influence of a beam of powerful electromagnetic radiation was considered. The main features of this phenomenon couldbe best studied in a simple model of a semi-infinite plasma with a sharp boundary and stationary ions for the case when electromagnetic radiation normally reached it in the form of a beam with an exponential intensity distribution in the frontal plane. The possibility of forming a cavity with a low electron density followedfrom the physical essence of the interaction of radiation with a plasma. This wasdue to the fact that, on the one hand, a powerful electromagnetic flux wasable to remove electrons from a certain volume and to hold the boundary in the equilibrium against forces of the thermal pressure and the charge separation field [7]. However, on the other hand, such cavities in the plasma couldacquire, under certain conditions, the properties of an electromagnetic resonator [8]. This occurredwhen the size of the cavity, the amplitude, the frequency and spatial structure of the electromagnetic field, the thermal pressure of electrons, and other characteristics reached certain resonant values, for which the stable state of the cavity couldbe maintained for a long periodin the absence of dissipation. The formation of the surface of the cavity and the spatial structure of the electromagnetic field inside the resonator wereinterrelated processes, the parameters of which maintained equilibrium by mutual correction of their values. At the same time, depending on the ratio of the characteristics of the task, the shape of the cavity couldbe either spherical, ellipsoidal, or cylindrical. In the latter case, a situation is possible when such a cylinder crosses the entire thickness of the plasma layer so that the radiation can pass through this layer of dense plasma into a region where it could not penetrate at low values of its intensity. At the same time, for small amplitudes of the electromagnetic signal, when the plasma boundary remains flat, a nonlinear surface charge couldbe formed on it under certain conditions [9], which createdan electrostatic field outside the plasma with a large localization region, when its amplitude decreasedwith a distance from the boundary and the center of the beam according to the power law, in contrast to the strength of the electromagnetic wave field. The possibilities of such a field may arouse interest, both from the point of view of the practical application (for example, for particle acceleration) and from the standpoint of the probable need to prevent undesirable effects.

2. Basic Equations

Consider a semi-infinite plasma consisting of electrons with mass m, density n_e, charge -e, and immobile ions with density n_i ($x \geq 0$), forming a sharp boundary, to which a beam of plane-polarized electromagnetic radiation with a frequency ω and a wave number k propagates along itsnormal on the axis $0X$ (Figure 1). In the region ($x \leq 0$) surrounding the plasma, the following expression can be written for the intensity of $\mathbf{E}_0$ = {0, E_0,0} of the electric field having auniform spatial distribution in azimuth in the front plane $0YZ$.

$$\mathbf{E}(\mathbf{r}, t) = -\nabla\varphi_e(\mathbf{r}) + \hat{\mathbf{y}}E_0\sin(\omega t - kx) + \hat{\mathbf{y}}E_{0r}\sin(\omega t + kx)\ , \quad (1)$$

where $\varphi_e(\mathbf{r})$ is the electrostatic potential of the surface charge formed at the plasma boundary [9–11], and E_{0r} is the amplitude of the reflected electromagnetic signal.

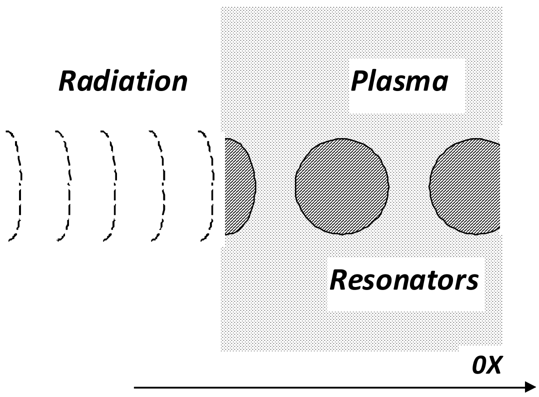

Figure 1. Scheme of interaction of the electromagnetic beam with the surface of the plasma.

The motion of electrons with the velocity $\mathbf{v}$ is described by the equation:

$$\partial_t\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \frac{e}{m}[\mathbf{E} + \mathbf{v}\times\mathbf{B}] - \frac{v_T^2}{n_0}\nabla n_e \quad (2)$$

where $\mathbf{B}$ is the strength of the magnetic field of external radiation, ${v_T}^2 = T_e/m$, T_e is the temperature of electrons, thermal pressure is taken into account in (2) only to estimate the parameters of the equilibrium state, and n_0 is the equilibrium density of plasma particles in the stationary state ($n_e = n_i \equiv n_0$).

The field strengths in (1), (2) satisfy Maxwell's equations:

$$\partial_t\mathbf{B} = -\nabla\times\mathbf{E}, \quad (3)$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2}\partial_t \mathbf{E} + \mu_0 e n_e \mathbf{v}, \tag{4}$$

$$\nabla \cdot \mathbf{E} = \mathrm{e}(n_\mathrm{e} - n_i)/\varepsilon_0. \tag{5}$$

Here, ε_0 is the dielectric density of a vacuum, and μ_0 is its magnetic permeability.

Due to the azimuthhomogeneity of the electromagnetic beam, it waspossible to use a cylindrical coordinate system with an axis of *0X* and coordinates ρ,χ in the frontal plane (z = ρcosχ, y = ρsinχ). In this case, the amplitude of $E_0(y, z)$ woulddepend only on the coordinate ρ, and it waspossible to consider different intensity distributions in the plane of the wave front, for example, an exponential one.

$$E_0(\rho) = E_a \exp\{-\rho/\rho_0\},\ \rho_0 = const,\ k\rho_0 >> 1. \tag{6}$$

For harmonic analysis, the velocity **v** should be divided into a fast-variable $\mathbf{v}_\mathrm{E}$ component and a static δ**v**(**r**) part ($\mathbf{v}_\mathrm{E}$ = e $\mathbf{E}_0/m\omega$). As a result, the following expression can be derived from Equation (2)

$$(\delta\mathbf{v} \cdot \nabla)\delta\mathbf{v} - \frac{1}{2}\nabla \mathrm{v}_\mathrm{E}^2 - \frac{e}{m}\nabla\varphi - \frac{v_\mathrm{T}^2}{n_0}\nabla n_e = 0 \tag{7}$$

Therefore, the function $F(x,\rho)$ defined by the formula.

$$F(x,\rho) = \frac{1}{2}\delta v^2 - \frac{1}{2}v_\mathrm{E}^2 - \frac{e}{m}\varphi - \frac{v_\mathrm{T}^2}{n_0}n_e \tag{8}$$

This is a continuous quantity both along the polar coordinate ρ and normally to the surface of the plasma (axis *0X*) in the case where the velocity δ**v** is determined by the potential ψ (δ**v** = ∇ψ). With its help, it waspossible to estimate the change in individual physical quantities, as compared to their values at selected points.

3. Analytical and Numerical Results

For high-power radiation, the continuity of the function $F(x,\rho)$ wasreduced to the balance of electromagnetic and thermal energy:

$$\frac{1}{2}v_\mathrm{E}^2(x,\rho) + \frac{v_\mathrm{T}^2}{n_0}n_e(x,\rho) \cong const. \tag{9}$$

From the equilibrium ratio (9), it followedthat the total pressure (the sum of radiation and heat) of electrons wasa continuous quantity, and this balance wasobserved everywhere, including along the *0X* axis and along the ρ axis. It also enabled us to understand how many electrons wereforced out of the cavern formed by the electromagnetic beam incident on the plasma. Along the polar radius ρ, the amplitude of E_0changedsmoothly, and when the density of $n_\mathrm{e}$reachedthe critical value of n_c, that is ω = ω_p($\omega_\mathrm{p}^2 = e^2 n_e/(m\cdot\varepsilon_0)$), the plasma becameopaque to this wave field, as a result of which it droppedexponentially rapidly into the dense plasma, the density of which, in turn, increasedas rapidly, according to (9). At this increase in density on the surface ($x = x_\mathrm{b}$, $\rho = \rho_\mathrm{b}$), thermal pressure v_T^2 dominated one side, and on the other, electromagnetic, characterized by v_E^2, which in a stationary state wouldbalance each other. Therefore, an approximate condition

$$v_\mathrm{E}^2 \approx v_\mathrm{T}^2 \tag{10}$$

wouldbe executed at this boundary to determinethe threshold value of the amplitude $E_0(x_\mathrm{b},\rho_\mathrm{b})$ for the formation of the cavern.

3.1. Conditions for the Formation of Globe-Shaped Resonators of the Electromagnetic Field

The dynamics of the development of the cavity were represented as follows. First, a small space formednear the surface of the plasma and close to the center of the beam with a boundary separating the bulk of the electrons and having a surface shape similar to function (6). As it moveddeeper into the plasma, this cavity wasformed in accordance with the values of the plasma and radiation parameters acting at each time. Since the ions remained stationary, a bulk electric charge formed in the cavern, creating an electric field φ_e, which attemptedto return the displaced electrons back to their positions. The shape of the cavern varied depending on the ratio of the characteristics of the task. However, for example, in the case of a spherical cavity, its radius R in the equilibrium state was determined bycondition (8), in which the potential φ_e that dependedonly on the radial coordinate $r$had tobe substituted from the solution of Equation (5) for the sphere, within which $n_e \approx 0$. In this case, one couldobtain from (5):

$$\varphi_e = -\frac{e}{6\varepsilon_0} n_i r^2 \quad (11)$$

By substituting (11) into condition (8) taken at the boundary $r = R$, it was possible to obtain an estimate of the magnitude of the cavity radius:

$$R = \frac{\sqrt{6}}{\omega_p}\sqrt{v_E^2 - v_T^2} \sim \frac{\sqrt{6}}{\omega_p} v_E \quad (12)$$

When a spherical resonator formed simultaneously with the electronic surface of the cavity, structural changes in the spatial distribution of electric (and magnetic) fields occurred, which beganto reflect from the curved boundary and, according to (3) and (4), were described by the equation:

$$\Delta \mathbf{E} + \frac{\omega^2}{c^2}\varepsilon(\omega)\mathbf{E} = 0,\ \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}. \quad (13)$$

The general solution of this equation wasgiven in [8] for a spherical coordinate system (r, ϑ, χ), beginning in the center of the cavity. It has a cumbersome appearance, but for a spherically symmetric case, it was written in a simple form for the radial intensity E_r of the electric field

$$E_r(r \le R) = E_a \frac{\sin kr}{kr}, k = \frac{\omega}{c}\sqrt{\varepsilon_1}, \varepsilon_1 = \varepsilon(\omega, r \le R). \quad (14)$$

$$E_r(r \ge R) = E_a \frac{1}{\kappa r} e^{-\kappa r}, \kappa = \frac{\omega}{c}\sqrt{\varepsilon_2}, \varepsilon_2 = -\varepsilon(\omega, r \ge R). \quad (14a)$$

The oscillations described by formulas (14) and (14a) did not have a wave structure along the surface of the sphere and hada frequency of ω_m (m = 1,2,3, …), as defined from the following dispersion equation:

$$\varepsilon_2 k e^{-\kappa R} = \varepsilon_1 \kappa \sin(kR). \quad (15)$$

For large values of the parameter $a_p = \omega_p R/c$, the approximate value of the frequency of natural oscillations was in the form $\omega_m = a_m c/R$, where the constant a_m is determined from the solution of the following transcendental equation:

$$a_p e^{-a_p} = a_m \sin a_m. \quad (16)$$

The expression (16) together with (12) allowed us to derive the value of the amplitude of the electric field and frequency, at which it was possible to form a spherical resonator with the parameters presented herein in the form of estimates. We determined that at the value of the velocity δv, the resonator moved deeply into the plasma. To do this, using expression (8), it was necessary to take the parameter values near the surface of the cavity close to the center of the beam where the velocity δv wasentirely directed along the 0X axis. The result wasthe following approximation:

$$\delta v^2 \sim V_E^2 - V_T^2 \quad (17)$$

It should be noted that for other resonant combinations between the parameters of plasma and external radiation, an ellipsoidal form of the resonator couldbe realized. In addition, when the thickness of the plasma along the *0X* axis wasnarrow, it could-have the appearance of a cylinder through which radiation wasable to penetrate through dense plasma.

3.2. Generation of Electrostatic Fields of Surface Charge near Plasma Space by a Beam of Electromagnetic Radiation

In the case when the force effect of the electromagnetic beam wassmall, as compared to the pressure of electrons, the surface of the plasma remainedflat when interacting with the radiation. However, as shown in [9–11], it formeda nonlinear surface charge associated with the electrostatic field $\varphi_e(\mathbf{r})$, which hada large localization region near the plasma boundary and couldaccelerate charged particles [12–14]. The description of this surface charge, performed in [9–11], was based on the theory of the potential [8,15], which could express the value of the potential $\varphi_e(\mathbf{r})$ throughout space viaits value on the surface of the plasma:

$$\varphi_e(\mathbf{r}) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} dy' \int\limits_{-\infty}^{\infty} dz' \frac{|x|\Phi(y',z')}{\left[(y-y')^2+(z-z')^2+x^2\right]^{3/2}},\ \Phi(y,z) = \varphi_e(x=0,y,z). \quad (18)$$

The integral in (18) should be interpreted as the principal value (p.v.). It indicatedthat in the limit $x \to \pm 0$, when the peculiar point appearedin (18), the path of the integration must have had the form of a small sphere that surrounded this point.

In polar coordinates (χ,ρ), Equation (18), after the integration at the azimuthal angle χ for $x \leq 0$ values not close to the plasma boundary, could be written as follows:

$$\varphi_e(x,\rho) = -\int\limits_0^{\infty} \frac{\Phi(\rho')x\rho' d\rho'}{[\rho^2+\rho'^2+x^2]^{3/2}}. \quad (19)$$

The value of the function $\Phi(\rho)$ couldbe obtained from the equation of motion (2) at the boundary (for $x = 0$) under conditions when nonlinear corrections from the stationary velocity of movement of electrons in the surface charge zone couldbe neglected, and the representation (6) was valid:

$$\Phi(\rho) \cong \frac{eE_0^2(\rho)}{m\omega^2} = \Phi_0 \exp\{-\rho/\rho_0\} \quad (20)$$

In this case, the expression (19) could be expressed as follows:

$$\varphi_e(x,\rho) = \pi\Phi_0 \frac{x}{\rho_0}\left\{\beta\mathbf{H}_0(\beta) - \beta N_0(\beta) - \frac{2}{\pi}\right\},\ \beta = \frac{\rho^2+x^2}{\rho_0^2}. \quad (21)$$

Here, $\mathbf{H}_0(x)$ and $N_0(x)$ are Struve and Neumann functions, respectively [16].

The asymptotic value of the potential in the region far from the boundary $|x| > \rho$ was described, as follows from (21), by the formula:

$$\varphi_e(x,\rho) \cong -2\Phi_0 \frac{x\rho_0}{\rho^2+x^2}. \quad (22)$$

Based on (22), the electrostatic field component along the plasma $E_\rho = -\partial_\rho\varphi_e$ decreased with increasing distance $|x|$ proportionally $1/|x|$ (component $E_x = -\partial_x\varphi_e$ fell $1/|x|^2$). As compared tothe amplitude of the electromagnetic beam, the magnitude of the electrostatic strength of the field decreasedat a distance from its center not according to the exponential but according to the power law, that is, the area of its localization was much larger.

As an example of the acceleration of charged particles in the electrostatic field of a surface charge (22), it was possible to consider the motion of a particle with a charge e_0 and a mass M from a point (x,ρ) and calculate the final velocity of its movement at infinity. From the equation of motion for the velocity $\mathbf{v}_\mathrm{p}$ of the particle:

$$\partial_t \mathbf{v}_\mathrm{p} + (\mathbf{v}_\mathrm{p} \cdot \nabla)\mathbf{v}_\mathrm{p} = -\frac{e_0}{M} \nabla \varphi_\mathrm{e} \tag{23}$$

one can write

$$V_\mathrm{p} = \frac{E_a}{\omega} \sqrt{\frac{e_0 e}{Mm}} \tag{24}$$

It followedfrom (24) that a particle with a mass M in the electrostatic field of the surface charge acquireda constant velocity, which in $(m/M)^{1/2}$ times wasless than the amplitude of electron oscillation at the center of the electromagnetic beam.

4. Summary and Conclusions

The electrostatic field of the surface charge that arosein the process of interaction of the electromagnetic radiation beam with the plasma appearedand affectedthe environment due to the specific movement of electrons [3,9–11] and the complex of conditions that supported its existence (e.g., sharp boundary, quasi-neutrality, absence of non-harmonic perturbations, etc.). The power law of the decrease inthis field in space for a distance from the boundary and from the axis of the electromagnetic beam determinedthe large size of the region of its localization. This circumstance couldbe useful for achieving practical application (e.g., for particle acceleration) or couldbe considered in cases where its effect is likely to have negative consequences. In its magnitude, the strength of this electrostatic field was comparable to the amplitude of electromagnetic oscillations, but it didnot have a spatial and temporal oscillatory structure.

For high intensities of the electromagnetic beam, when the rate of oscillation of electrons wascomparable to their thermal velocity in the plasma, the flat boundary of the electrons wascurved, which in the model of stationary ions ledto the appearance of a charge separation field. As a result of the self-consistent deformation of the surface of electrons and the spatial structure of the electromagnetic field, it waspossible, under certain conditions, to form a cavity, which wasan electromagnetic resonator where the shape of its surface and the structure of the field could exist together for a long period, unchanged. Such conditions werefound in the present work for a resonator in aspherical shape. However, under other conditions, ellipsoidal cavities and even cylindrical cavities can occur. The latter, in the case of a relatively narrowthickness of the plasma layer, wereable to ensure the passage of radiation through a non-transparent medium (in other words, burn through it). The movement of electromagnetic resonators of various shapes also contributedto the penetration of the electromagnetic radiation deeply into the dense plasma and couldbe used to create a number of special nonlinear interactions [3,17–20]. It should be noted that the appearance of resonators waspossible not only in plasma, but hasalso been actively investigated in plasmonic materials, such as hyperbolic metamaterials with giant enhancements [21], metamaterial cavities with broadband strong coupling, and metamaterials with large index sensitivities [22]. The results obtained in these and other similar works couldbe useful for continuing research in plasma with similar configurations.

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