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Analog Replicator of Long Chaotic Radio Pulses for Coherent Processing

Lev Kuzmin *, Elena Efremova, Pavel Vladyka and Vadim Itskov

Kotelnikov Institute of Radioengineering and Electronics of RAS, Mokhovaya St., 11/7, 125009 Moscow, Russia; efremova@cplire.ru (E.E.); pavvladyka@yandex.ru (P.V.); mr.d4de@gmail.com (V.I.)
* Correspondence: lvk@cplire.ru

Abstract: The relative structural simplicity of chaotic oscillators and the possibility of obtaining signals with a large dimension is of great interest for wireless data transmission and processing. The diversity of signal waveforms from the same source of chaos is provided by a fundamental property of chaotic oscillations: sensitivity to the choice of initial conditions. In this paper, this sensitivity is employed in the proposed method for forming analog chaotic radio pulses of arbitrary (specified) duration using an analog oscillator in such a way that the pulse shape can be changed and repeated from pulse to pulse. To repeat the shape of oscillations for an arbitrarily long period of time is not a problem for digital chaotic oscillators, but for analog systems, this is a challenge due to the impossibility of controlling the initial conditions and the evolution of the analog trajectory. In this paper, a new method for generating chaos is proposed, which can both change and repeat the shape of a chaotic signal of arbitrary duration, i.e., long chaotic radio pulses. The generator acts as a reservoir and as a replicator from which, under external influence, a signal of a certain shape can be extracted, and this shape can be reproduced. The term "long" in this case means that the duration of chaotic radio pulses is many times greater than the characteristic time of divergence of chaotic trajectories. To prove the correctness of the proposed generation method, the results of its experimental implementation in the frequency range of 100 to 500 MHz are given. Examples of forming equal pulses with a duration of about 20 to 200 quasi-periods of oscillations (up to 500 ns) are given. The proposed method provides the technical possibility of forming pulses whose dimensions can vary in a wide range, which is important for implementing large processing gains in various wireless applications. The method can be implemented in various frequency ranges in the class of analog generators of chaotic oscillations, since the employed generation method, i.e., modulation of a transistor generator by supply voltage, is natural for radio engineering.

Keywords: chaos; nonlinear dynamics; chaotic radio pulses; chaotic pulse shaping; coherent processing; wireless communications; UWB chaotic signals; UWB signals

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1. Introduction

Auto- and cross-correlation properties of chaotic signals, which are similar to the properties of noise, and the method for generating those using relatively simple devices continually stimulate us to develop new methods for using them in data communication and processing problems. One such problem, which once raised a wave of interest in chaotic signals, was wireless communications [1–4] in underwater communication systems [5–7] and in radar systems based on noise-like signals [8–11].

In wireless applications, the key idea is to synthesize noise-like signals with a delta-shaped autocorrelation function that would be immune to multipath fading and could at least have the potential to retrieve the useful signal that had passed through the multipath channel.

Chaotic signals can be used as a data carrier directly in an RF band or as spreading sequences [12], to whiten the spectrum and organize the signal processing gain [13–15], in communication schemes with frequency hopping [16], in multiple access schemes [17], etc.

Chaotic systems are used to form a radio carrier, e.g., a pulsed signal with chaotically changing intervals between pulses [18,19]. Such signals are also used in UWB systems [20–22] to form sequences of ultrashort pulses with a chaotic interval between them.

The use of spreading sequences implies a coherent receiver, which means the possibility of reproducing a signal of the same shape both in the transmitter and receiver. Therefore, in most such applications, digital generators of chaotic oscillations are used. The range of problems that are solved in the synthesis of digital chaotic signals is very wide, since, unlike regular signals, for which there is a developed theory of their synthesis (m-sequences, Gold sequences, Walsh functions, etc.), for chaotic signals, these issues are still at the development stage. Auto- and cross-correlation properties of discrete chaotic sequences and ways to improve them are analyzed in [23–30].

A significant problem of digital systems is the deterioration of the properties of chaotic signals due to the finiteness of integer arithmetic by digital generation of chaotic sequences, so this aspect is also investigated in a number of works [31–33]. The problems of generating chaotic sequences and methods for solving them in high-rate digital systems are considered in [34,35]. Different versions of digital generators are proposed, both nonautonomous [36] and autonomous [37–41]. Examples of the generation of chaotic digital sequences in laser systems [42], as well as in CMOS microelectronic circuits [43], are given.

The properties of chaotic signals obtained with maps are verified in various randomness tests on FPGA [44–47] or general-purpose microcontrollers [48,49]. There are also works in which the problem of synthesizing discrete chaotic sequences is solved using continuous-time systems [50–55].

From a practical point of view, the reason for the great interest in the digital generation of chaos is absolutely clear and is related to the combination of two contradictory qualities: the possibility of obtaining signals of a noise-like form and at the same time reproducing this form by repeating the initial conditions of the chaotic trajectory. This is the fundamental reason why the problem of synthesizing chaotic signals with a repeating oscillation shape in the class of analog generators has not yet gained such great popularity and a wide range of research topics as in the class of digital generators. At the same time, there are at least two reasons why analog chaotic systems and signals are interesting for wireless applications: first, analog systems are free from the problems associated with digital signal degradation, and second, analog oscillators can generate a signal directly in the required frequency range, so there is no need to convert it from the baseband. Perhaps the only direction where the synthesis of chaotic trajectories in continuous-time systems has been developed is the topic of chaos control [56].

In analog generators, when trying to repeat the shape of oscillations, we encounter the same basic difficulty as in digital generators: high sensitivity of chaotic oscillations to initial conditions, but unlike digital systems, in analog systems it is more difficult to ensure the start of the trajectory from the same initial conditions and also to ensure repeatability of the parameters of the dynamic system.

It is shown that within certain limits this difficulty can be overcome and the reproduction of the initial section of the chaotic signal can be achieved if the oscillations are generated starting from the same initial conditions, actually from an equilibrium state [57–61]. Using this approach, it is possible to generate repeating pulses whose shape can be changed by changing the generator supply voltage. In [61], the fundamental feasibility of the idea of generating pulses of the same shape in analog chaotic systems was demonstrated. However, the duration of the coinciding sections of the pulses obtained in this way has a natural restriction: it is limited by the time of divergence of the chaotic trajectories. The divergence inevitably arises due to the mismatch of the initial conditions at the beginning of each pulse.

The experimental studies described in [61] were performed on a panel of generators implemented on a single board and controlled and modulated by a single power supply and modulation system. For practical use in wireless systems, it is necessary to ensure that generators made as separate devices and having individual power supply and modulation systems can also generate identical pulses.

The novelty of this work is that, first, here the sources of chaos are made as separate devices, which are studied separately and compared with each other. For chaotic dynamics, this is not an idle question, since one of the cliches in the science of chaos is the mantra about the extreme sensitivity of the dynamics of a chaotic system to the spread of parameters and the inaccuracy of setting the initial conditions. Whether this is true or not in this case can only be verified experimentally.

Second, an interesting question that also remains outside the scope of [61] is as follows: Is it possible to increase the pulse duration, and if it is, by how much? Is it possible to obtain an arbitrarily long chaotic signal with a repeating shape, as is easily achieved in digital chaos generation systems and has not yet been achieved in analog systems?

To summarize, the goals of this work are as follows:

- Development of an approach to the formation of identical chaotic radio pulses of arbitrary duration, overcoming the limit associated with the instability of chaotic trajectories;
- Experimental verification of the possibility of generating long identical radio pulses in separate samples of the generators.

The proposed method for generating chaotic radio pulses transforms a chaotic oscillator into a replicator of chaotic radio pulses. The solution is based on modulating the generator with a regular signal, namely a sequence of rectangular pulses with constant amplitude, the parameters of which (pulse repetition period and duty cycle) are set in such a way that the generator forms a radio pulse with a noise-like shape and a prescribed duration. The shape of the pulses can be controlled, and this shape is preserved both in one generator and in different copies of this generator if they are made of radio components with a small spread of ratings.

The method of forming repeating pulses with a duration of tens and hundreds of quasi-periods of oscillations (a quasi-period is the average time interval between two consecutive local maxima of the signal waveform) is of practical interest for the implementation of processing gain in various wireless technologies: communication, navigation, or radar.

The structure of this paper is as follows: Section 2 describes the layout of the chaos generator and the generator circuit. Section 3 shows that chaotic oscillations take place in this type of generator and that the generator has wide zones of chaotic generation. Section 4 describes the setup used in the experimental studies. Section 5 describes the proposed modulation method and also presents the results of experimental studies on the generation of short chaotic radio pulses with a repeating shape. Section 6 describes a method for increasing the duration of chaotic pulses with a repeating shape, confirming the goals stated in the paper. Finally, Section 7 presents a method for obtaining pulses with the same shape of arbitrary duration and confirms it with the experimental data. Section 8 presents data on the auto- and cross-correlation properties of chaotic radio pulses generated by the proposed method. Section 9 is devoted to a discussion of the results obtained, and Section 10 draws the conclusions.

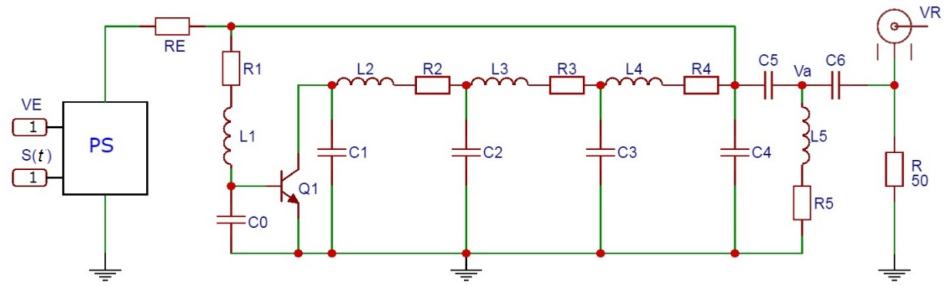
2. Generator Layout

This study uses chaos generators described in [61]. The generators form oscillations in 100–500 MHz frequency band. The essential difference between these generators and the generators described in [61] is that the latter were implemented on a single printed circuit board, with a single power supply and were intended for an initial cycle of studies on the formation of UWB chaotic radio pulses of the same shape.

In this paper, the following step is taken: based on the generator circuit in [61], eight autonomous generators are manufactured, each equipped with its own power source and modulation system. The generator schematic is shown in Figure 1a. A photo of the generators is shown in Figure 1b.

The theoretical mechanism and method for generating short chaotic radio pulses of similar shape in such generators were previously studied based on the generator model (in particular, using numerical simulation and experimental prototyping); the results are published in [61]. The spread of the nominal values of the radio components of the generators is within 2%. General-purpose radio components available on the market were used. It was experimentally shown that this is sufficient to achieve the goal of generating chaotic radio pulses with an identical initial section [61].

Based on this achievement and firmly established fact, in this work, the generators were made as eight separate devices (Figure 1b), with the above tolerances for the spread of nominal values.



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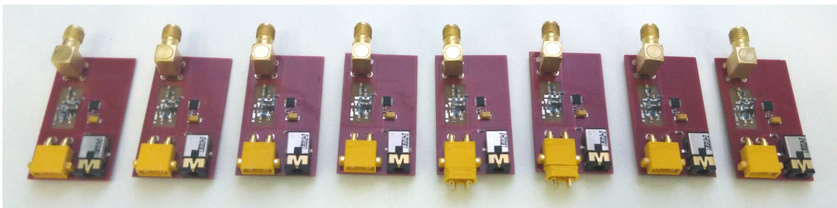


(**b**)

Figure 1. (**a**) Schematic of the chaotic oscillator: PS—modulator of the power supply, V_E—power supply amplitude, $s(t)$—modulation signal (video pulses), V_R—output signal of the generator. (**b**) Eight generator samples.

The idea of generating pulses of the same shape, presented in [61], is that the dynamic system that describes the generator in Figure 1 has a single equilibrium position (fixed point) from which the trajectories start when the generator is turned on (when power is applied). Due to this, all trajectories have the same initial section for a certain period of time, until the instability inherent in the chaotic system separates the trajectories.

When the generator is switched off, the phase trajectory returns to this equilibrium position. The results of this paper are based on this property.

3. Generator Operation Modes

The design of the chaos generators considered in this work is new; therefore, before studying the possibilities of forming identical individual pulses, it is necessary to have an idea of their continuous oscillation modes.

At the first stage, a constant supply voltage V_E was applied and the reproducibility of the obtained stationary mode from generator to generator was studied.

To understand the operation modes of the generators, a bifurcation diagram was experimentally obtained (Figure 2) using a digitally controlled power source, an oscilloscope, and a PC. The oscilloscope recorded waveforms in an array of 10^6 samples with a sampling frequency of 2.5 GHz at a given supply voltage V_E; the power supply voltage was swept from the minimum (0.8 V) to the maximum (5.0 V) value of a given range with a step of 0.01 V, then the data were transferred to a PC to find local maxima, as is usually carried out to plot a bifurcation diagram.

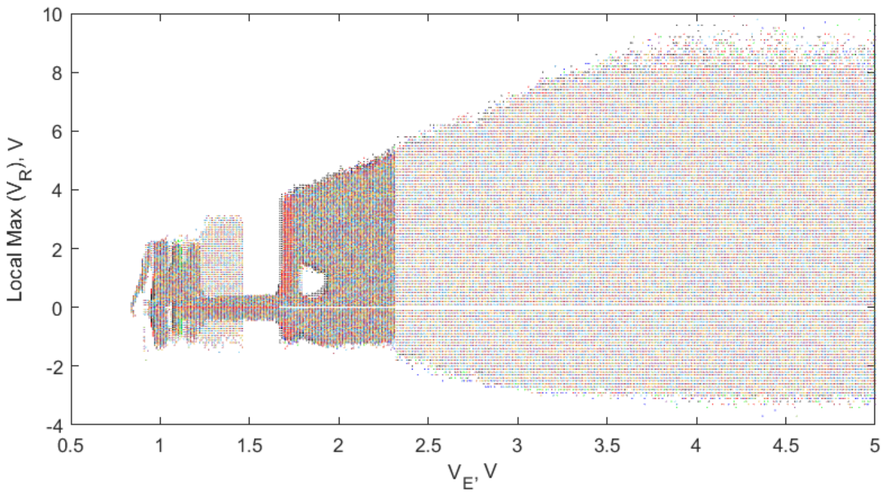


Figure 2. Bifurcation diagram: local maxima V_R as a function of supply voltage V_E.

The bifurcation diagram in Figure 2 depicts the evolution of the oscillation modes in the supply voltage range of 0.8 to 5 V. As can be seen in the bifurcation diagram, for different examples of generators, the bifurcation diagrams correspond well to each other, which indicates the identity of the dynamic modes in these generators. The diagram also shows that the generators have wide continuous zones of chaotic generation by supply voltage, which provides the formation of various waveforms of chaotic signals. The oscillation modes of the designed generators do not have periodicity windows, except in the range of 1.45 V to 1.65 V. At a constant supply voltage of 5 V, the power spectrum of the chaotic signals is continuous and the signal power of each generator is about 1.5 mW (Figure 3). The spectral power densities of eight generators (Figure 3) coincide, which indicates good repeatability of the generators' regimes.

The oscillation modes from the range of 4 V to 5 V, in which their amplitude reaches a constant value (see the bifurcation diagram in Figure 2), were also compared. As an example, Figure 4 shows the oscillation waveforms for supply voltages $V_E = 4$ V and $V_E = 5$ V (generator CS_1). The given signal waveforms and their spectra were obtained using a digital oscilloscope.

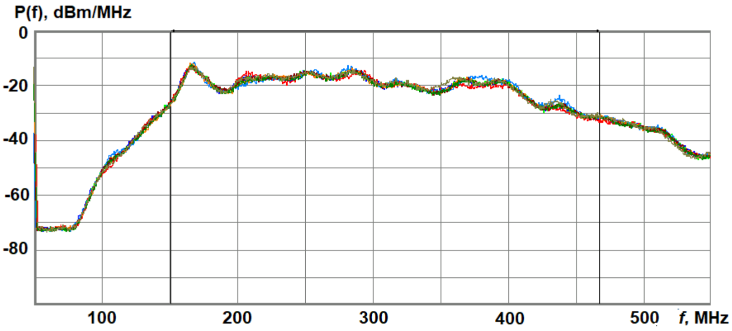


Figure 3. Spectra of considered generators superimposed: blue—CS_1, yellow-green—CS_2, red—CS_3, light-blue—CS_4, orange—CS_5, purple—CS_6, green—CS_7, olive—CS_8.

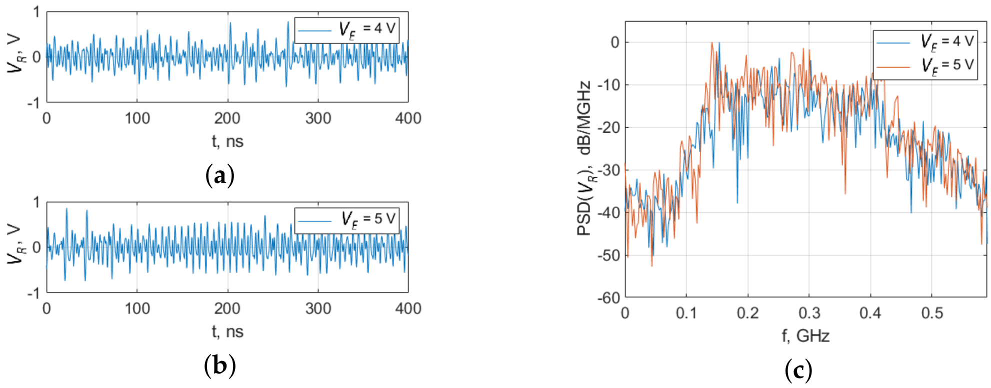


Figure 4. Signal waveforms for supply voltages $V_E = 4$ V (**a**) and $V_E = 5$ V (**b**); power spectra of these modes (**c**).

4. Experimental Sample for Generation of Equal-Shape Signals

An experiment on the formation of chaotic radio pulses of the same shape and their reproduction (replication) by different generator samples was carried out according to the scheme of Figure 5a on the setup (Figure 5b).

The setup consists of a source of modulating video pulses implemented on the FPGA DE-10 Lite debug board); the pulses are fed to $s(t)$ generator's input via a backplane.

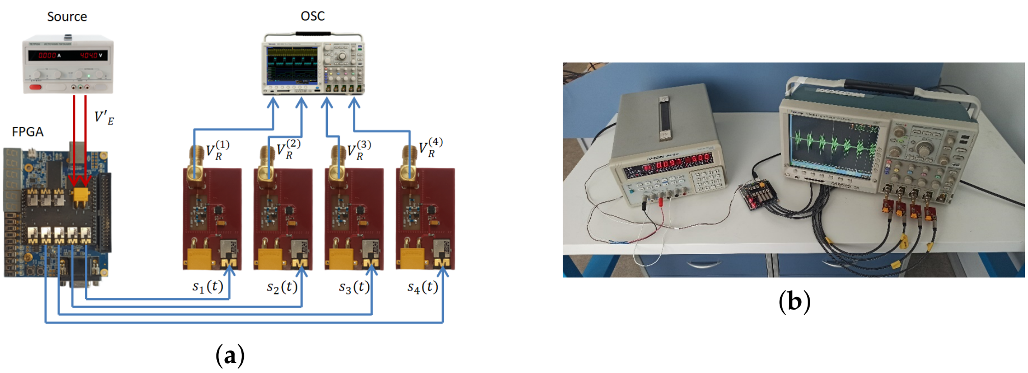


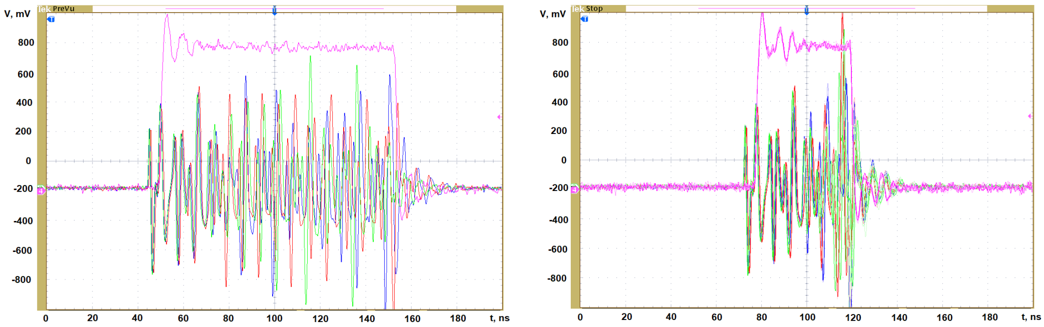
Figure 5. (**a**) Setup layout: Source—power supply, FPGA—debug board with a splitter, OSC—4-channel oscilloscope, $s_1(t) \ldots s_4(t)$—modulation signals, V_E—power supply, $V_R^{(i)}$—generators' G_i outputs, $i = 1 \ldots 4$; (**b**)—experimental setup.

Voltage V_E is applied to the oscillator if $s(t) = 0$, and it is not applied otherwise. The oscillation mode of the generator depends on the supply voltage V_E. The modulation video signal is generated with the FPGA debug board. The lengths of the cables carrying the modulation signal to the input of each oscillator are identical to ensure that the oscillators are turned on simultaneously. The output signal of each generator is fed to the corresponding input of the 4-channel oscilloscope.

5. Formation of Chaotic Radio Pulses of the Same Shape

Below are the results of experiments on the formation of chaotic radio pulses of the same shape on the described experimental setup.

The goal of the first series of experiments was to reproduce the results of [61] for standalone samples of the chaotic generator. When a modulating signal in the form of 100 ns video pulses is applied, the generator produces chaotic radio pulses, shown in Figure 6. Oscilloscope screenshots for the signals from the generators modulated by 100 ns and 35 ns video pulses are shown in (Figure 6a) and (Figure 6b), respectively.



(**a**) (**b**)

Figure 6. Oscilloscope screenshots: (**a**) signals $V_R^{(1,2,3)}$ of three generators CS_1, CS_2, CS_3 and 100 ns modulation video pulse $s(t)$; (**b**) signals of four generators CS_1, CS_2, CS_3, CS_4, modulated by 35 ns video pulses. $V_E = 4$ V.

Figure 6a demonstrates a basic feature of chaotic trajectories: divergence after starting from slightly different initial conditions. In this case, the trajectories begin to diverge after ≈35 ns of evolution. As a result, if the duration of the modulating video pulses is greater than the time of divergence (in this case, 100 ns), then the radio pulses of different generators, which coincide in shape in the initial section, subsequently become different. On the contrary, if the duration of the modulating pulses is less than 35 ns, it is possible to extend the interpulse interval in a way that allows the system to relax to a stable equilibrium position (rest state); then, upon arrival of the next modulating video pulse, the generator begins oscillation from approximately the same initial conditions that lead to the coincidence of the shape of the initial sections of the pulses (Figure 6b).

These results confirm the mechanism and method of generating identical radio pulses by modulating the generator power source with a video signal [58,60,61] when the generator is abruptly transferred from the "rest" mode to the chaotic generation mode by switching the power source from "off" to "on" mode by means of a video pulse. Then, over time, the trajectories begin to diverge.

The key question posed in this paper is whether the initial sections of the trajectories of completely separated generators will be repeated. To answer this question, we directly compared the initial sections of chaotic radio pulses obtained from different generators, similar to the technique of [61]. The results of the comparison are presented in Figure 7, which shows the time-aligned initial fragments of radio pulses for four generators. As can be seen from the figures, over a period of approximately ≈35 ns, the initial sections coincide both for the same generator instance and for different instances.

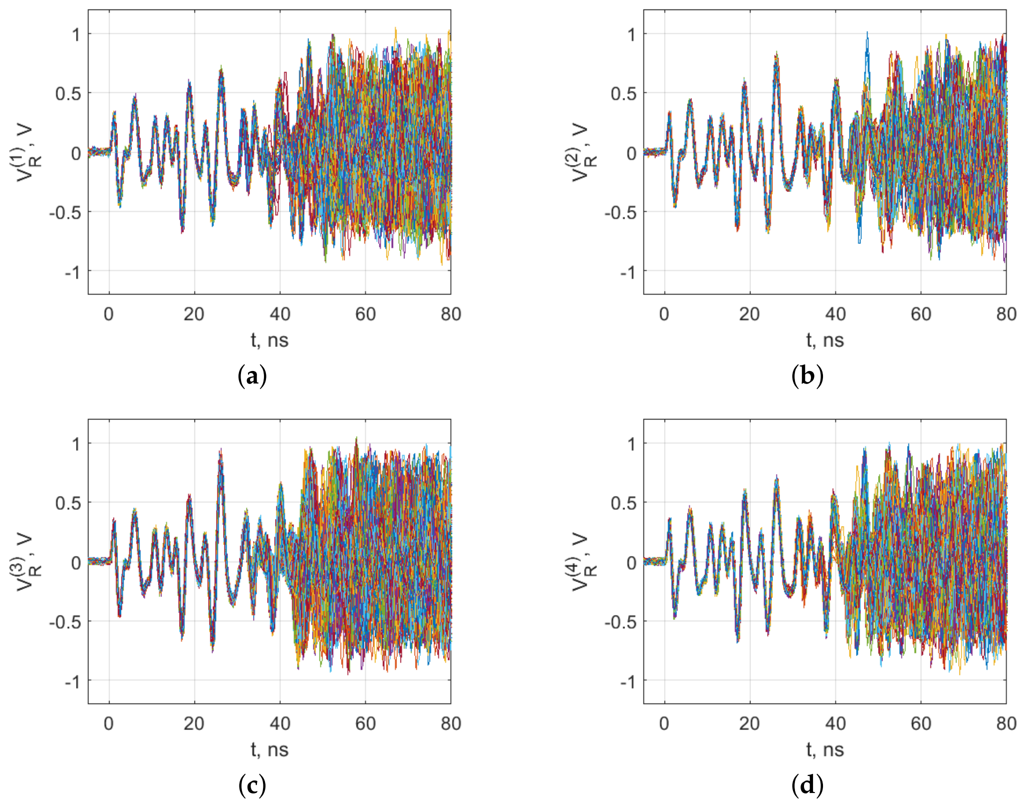


Figure 7. Initial section waveforms of 1000 pulses of generators CS_1 (**a**), CS_2 (**b**), CS_3 (**c**), CS_4 (**d**), at the supply voltage $V_E = 4$ V. The signals were sampled using an oscilloscope with 2.5 GHz sampling frequency.

As a result, the possibility of generating identical pulses with a duration of 35 ns was experimentally confirmed. This time interval $T_{Lim} = 35$ ns can be conventionally called the divergence time.

The question arises: is it possible to increase the duration of repeating sections while maintaining the ability to control their shape?

6. Formation of Long Repetitive Pulses

The approach to the formation of long identical pulses with a controlled shape is determined by two things: the fundamental properties demonstrated above of the exponential instability of a chaotic trajectory with respect to the uncertainty of the initial conditions and the reproducibility of oscillations of a similar shape over a limited time interval. These two properties are complementary in the context of the task.

The term "long" in this case means that the pulse duration is longer than the trajectory divergence time T_{Lim} when starting from the same initial conditions. Returning to Figure 7, the time is set at $T_{Lim} = 35$ ns. Thus, the task is to obtain pulses with duration $T_P >> T_{Lim}$.

The idea is to generate radio pulses each time from new initial conditions in a controlled manner using modulation with video pulses. The idea of controlled formation of radio pulses is based on alternating periods of switching on (high level of video pulse) and switching off (low level of video pulse) of the generator on time intervals of limited duration. On the time interval corresponding to the high level of video pulse, the amplitude of oscillations increases exponentially. On the time interval corresponding to the low level of video pulse, the reverse process occurs, i.e., exponential attenuation of the amplitude of oscillations. Setting a specific duration of video pulses and interpulse intervals allows us to choose the initial conditions from which the trajectory starts at the beginning of each interval.

In this case, two conditions must be met: (1) the duration of the interval with a high level of video pulses must be less than the time of divergence of the trajectories; (2) the duration of the interval with a low level of video pulses must be less than the time of return of the trajectory to the fixed point of the dynamic system.

Fulfillment of condition (1) allows the trajectory to be repeated from pulse to pulse, and fulfillment of condition (2) allows the shape of the radio pulse to be changed by choosing different initial conditions.

For example, if only condition (1) is met and condition (2) is not, then we obtain a sequence of identical radio pulses, as shown earlier in [61]. If only condition (2) is met and condition (1) is not met, then we obtain a sequence of pulses with unpredictable shapes.

Figure 8 schematically illustrates this idea: it conventionally depicts a modulating signal (the blue curve is a sequence of video pulses of different durations and different gaps between them), and the phase trajectory of the generator starting from constant initial conditions (the video pulse sequence begins in the equilibrium position).

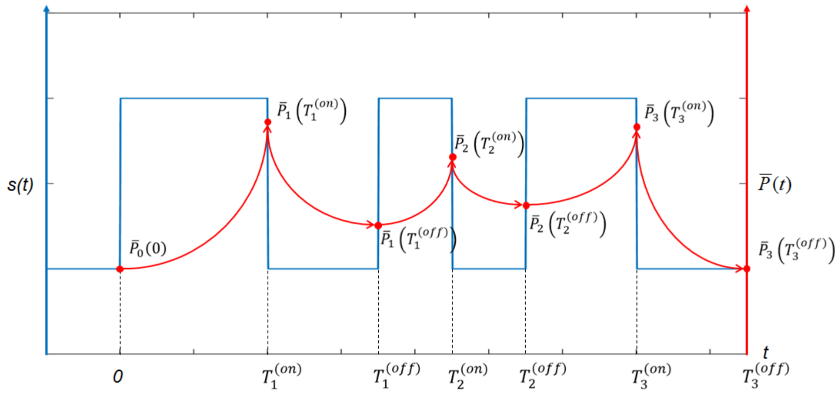


Figure 8. The modulating signal (sequence of video pulses) to form repeating chaotic radio pulses of arbitrary duration: blue—video pulse signal; red—conventionally depicted phase trajectory of the generator.

By turning the generator on for a limited time interval, which is less than T_{Lim}, i.e., less than the interval after which the oscillations lost repeatability, we select the starting point of the trajectory for the next switch-on interval after the current one, as shown in Figure 8: The generator is turned on for the time $[0, T_1^{(on)}]$, $T_1^{(on)} < T_{Lim}$, during which the oscillations develop from the fixed point $\overline{P}_0$ to some value $\overline{P}(T_1^{(on)})$, then the power is turned off. Over the time interval $T_1^{(off)}$ the oscillations decay, after which the phase trajectory of the dynamic system takes some new value $\overline{P}(T_1^{(off)})$. Then, everything is repeated for the next pair of intervals $[T_1^{(off)}, T_2^{(on)}]$ and $[T_2^{(on)}, T_2^{(off)}]$, where $T_2^{(on)} - T_1^{(off)} < T_{Lim}$.

Thus, on the interval $[0, T_1^{(off)}]$, there is a subinterval $[0, T_1^{(on)}]$, on which the oscillation amplitude grows exponentially, and a subinterval $[T_1^{(on)}, T_1^{(off)}]$, on which it exponentially decreases (dissipating energy). By changing the duration of both subintervals, we can controllably choose the initial conditions for each subinterval within certain limits (i.e., change the values of $\overline{P}(T_1^{(on)})$ and $\overline{P}(T_1^{(off)})$), on which the trajectory amplitude will again grow/decrease exponentially. Due to the change in the initial values on each interval, the shape of the oscillations will be different each time from interval to interval. In this case, if we do not go beyond the limits of $T_{(Lim)}$ (i.e., fulfill the condition $T_1^{(on)} < T_{Lim}$), which limits the repeatability interval (coherence interval) of the oscillation shape, then the required result will be achieved: control of the shape of the oscillations with the possibility of repeating this shape.

Even if the duration of the intervals $[T_1^{(off)}, T_2^{(on)}]$ and $[T_2^{(on)}, T_2^{(off)}]$ is the same, the trajectory at the beginning of each active interval (i.e., $[T_1^{(off)} or T_2^{(on)}]$) starts from new initial conditions, which each time gives signals with different shapes.

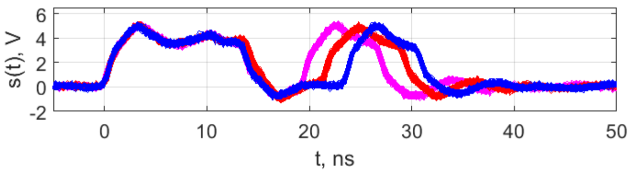
It was hypothesized that the described process could be repeated indefinitely with variations in the duration of the (sub)intervals so that theoretically it would be possible to generate a signal of one form or another for an arbitrarily long time.

To confirm this hypothesis, a series of below-described experiments were conducted.

7. Results of Experiments

According to the proposed method, the first series of experiments included modulation with signals of two types: containing two or three active intervals (generator switch-on intervals).

First, we tested the idea that with a change in the initial conditions at the beginning of each active interval, the pulse form on this interval would change. For this, the generator was modulated by a two-video pulse signal. The durations of the video pulses were 12 and 6 ns, respectively, and the interval between the leading fronts of the pulses took values of 18, 20, or 22 ns, i.e., the passive pause between the pulses was 6, 8, or 10 ns Figure 9a. The effect of such a signal on the generator is shown in Figure 9b.



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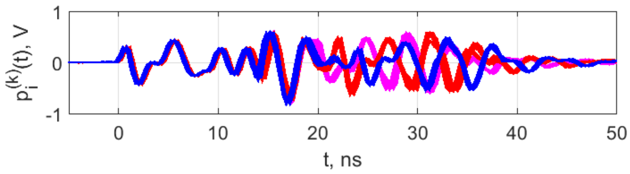


(**b**)

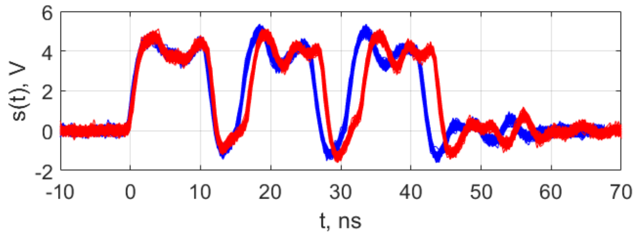
Figure 9. (**a**)Superimposed on each other 100 waveforms of two-video pulse (12 ns and 6 ns) modulating signal. (**b**) Superimposed on each other 100 waveforms of radio pulses generated by a chaotic generator CS_1, CS_2, CS_3 under control of the modulating signals (**a**). Generator supply voltage is 4 V.

The experiment clearly demonstrates that within the first pulse, where the initial conditions are the same for the three types of modulating signals, we have the same signal shape. The waveforms within the position of the second modulating pulse differ significantly since the trajectories start from different initial conditions.

This experiment confirmed the possibility of selecting initial conditions by choosing the moment of switching on the generator.

The signal of the second type consisted of a sequence of three pulses, each of a duration of 10 ns (Figure 10a). The interpulse interval took two values: 5 ns and 7 ns (the power supply voltage was constant $V_E = 4$ V).

The result is the same: we have the pulses with completely repeating shapes (Figure 10b). Moreover, this experiment demonstrated the possibility of using the exponential divergence of the trajectory to control the pulse shape, even if the oscillator is modulated by a periodic signal.



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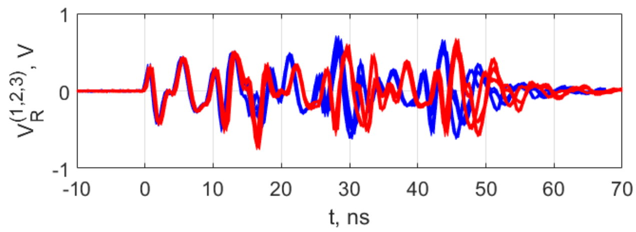


(**b**)

Figure 10. (**a**) Superimposed on each other with 100 waveforms of modulating signal. The passive interval between the 10 ns modulating video pulses is: 5 ns (blue line) or 7 ns (red line). (**b**) A total of 100 superimposed radio pulses generated by chaotic oscillation generators (CS_1, CS_2, CS_3) under the control of modulating video pulses (a). Generator supply voltage is 4 V.

Modulation of the generators with such signals confirmed the above hypothesis: it is possible to form completely reproducible pulses of various shapes even for different instances of the generators (Figure 9).

To answer the question of whether it is possible to indefinitely continue the process of increasing the duration of repeatable chaotic radio pulses, a task was set to synthesize a sequence including several dozen video pulses (packets of video pulses) with variable pulse duration in a packet. The total duration of a chaotic radio pulse with this approach will be several hundred quasi-periods of chaotic oscillations.

The proposed solution is based on the synthesis of a modulating signal by means of combining two periodic rectangular sequences with slightly different pulse repetition frequencies $f_1 = 1/T_1$ and $f_2 = 1/T_2$ and with duty cycles $D_1 = T_1/\tau_1$ and $D_2 = T_2/\tau_2$ (Figure 11).

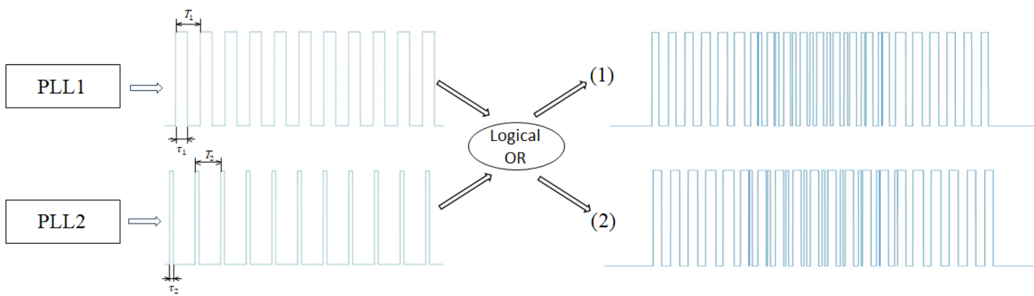


Figure 11. Two types of modulating signals $s(t)$ for generation of long chaotic radio pulses: signal (1) of the 1st type, $T_1 = 24$ ns, $T_2 = 25$ ns, $\tau_1 = 10$ ns, $\tau_2 = 5$ ns; signal (2) of the 2nd type, $T_1 = 25$ ns, $T_2 = 26$ ns, $\tau_1 = 12$ ns, $\tau_2 = 4$ ns.

The combination (by logical OR) of these sequences gives a signal consisting of packets of rectangular meanders, in which the packet length is $T_{Pack} = T_1 T_2/(T_2 - T_1)$. The maximum duration of a rectangular pulse in the packet is $\tau_1 + \tau_2$, and the minimum is $min(\tau_1, \tau_2)$.

The duration of each individual meander in the sequence with frequencies f_1 and f_2 is less than $T_{Lim} = 35$ ns, i.e., less than the time required for the trajectories to diverge. Consequently, one can expect the signal to be reproducible.

Below, the modulating signal $s(t)$ for the set of parameters $T_1 = 24$ ns, $T_2 = 25$ ns, $\tau_1 = 10$ ns, $\tau_2 = 5$ ns is called the signal of the 1st type ($s_1(t)$), and for the set of parameters $T_1 = 25$ ns, $T_2 = 26$ ns, $\tau_1 = 12$ ns, $\tau_2 = 4$ ns is called the signal of the 2nd type ($s_2(t)$). The effect of such signals on the chaos generator is shown in Figure 12. Chaotic radio pulses formed under the effect of modulating signals of the 1st and 2nd types are also called chaotic radio pulses of the 1st and 2nd types by analogy.

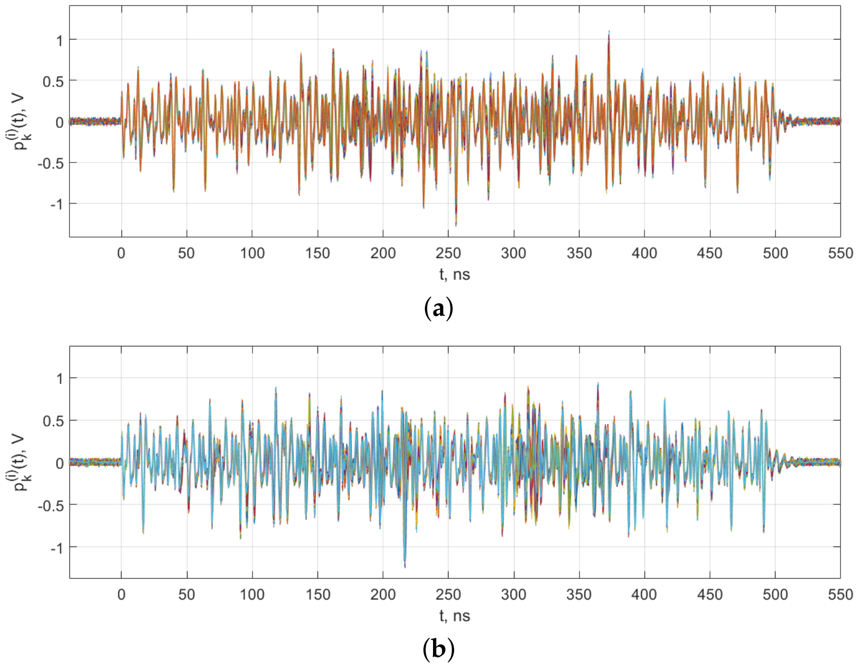


Figure 12. Superimposed 240-long chaotic radio pulses $p_i^{(k)}$, $i = 1 \ldots 30$, $k = 1 \ldots 8$ of the same shape, generators $CS_1 \ldots CS_8$. (**a**) Chaotic radio pulse of the 1st type; (**b**) chaotic radio pulse of the 2nd type. Supply voltage 4 V.

Figure 12 shows the waveforms of 240 chaotic radio pulses formed by eight generators $CS_1 \ldots CS_8$ (each generator produced 30 pulses), aligned by start time. It is evident that (1) the hypothesis about the possibility of repetition of the pulse shape over ≈200 quasi-periods of oscillations is confirmed (the pulse has a duration of 500 ns), and (2) the pulse shape changes with a change in the shape of the modulating video signal.

Thus, the above assumptions have been experimentally confirmed: the effect on a chaotic system of a modulating video signal, the periods of which last less than the characteristic time of divergence of the trajectories, allows the formation of pulses that coincide in shape, and the shape of such pulses can be changed.

8. Auto- and Cross-Correlation Properties of Pulses

Visual comparison of the pulse shapes obtained in the experiments shows a high degree of coincidence (Figure 12). However, the formal criterion of their identity is the mutual correlation coefficient, whereas the autocorrelation function of each pulse will show the degree of its noise-like character.

Ideally, identical pulses should have a correlation coefficient equal to one, whereas the correlation coefficient of different-shaped pulses should be equal to zero (orthogonality) or minus one (antipodality), which is also extremely attractive for coherent communications based on chaotic signals. Of course, strictly orthogonal and, especially, antipodal pulses cannot be obtained in this way, but it is necessary to estimate the degree of their nonorthogonality. Antipodality is theoretically possible if multiplication by minus one is technically permitted.

To evaluate the autocorrelation (noise-like) properties of chaotic radio pulses, the autocorrelation function $A_c(\Delta t)$ of a chaotic signal consisting of tens of thousands of quasi-periods was first calculated (the duration of the signal fragment recorded in the experiment with an oscilloscope is 40 µs, as in Figure 13).

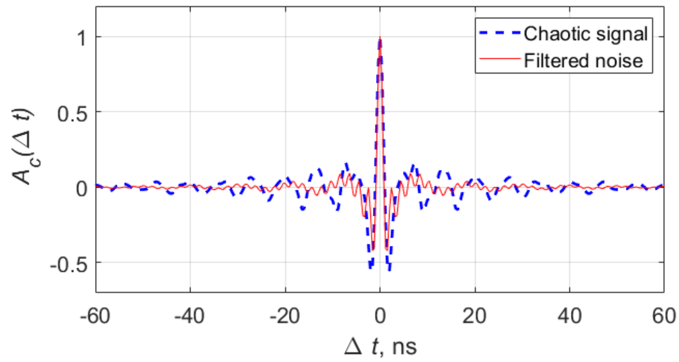


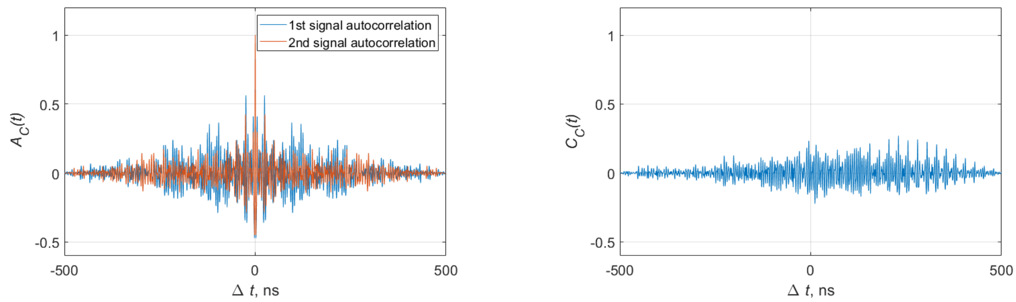
Figure 13. Autocorrelation function $A_c(\Delta t)$ of a continuous chaotic signal of 40 µs duration.

For comparison, the same Figure 13 shows the autocorrelation function of a signal by filtering random independent samples passed through a bandpass filter with a passband equal to the bandwidth of the chaotic signal at a level of −10 dB from the maximum spectral power density, i.e., in the range of 100 to 500 MHz.

The half-width of the main lobe of the autocorrelation function is $\approx 3 \times 10^{-9}$ ns, i.e., it is inversely proportional to the bandwidth of the chaotic signal. This comparison convinces us that "at infinity" the autocorrelation properties of the chaotic signal differ only slightly from the properties of the noise-like signal of the same bandwidth.

What happens if we consider impulses instead of an infinite continuous chaotic signal?

Auto- and cross-correlation functions of pulses were analyzed, the shape of which is presented in Figure 12. The results of calculating auto- and cross-correlation functions are presented in Figure 14.



(**a**) (**b**)

Figure 14. (**a**)Autocorrelation functions $A_C(\Delta t)$ for long pulses of the 1st and 2nd types: blue is for radio pulses of the 1st type, red is for radio pulses of the 2nd type; (**b**) cross-correlation function $C_C(\Delta t)$ for long pulses of the 1st and 2nd types.

With increasing Δt, the level of the side lobes drops, which proves directly that the pulses obtained by the method proposed in the work are not correlated and retain their noise-like properties.

To quantitatively assess the degree of identity of pulses both among themselves (pulses obtained from one generator) and between pulses obtained from different generators, Pearson correlation coefficients $r_{ij}^{(k,p)}$ between pulses $P_i^{(k)}$ and $P_j^{(p)}$ were calculated, where k, p are the generator numbers and i, j are the numbers of pulse waveforms of k-th and p-th generators, respectively. The correlation coefficient is equal to one if the pulse shapes

P_1 and P_2 completely coincide, $P_1 = P_2$, and to minus one if the signals are antipodal, i.e., $P_1 = -P_2$.

$$r_{ij}^{(k,p)} = \frac{\sum(P_i^{(k)} - \overline{P_i^{(k)}})(P_j^{(p)} - \overline{P_j^{(p)}})}{\sqrt{\sum(P_i^{(k)} - \overline{P_i^{(k)}})^2 \sum(P_j^{(p)} - \overline{P_j^{(p)}})^2}} \quad (1)$$

where $\overline{P_j^{(k)}}$ stands for the mean value of the pulse waveform, i.e., $\overline{P_j^{(k)}} = \frac{1}{N_P}\sum_1^{N_P} P_j^{(k)}$. Here, it is assumed that the pulses are represented by a set of discrete samples with the oscilloscope sampling frequency f_s: $P_j^{(k)} = \{p_j^{(k)}(t)\}$, $t = 1 \ldots N_P$. The number of samples per pulse is $N_P = T_P f_s$, where $f_s = 2.5$ GHz, $T_P = 500$ ns - pulse duration.

Calculated distributions $D(r_{ij}^{(k,p)})$ of the correlation coefficients for pulses from different pairs of generators are shown in Figure 15.

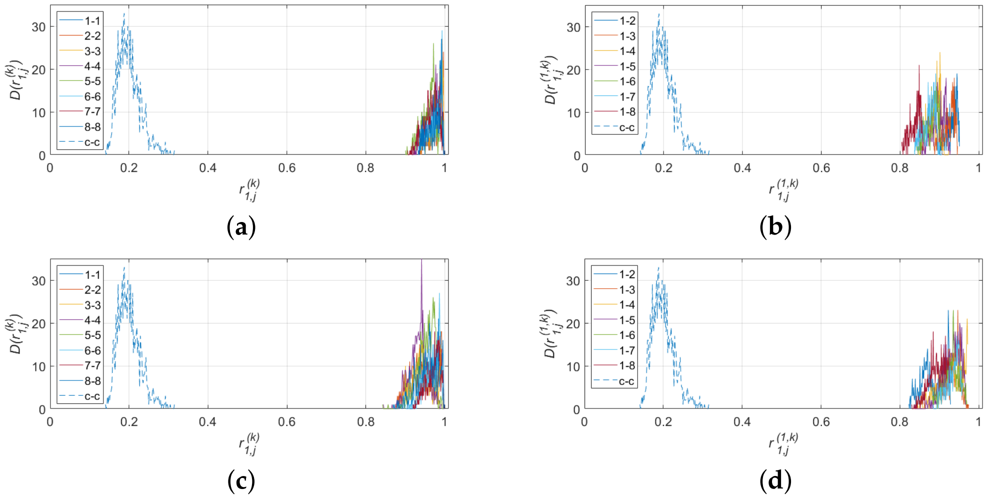


Figure 15. Distributions D of the correlation coefficients r_{ij} at $V_E = 4$ V for signals with 1st-type modulation (**a**,**b**) and 2nd-type modulation (**c**,**d**); (**a**,**c**) are the results of cross-correlation of the one pulse from specified generator with all other pulses from the same generator; (**b**,**d**) are the results of cross-correlation of the one pulse from generator CS_1 with pulses from generators $CS_2 \ldots CS_8$. The color lines represent the distributions of the correlation coefficients for a given pair of pulses. The dashed line (c-c) denotes the distribution of the maximum value of cross-correlation function between different fragments of a continuous chaotic signal, the fragment duration is equal to the pulse duration $T_P = 500$ ns.

The pulse correlation coefficients are concentrated in the range of 0.8 to 1. For comparison, the dashed line (c-c) shows the distribution of the correlations of different fragments of a continuous chaotic signal. The duration of the fragments is equal to the duration of the chaotic radio pulses (Figure 12).

9. Discussion

Ways of using chaotic signals in various types of communication systems to improve noise immunity have long been discussed in the literature. Most of the work is devoted to the use of digital signals in circuits with coherent processing. Here the first stage of coherent processing of an analog chaotic signal is carried out, i.e., the formation of signals with identical waveforms that can be replicable in the different samples of chaotic generators. Control of the shape of an analog signal by time parameters allows us to implement a set of independent controlled chaotic sources, coherently emitting signals of complex shape.

In the proposed approach, instability plays a decisive and constructive role, which allows controlling the pulse shape. This quality has not previously been used explicitly for the purposes of synthesizing analog chaotic signals. The considered generator is an autonomous dynamic system with chaotic behavior. The effect of an external video signal on it implements control of an autonomous dynamic system (generator control).

The exponential divergence property, characteristic of chaotic systems, is preserved here and is directly used to obtain pulses of various shapes. That is, the system remains a chaos generator, not just a nonlinear filter, which may or may not have the property of exponential divergence of the signal in response to an external action. In our case, the external action does not affect the internal dynamics of the nonlinear dynamic system. External dynamics are not imposed on the system, as can happen in nonautonomous dynamic systems, and, in particular, in nonlinear filters. The action does not change the dynamic modes of the system, but only turns them on or off at the required moments in time.

Analog chaotic generators become universal signal sources, with the help of which it is possible to obtain, among other things, UWB signals of the microwave range of various shapes. This is very promising for creating sources of complex oscillations in various frequency bands.

The results presented in the work expand the spectrum of the potential application of analog chaotic signals to fields where it has not been considered yet, namely beam forming, due to the possibility of coherently summing chaotic signals of identical forms, or in problems of coherent reception of chaotic signals to implement coherent processing gain.

10. Conclusions

A method for generating long pulses by modulating the supply voltage of a chaotic generator is proposed and experimentally confirmed. It is established that the auto- and cross-correlation properties of such pulses retain their noise-like properties. Moreover, due to the sensitivity of chaos generators to initial conditions, this occurs naturally; in analog generators, there is no need to worry about maintaining a weak correlation between pulses.

The practical possibility of forming ultra-wideband chaotic radio pulses, the shape of which can be reproduced from pulse to pulse, is demonstrated, as well as the fundamental and technical possibility of making the duration of such pulses arbitrarily large, which opens wide possibilities for implementing coherent processing gain in various wireless applications.

Further, we will consider the issues of receiving chaotic radio pulses of the same shape and the influence of the carrier signal transformations in the channel and in the receiver circuits on the shape and properties of such signals.

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Abbreviations

The following abbreviations are used in this manuscript:

CMOS	Complementary Metal-Oxide Semiconductor
FPGA	Field-Programmable Gate Array
RF	Radio Frequency
UWB	Ultra-Wide Band

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