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A New Control Chart for Monitoring the Process Mean Using Successive Sampling and Multiple Dependent State Repetitive Sampling

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Abstract: In this paper, a new control chart scheme has been developed for monitoring the production process mean using successive sampling over two occasions. The proposed chart reduces to three different existing control charts under different assumptions and is compared with these three existing control charts for monitoring the process average. It has been observed that the proposed control chart performs better than the other existing control charts in terms of average run length (*ARL*). A simulation study using an artificial data set was included for demonstrating the process shift detection power of the proposed control chart.

Keywords: successive sampling; auxiliary information; control chart; average run length; process mean

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1. Introduction

Control charts are being used extensively for monitoring the manufacturing processes to detect any unusual change in the quality characteristic of interest. Timely and speedily investigation of the process shift and the corrective actions, communicated by the control charts, are useful measures toward bringing the process back into statistical control. The control chart is an effective on-line monitoring scheme extensively used for this purpose [1,2]. Two types of control charts are most commonly used in the literature of the process control: one is used for monitoring the process mean or location (for example, $\overline{X}-$ Chart), and the other is used for monitoring the variability or dispersion (for example, $R-$ Chart) [3]. In the beginning, the design structure of these charts was based upon simple random sampling, but recently, many techniques have been proposed regarding the designs and sampling schemes.

Successive sampling has been used extensively in applied and social sciences for estimating the mean of the finite population and attracted the attention of many researchers during the last two decades, for more details, see [1,4–11]. The idea of successive sampling on two occasions was introduced by [12]. The use of successive sampling in the area of control charts increases its monitoring ability substantially (see for example [13]). The preparation of surveying the population for estimating the population parameter at different time points is called sampling over successive occasions [4]. In sampling over successive occasions on the matched portion of the sample, the information about the target/interested quality characteristics is collected from the sample of the current occasion, and the information from the preceding samples is used as the auxiliary information. The utilization of the auxiliary information for estimating the parameter using the current situation of population only on two successive occasions have been explored by many authors, including [4,7,14].

The performance evaluation of any proposed chart is examined by Average Run Length (ARL), which may be defined as the average number of samples before the process indicates an out-of-control process [2]. The ARL is an important tool of process design and performance evaluation [15]. There are several methods including the Markov Chain approach, integral equation approach, and Monte Carlo simulations to calculate the in-control ARL (ARL_0) and ARL of the out-of-control (ARL_1) processes [16]. The value of ARL_0 is considered to be the higher, as the process is in a state of in-control, while a smaller value of ARL_1 is known to be better for the efficient monitoring of the process, as the out-of-control process is indicated quickly to avoid losses of scrap and/or rework. Several researchers used the ARL calculation for examining the performance of the proposed scheme, including [17–20].

In this article, a control chart design has been proposed for monitoring the process mean using successive sampling over two occasions. In sampling over successive occasions on the matched portion of the sample, the information about the target/interested quality characteristics is collected from the sample of the current occasion, and the information from the preceding samples is used as the auxiliary information. The correlation coefficient here is the one between the main variable with an auxiliary one. One of the objectives in this study is to investigate the performance of the proposed chart according to the magnitude of this correlation coefficient. The rest of the article is organized as follows: The designing of the proposed chart is given in Section 2. In Section 3 the probability of in-control and out-of-control processes is described. The methodology of ARL of the in-control and the out-of-control is given in Section 4. In Section 5, a comparison of proposed chart with four existing charts has been discussed. Concluding remarks are given in the last Section.

2. Designing of Proposed Control Chart

Suppose that the main quality characteristic of interest is Y with mean μ_Y and variance σ_Y^2, and that an auxiliary variable X having mean μ_X and variance σ_X^2 is also measured from sampling. The correlation coefficient between Y and X is denoted by ρ. It is assumed that $\sigma_X^2 = \sigma_Y^2 = \sigma^2$ for simplicity. It is also to be noted that our main variable of interest to be monitored here is the Y variable through discovered information of the X variable. Here, we would like to improve the efficiency of the estimator of μ_Y using successive sampling over two occasions.

We draw two samples of size n each at the first and the second occasions. Suppose that there are m common (called matched) units from two occasions so that there are u $(n - m)$ unmatched units from the second occasion. Let $\overline{x}_m$ and $\overline{y}_m$ show the sample means of the matched units for X and Y variables, respectively. Let $\overline{x}_u$ and $\overline{y}_u$ be the sample means for the unmatched units for X and Y variables, respectively. Then, Mukhopadhyay proposed the following estimator of μ_Y:

$$\hat{\mu_Y} = a\overline{x}_u + b\overline{x}_m + c\overline{y}_m + d\overline{y}_u \tag{1}$$

where a, b, c, and d are constant satisfying $a + b = 0$ and $c + d = 1$. A more details about the estimation of these constants can be seen in [11].

The variance of estimator under the assumption that population variances are equal is given by

$$Var(\hat{\mu_Y}) = Var\{a(\overline{x}_u - \overline{x}_m) + c\overline{y}_m + (1 - c)\overline{y}_u\} \tag{2}$$

According to [21–23], the distribution of $\hat{\mu_Y}$ is given as

$$\hat{\mu_Y} \sim N\left[\mu_Y, \frac{\sigma^2}{2n}\left(1 + \sqrt{1 - \rho^2}\right)\right]$$

The steps of the proposed control chart will be as follows:

Step 1

We draw two samples of size n each at the first and the second occasions. Suppose that there are m common (called matched) units from two occasions so that there are u $(n - m)$ unmatched units from the second occasion. Let $\overline{x}_m$ and $\overline{y}_m$ be the sample means of the matched units for X and Y variables, respectively. Let $\overline{x}_u$ and $\overline{y}_u$ be the sample means for the unmatched units for X and Y variables, respectively.

Step 2

Compute the value of the estimator

$$\hat{\mu_Y} = a\overline{x}_u + b\overline{x}_m + c\overline{y}_m + d\overline{y}_u$$

(The algorithm of determining a, b, c and d is given in Section 3.)

Step 3 (Decision State):

Declare the process as in-control if $LCL_2 \leq \hat{\mu_Y} \leq UCL_2$ and as out-of-control if $\hat{\mu_Y} \geq UCL_1$ or $\hat{\mu_Y} \leq LCL_1$. Otherwise, go to Step 4

Step 4 (Indecision State):

If $UCL_2 \leq \hat{\mu_Y} \leq UCL_1$ or $LCL_1 \leq \hat{\mu_Y} \leq LCL_2$. The process is declared as in-control if i proceeding subgroups have been declared as in-control. Otherwise, repeat Step 1.

Where LCL and UCL show lower control limit and upper control limit, respectively.

The proposed control chart consists of four control limits according to the successive sampling estimator are namely LCL_1, LCL_2, UCL_1, UCL_2 having two control limits coefficients k_1 and k_2. The proposed control chart is an extension of several existing control charts. The proposed control chart reduces to the chart by [18] when i = 0 and to the chart by [24] when $k_1 = k_2$. The proposed chart becomes the chart by [25] when i = 0 and $k_1 = k_2$.

The two outer control limits for the proposed control chart are given as

$$LCL_1 = \mu_0 - k_1\sqrt{\frac{\sigma^2}{2n}\,(1+\sqrt{1-\rho^2})} \tag{3}$$

$$UCL_1 = \mu_0 + k_1\sqrt{\frac{\sigma^2}{2n}\,(1+\sqrt{1-\rho^2})}$$

The two inner control limits are given as

$$LCL_2 = \mu_0 - k_2\sqrt{\frac{\sigma^2}{2n}\,(1+\sqrt{1-\rho^2})} \tag{4}$$

$$UCL_2 = \mu_0 + k_2\sqrt{\frac{\sigma^2}{2n}\,(1+\sqrt{1-\rho^2})}$$

Here, μ_0 is the population mean when the process is in control.

3. Average Run Lengths

The probability that the process is declared as in-control based on a single sample is given as follows:

$$\begin{aligned} P^0_{in,1} = \; & P(LCL_2 \leq \hat{\mu_Y} \leq UCL_2) \\ & +\{P(LCL_1 < \hat{\mu_Y} < LCL_2) \\ & +P(UCL_2 < \hat{\mu_Y} < UCL_1)\}\{P(LCL_2 \leq \hat{\mu_Y} \leq UCL_2)\}^i \end{aligned} \tag{5}$$

When the plotting statistic is in-decision state, the process is repeated as stated in Step 4 of proposed control chart. Let P^0_{rep} denote the probability for this area. Then, it is given as follows:

$$P^0_{rep} = \{P(LCL_1 < \hat{\mu_Y} < LCL_2) + P(UCL_2 < \hat{\mu_Y} < UCL_1)\}(1 - [P\{LCL_2 \leq \hat{\mu_Y} \leq UCL_2\}]^i) \tag{6}$$

Let us define

$$A^0_1 = P\{LCL_2 < \hat{\mu_Y}\langle UCL_2 |\, \mu_Y = \mu_0\} = 2\Phi(k_2) - 1 \tag{7}$$

$$A^0_2 = P(LCL_1 < \hat{\mu_Y}\langle LCL_2\, |\mu_Y = \mu_0) = \Phi(k_1) - \Phi(k_2) \tag{8}$$

$$A^0_3 = P(UCL_2 < \hat{\mu_Y}\langle UCL_1 |\, \mu_Y = \mu_0) = \Phi(k_1) - \Phi(k_2) \tag{9}$$

Then, the probability $P^0_{in,1}$ given in Equation (10) can be written as follows:

$$P^0_{in,1} = A^0_1 + \{A^0_2 + A^0_3\}\{A^0_1\}^i \tag{10}$$

or

$$P^0_{in,1} = (2\Phi(k_2) - 1) + 2\{\Phi(k_1) - \Phi(k_2)\}\{2\Phi(k_2) - 1\}^i \tag{11}$$

The probability of repetition is given as

$$P^0_{rep} = 2\{\Phi(k_1) - \Phi(k_2)\}(1 - [2\Phi(k_2) - 1]^i) \tag{12}$$

or

$$P^0_{rep} = \{A^0_2 + A^0_3\}(1 - [A^0_1]^i) \tag{13}$$

The probability that the process is declared as in control for the proposed chart is given as follows

$$P^0_{in} = \frac{P^0_{in,1}}{1 - P^0_{rep}} = \frac{A^0_1 + \{A^0_2 + A^0_3\}\{A^0_1\}^i}{1 - \{A^0_2 + A^0_3\}(1 - [A^0_1]^i)} \tag{14}$$

The average run length (*ARL*) is one of the most useful performance measures for evaluating the efficiency of a control chart.

The *ARL* of the proposed control chart when the process is in control is defined as follows

$$ARL_0 = \frac{1}{1 - P^0_{in}} \tag{15}$$

Now, we suppose that the process is shifted from μ_0 to $\mu_1 = \mu_0 + \sigma f$; where 'f' indicates shift constant. Let $P^1_{in,1}$ denote the probability that the process is declared as in-control for the shifted process based on a single sample is given as

$$\begin{aligned} P^1_{in,1} = \; & P(\, LCL_2 \leq \hat{\mu_Y} \leq UCL_2|\mu_1\,) \\ & + \{P(LCL_1 < \hat{\mu_Y}\langle LCL_2|\mu_1) \\ & + P(UCL_2 < \hat{\mu_Y} \\ & < UCL_1|\mu_1\,)\}\{P(\, LCL_2 \leq \hat{\mu_Y} \leq UCL_2|\mu_1\,)\}^i \end{aligned} \tag{16}$$

The probability of repeated sampling, say P^1_{rep} at μ_1 is given as

$$P^1_{rep} = \{P(LCL_1 < \hat{\mu_Y}\langle LCL_2|\mu_1) + P(UCL_2 < \hat{\mu_Y}\langle UCL_1|\mu_1)\}(1 - [P\{LCL_2 \leq \hat{\mu_Y} \leq UCL_2|\mu_1\}]^i) \tag{17}$$

Let us define

$$\begin{aligned} A^1_1 = P(LCL_2 \; & \leq \hat{\mu_Y} \leq UCL_2|\mu_1) \\ & = \Phi\left(k_2 - \frac{f}{\sqrt{\frac{1}{2n}\;(1+\sqrt{1-\rho^2})}}\right) \\ & +\Phi\left(k_2 + \frac{f}{\sqrt{\frac{1}{2n}\;(1+\sqrt{1-\rho^2})}}\right) - 1 \end{aligned} \tag{18}$$

$$
\begin{aligned}
A_2^1 = P(LCL_1 &< \hat{\mu_Y}\langle LCL_2|\mu_1) \\
&= \Phi\left(k_1 + \frac{f}{\sqrt{\frac{1}{2n}\ (1+\sqrt{1-\rho^2})}}\right) \\
&-\Phi\left(k_2 + \frac{f}{\sqrt{\frac{1}{2n}\ (1+\sqrt{1-\rho^2})}}\right)
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
A_3^1 = P(UCL_2 &< \hat{\mu_Y}\langle UCL_1|\mu_1) \\
&= \Phi\left(k_1 + \frac{f}{\sqrt{\frac{1}{2n}\ (1+\sqrt{1-\rho^2})}}\right) \\
&-\Phi\left(k_2 + \frac{f}{\sqrt{\frac{1}{2n}\ (1+\sqrt{1-\rho^2})}}\right)
\end{aligned} \tag{20}
$$

Then, Equation (22) can be written as

$$
P_{in,1}^1 = A_1^1 + \ \{A_2^1 + A_3^1\}\{A_1^1\}^i \tag{21}
$$

or

$$
\begin{aligned}
P_{in}^1 = &\left(\Phi(k_2 - \frac{f}{\sqrt{\frac{1}{2n}\ (1+\sqrt{1-\rho^2})}}\right) + \Phi\left(k_2 + \frac{f}{\sqrt{\frac{1}{2n}\ (1+\sqrt{1-\rho^2})}}\right) - 1\Bigg) \\
&+2\Bigg\{\left(k_1 + \frac{f}{\sqrt{\frac{1}{2n}\ (1+\sqrt{1-\rho^2})}}\right) \\
&-\Bigg(k_2 \\
&+\frac{f}{\sqrt{\frac{1}{2n}\ \left(1+\sqrt{1-\rho^2}\right)}}\Bigg)\Bigg\}\Bigg\{\left(\Phi\left(k_2 - \frac{f}{\sqrt{\frac{1}{2n}\ \left(1+\sqrt{1-\rho^2}\right)}}\right)\right. \\
&\left.+\ \Phi\left(k_2 + \frac{f}{\sqrt{\frac{1}{2n}\ \left(1+\sqrt{1-\rho^2}\right)}}\right) - 1\right)\Bigg\}^i
\end{aligned} \tag{22}
$$

The probability of repetition P_{rep}^1 is given as

$$
\begin{aligned}
P_{rep}^1 = 2\Bigg\{&\left(k_1 + \frac{f}{\sqrt{\frac{1}{2n}\ (1+\sqrt{1-\rho^2})}}\right) - \left(k_2 + \frac{f}{\sqrt{\frac{1}{2n}\ (1+\sqrt{1-\rho^2})}}\right)\Bigg\}\Bigg(1 \\
&-\left[\Phi\left(k_2 - \frac{f}{\sqrt{\frac{1}{2n}\ (1+\sqrt{1-\rho^2})}}\right) + \Phi\left(k_2 + \frac{f}{\sqrt{\frac{1}{2n}\ (1+\sqrt{1-\rho^2})}}\right) - 1\right]^i\Bigg)
\end{aligned} \tag{23}
$$

Hence, the probability that the process is declared as in control for the shifted process is given as follows

$$
P_{in}^1 = \frac{P_{in,1}^1}{1 - P_{rep}^1} = \frac{A_1^1 + \ \{A_2^1 + A_3^1\}\{A_1^1\}^i}{1 - \{A_2^1 + A_3^1\}(1 - [A_1^1]^i)} \tag{24}
$$

So, the ARL for the shifted process is given as follows

$$
ARL_1 = \frac{1}{1 - P_{in}^1} \tag{25}
$$

Using the above mentioned equations an R-language code program was written and run under the Monte Carlo simulation procedure. The $ARLs$ for in-control and shifted processes were estimated for mean monitoring under the normal distribution for different process settings. The ARL analysis of the proposed scheme for different process settings with control chart coefficients has been given in Tables 1–6. Tables 1–3 are for the cases of $ARL_0 = 300$ and Tables 4–6 are for the cases of $ARL_0 = 370$.

Table 1. The average run length (*ARL*) analysis for 300 with n = 5.

f	$\rho = 0.3$					$\rho = 0.6$					$\rho = 0.9$				
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$
	k_1				k	k_1				k	k_1				k
	2.9523	**2.9701**	**2.9867**	**2.9953**		**2.9607**	**2.9861**	**3.0027**	**3.0211**		**2.9552**	**2.9706**	**3.0084**	**3.0170**	
	k_2				**2.9352**	k_2				**2.9352**	k_2				**2.9352**
	1.1900	**1.1537**	**1.1238**	**1.1316**		**1.0728**	**1.0345**	**1.0323**	**1.0030**		**1.1447**	**1.1493**	**1.0040**	**1.0214**	
0.0000	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00
0.0005	263.81	260.44	257.26	256.54	300.00	256.78	251.97	249.60	245.65	300.00	256.28	254.58	242.48	241.99	300.00
0.0010	235.41	230.11	225.20	224.10	299.99	224.44	217.21	213.73	208.01	299.99	223.69	221.12	203.50	202.82	299.99
0.0015	212.53	206.11	200.25	198.96	299.98	199.35	190.89	186.89	180.40	299.98	198.46	195.44	175.34	174.59	299.98
0.0020	193.71	186.65	180.29	178.91	299.97	179.31	170.27	166.05	159.28	299.97	178.34	175.11	154.04	153.28	299.96
0.0025	177.95	170.56	163.96	162.53	299.95	162.93	153.68	149.41	142.60	299.95	161.93	158.62	137.37	136.63	299.94
0.0030	164.57	157.02	150.35	148.92	299.93	149.29	140.04	135.81	129.10	299.93	148.29	144.97	123.97	123.25	299.91
0.0035	153.06	145.48	138.83	137.41	299.91	137.77	128.63	124.49	117.94	299.90	136.77	133.49	112.96	112.27	299.88
0.0040	143.05	135.52	128.96	127.56	299.88	127.90	118.95	114.91	108.57	299.87	126.91	123.70	103.76	103.09	299.84
0.0050	126.51	119.21	112.91	111.58	299.82	111.87	103.39	99.61	93.70	299.80	110.93	107.88	89.24	88.63	299.75
0.0060	113.41	106.41	100.42	99.18	299.74	99.42	91.45	87.92	82.43	299.72	98.52	95.66	78.30	77.75	299.65
0.0070	102.76	96.10	90.44	89.27	299.64	89.47	81.99	78.70	73.60	299.61	88.62	85.93	69.76	69.27	299.52
0.0080	93.95	87.62	82.26	81.16	299.54	81.33	74.31	71.24	66.49	299.50	80.52	78.00	62.92	62.46	299.37
0.0090	86.52	80.51	75.45	74.42	299.41	74.55	67.95	65.08	60.65	299.36	73.78	71.42	57.31	56.89	299.20
0.0100	80.19	74.48	69.69	68.72	299.28	68.82	62.60	59.91	55.76	299.21	68.09	65.87	52.62	52.24	299.02
0.0300	32.58	29.92	27.75	27.35	293.59	27.18	24.47	23.35	21.61	293.05	26.81	25.89	20.20	20.09	291.34
0.0500	20.46	18.79	17.44	17.21	282.78	16.98	15.31	14.65	13.58	281.39	16.72	16.18	12.65	12.61	277.00
0.1000	10.60	9.79	9.14	9.06	240.41	8.80	8.01	7.72	7.22	236.29	8.63	8.40	6.68	6.69	223.85
0.2000	5.38	5.03	4.76	4.75	145.54	4.50	4.18	4.08	3.87	138.84	4.37	4.31	3.55	3.58	120.48
0.3000	3.57	3.38	3.24	3.24	82.17	3.03	2.86	2.82	2.71	76.50	2.92	2.89	2.47	2.50	61.98
0.4000	2.66	2.55	2.46	2.47	47.11	2.30	2.19	2.18	2.11	43.07	2.19	2.18	1.93	1.95	33.21
0.5000	2.12	2.05	1.99	2.00	28.06	1.86	1.80	1.79	1.75	25.31	1.76	1.76	1.60	1.61	18.84

Table 2. The *ARL* analysis for 300 with n = 30.

f	ρ = 0.3					ρ = 0.6					ρ = 0.9				
	i = 1	i = 2	i = 3	i = 4	i = 0	i = 1	i = 2	i = 3	i = 4	i = 0	i = 1	i = 2	i = 3	i = 4	i = 0
	k_1				k	k_1				k	k_1				k
	2.9623	**2.9825**	**3.0048**	**3.0208**		**2.9567**	**2.9757**	**3.0031**	**3.0115**		**2.9596**	**2.9774**	**2.9905**	**3.0179**	
	k_2				**2.9352**	k_2				**2.9352**	k_2				**2.9352**
	1.0556	**1.0581**	**1.0223**	**1.0047**		**1.1244**	**1.1075**	**1.0309**	**1.0475**		**1.0874**	**1.0945**	**1.1003**	**1.0174**	
0.0000	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00
0.0010	213.49	209.53	202.35	197.59	299.99	216.78	211.79	200.61	199.93	299.99	206.55	203.28	200.87	188.58	299.99
0.0015	165.73	161.03	152.74	147.40	299.96	169.72	163.71	150.76	150.01	299.95	157.52	153.77	151.05	137.61	299.94
0.0020	135.45	130.79	122.71	117.60	299.90	139.46	133.45	120.80	120.09	299.89	127.31	123.68	121.07	108.40	299.87
0.0025	114.53	110.13	102.58	97.87	299.83	118.36	112.65	100.81	100.16	299.81	106.83	103.46	101.05	89.46	299.76
0.0030	99.22	95.13	88.15	83.84	299.73	102.82	97.47	86.52	85.94	299.71	92.04	88.94	86.74	76.19	299.63
0.0035	87.53	83.73	77.29	73.34	299.61	90.88	85.91	75.79	75.27	299.58	80.85	78.01	75.99	66.37	299.47
0.0040	78.30	74.78	68.83	65.20	299.47	81.44	76.80	67.45	66.97	299.42	72.09	69.47	67.62	58.81	299.28
0.0045	70.84	67.57	62.06	58.70	299.30	73.77	69.45	60.77	60.34	299.25	65.05	62.63	60.93	52.81	299.05
0.0050	59.51	56.66	51.87	48.98	298.91	62.09	58.31	50.75	50.38	298.82	54.42	52.34	50.88	43.89	298.53
0.0060	51.31	48.80	44.58	42.04	298.44	53.61	50.26	43.59	43.28	298.31	46.79	44.97	43.69	37.57	297.88
0.0070	45.10	42.87	39.10	36.85	297.88	47.17	44.17	38.22	37.95	297.70	41.04	39.43	38.30	32.87	297.12
0.0080	40.24	38.23	34.83	32.81	297.23	42.11	39.41	34.04	33.80	297.00	36.55	35.11	34.10	29.23	296.25
0.0090	36.32	34.50	31.42	29.59	296.51	38.04	35.58	30.69	30.48	296.21	32.95	31.65	30.75	26.33	295.27
0.0100	33.11	31.44	28.62	26.95	295.70	34.69	32.43	27.95	27.77	295.34	30.00	28.82	28.00	23.96	294.18
0.0300	12.01	11.48	10.51	9.94	264.92	12.60	11.83	10.24	10.22	262.26	10.80	10.45	10.21	8.81	254.02
0.0500	7.37	7.10	6.56	6.25	218.00	7.71	7.29	6.39	6.40	212.83	6.60	6.44	6.33	5.54	197.59
0.1000	3.77	3.69	3.48	3.36	112.69	3.91	3.76	3.38	3.41	106.24	3.36	3.32	3.30	2.98	89.15
0.2000	1.94	1.93	1.87	1.84	29.51	1.97	1.94	1.82	1.84	26.65	1.73	1.73	1.73	1.64	19.89
0.3000	1.37	1.37	1.35	1.34	9.85	1.37	1.36	1.32	1.33	8.74	1.24	1.25	1.25	1.22	6.26
0.4000	1.14	1.14	1.13	1.13	4.23	1.13	1.13	1.11	1.12	3.76	1.07	1.07	1.07	1.06	2.75
0.5000	1.05	1.05	1.04	1.04	2.30	1.04	1.04	1.03	1.03	2.08	1.02	1.02	1.02	1.01	1.62

Table 3. The *ARL* analysis for 300 with n = 60.

f	$\rho = 0.3$					$\rho = 0.6$					$\rho = 0.9$				
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$
	k_1				k	k_1				k	k_1				k
	2.9590	**2.9752**	**2.9953**	**3.0033**		**2.9574**	**2.9849**	**3.0039**	**3.0150**		**2.9618**	**2.9804**	**2.9915**	**2.9984**	
	k_2				**2.9352**	k_2				**2.9352**	k_2				**2.9352**
	1.0940	**1.1113**	**1.0727**	**1.0884**		**1.1152**	**1.0424**	**1.0264**	**1.0307**		**1.0612**	**1.0727**	**1.0948**	**1.1143**	
0.0000	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00
0.0005	194.36	192.08	184.45	183.81	299.98	193.57	181.61	175.88	173.54	299.98	180.38	176.89	176.05	175.99	299.97
0.0010	143.77	141.32	133.25	132.60	299.91	142.90	130.29	124.51	122.20	299.91	128.99	125.49	124.67	124.62	299.88
0.0015	114.09	111.81	104.35	103.77	299.80	113.28	101.63	96.43	94.38	299.79	100.41	97.28	96.56	96.53	299.73
0.0020	94.58	92.52	85.79	85.28	299.65	93.83	83.33	78.72	76.92	299.62	82.21	79.45	78.83	78.82	299.53
0.0025	80.78	78.92	72.86	72.41	299.46	80.09	70.64	66.54	64.95	299.41	69.61	67.16	66.63	66.63	299.26
0.0030	70.50	68.82	63.33	62.94	299.22	69.87	61.31	57.64	56.23	299.15	60.36	58.18	57.71	57.72	298.94
0.0035	62.54	61.02	56.02	55.67	298.94	61.96	54.17	50.86	49.59	298.85	53.29	51.33	50.92	50.93	298.55
0.0040	56.20	54.82	50.24	49.92	298.61	55.67	48.53	45.51	44.37	298.49	47.71	45.93	45.57	45.59	298.11
0.0050	46.74	45.57	41.66	41.41	297.84	46.28	40.19	37.64	36.68	297.65	39.45	37.96	37.67	37.70	297.06
0.0060	40.01	39.00	35.61	35.40	296.89	39.60	34.31	32.11	31.29	296.63	33.63	32.37	32.13	32.17	295.78
0.0070	34.98	34.10	31.11	30.93	295.78	34.61	29.94	28.02	27.30	295.43	29.32	28.22	28.02	28.07	294.29
0.0080	31.07	30.30	27.63	27.48	294.51	30.74	26.57	24.86	24.23	294.05	25.99	25.03	24.86	24.91	292.58
0.0090	27.96	27.27	24.86	24.73	293.09	27.66	23.89	22.36	21.80	292.51	23.35	22.49	22.35	22.40	290.66
0.0100	25.41	24.80	22.60	22.49	291.51	25.13	21.71	20.32	19.82	290.80	21.19	20.42	20.30	20.36	288.55
0.0300	9.05	8.91	8.20	8.21	236.55	8.93	7.81	7.39	7.26	232.24	7.50	7.31	7.33	7.39	219.25
0.0500	5.52	5.47	5.09	5.12	168.68	5.44	4.83	4.62	4.57	162.16	4.58	4.50	4.54	4.60	143.87
0.1000	2.80	2.81	2.68	2.70	63.22	2.75	2.52	2.46	2.46	58.32	2.34	2.34	2.37	2.41	46.09
0.2000	1.47	1.48	1.45	1.46	11.67	1.43	1.38	1.37	1.37	10.37	1.28	1.29	1.30	1.31	7.45
0.3000	1.12	1.12	1.11	1.11	3.58	1.10	1.09	1.08	1.08	3.19	1.05	1.05	1.05	1.05	2.36
0.4000	1.02	1.02	1.02	1.02	1.73	1.02	1.01	1.01	1.01	1.59	1.00	1.00	1.01	1.01	1.31
0.5000	1.00	1.00	1.00	1.00	1.19	1.00	1.00	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.05

Table 4. The *ARL* analysis for 370 with n = 5.

f	$\rho = 0.3$					$\rho = 0.6$					$\rho = 0.9$				
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$
	k_1				k	k_1				k	k_1				k
	3.0175	**3.0326**	**3.0469**	**3.0759**		**3.0104**	**3.0210**	**3.0335**	**3.0694**		**3.0087**	**3.0278**	**3.0586**	**3.0706**	
	k_2				**2.9997**	k_2				**2.9997**	k_2				**2.9997**
	1.1753	**1.1665**	**1.1469**	**1.0418**		**1.3191**	**1.2952**	**1.2528**	**1.0740**		**1.3652**	**1.2146**	**1.0737**	**1.0678**	
0.0000	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
0.0005	315.26	312.36	308.82	297.06	370.00	322.58	319.72	315.36	297.97	370.00	320.73	308.39	293.40	290.51	370.00
0.0010	274.63	270.26	265.02	248.18	369.99	285.93	281.47	274.79	249.45	369.99	283.05	264.39	243.11	239.18	369.99
0.0015	243.29	238.18	232.12	213.15	369.98	256.76	251.41	243.49	214.54	369.98	253.29	231.38	207.57	203.30	369.97
0.0020	218.37	212.91	206.49	186.80	369.96	233.00	227.15	218.59	188.23	369.96	229.19	205.71	181.11	176.80	369.95
0.0025	198.08	192.50	185.97	166.27	369.94	213.26	207.16	198.32	167.68	369.94	209.28	185.17	160.65	156.43	369.92
0.0030	181.24	175.66	169.17	149.81	369.92	196.60	190.41	181.50	151.19	369.91	192.55	168.37	144.35	140.28	369.89
0.0035	167.05	161.54	155.16	136.34	369.89	182.36	176.17	167.31	137.66	369.88	178.30	154.36	131.07	127.17	369.84
0.0040	154.92	149.52	143.30	125.09	369.85	170.04	163.91	155.18	126.37	369.84	166.02	142.51	120.03	116.31	369.80
0.0050	135.27	130.16	124.31	107.40	369.77	149.81	143.90	135.55	108.57	369.75	145.91	123.55	102.74	99.36	369.68
0.0060	120.05	115.25	109.78	94.12	369.66	133.88	128.24	120.33	95.19	369.64	130.14	109.05	89.83	86.75	369.54
0.0070	107.91	103.41	98.30	83.77	369.54	121.01	115.66	108.20	84.76	369.50	117.45	97.61	79.81	76.99	369.38
0.0080	98.01	93.79	89.00	75.49	369.40	110.40	105.33	98.29	76.41	369.35	107.02	88.34	71.82	69.22	369.19
0.0090	89.77	85.80	81.32	68.71	369.25	101.50	96.70	90.05	69.56	369.18	98.29	80.68	65.29	62.89	368.97
0.0100	82.81	79.08	74.86	63.06	369.07	93.93	89.38	83.09	63.85	368.99	90.87	74.25	59.85	57.63	368.74
0.0300	32.52	30.91	29.11	24.10	361.78	37.70	35.61	32.77	24.42	361.09	36.21	28.72	22.67	21.81	358.89
0.0500	20.26	19.28	18.18	15.07	347.94	23.57	22.27	20.49	15.26	346.16	22.58	17.86	14.12	13.61	340.55
0.1000	10.44	9.99	9.48	7.95	294.01	12.13	11.50	10.63	8.03	288.80	11.57	9.21	7.38	7.17	273.09
0.2000	5.28	5.11	4.90	4.22	175.21	6.06	5.80	5.42	4.24	166.93	5.72	4.67	3.87	3.79	144.30
0.3000	3.52	3.43	3.32	2.93	97.46	3.96	3.82	3.61	2.93	90.57	3.70	3.10	2.65	2.62	73.00
0.4000	2.63	2.58	2.52	2.27	55.12	2.90	2.82	2.69	2.25	50.28	2.68	2.31	2.04	2.02	38.51
0.5000	2.10	2.07	2.03	1.87	32.40	2.27	2.22	2.13	1.84	29.15	2.08	1.85	1.67	1.66	21.52

Table 5. The *ARL* analysis for 370 with n = 30.

f	$\rho = 0.3$					$\rho = 0.6$					$\rho = 0.9$				
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 0$
	k_1				k	k_1				k	k_1				k
	3.0131	**3.0248**	**3.0473**	**3.0559**		**3.0088**	**3.0276**	**3.0482**	**3.0714**		**3.0124**	**3.0318**	**3.0509**	**3.0801**	
	k_2				**2.9997**	k_2				**2.9997**	k_2				**2.9997**
	1.2566	**1.2484**	**1.1445**	**1.1498**		**1.3626**	**1.2171**	**1.1381**	**1.0641**		**1.2719**	**1.1746**	**1.1203**	**1.0221**	
0.0000	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
0.0005	268.36	264.68	248.82	246.99	369.99	276.66	257.79	244.58	230.95	369.98	258.36	243.33	232.51	214.40	369.98
0.0010	210.53	206.06	187.50	185.44	369.94	220.93	197.83	182.73	167.96	369.94	198.49	181.32	169.59	151.05	369.92
0.0015	173.22	168.72	150.46	148.49	369.87	183.89	160.53	145.89	132.04	369.86	161.15	144.52	133.52	116.68	369.83
0.0020	147.14	142.85	125.67	123.86	369.78	157.49	135.07	121.44	108.81	369.76	135.64	120.16	110.14	95.10	369.70
0.0025	127.89	123.86	107.92	106.26	369.65	137.72	116.60	104.03	92.57	369.62	117.10	102.84	93.75	80.29	369.52
0.0030	113.10	109.34	94.57	93.06	369.50	122.36	102.57	91.00	80.56	369.45	103.03	89.89	81.62	69.49	369.32
0.0035	101.38	97.87	84.18	82.79	369.32	110.08	91.57	80.89	71.33	369.26	91.97	79.85	72.28	61.28	369.07
0.0040	91.86	88.58	75.85	74.57	369.11	100.04	82.70	72.81	64.02	369.03	83.06	71.83	64.88	54.82	368.79
0.0050	77.34	74.46	63.34	62.24	368.61	84.61	69.30	60.70	53.15	368.49	69.59	59.83	53.86	45.30	368.11
0.0060	66.78	64.24	54.39	53.44	368.00	73.30	59.64	52.06	45.46	367.83	59.87	51.28	46.06	38.63	367.28
0.0070	58.77	56.48	47.67	46.83	367.28	64.66	52.35	45.60	39.74	367.05	52.54	44.88	40.25	33.69	366.30
0.0080	52.47	50.41	42.44	41.69	366.45	57.84	46.66	40.57	35.31	366.15	46.81	39.90	35.76	29.89	365.19
0.0090	47.39	45.52	38.26	37.58	365.52	52.32	42.09	36.55	31.78	365.14	42.21	35.92	32.17	26.87	363.92
0.0100	43.21	41.49	34.83	34.21	364.48	47.76	38.33	33.26	28.90	364.02	38.43	32.67	29.25	24.42	362.53
0.0300	15.63	15.05	12.63	12.45	325.15	17.37	13.85	12.04	10.52	321.76	13.76	11.71	10.54	8.89	311.28
0.0500	9.52	9.20	7.80	7.72	265.71	10.55	8.47	7.43	6.57	259.21	8.34	7.17	6.51	5.58	240.06
0.1000	4.75	4.63	4.03	4.01	134.74	5.19	4.28	3.84	3.48	126.83	4.12	3.64	3.38	3.00	105.95
0.2000	2.30	2.27	2.07	2.07	34.12	2.42	2.12	1.98	1.86	30.73	1.98	1.83	1.76	1.64	22.76
0.3000	1.52	1.51	1.43	1.44	11.04	1.55	1.43	1.38	1.33	9.76	1.34	1.29	1.26	1.22	6.92
0.4000	1.20	1.20	1.16	1.17	4.61	1.21	1.16	1.14	1.12	4.08	1.10	1.09	1.08	1.06	2.95
0.5000	1.07	1.07	1.05	1.06	2.44	1.07	1.05	1.04	1.04	2.20	1.03	1.02	1.02	1.01	1.69

Table 6. The *ARL* analysis for 370 with $n = 60$.

f	ρ = 0.3					ρ = 0.6					ρ = 0.9				
	i = 1	i = 2	i = 3	i = 4	i = 0	i = 1	i = 2	i = 3	i = 4	i = 0	i = 1	i = 2	i = 3	i = 4	i = 0
	k_1				k	k_1				k	k_1				k
	3.0161	**3.0418**	**3.0597**	**3.0705**		**3.0093**	**3.0220**	**3.0300**	**3.0548**		**3.0218**	**3.0464**	**3.0652**	**3.0825**	
	k_2				**2.9997**	k_2				**2.9997**	k_2				**2.9997**
	1.1988	**1.0905**	**1.0675**	**1.0686**		**1.3468**	**1.2827**	**1.2855**	**1.1562**		**1.1118**	**1.0569**	**1.0372**	**1.0110**	
0.0000	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
0.0005	233.92	215.25	207.54	204.36	369.97	248.56	237.87	235.91	214.44	369.97	209.48	196.44	188.70	180.84	369.96
0.0010	171.04	151.85	144.32	141.30	369.89	187.14	175.31	173.21	151.07	369.88	146.13	133.80	126.78	119.83	369.85
0.0015	134.82	117.34	110.69	108.05	369.75	150.06	138.82	136.87	116.68	369.73	112.22	101.50	95.53	89.69	369.66
0.0020	111.27	95.64	89.81	87.52	369.55	125.25	114.92	113.16	95.08	369.51	91.10	81.80	76.68	71.73	369.39
0.0025	94.72	80.73	75.59	73.58	369.30	107.48	98.05	96.47	80.26	369.24	76.68	68.52	64.08	59.80	369.05
0.0030	82.47	69.86	65.27	63.49	369.00	94.12	85.51	84.08	69.46	368.91	66.20	58.97	55.06	51.30	368.63
0.0035	73.02	61.58	57.45	55.86	368.63	83.72	75.82	74.51	61.23	368.52	58.25	51.77	48.28	44.94	368.14
0.0040	65.52	55.06	51.32	49.88	368.22	75.39	68.10	66.91	54.76	368.07	52.01	46.14	43.00	40.00	367.58
0.0050	54.36	45.46	42.31	41.11	367.22	62.88	56.59	55.59	45.23	366.99	42.84	37.92	35.31	32.82	366.23
0.0060	46.45	38.72	36.02	34.99	366.01	53.92	48.42	47.55	38.55	365.67	36.42	32.21	29.98	27.86	364.59
0.0070	40.55	33.74	31.37	30.48	364.59	47.20	42.32	41.56	33.61	364.13	31.69	28.00	26.07	24.22	362.67
0.0080	35.99	29.90	27.80	27.02	362.96	41.97	37.58	36.91	29.80	362.37	28.04	24.78	23.07	21.44	360.48
0.0090	32.35	26.85	24.97	24.27	361.13	37.78	33.80	33.20	26.78	360.39	25.15	22.23	20.70	19.25	358.03
0.0100	29.38	24.38	22.67	22.04	359.11	34.35	30.72	30.18	24.32	358.20	22.81	20.16	18.78	17.47	355.32
0.0300	10.37	8.68	8.15	7.99	289.14	12.13	10.89	10.74	8.75	283.68	7.99	7.17	6.77	6.38	267.29
0.0500	6.28	5.34	5.06	4.99	203.94	7.28	6.59	6.53	5.41	195.84	4.85	4.42	4.22	4.02	173.16
0.1000	3.12	2.75	2.66	2.65	74.50	3.50	3.24	3.23	2.79	68.59	2.45	2.30	2.25	2.19	53.89
0.2000	1.55	1.46	1.44	1.44	13.15	1.63	1.57	1.57	1.46	11.65	1.31	1.28	1.27	1.26	8.28
0.3000	1.14	1.12	1.11	1.11	3.87	1.16	1.14	1.14	1.11	3.44	1.05	1.05	1.05	1.04	2.51
0.4000	1.03	1.02	1.02	1.02	1.81	1.03	1.02	1.02	1.02	1.65	1.01	1.00	1.00	1.00	1.34
0.5000	1.00	1.00	1.00	1.00	1.22	1.00	1.00	1.00	1.00	1.16	1.00	1.00	1.00	1.00	1.06

The following trends can be noted from Tables 1–6:

1. For all other fixed parameters, the values of ARL decrease as the sample size increases. For example, when $ARL_0 = 300,\ i = 1, \rho = 0.3,\ f = 0.0005$, the ARL = 263 for n = 5, and ARL = 213 for n = 30.
2. For all other fixed parameters, the values of ARL decrease as i increases from 1 to 4.
3. It is found that the performance becomes better as this correlation gets stronger.
4. The performance in terms of ARL becomes better as the subgroup size increases.

Figure 1 shows the $ARLs$ according to the shift constant f for different values of n when $ARL_0 = 370$, $i = 1$ and $\rho = 0.6$. It shows that $ARLs$ decrease faster as the sample size increases. Figure 2 shows the $ARLs$ according to the shift constant f for different values of n when $ARL_0 = 370$, $i = 3$, and $\rho = 0.6$. It is seen that ARL curves may meet each other as the sample size increases. It means that the difference between $ARLs$ decreases as the sample size increases for higher values of i.

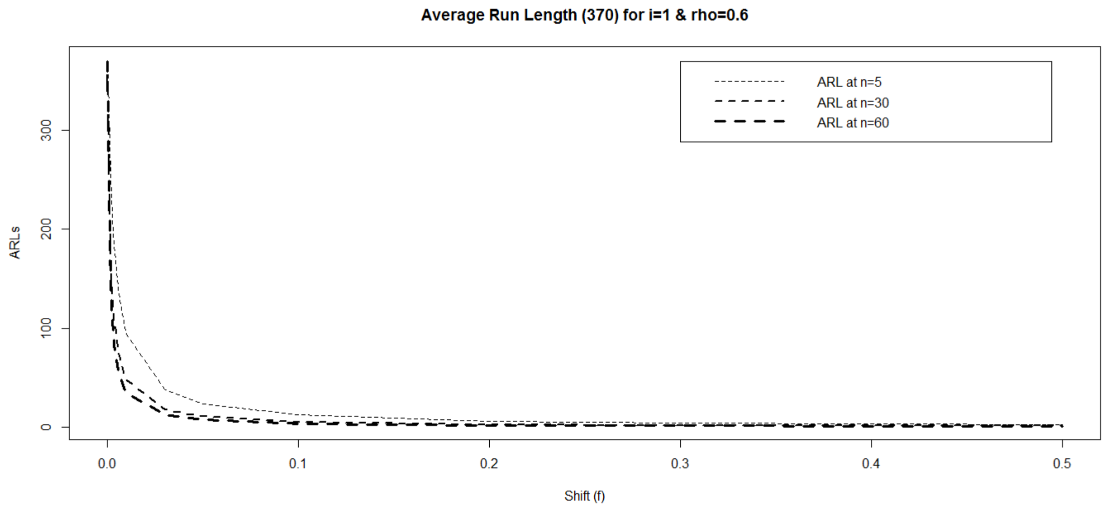

Figure 1. Trend of $ARLs$ according to shift constant when $ARL_0 = 370$, $i = 1$ and $\rho = 0.6$ with $n = 5,\ 30,\ 60$.

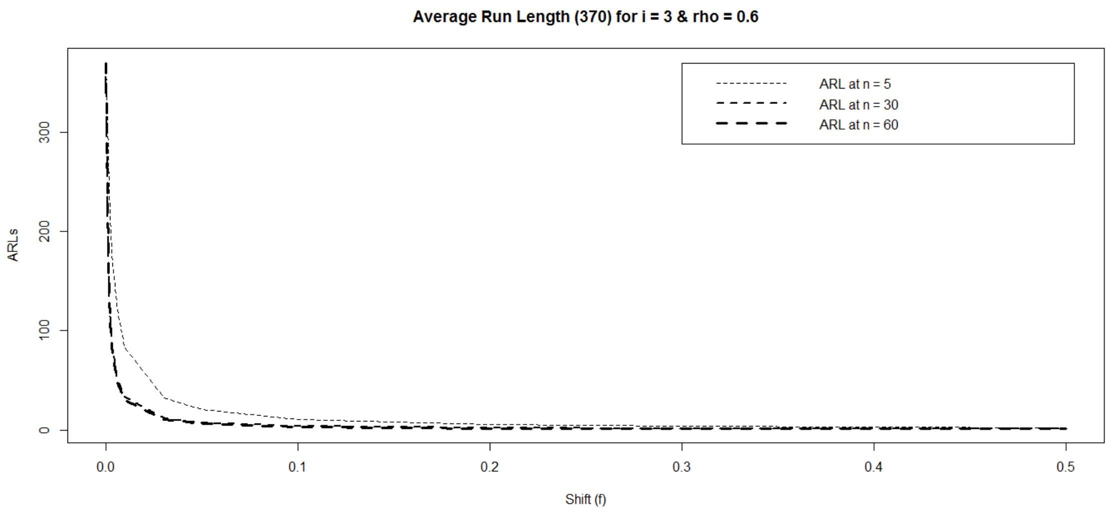

Figure 2. Trend of $ARLs$ according to shift constant when $ARL_0 = 370$, $i = 3$ and $\rho = 0.6$ with $n = 5,\ 30,\ 60$.

4. Simulation Study

In this section, we will compare the efficiency of the proposed control chart over the existing control charts using the repetitive sampling and multiple dependent state (MDS) sampling through the simulated data. The comparisons between charts will be given for the same values of control chart parameters.

The methodology of developing the proposed control chart using MDS repetitive sampling for process mean monitoring will be further explained via simulation data. In this simulation study, we consider the case of $ARL_0 = 370$, $n = 30$, $i = 2$, $k_1 = 3.0275$, $k_2 = 1.2171$, and $\rho = 0.6$. A simulation data have been generated for constructing the control chart using the above mentioned parametric values. First 20 subgroups are generated from the in-control state of the process with mean zero and standard deviation 3, while the next 18 subgroups are generated from an out-of-control process with $f = 0.01$.

Figure 3 shows the proposed control chart, where an out-of-control signal appears at 38th subgroup. The same data is also plotted on an control chart using repetitive sampling in Figure 4. From Figure 4, it is noted that all points lie between LCL_1 and UCL_1, which cannot detect a shift in the process. The Figure 5 shows the control chart based on the MDS sampling. The control statistics are also plotted on this chart, which again cannot detect a shift in the process. So, by comparing the proposed chart with existing charts, it can be seen that the proposed chart has ability to detect a shift in the process as compared to other charts.

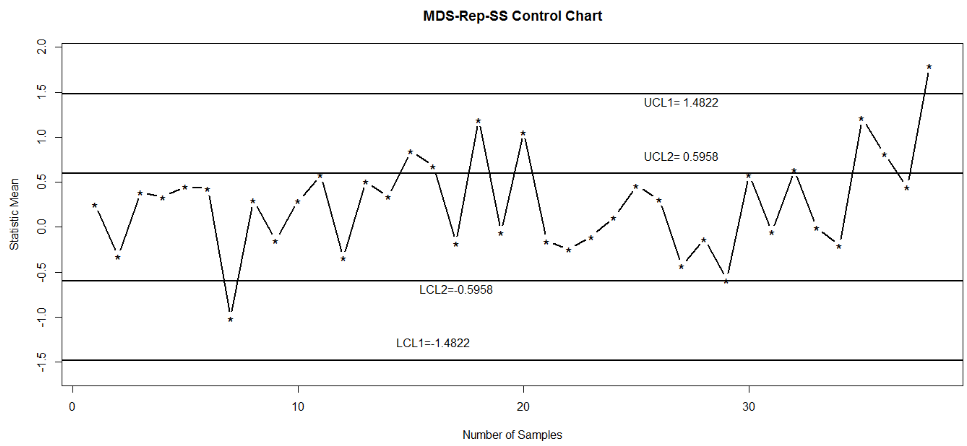

Figure 3. Proposed control chart for simulation data: ARL_0 = 370 for $i = 2$ and $\rho = 0.6$ with $n = 30$.

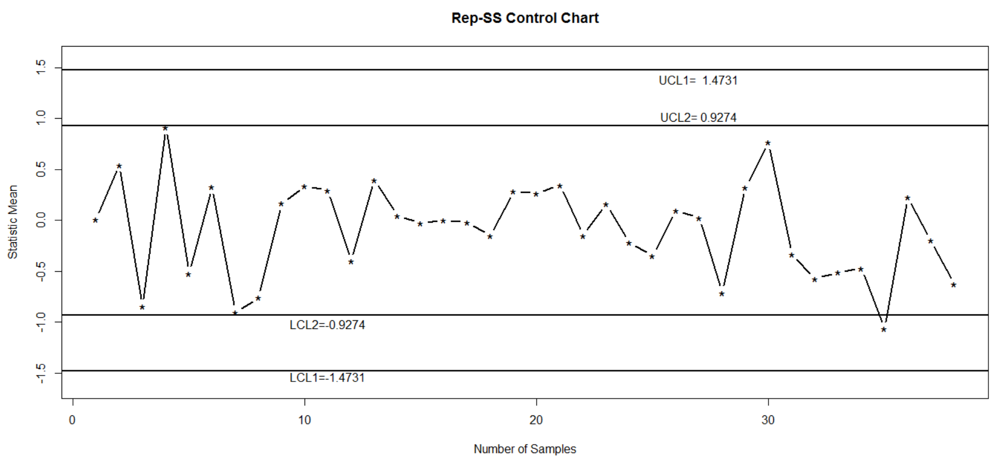

Figure 4. Control chart using repetitive sampling for simulation data: ARL_0 = 370 for $i = 2$ and $\rho = 0.6$ with $n = 30$.

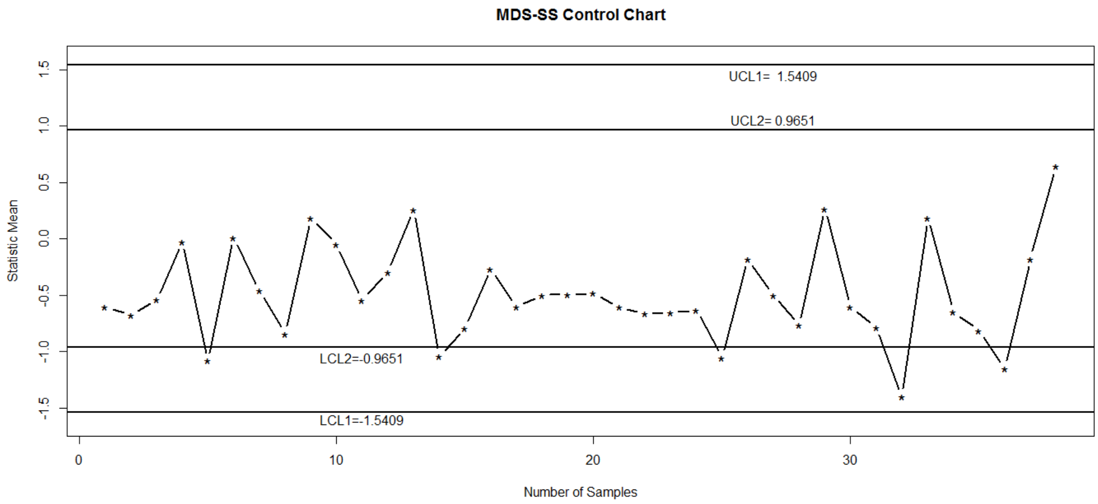

Figure 5. Control chart using MDS sampling for simulation data: ARL_0 = 370 for $i = 2$ and $\rho = 0.6$ with $n = 30$.

5. Comparison of Proposed Chart with Existing Charts

In this section, we compare the performance of the proposed chart with some existing charts including Shewhart control chart, repetitive sampling control chart, and the multiple dependent state sampling control chart using successive sampling.

Tables 7–9 have been prepared for $\rho = 0.3,\ 0.6$ and 0.9 when ARL_0 = 300, n = 5. The control chart coefficients were given. The *ARL* values of the existing charts and the proposed chart for different shift levels $f = 0.00,\ 0.10,\ 0.20,\ 0.30,\ 0.40,$ and 0.50 have been estimated and given in Tables 7–9.

Table 7. *ARL* comparison when $ARL = 300,\ n = 5, rho = 0.3$.

f	Shewhart Control Chart	Repetitive Sampling Control Chart	MDS Sampling Chart	Proposed Chart
			i = 2	i = 2
	k = 2.9352	k_1 = 2.9394	k_1 = 3.5987	k_1 = 2.9701
		k_2 = 2.3999	k_2 = 2.0604	k_2 = 1.1537
0.0	300.08	300	300	300
0.1	241.6	239.91	234.75	9.79
0.2	147.47	144.2	129.56	5.03
0.3	83.83	80.46	63.2	3.38
0.4	48.32	45.36	31.04	2.55
0.5	28.89	26.42	16.19	2.05

Table 8. *ARL* comparison when $ARL = 300,\ n = 5, rho = 0.6$.

f	Shewhart Control Chart	Repetitive Sampling Control Chart	MDS Sampling Chart	Proposed Chart
			i = 2	i = 2
	k = 2.9352	k_1 = 2.9394	k_1 = 3.5881	k_1 = 2.9861
		k_2 = 2.3999	k_2 = 2.0612	k_2 = 1.0345
0	300.08	300	300	300
0.1	241.6	235.76	230.13	8.01
0.2	147.47	137.45	122.21	4.18
0.3	83.83	74.77	57.66	2.86
0.4	48.32	41.33	27.69	2.19
0.5	28.89	23.71	14.26	1.8

Table 9. *ARL* comparison when $ARL = 300,\ n = 5, rho = 0.9$.

f	Shewhart Control Chart	Repetitive Sampling Control Chart	MDS Sampling Chart	Proposed Chart
			i = 2	i = 2
	k = 2.9352	k_1 = 2.9394	k_1 = 2.9579	k_1 = 2.9706
		k_2 = 2.3999	k_2 = 2.4959	k_2 = 1.1493
0	300.08	300	300	300
0.1	241.6	223.2	220.97	8.4
0.2	147.47	118.96	114.85	4.31
0.3	83.83	60.23	56.16	2.89
0.4	48.32	31.52	28.32	2.18
0.5	28.89	17.34	15.1	1.76

It can be found from these tables that the proposed chart is much faster in detecting an out-of-control process. For instance, if a process faces a shift of 0.30 then the Shewhart chart will indicate the out-of-control process after an average of 83.83 samples, Shewhart control chart 82.19, repetitive sampling control chart 80.46 and multiple dependent state sampling chart will indicate an out-of-control process in 63.20 samples while the proposed successive sampling scheme detects the same process in only 3.38 average samples. The similar detecting ability of the proposed chart for $\rho = 0.6$ and 0.9 can be observed in Tables 7–9.

6. Concluding Remarks

In this article, we presented a control chart for process mean monitoring using successive sampling over two occasions and MDS sampling. The coefficients of the proposed control charts have been estimated for the target in-control *ARL*. Extensive tables have been constructed for different process settings to evaluate the monitoring ability of the proposed scheme. It has been observed that the proposed chart is comparatively efficient than four other existing charts in terms of the out-of-control *ARLs*. The proposed chart using successive sampling works well when the subgroup size is large. The performance in terms of *ARL* becomes better as the subgroup size increases. It is found that the performance becomes better as this correlation gets stronger. The proposed chart will be an efficient addition in the toolkit of the quality control personnel. The proposed control chart can be only used when the quality of interest follows the normal distribution. The proposed control chart for multivariate distribution can be considered as future research.

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