A. Supplemantary Materials

i. Firm, first optimization problem

First, for any given level of labor costs, they have to find the output-maximizing combination of labor.

$$\max_{N_{ijt}} \left(\int_0^1 N_{ijt}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \equiv N_t$$
$$s.t \quad \int_0^1 W_{jt} N_{ijt} dj \leq Z_t$$

$$\mathscr{L} = \left(\int_0^1 N_{ijt}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}} + \left(Z_t - \int_0^1 W_{jt} N_{ijt} dj \right)$$

Solve this problem,

$$\frac{\partial \mathscr{L}}{\partial N_{ijt}} = \frac{\varepsilon_w}{\varepsilon_w - 1} \left(\int_0^1 N_{ijt}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1} - 1} \frac{\varepsilon_w - 1}{\varepsilon_w} N_{ijt}^{-\frac{1}{\varepsilon_w}} - \lambda_t W_{jt} = 0$$

$$\Rightarrow \quad N_t^{\frac{1}{\varepsilon_w}} N_{ijt}^{-\frac{1}{\varepsilon_w}} = \lambda_t W_{jt}$$
$$N_t^{\frac{1}{\varepsilon_w}} N_{ikt}^{-\frac{1}{\varepsilon_w}} = \lambda_t W_{kt}$$

$$\Rightarrow \quad \left(\frac{N_{ijt}}{N_{ikt}} \right)^{-\frac{1}{\varepsilon_w}} = \frac{W_{jt}}{W_{kt}} \quad \Rightarrow \quad N_{ijt} = \left(\frac{W_{jt}}{W_{kt}} \right)^{\varepsilon_w} \cdot N_{ikt}$$

Insert N_{ijt} we got to the constraint mentioned above, then

$$Z_t = \int_0^1 W_{jt} N_{ijt} dj = \int_0^1 W_{jt} \left(\frac{W_{jt}}{W_{kt}} \right)^{\varepsilon_w} N_{ikt} dj = \int_0^1 W_{jt}^{1-\varepsilon_w} dj W_{kt}^{\varepsilon_w} N_{ikt}$$

$$\Rightarrow \quad N_{ikt} = \frac{Z_t W_{kt}^{-\varepsilon_w}}{\int_0^1 W_{jt}^{1-\varepsilon_w} dj}$$

We insert N_{ikt} again to N_t,

$$N_t = \left(\int_0^1 N_{ijt}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}} = \left[\int_0^1 \left(\frac{Z_t W_{jt}^{-\varepsilon_w}}{\int_0^1 W_{jt}^{1-\varepsilon_w} dj} dj \right)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} = Z_t \left[\int_0^1 \frac{W_{jt}^{1-\varepsilon_w}}{\left(\int_0^1 W_{jt}^{1-\varepsilon_w} dj \right)^{\frac{\varepsilon_w - 1}{\varepsilon_w}}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

$$\Rightarrow \quad Z_t \left[\left(\int_0^1 W_{jt}^{1-\varepsilon_w} dj \right)^{1 - \frac{(\varepsilon_w - 1)}{\varepsilon_w}} \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} = Z_t \left(\int_0^1 W_{jt}^{1-\varepsilon_w} dj \right)^{\frac{1}{\varepsilon_w - 1}} \equiv 1$$

Since $W_t \int_0^1 N_{it} di = Z_t \quad \Rightarrow \quad W_t = Z_t$, thus

$$\therefore \;\; W_t = \left(\int_0^1 W_{jt}^{1-\varepsilon_w} dj \right)^{\frac{1}{1-\varepsilon_w}}$$

We insert N_{ijt} again to Z_t, to get the demand of labor for firm i

$$Z_t = \int_0^1 W_{jt} N_{ijt} dj = \int_0^1 W_{jt} \left(\frac{W_{jt}}{W_{kt}} \right)^{-\varepsilon_w} N_{ikt} dj = N_{ikt} W_{kt}^{\varepsilon_w} \int_0^1 W_{jt}^{1-\varepsilon_w} dj$$

$$\Rightarrow \quad Z_t = N_{ikt} W_{kt}^{\varepsilon_w} W_t^{1-\varepsilon_w} \Rightarrow \quad N_{ikt} = \left(\frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} \frac{Z_t}{W_t} = \left(\frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} N_{it}$$

$$\therefore \;\; N_{ijt} = \left(\frac{W_{jt}}{W_t} \right)^{-\varepsilon_w} N_{it}$$

ii. Firm, second optimization problem

Second, given this optimal labor vector, firms have to set prices such that profit is maximized.

$$\max_{P_t^*} \left\{ \sum_{k=0}^{\infty} \theta_p^k E_t \left[Q_{t,t+k} \left(P_t^* Y_{it+k|t} - TC_{it+k|t}^n \left(Y_{it+k|t} \right) \right) \right] \right\}$$
$$s.t. \;\; Y_{it+k|k} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon_p} C_{t+k}$$

$$\Rightarrow \max_{P_t^*} \left\{ \sum_{k=0}^{\infty} \theta_p^k E_t \left[\beta^k (1-\lambda) \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \left(P_t^* \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon_p} C_{t+k} - TC_{it+k|t}^n \left(\left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon_p} C_{t+k} \right) \right) \right] \right\}$$

Find the optimal price P_t^*

$$P_t^* = \frac{\varepsilon_p}{(\varepsilon_p - 1)(1-\lambda)} \frac{E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon_p} MC_{t+k|t}^r}{E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon_p - 1}}$$

Log-linearized P_t^*

$$\frac{P_t^*}{P_{t-1}} = \frac{\varepsilon_p}{(\varepsilon_p - 1)(1-\lambda)} \frac{E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon_p} MC_{t+k|t}^r}{E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon_p - 1}} \frac{1}{P_{t-1}}$$

$$\Rightarrow \frac{P_t^*}{P_{t-1}} E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon_p - 1} = \frac{\varepsilon_p}{(\varepsilon_p - 1)(1-\lambda)} E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon_p} MC_{t+k|t}^r \frac{1}{P_{t-1}}$$

A first-order Taylor expansion of the LHS:

$$\sum_{k=0}^{\infty}\theta_p^k\beta^k C^{1-\sigma}P^{\varepsilon_p-1}+\frac{1}{P}E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k C^{1-\sigma}P^{\varepsilon_p-1}\left(P_t^*-P\right)$$
$$-\frac{P}{P^2}E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k C^{1-\sigma}P^{\varepsilon_p-1}(P_{t-1}-P)$$
$$+E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k C^{1-\sigma}(\varepsilon_p-1)P^{\varepsilon_p-2}(P_{t+k}-P)$$
$$+E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k(1-\sigma)C^{-\sigma}P^{\varepsilon_p-1}(C_{t+k}-C)$$

$$=\; C^{1-\sigma}P^{\varepsilon_p-1}E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k\left[1+\hat{p}_t^*-\hat{p}_{t-1}+(\varepsilon_p-1)\hat{p}_{t+k}+(1-\sigma)\hat{c}_{t+k}\right]$$

A first-order Taylor expansion of the RHS:

$$\frac{\varepsilon_p}{(\varepsilon_p-1)(1-\lambda)}\sum_{k=0}^{\infty}\theta_p^k\beta^k C^{1-\sigma}P^{\varepsilon_p-1}MC^r-\frac{\varepsilon_p}{(\varepsilon_p-1)(1-\lambda)}\sum_{k=0}^{\infty}\theta_p^k\beta^k C^{1-\sigma}P^{\varepsilon_p}MC^r\frac{1}{P^2}(P_{t-1}-P)$$
$$+\frac{\varepsilon_p}{(\varepsilon_p-1)(1-\lambda)}E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k C^{1-\sigma}\varepsilon_p P^{\varepsilon_p-1}MC^r\frac{1}{P}(P_{t+k}-P)$$
$$+\frac{\varepsilon_p}{(\varepsilon_p-1)(1-\lambda)}E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k(1-\sigma)C^{-\sigma}P^{\varepsilon_p}MC^r\frac{1}{P}(C_{t+k}-C)$$
$$+\frac{\varepsilon_p}{(\varepsilon_p-1)(1-\lambda)}\sum_{k=0}^{\infty}\theta_p^k\beta^k C^{1-\sigma}P^{\varepsilon_p-1}MC^r\frac{1}{P}(MC^r_{t+k|t}-MC^r)$$

$$=\; C^{1-\sigma}P^{\varepsilon_p-1}E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k\left[1+-\hat{p}_{t-1}+\varepsilon_p\hat{p}_{t+k}+(1-\sigma)\hat{c}_{t+k}+\hat{mc}^r_{t+k|t}\right]$$

The expressions on the LHS and RHS can be rearranged to appear as follows.

$$\Rightarrow\; E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k(\hat{p}_t^*-\hat{p}_{t+k})=E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k(\hat{mc}^r_{t+k|t})$$

$$\Rightarrow\; \frac{\hat{p}_t^*}{1-\theta_p\beta}=E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k\left(\hat{mc}^r_{t+k|t}+\hat{p}_{t+k}\right)$$

$$\therefore\,\hat{p}_t^*=(1-\theta_p\beta)E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k(mc^{\hat{r}}_{t+k|t}+\hat{p}_{t+k})$$

B. Supplemantary Materials

i. Household Optimization

Unrestricted households (represented by the share of λ) maximize their lifetime utility while discounting the future with a factor $\beta \in (0, 1)$.

$$\max_{C_{t,s}, N_{t,s}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(C_{t,s}, N_{t,s}\right) \right\} \tag{1}$$

$$s.t. \quad \int_0^1 P_t C_{t,s} + Q_t B_{t,s} \leq B_{t-1,s} + W_t N_{t,s} + T_t \tag{2}$$

F.O.Cs

$$1 = \beta Q_t^{-1} E_t \left\{ \left(\frac{C_{t+1,s}}{C_{t,s}} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \tag{3}$$

$$C_{t,s}^{\sigma} N_{t,s}^{\varphi} = \frac{W_t}{P_t} \tag{4}$$

Log-linearization yields:

$$\hat{c}_{t,s} = E_t \hat{c}_{t+1,s} - \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1}) \tag{5}$$

$$\sigma \hat{c}_{t,s} + \varphi \hat{n}_{t,s} = \hat{\omega}_t \tag{6}$$

Where ω_t indicates a real wage $\frac{W_t}{P_t}$ and $\hat{\omega}_t$ denotes a linear form of real wage.

ii. Rule of thumb Optimization

Restricted households (represented by the share of $1 - \lambda$) also maximize their lifetime utility in the same way as unrestricted households. However, their budget constraint is different because they don't consider inter-temporal problems.

$$\max_{C_{t,h}, N_{t,h}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(C_{t,h}, N_{t,h}\right) \right\} \tag{7}$$

$$s.t. \quad \int_0^1 P_t C_{t,h} \leq W_t N_{t,h} + T_t \tag{8}$$

Due to their inability to maximize consumption inter-temporally, we can deduce the following:

$$C_{t,h} = \frac{1}{P_t} \left(W_t N_{t,h} + T_t\right) \tag{9}$$

$$C_{t,h}^{\sigma} N_{t,h}^{\varphi} = \frac{W_{t,h}}{P_t} \tag{10}$$

Log-linearized yields:

$$\hat{c}_{t,h} = \hat{\omega}_t + \hat{n}_{t,h} \tag{11}$$

$$\sigma\hat{c}_{t,h} + \varphi\hat{n}_{t,h} = \hat{\omega}_t \tag{12}$$

iii. Optimal wage selection

Rewrite the flow budget constraints for $C_{t+k|t}$

$$C_{t+k|t} = \frac{1}{P_{t+k}}\left(D_{t+k|t} + W_t^* N_{t+k|t} + T_{t+k} - E_{t+k}Q_{t+k,t+k+1}D_{t+k+1|t}\right)$$

Insert $C_{t+k|t}$ and $N_{t+k|t}$ we derived in firms part to the utility function:

$$E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k u\left(C_{t+k|t}, N_{t+k|t}\right)\right\}$$

$$= E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k u\left(\left[\frac{1}{P_{t+k}}\left(D_{t+k|t} + W_t^* N_{t+k|t} + T_{t+k} - E_{t+k}Q_{t+k,t+k+1}D_{t+k+1|t}\right)\right], \left[\left(\frac{W_t^*}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}\right]\right)\right\}$$

$$= E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k u\left(\left[\frac{1}{P_{t+k}}\left(D_{t+k|t} + W_t^* \left(\frac{W_t^*}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k} + T_{t+k} - E_{t+k}Q_{t+k,t+k+1}D_{t+k+1|t}\right)\right], \left[\left(\frac{W_t^*}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}\right]\right)\right\}$$

$$\max_{W_t^*} E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k u\left(\left[\frac{1}{P_{t+k}}\left(D_{t+k|t} + W_t^* \left(\frac{W_t^*}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k} + T_{t+k} - E_{t+k}Q_{t+k,t+k+1}D_{t+k+1|t}\right)\right], \left[\left(\frac{W_t^*}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}\right]\right)\right\}$$

Solve the problem. FOC:

$$E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k \left[u_C\left(C_{t+k|t}, N_{t+k|t}\right)\frac{1}{P_{t+k}}(1-\varepsilon_w)\frac{W_t^*}{W_{t+k}}^{-\varepsilon_w} N_{t+k} - u_N\left(C_{t+k|t}, N_{t+k|t}\right)\varepsilon_w \frac{W_t^*}{W_{t+k}}^{-\varepsilon_w - 1}\frac{1}{W_{t+k}}N_{t+k}\right]\right\}$$

$\Rightarrow$

$$E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k \left[u_C\left(C_{t+k|t}, N_{t+k|t}\right)\frac{1}{P_{t+k}}(1-\varepsilon_w)\frac{W_t^*}{W_{t+k}}^{-\varepsilon_w} N_{t+k} - u_N\left(C_{t+k|t}, N_{t+k|t}\right)\varepsilon_w \frac{1}{W_t^*}\frac{W_t^*}{W_{t+k}}^{-\varepsilon_w} N_{t+k}\right]\right\}$$

$$= E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k \left[u_C\left(C_{t+k|t}, N_{t+k|t}\right)\frac{1}{P_{t+k}}(1-\varepsilon_w)N_{t+k|t} - u_N\left(C_{t+k|t}, N_{t+k|t}\right)\varepsilon_w \frac{1}{W_t^*}N_{t+k|t}\right]\right\} = 0$$

Multiply both sides by $\frac{W_t^*}{1-\varepsilon_w}$:

$$E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k \left[u_C\left(C_{t+k|t}, N_{t+k|t}\right)\frac{1}{P_{t+k}}N_{t+k|t}W_t^* + \frac{\varepsilon_w}{\varepsilon_w - 1}u_N\left(C_{t+k|t}, N_{t+k|t}\right)N_{t+k|t}\right]\right\} = 0$$

Define $MRS_{t+k|t} \equiv -\frac{u_N\left(C_{t+k|t}, N_{t+k|t}\right)}{u_C\left(C_{t+k|t}, N_{t+k|t}\right)}$ as the marginal rate of substitution between consumption and hours in period t+k for a household resetting the wage in period t.

$$E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k \left[N_{t+k|t}u_C\left(C_{t+k|t}, N_{t+k|t}\right)\left(\frac{W_t^*}{P_{t+k}} - \frac{\varepsilon_w}{\varepsilon_w - 1}MRS_{t+k|t}\right)\right]\right\} = 0$$

$\Rightarrow$

$$E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[\left(\frac{W_t^*}{W_{t+k}} \right)^{-\varepsilon_w} N_{t+k} u_C \left(C_{t+k|t}, N_{t+k|t} \right) \frac{W_t^*}{P_{t+k}} \right] \right\} = \frac{\varepsilon_w}{\varepsilon_w - 1} E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[\left(\frac{W_t^*}{W_{t+k}} \right)^{-\varepsilon_w} N_{t+k} u_C \left(C_{t+k|t}, N_{t+k|t} \right) MRS_{t+k|t} \right] \right\}$$

$\Rightarrow$

$$W_t^{*1-\varepsilon_w} E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{1}{W_{t+k}^{-\varepsilon_w}} u_C \left(C_{t+k|t}, N_{t+k|t} \right) \frac{N_{t+k}}{P_{t+k}} \right\} = \frac{\varepsilon_w}{\varepsilon_w - 1} W_t^{*-\varepsilon_w} E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[\frac{1}{W_{t+k}^{-\varepsilon_w}} N_{t+k} u_C \left(C_{t+k|t}, N_{t+k|t} \right) MRS_{t+k|t} \right] \right\}$$

$$\therefore \quad W_t^* = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k W_{t+k}^{\varepsilon_w} N_{t+k} u_C \left(C_{t+k|t}, N_{t+k|t} \right) MRS_{t+k|t}}{E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k W_{t+k}^{\varepsilon_w} u_C \left(C_{t+k|t}, N_{t+k|t} \right) \frac{N_{t+k}}{P_{t+k}}}$$

And the linearization process aligns with the goods price.

C. Supplemantary Materials

i. Aggregate dynamic IS curve

We will derive the necessary equations by using equations (18), (19), (22), (27), (28), (33), and (34). First, we use the labor aggregate equation for getting the $\hat{n}_{t,s}$

$$\begin{aligned}
\hat{n}_t &= \lambda \hat{n}_{t,h} + (1-\lambda)\hat{n}_{t,s} \\
\Rightarrow \hat{n}_{t,s} &= \frac{1}{1-\lambda}(\hat{n}_t - \lambda \hat{n}_{t,h}) \\
&= \frac{1}{1-\lambda} \left(\frac{1}{1-\alpha} (\hat{y}_t - \hat{a}_t) - \lambda \hat{n}_{t,h} \right) \qquad (\because \hat{y}_t = \hat{a}_t + (1-\alpha)\hat{n}_t)
\end{aligned}$$

Second, use again the consumption aggregate equation for getting the $\hat{c}_{t,s}$;

$$\begin{aligned}
\hat{c}_t &= \lambda \hat{c}_{t,h} + (1-\lambda)\hat{c}_{t,s} \\
\Rightarrow \hat{c}_{t,s} &= \frac{1}{1-\lambda}(\hat{c}_t - \lambda \hat{c}_{t,h}) \\
&= \frac{1}{1-\lambda} (\hat{y}_t - \lambda \hat{c}_{t,h}) \qquad (\because \hat{y}_t = \hat{c}_t)
\end{aligned}$$

Third, we utilize the equation $\sigma\hat{c}_{t,s} + \varphi\hat{n}_{t,s} = \hat{\omega}_t = \sigma\hat{c}_{t,h} + \varphi\hat{n}_{t,h}$ and rephrase it as $\hat{n}_{t,h}$;

$$\begin{aligned}
\sigma\hat{c}_{t,h} + \varphi\hat{n}_{t,h} &= \sigma\hat{c}_{t,s} + \varphi\hat{n}_{t,s} \\
&= \frac{\varphi}{1-\lambda}\left[\frac{1}{1-\alpha}\left(\hat{y}_t - \hat{a}_t\right) - \lambda\hat{n}_{t,h}\right] + \frac{\sigma}{1-\lambda}\left(\hat{y}_t - \lambda\hat{c}_{t,h}\right) \\
\Rightarrow \frac{\varphi}{1-\lambda}\hat{n}_{t,h} + \frac{\sigma}{1-\lambda}\hat{c}_{t,h} &= \frac{\varphi + (1-\alpha)\sigma}{(1-\lambda)(1-\alpha)}\hat{y}_t - \frac{\varphi}{(1-\lambda)(1-\alpha)}\hat{a}_t \\
\Rightarrow \varphi\hat{n}_{t,h} + \sigma\hat{c}_{t,h} &= \frac{\varphi + (1-\alpha)\sigma}{1-\alpha}\hat{y}_t - \frac{\varphi}{1-\alpha}\hat{a}_t
\end{aligned}$$

Rearrange the above equation as $\hat{n}_{t,h} \Rightarrow \hat{n}_{t,h} = \frac{1}{\varphi}\left[\frac{\varphi + (1-\alpha)\sigma}{1-\alpha}\hat{y}_t - \frac{\varphi}{1-\alpha}\hat{a}_t - \sigma\hat{c}_{t,h}\right]$

Fourth, we are going to express $\hat{c}_{t,h}$ as a function of aggregate;

$$\begin{aligned}
\hat{c}_{t,h} &= \hat{\omega}_t + \hat{n}_{t,h} \\
&= \sigma\hat{c}_{t,h} + (1+\varphi)\hat{n}_{t,h} \qquad (\because \hat{\omega}_t = \sigma\hat{c}_{t,h} + \varphi\hat{n}_{t,h}) \\
&= \sigma\hat{c}_{t,h} + (1+\varphi)\frac{1}{\varphi}\left[\frac{\varphi + (1-\alpha)\sigma}{1-\alpha}\hat{y}_t - \frac{\varphi}{1-\alpha}\hat{a}_t - \sigma\hat{c}_{t,h}\right]
\end{aligned}$$

And rearrange the above equation as $\hat{c}_{t,h}$;

$$\begin{aligned}
\hat{c}_{t,h} &= \frac{(1+\varphi)}{\varphi+\sigma} \cdot \frac{\varphi + (1-\alpha)\sigma}{1-\alpha}\hat{y}_t - \frac{\varphi(1+\varphi)}{(\varphi+\sigma)(1-\alpha)}\hat{a}_t \\
\therefore \quad \hat{c}_{t,h} &= \chi\hat{y}_t - \frac{\varphi(1+\varphi)}{(\varphi+\sigma)(1-\alpha)}\hat{a}_t\hat{a}_t \qquad (\because \ \chi = \frac{(1+\varphi)}{\varphi+\sigma} \cdot \frac{\varphi + (1-\alpha)\sigma}{1-\alpha})
\end{aligned}$$

Fifth, we continue to derive $\hat{c}_{t,s}$ as a function of aggregate;

$$\begin{aligned}
\hat{c}_t &= \lambda\hat{c}_{t,h} + (1-\lambda)\hat{c}_{t,s} \\
\Rightarrow \hat{c}_{t,s} &= \frac{1}{1-\lambda}(\hat{c}_t - \lambda\hat{c}_{t,h}) \\
&= \frac{1}{1-\lambda}\left(\hat{y}_t - \lambda\chi\hat{y}_t + \frac{\lambda\varphi(1+\varphi)}{(\varphi+\sigma)(1-\alpha)}\hat{a}_t\right) \\
&= \frac{1-\lambda\chi}{1-\lambda}\hat{y}_t + \frac{\lambda\varphi(1+\varphi)}{(1-\lambda)(\varphi+\sigma)(1-\alpha)}\hat{a}_t
\end{aligned}$$

Sixth, we will now derive the DIS curve in the two-agent model;

$$\hat{c}_{t,s} = E_t \hat{c}_{t+1,s} - \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1})$$

$$\Rightarrow \frac{1-\lambda\chi}{1-\lambda}\hat{y}_t + \frac{\lambda\varphi(1+\varphi)}{(1-\lambda)(\varphi+\sigma)(1-\alpha)}\hat{a}_t = \frac{1-\lambda\chi}{1-\lambda}E_t\hat{y}_{t+1} + \frac{\lambda\varphi(1+\varphi)}{(1-\lambda)(\varphi+\sigma)(1-\alpha)}\hat{a}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1})$$

After rearranging for $\hat{y}_t$, we can finally obtain the aggregate IS curve.

$$\therefore \quad \hat{y}_t = E_t\hat{y}_{t+1} - \frac{(1-\lambda)}{\sigma(1-\lambda\chi)}(\hat{i}_t - E_t\hat{\pi}_{t+1}) + \frac{\lambda\varphi(1+\varphi)}{(1-\lambda)(\varphi+\sigma)(1-\alpha)}(\rho_a - 1)\hat{a}_t$$