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One Who Hesitates Is Lost: Monetary Policy Under Model Uncertainty and Model Misspecification

Viktors Ajevskis

Institute of Applied Mathematics, Riga Technical University, Zunda krastmala 10, LV-1048 Rīga, Latvia; viktors.ajevskis@rtu.lv; Tel.: +371-29484773

Abstract: This paper investigates how different parametrisations of the monetary policy reaction function and different mechanisms of expectation formation shape the macroeconomic outcomes in the estimated Smets–Wouters type of DSGE model. The initial macroeconomic conditions of the simulations correspond to the high inflation environment of early 2022. The simulation results show that, under the hybrid expectations, the terminal monetary policy rate is significantly higher than under the rational expectations for all Taylor rule parametrisations. Under hybrid expectations, the inflation rate is much more persistent than under the rational expectations; three years is not enough to reach the inflation target of two percent, even for the quite hawkish calibration of the Taylor rule. In the modelled economy, relatively fast inflation stabilisation for the hawkish Taylor rule has its own price in form of the cumulative output loss when compared with the dovish Taylor rule. Simulations are also performed for the case where the central bank misspecifies the expectation formation mechanism in the DSGE model and follows an interest rate path implied by a false model. The results show that the hawkish reaction is preferable for both correctly and incorrectly specified models.

Keywords: monetary policy; expectations; inflation; GDP growth; DSGE; loss function

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1. Introduction

Surging inflation in developed economies during 2021–2023 presented a significant challenge to central banks in their pursuit of price stability. In such circumstances, answers to questions about how rapidly the inflation rate would return to the target level and the price to pay in the form of cumulative output loss depended on the strength of the central bank's policy response, i.e., how hawkish or dovish the central bank was when increasing the interest rate. At the same time, the economic outcomes of monetary policy actions also depend on how the economic agents react to various shocks and the actions of the central bank, which, in turn, are conditional on the expectation formation mechanism of the economic agents.

Dynamic stochastic general equilibrium (DSGE) models allow an understanding of the transmission mechanisms of monetary policy actions and the acquisition of quantitative estimations of the macroeconomic impacts of monetary policy decisions. Therefore, DSGE models are widely used for policy simulations. However, the use of models is not without caveats, particularly considering the uncertainty of economic models and their forecasts, which can present a problem in the decision-making processes of monetary policy. Among the uncertainties related to DSGE models is the specification of the means by which agents form their expectations and the potential variations in the parameters over time. Since different assumptions can significantly alter the predicted macroeconomic outcomes, understanding these sensitivities is critical for policymakers when using such models as input.

Studies investigating the impact of uncertainty surrounding the model coefficients on the optimal monetary policy date back to Brainard (1967), who showed that, when the parameter that links the policy instrument to the target variable is uncertain, the policy should be less aggressive. However, Brainard's results were obtained under a rather simple setup, with only one parameter being uncertain. The subsequent literature considers the uncertainty surrounding multiple parameters and finds the opposite to be true; see Soderstrom (2002), Kimura and Kurozumi (2007), and Cateau and Murchison (2010), among others. They argue that central banks should respond more aggressively when they are uncertain about the model parameters.

Another type of uncertainty that central banks have to deal with is the uncertainty regarding the mechanism of expectation formation. Although the rational expectation (RE) assumption is still widely used in DSGE modelling, the empirical literature finds evidence of deviation from it. Specifically, Landier et al. (2017), in an experimental study, find that rational expectations are rejected by the data for most participants in the experiment. Moreover, expectations are influenced by previous forecasts and tend to exaggerate the impact of the most recent shocks. Pfajfar and Zoakeli (2014) find that expectations are heterogeneous, with some subjects behaving in line with RE and others adhering to adaptive learning methods. Using the Survey of Professional Forecasters, Coibion and Gorodnichenko (2015) find the underreaction of the consensus forecast relative to the predictions of the RE model. Broer and Kohlhas (2018) also analyze survey data and find that forecasters revise their forecasts more than what is implied by the RE model. This has direct implications for practical policymaking: whereas, in the world of rational expectations, temporary inflation shocks can be "looked through" as they do not affect agents' medium-term inflation expectations in a meaningful way, such temporary shocks can turn out to be more persistent and lead to potentially more significant deviations from central banks' targets if the agents are less forward-looking and pay more attention to the current inflation rate when making their consumption and saving decisions.

Finally, as is obvious from the rather poor recent track record in terms of inflation forecasting across almost all major central banks, we have to acknowledge in this analysis the fact that the models used by most central banks are not always perfect. In order to address the problem of model uncertainty, two approaches are often used in the literature: the first is the Bayesian approach Cogley et al. (2011), which weighs each possibility of model specification by its prior probability; the second is the robust control or minimax method Hansen and Sargent (2007), where a policymaker aims to minimise the outcome of the worst-case scenario.

This paper investigates how assumptions regarding these uncertainties and different parameterisations of the monetary policy reaction function shape macroeconomic outcomes in the standard Smets–Wouters-type DSGE model; see Smets and Wouters (2003) and Smets and Wouters (2007). We first compare the optimal policy response given different forms of expectation formation, namely backward-looking expectations with elements of learning and RE.[1] Second, in a similar way as in robust control theory, we assume that the central bank may use incorrect models of expectation formation to define a path for the future interest rate, and we then consider what this means for inflation and the output. We also consider cases where the central bank is "learning" about the true state of the economy together with other economic agents, and, after observing the actual incoming data and comparing them to previous model predictions, it is ready and willing to adjust its views.

In particular, we consider the case in which the central bank realises that it has used an incorrect model, and it can switch to the correct model after some time. Third, we analyse different forms of potential non-linearity in the conduct of monetary policy. In a high-inflation environment, a central bank may raise the interest rate using the conventional linear Taylor rule until the inflation is reduced considerably or commit to keeping the interest rate at a somewhat lower level but for a longer time. The latter circumstance is often called the "higher-for-longer" approach and can be modelled by the Taylor rule with a threshold.

In general, our results show that the degree to which a central bank should be aggressive in the face of high inflation depends on the weight that it assigns to output loss in its objective function. These results indicate that a central bank with a strict price stability mandate, i.e., the output loss weight in the objective function is low, should be more hawkish in its conduct of monetary policy. This holds true irrespective of whether the model used by the central bank is the correct one or not. Our findings also reveal that, compared to the policy implied by a linear Taylor rule, the higher-for-longer policy provides a noticeable reduction in the cumulative output loss with a very small increase in cumulative inflation.

Among the works devoted to monetary policy analysis during the recent surge in inflation in industrial countries, the closest to ours are the series of papers by the IMF: Alvarez and Diziol (2023), Dizioli (2023), and WEO (2023). They develop a DSGE model with a mixture of forward- and backward-looking agents. They highlight the trade-off in reducing inflation quickly and avoiding a significant loss in output. Their results also reveal that, with a larger share of backward-looking agents in the economy, inflation is prolonged, the monetary policy weakens, and the output costs of monetary tightening rise. Our simulations also support these findings.

Our paper is different from the literature, such as (Alvarez and Diziol, 2023; Bartocci et al., 2023; Darracq Pariès et al., 2023; Dizioli, 2023; WEO, 2023), in several ways. First, most of the papers focus on impulse response functions for demand, supply, and monetary policy shocks, while we consider the forecast dynamics of the economy with high initial inflation and filtered values for the initial values of all state variables. This approach allows us to analyse the development of the macrovariables in a more comprehensive way. Second, the loss function in our welfare analysis is the deviation of inflation from the target and cumulative output loss after three years, but the IMF papers focus on the usual quadratic loss function. The loss function introduced in our study may reflect the preferences of policymakers more appropriately. Third, we also provide a welfare analysis for two types of uncertainty regarding (1) the expectation formation mechanism and (2) whether the correct or incorrect model is used by the central bank.

This paper is organised as follows. Section 2 describes the DSGE model used in the simulations. Section 3 discusses expectation formation mechanisms in DSGE models. Section 4.1 presents simulation results for models with rational and hybrid expectations and various parametrisations of the Taylor rule; Section 4.2 considers simulations for the case where the central bank uses an incorrect model. Sections 4.3 and 4.4 address the implications of the simulation results in terms of the loss function and the expected loss function, respectively. Section 4.5 discusses the consequences of the delay in the monetary policy response if a central bank uses an incorrect model. Section 4.6 compares non-linear policy responses, such as the higher-for-longer one, to linear Taylor rule policies. Section 5 provides the conclusions of this study.

2. Modelling Setup

This paper uses the Smets–Wouters Smets and Wouters (2003), Smets and Wouters (2007) and Linde et al. (2016) model. In this model, there is a continuum of households who supply household-specific labour in monopolistic competition and set wages. There is a continuum of intermediate goods firms who supply intermediate goods in monopolistic competition and set prices. Final goods use intermediate goods and are produced in perfect competition. To provide a reasonable fit of the model to the data for the euro area, a number of real and nominal frictions are introduced: staggered prices and wages, price and wage indexation, Kimball aggregation, an investment adjustment cost, and habit formation in consumption. The following shocks affect the economy: total factor productivity, investment-specific technology, household preferences, exogenous spending, price mark-up, wage mark-up, monetary policy, and risk premiums.

The monetary policy reaction function is defined by the Taylor rule

$$R_t = \rho R_{t-1} + (1-\rho)\left(r^* + \pi^* + \psi_p\left(\pi_t^{(4)} - \pi^*\right) + \psi_y y_t^{gap}\right) \tag{1}$$

where R_t is the annual nominal interest rate in time t; r^* is the annual real natural rate of interest fixed at 0.5% (inverse discount factor minus one), $\pi^* = 2.0\%$ is the annual steady state inflation, and $\pi_t^{(4)} = \sum_{i=1}^{4} \pi_{t-i+1}$ is the annual inflation. The output gap, y_t^{gap}, is defined as the difference between the actual output and the potential one that corresponds to the equilibrium of flexible prices. The interest smoothing parameter is calibrated as $\rho = 0.85$, which corresponds to the value used in Cecion et al. (2021). The benchmark specification assumes an output gap coefficient $\psi_y = 1$ and an inflation coefficient $\psi_p = 1.5$. As a counterfactual, a more hawkish reaction function is considered with $\psi_p = 2$, 4, and 7.

The model parameters are obtained by applying Bayesian estimation to the linearised model and using data spanning 1999Q1–2014Q2. This sample covers the period from the inception of the euro until the euro area policy interest rate breached the zero lower bound (ZLB). This sample is chosen to avoid the issue of non-linearity implied by the ZLB, thus allowing the use of the linearised model for estimation. The euro area macroeconomic time series include the same observable variables as in Smets and Wouters (2003): the real GDP, real consumption, real investment, GDP deflator, real wages, employment, and nominal interest rate.

Most of the calibrated parameters are set to the same values as in Smets and Wouters (2007). Specifically, the depreciation rate is 0.0025 per quarter, and the gross mark-up on wages is 1.5. The share of government spending in the output is 0.18. The curvature of the Kimball aggregator for wages and prices is set at 10. Exceptions include the steady-state inflation, which is set at 2%, and the discount rate, which equals 0.125, implying a discount factor of 0.99875. Additionally, we calibrate the parameters of the Taylor rule as described above. The remaining parameters are estimated. Information regarding the prior distribution, as well as the estimated mean, standard deviation, and posterior density 90% intervals for the parameters, is provided in Appendix A.

3. Expectation Formation in DSGE Models

The standard assumption in DSGE modelling posits that agents have RE, implying that they have complete knowledge of the underlying structure of the economy and that they make optimal decisions. Moreover, they are able to solve and estimate a DSGE model and, based on the obtained solution, make their forecasts regarding the true probabilistic expectations of the model's variables. However, as mentioned in the Introduction, empirical studies show that the RE hypothesis is rejected for most individuals and that expectations are influenced by previous forecasts and tend to exaggerate the impact of recent shocks. Moreover, the assumption of RE in DSGE models can produce peculiar outcomes, such as the forward guidance puzzle, i.e., an overly effective impact on the economy resulting from an announced future interest rate change by the central bank Del Negro et al. (2012). To address these issues, an HE formation mechanism has been proposed (see Gertler, 2017; Walsh, 2019), which incorporates past observations and model-based forecasts into agents' expectations. This approach produces better out-of-sample forecast properties than the RE assumption. This paper examines a particular specification of the HE mechanism proposed in Cecion et al. (2021):

$$\mathbb{E}_t x_{t+1} = \alpha \mathbb{E}_t^{RE} x_{t+1} + (1-\alpha)\mathbb{E}_t^{AE} x_{t+1} \quad (2)$$

$$\mathbb{E}_t^{AE} x_{t+1} = \delta \mathbb{E}_{t-1}^{AE} x_t + (1-\delta) x_t \quad (3)$$

where x_t is a forward-looking variable of interest, $\mathbb{E}_t^{RE}$ is an expectation operator under RE, $\mathbb{E}_t^{AE}$ is an expectation operator under autoregressive expectations, and α is a fraction of agents who understand the model and forecast the variable x_t according to the RE solution. The fraction $(1-\alpha)$ uses a learning scheme with an autoregressive component. In addition, these agents also update their beliefs according to the actual realisation of the variables of interest, $(1-\delta)x_t$. If $\alpha = 1$, the expectations are fully rational, and, if $\alpha = 0$, the expectations are fully backward-looking. A degree of backward-looking behaviour of 0.8 is chosen for both parameters (α and δ), as in Cecion et al. (2021). The mixture of rational and adaptive expectations is applied to prices. The application of a mixture of expectations to wages does not significantly change the results.

4. Results

4.1. Model Simulations Under Rational and Hybrid Expectations and Different Parametrisations of the Taylor Rule

The conventional DSGE modelling involves assuming that an economy is in its steady state before being impacted by a shock. Impulse response functions are used to illustrate how the economy adjusts back to its steady state following the shock. In fact, these functions are forecasts of the deviations in endogenous variables from their steady states, under the initial conditions set as one standard deviation for a shock and as the steady state for all other endogenous variables. Various forms of impulse response functions, even for the same variable but to different shocks, illustrate the significant influence of the initial economic conditions on the forecast paths of macrovariables.

Instead of simulating impulse response functions for different shocks at the steady state of the economy, we focus on the dynamics of macrovariables with the initial conditions obtained by employing the Kalman filter, as implied by the DSGE model, to euro area data up to 2022Q2. Specifically, the initial value for inflation is set at 8.6% and for output growth at 1.1%. With these initial conditions established, we proceed to compute the inflation, output growth, and nominal interest rates using the DSGE model. We consider three types of monetary policy reaction functions with different inflation coefficients: (a) benchmark ψ_p = 1.5; (b) hawkish ψ_p = 4; and (c) super-hawkish ψ_p = 7. The last coefficient is chosen to guarantee inflation being close to the target after eight quarters under the HE models.

Table 1 shows the results of the simulations. Columns 3 and 4 indicate the inflation rates at the end of 8 and 12 quarters, respectively. Under RE, only the hawkish reaction functions, ψ_p = 4, can reach the inflation target after eight quarters, with inflation being 2.01%. After 12, quarters, the target of 2% is reached in nearly all forms of reaction function. Under HE, the terminal rates are higher than for RE by 1.7 times for the benchmark and 1.3 for the aggressive reaction functions, ψ_p = 4. After eight quarters, both the benchmark and the hawkish reaction functions do not provide inflation close to the target, at 4.91% and 2.685%, respectively. After 12 months, the aggressive reaction function, ψ_p = 4, entails the inflation rate being relatively close to the target, namely 2.18%; meanwhile, the benchmark reaction function provides an inflation rate of 3.72%. The cumulative output loss is about two times higher for HE than for RE for each reaction function.

Table 1. Model-implied outcomes with different parametrisations of the Taylor rule and different forms of expectation. Notes: Rows represent scenarios assuming different values of the inflation coefficient in the Taylor rule, $\psi_p = 1.5, 2, 4$, and 7. HE features backward-looking expectations and elements of learning.

Inflation Coefficient	Terminal Monetary Policy Rate	Inflation After 8 Quarters	Inflation After 12 Quarters	Cumulative Output Loss
Rational expectations				
ψ_p = 1.5	3.41	2.44	2.15	1.38
ψ_p = 2	3.73	2.29	2.1	1.72
ψ_p = 4	5.51	2.01	2.02	2.61
ψ_p = 7	8.47	1.85	2.01	3.42
Hybrid expectations				
ψ_p = 1.5	5.76	4.91	3.72	2.90
ψ_p = 2	5.68	4.03	3.04	3.40
ψ_p = 4	7.19	2.68	2.18	5.00
ψ_p = 7	10.41	2.03	1.92	6.35

Under the super-hawkish reaction function, the inflation reaches the target for HE, but at the cost of a doubled output loss compared with the benchmark case for both RE and HE. The terminal rate for ψ_p = 7 is also much higher than for the benchmark and hawkish reaction functions—8.47% for RE and 10.41% for HE. Under the super-hawkish reaction function, inflation undershoots after two quarters for RE and after three quarters for HE. The reasoning behind these results is as follows. The presence of agents with backward-looking expectations in the HE model implies the higher persistence of inflation, as deviations from the central bank's target are not expected to automatically disappear but are instead gradually morphing into higher inflation expectations, thus potentially creating a self-enforcing inflationary loop. This results in the significantly slower decay of inflation than under RE. Consequently, the central bank has to raise the interest rate to a higher degree under RE. As a result, the terminal monetary policy rate is higher. The forward-looking agents in the HE model internalise this information and reduce their consumption and investment to a larger extent than in the RE model. For a lower degree of backward-looking behaviour, $\alpha = \delta = 0.5$, the results obtained are somewhere in between RE and HE with $\alpha = \delta = 0.8$ (see Appendix B).

4.2. Model Simulations with Uncertainty About the Central Bank Assumptions

In Section 4.1, we examine scenarios where the central bank employs the correct model. Here, we assume that the central bank instead uses an incorrect model to determine the future path of the nominal interest rate. Once the central bank sets this path, it adheres to it, disregarding the nominal rate suggested by the Taylor rule. At the same time, the central bank communicates this projected interest rate path to rational economic agents, who incorporate this information into their decision-making, assuming that they know the correct model. This setup parallels Type I ambiguity as defined by Hansen and Sargent (2012), where private agents know the correct probability model, while a central authority—represented here by the central bank—does not. We add another dimension, assuming that the central bank can be either dovish (ψ_p = 1.5) or hawkish (ψ_p = 4). For the technical details of the scenarios' implementation, see Appendix D.

Figure 1 shows the model-implied inflation paths under different central bank reactions in the case in which the HE model is correct, i.e., there is a significant share of agents in the economy that form inflation expectations based on the current inflation rates. The blue line represents an inflation path for the dovish central bank, which has mistakenly assumed that most agents in the economy are fully rational and has therefore followed the interest rate path implied by the RE model and reacts (ex post) too weakly to the inflation's deviation from the target. As a result, after 12 quarters, the inflation remains notably above the target of 2% and is higher than the inflation rate implied if the dovish central bank had used the correct HE model and acted more forcefully (gray line vs. blue line).

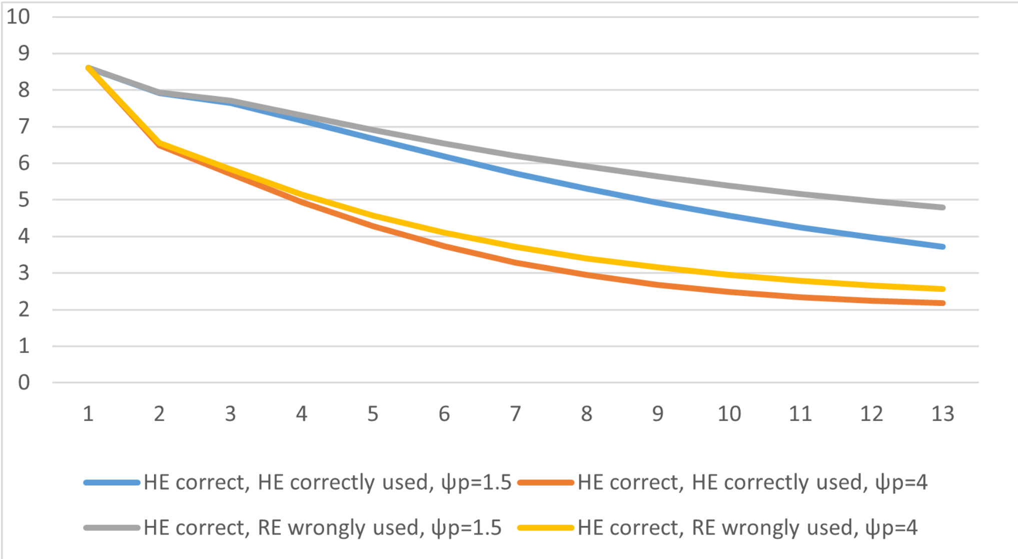

Figure 1. Model-implied inflation rate paths in the HE world and under different monetary policy rules. Notes: ψ_p = 1.5 represents a dovish central bank and ψ_p = 4 represents a hawkish central bank. Correct/false indicates whether the central bank has used the correct model (in this case, HE) to determine the interest rate path.

The hawkish central is able to bring down inflation to 2.18% in the third year if the central bank uses the correct HE model (orange line) and 2.56% if it uses the false RE model (yellow line). Thus, using the false RE model does not allow the target to be reached within a period of three years, although the difference in inflation between the correct and false models used is much smaller than in the case of a dovish central bank. In other words, being more hawkish brings a lower penalty, in the form of inflation overshot, when using the incorrect model in an HE environment. Thus, for an inflation-targeting central bank, uncertainty about whether the inflation expectations follow an RE or HE model would imply a bias towards a more "hawkish" policy stance, as it would allow one to minimise potential policy mistakes (in terms of larger and longer-lasting inflation deviations from the target).

If we assume that an RE model is correct (Figure 2), but the central bank uses the HE model's interest rate path, the inflation rate declines relatively quickly, reaching 2% after five quarters under a hawkish reaction function (yellow line) and after seven quarters under a dovish reaction function (orange line). In both cases, inflation undershoots the target level of 2% afterwards, but, for the hawkish reaction function, inflation bottoms out earlier, tending then towards the steady state of 2%, whereas, for the dovish reaction function, it bottoms out later. As a result, after 12 quarters, inflation under the hawkish regime is slightly higher than under the dovish one, at 1.79% vs. 1.50%.

Overall, however, being in the RE world is much more beneficial for the central bank, as potential policy mistakes produce much smaller inflation deviations from the target.

Thus, again, if, in the RE world, it does not matter much which type of model the central banks are using, while it does matter in the HE world, where potential policy mistakes related to "dovish" biases are more significant than those of "hawkish" biases, then, from a risk management perspective (if the central bank wishes to avoid the worst-case scenario), it can be assumed, as the default option, that we might be living in the HE world, and the correct approach for the central bank would be to be more "hawkish", at least initially, until a clearer picture emerges. Initially, the interest rate is considerably higher under the hawkish reaction function than under the dovish one (Figure 3). The terminal rate is higher by more than 1 pp. As a result, the inflation rate decreases more rapidly for an aggressive reaction function.

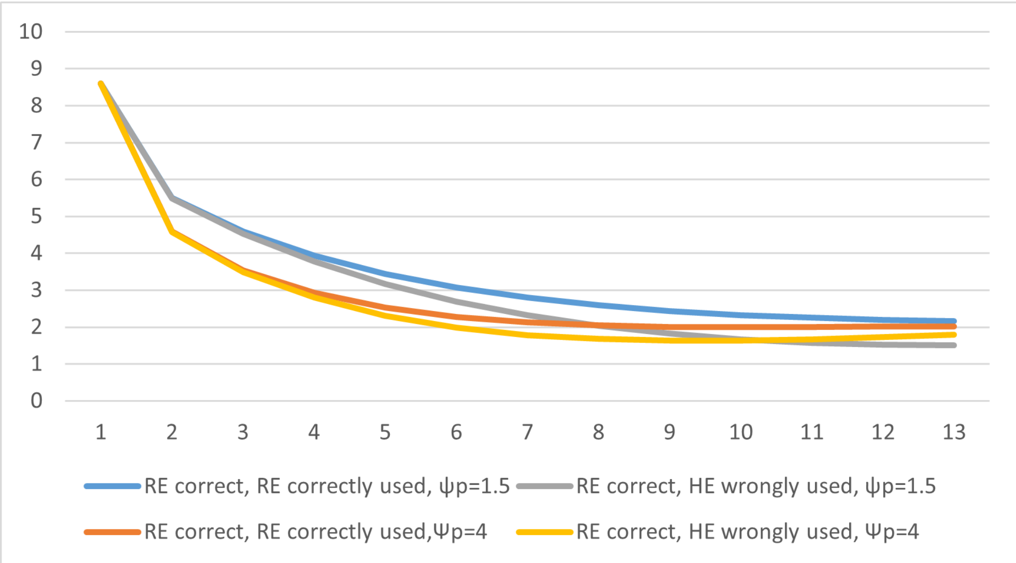

Figure 2. Model-implied inflation rate paths in the RE world and under different monetary policy rules. Notes: ψ_p = 1.5 represents a dovish central bank and ψ_p = 4 represents a hawkish central bank. Correct/false indicates whether the central bank has used the correct (in this case RE) model when determining the interest rate path.

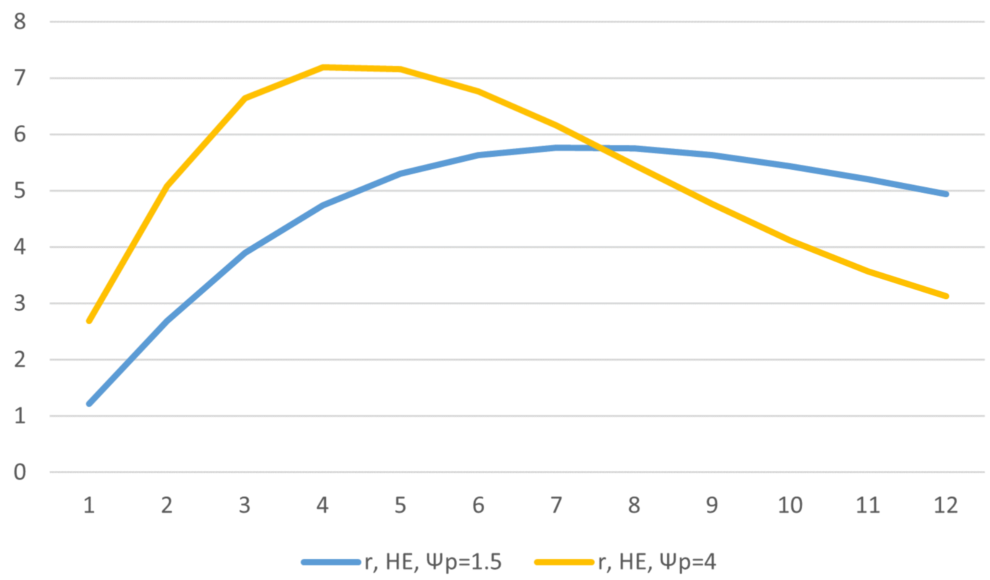

Figure 3. Model-implied interest rate paths in the HE world and under different monetary policy rules.

Table 2 summarises key variables from all scenarios analysed in this section, with an additional case of a super-hawkish monetary policy defined in Section 4, $\psi_p = 7$. We are particularly interested in the penalty when using the false model with an incorrect assumption of expectation formation, i.e., setting the nominal rates based on the HE model in an RE world and vice versa. It follows that falsely using the HE model when the real world is better represented by the RE model leads to the undershooting of inflation and noticeable loss in the cumulative output. This holds true for both the hawkish and the dovish reactions. In turn, falsely using the RE model instead of the HE model results in notably higher inflation and failure to reach the inflation target after 12 quarters, even for the hawkish monetary policy rule. In this scenario, only the super-hawkish reaction function, $\psi_p = 7$, allows one, to some extent, to reach the inflation target after 12 quarters—again, at the cost of a loss in cumulative output.

Table 2. Model-implied inflation after 8 and 12 quarters and cumulative output loss under different model specifications and monetary policy rules. Notes: Rows represent scenarios assuming different values of inflation coefficients in the Taylor rule: $\psi_p = 1.5$ (dovish reaction), $\psi_p = 4$ (hawkish reaction), and $\psi_p = 7$ (super-hawkish reaction). HE features backward-looking expectations and elements of learning. Upper and lower panes identify which model describes the economy correctly, i.e., RE world or HE world. RE model and HE model indicate which model is used by the central bank to set the nominal interest rate.

Inflation Coefficient	Inflation After 8 Quarters	Inflation After 12 Quarters	Cumulative Output Loss
Rational expectations			
$\psi_p = 1.5$			
RE model	2.44	2.15	1.38
HE model	1.82	1.50	2.88
$\psi_p = 4$			
RE model	2.01	2.02	2.61
HE model	1.63	1.77	3.37
$\psi_p = 7$			
RE model	1.85	2.01	3.42
HE model	1.52	1.78	4.36
Hybrid expectations			
$\psi_p = 1.5$			
HE model	4.91	3.72	2.89
RE model	5.64	4.79	1.20
$\psi_p = 4$			
HE model	2.68	2.18	5.00
RE model	3.06	2.53	4.21
$\psi_p = 7$			
HE model	2.03	1.92	6.35
RE model	2.39	2.13	5.48

4.3. Loss Function Considerations

To compare different monetary policy reaction functions under different (mis)specifications of expectation formation, we introduce the loss function as a weighted sum of the absolute value for the deviation in inflation from the target (2%) at the 12th quarter and the cumulative output loss over 12 quarters:

$$Loss = |\pi_{12} - 2| + w_y * y_{loss}, \quad (4)$$

where π_{12} is inflation after 12 quarters, y_{loss} is the cumulative output loss, and w_y is the weight assigned to the cumulative output loss in the loss function. Figure 4 shows the computed losses as a function of the weight w_y attached to the output loss for correctly specified models, i.e., in those scenarios in which the central bank faces no uncertainty about how expectations are formed. Under RE, the dovish policy is better than the hawkish policy, except for the functions that assign very small weights to the cumulative output loss. In these cases, the hawkish policy is somewhat better. Under HE, the hawkish policy is much better than the dovish one for all reasonable scenarios.

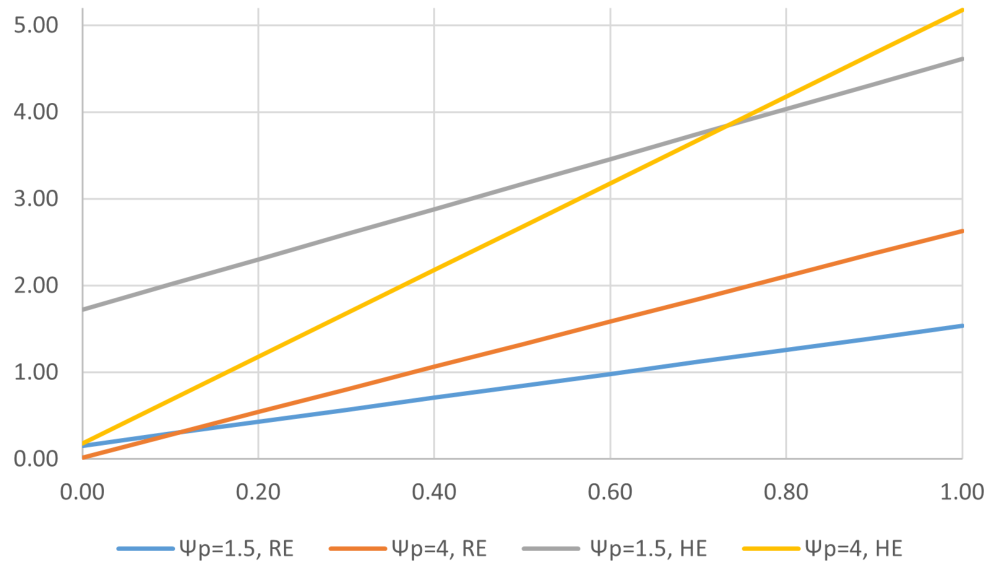

Figure 4. Loss functions depending on the weights assigned to the cumulative output loss when the central banks use false models. Notes: ψ_p = 1.5 (dovish reaction) and ψ_p = 4 (hawkish reaction). HE features backward-looking expectations and elements of learning; RE stands for rational expectations.

Figure 5 plots the loss function under the assumption that the central bank uses a model with the incorrect expectation formation mechanism. When incorrectly using the HE model instead of the RE model, the hawkish central bank generates lower losses than the dovish one if the weight of the cumulative output loss is less than roughly 0.5; however, the difference between the two is small (the blue line corresponds to the dovish CB, and the orange line corresponds to the hawkish CB). In the world of HE, when incorrectly using the RE model, the hawkish CB generates much lower losses than the dovish one in most of the cases, while, only for quite large weights (>0.75), the dovish reaction is preferable. Overall, in both cases, for models specified correctly or incorrectly, the hawkish reaction is the better choice if the weight assigned to the cumulative output loss is small.

4.4. Policymaking Under Double Uncertainty

In the modelling framework described above, the central bank deals with two types of uncertainty: (a) which expectation formation mechanism is correct, RE or HE; (b) whether the central bank uses the correct model. Thus, there are four outcomes of these uncertainties: (1) the RE world and the central bank uses the RE model; (2) the RE world and the central bank uses the HE model; (3) the HE world and the central bank uses the RE model; (4) the HE world and the central bank uses the HE model. Under the assumption of an equal prior probability for each outcome, the expectations of the loss function may be written as

$$E(loss) = \omega_1 L_{RE}^{RE} + \omega_2 L_{HE}^{RE} + \omega_3 L_{RE}^{HE} + \omega_4 L_{HE}^{HE}, \quad (5)$$

where ω is the probability of a given outcome (we assume that $\omega_1 = \omega_2 = \omega_3 = \omega_4$ = 0.25). Figure 6 plots the expected loss as a function of the weight w_y for the hawkish (orange line) and dovish (blue line) central banks. If the weight for the output loss is less than 0.6, the expected loss of the hawkish central bank is less than that of a dovish one. An aggressive monetary policy is much better than a moderate one for small weights assigned to the output loss, as is supposed to be the case for central banks with the main mandate of price stability.

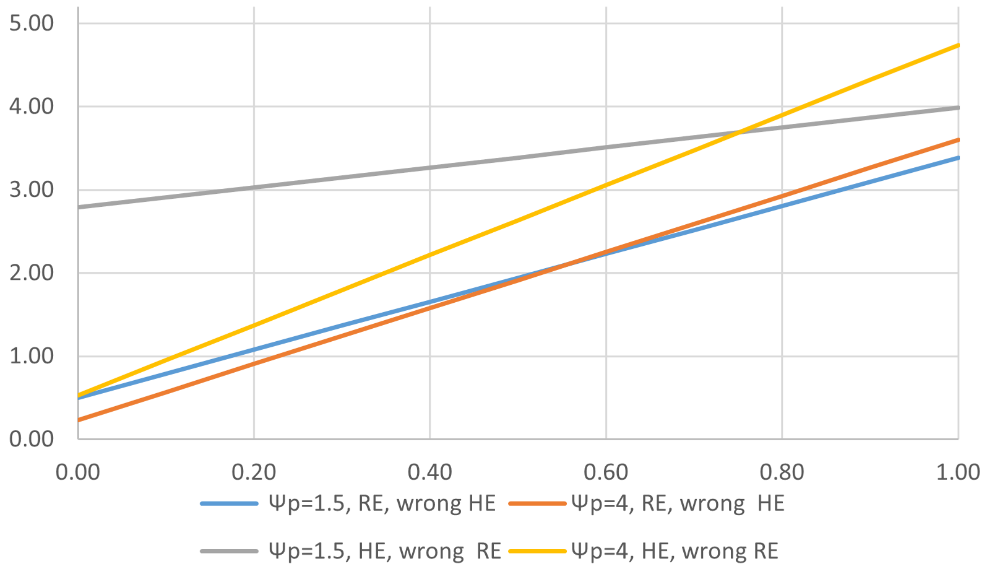

Figure 5. Loss functions depending on the weights assigned to the cumulative output loss when the central banks use false models. Notes: ψ_p = 1.5 (dovish reaction) and ψ_p = 4 (hawkish reaction). HE features backward-looking expectations and elements of learning; RE stands for rational expectations.

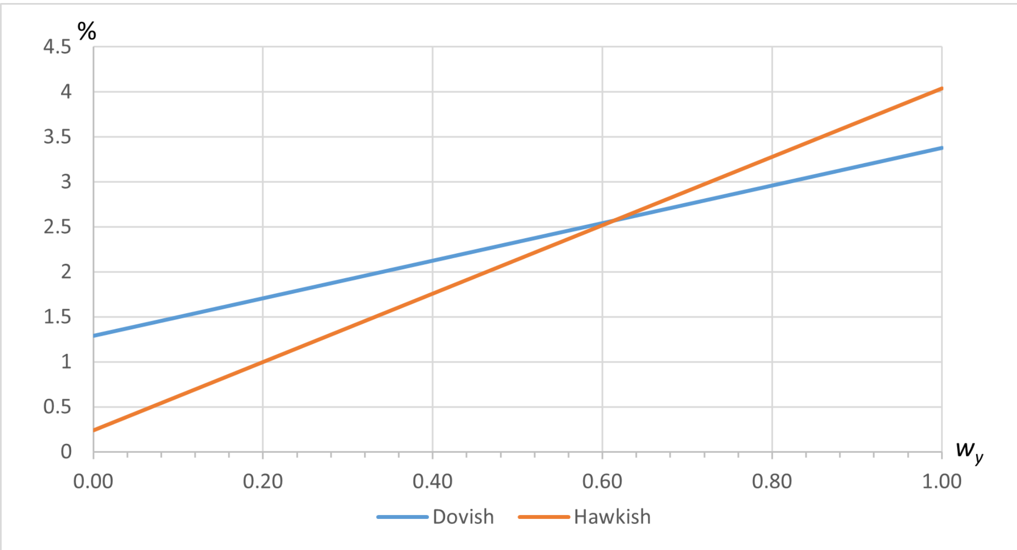

Figure 6. The expected loss functions by the weight attached to the cumulative output loss for hawkish and dovish central banks. Notes: ψ_p = 1.5 (dovish reaction) and ψ_p = 4 (hawkish reaction).

4.5. Switching the Model and Reaction Function

Assume that the dovish central bank uses the incorrect RE model to determine the future path of the nominal interest rate. However, after four or eight quarters, it realises that inflation is too high and starts to be hawkish and uses the correct HE model instead. This represents a case in which the central bank changes the model used and adjusts its reaction function. Next, we consider the following five cases: Case 1, as a benchmark, in which the dovish central bank uses the incorrect RE model over the whole horizon; Case 2, as a second benchmark, where the hawkish central bank uses the correct HE model from the start; Case 3, where the central bank starts out with an incorrect RE model and dovish reaction but switches to the HE model and being hawkish after four quarters (however, the bank must be very hawkish at this point, or it risks the serious de-anchoring of the inflation expectations and the permanent deviation of inflation from the target); and Case 4, where the dovish central bank has used the incorrect RE model for eight quarters—given that the reaction from the central bank comes with a larger lag, the size of the adjustment of the policy stance has to be even larger than in the previous case (ψ_p = 10). For the technical details of the scenarios' implementation, see Appendix D.

The modelling results are summarised in Table 3. In the case of switching to the aggressive reaction function and correct model after four quarters (Case 3), the central bank reaches inflation of 2.26% after 12 quarters. Interestingly, after 16 quarters, the inflation rate is 2.07%, which is even lower than in the case of following the correct HE model and an aggressive monetary policy from the beginning (Case 2). This can be explained by the fact that, for the former case, the much higher interest rate after four quarters reduces inflation more strongly than for the latter case. The cumulative output loss for Case 3 is 1.3 pp higher than for Case 2.

Table 3. Terminal rate, inflation, and output loss under switching scenarios. Notes: Rows represent scenarios assuming different values of inflation coefficients in the Taylor rule, ψ_p = 1.5 (dovish reaction) and ψ_p = 4 (hawkish reaction). HE features backward-looking expectations and elements of learning. Correct/false indicates whether the model used by the central bank describes the economy correctly.

Scenario	Terminal Monetary Policy Rate	Inflation After 12 Quarters	Inflation After 16 Quarters	Cumulative Output Loss
Case 1. (ψ_p = 1.5, RE false)	3.41	4.77	4.16	1.21
Case 2. (ψ_p = 4, HE correct)	7.19	2.18	2.11	5.00
Case 3. (ψ_p = 1.5, RE false and after Q4 ψ_p = 4, HE correct)	7.98	2.26	2.07	6.32
Case 4. (ψ_p = 1.5, RE false and after Q8 ψ_p = 10, HE correct)	11.7	2.32	1.77	8.48

In the case of switching to the aggressive reaction function and the correct model after eight quarters (Case 4), the inflation rate after 12 quarters is relatively high (2.86%) and reaches 2.18% after 16 quarters. The cumulative output loss for Case 4 is lower than for Case 3 and even lower than for Case 2, since the hawkish policy response is activated rather late in the horizon of the scenario. Finally, in Case 5, where the dovish central bank has used the incorrect RE model for eight quarters and then tries to bring inflation close to the target by the 12th quarter, inflation is reduced to 2.32% after 12 quarters, but at the cost of a terminal rate of 11.7% and an output loss of 8.48%, which is much higher than for all other scenarios (Figures A2 and A3 show the path of inflation and the interest rate under different cases).

So, what are the main conclusions from this exercise so far? First, quite obviously, it is always beneficial for welfare if, despite all uncertainties with regard to the way in which the economy operates and economic agents behave, the central banks select the correct model. Secondly, if they are wrong from the beginning, the central banks can still achieve outcomes that are close to optimal if, upon seeing that their initial assumptions do not hold, they change them and also modify their policy stance according to the new assumptions. The sooner they adjust their policy to the new reality, the smaller the potential welfare loss. Conversely, maintaining the initial policy stance in the face of changing evidence regarding the state of the economy implies the need for larger policy adjustments when they can no longer be avoided, and these larger policy adjustments come with the risk of more significant welfare losses.

4.6. Higher for Longer or Further Hike?

In the previous section, we mostly discussed the potential risks of central bank policy reactions that are too late and too large. However, the analytical framework also allows for the analysis of a different approach to monetary policy reaction: namely a "higher for longer" option. Facing high inflation, central banks have two possibilities: (1) to raise the interest rate until inflation declines to an acceptable level or (2) to commit to keeping the interest rate at a lower but still restrictive level for a longer period. The second option may be preferable from a financial stability perspective, as high interest rates may create problems for the financial system by reducing the value of fixed income assets. The failure of some regional US banks at the beginning of 2023 is an example of such a situation.

We simulate scenarios where the central bank keeps the maximum interest rate, R_{max}, at 3.5% and 4% until the period in which the interest rate implied by the conventional Taylor rule is lower than these levels. We consider cases of $\psi_p = 2$ and $\psi_p = 4$ in the RE model.[2] Figure 7 shows the paths of the interest rate under the benchmark scenario and scenarios for which the interest rate does not exceed 3.5% and 4%.

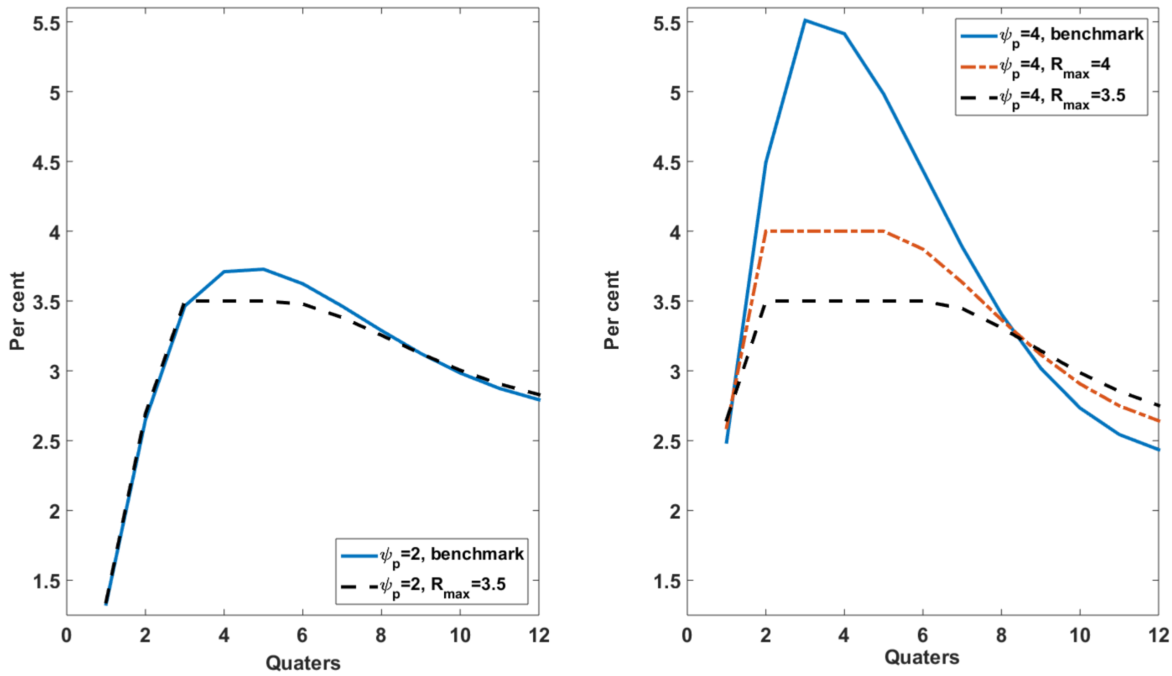

Figure 7. Interest rate paths for higher-for-longer and linear Taylor rule scenarios. Notes: $\psi_p = 2$ (dovish reaction) and $\psi_p = 4$ (hawkish reaction). A correctly chosen RE model is assumed. R_{max} represents a ceiling for the monetary policy rate (set at 3.5% and 4%), which is kept until the interest rate implied by the conventional Taylor rule is lower than these levels.

Table 4 shows that the changes in cumulative inflation[3] for 12 quarters (log-differences in prices for a period of 3 years) are not considerable across either of the scenarios. In terms of the output loss, the higher-for-longer policies seem to perform better compared with linear Taylor rule policies. In summary, in a model economy, the higher-for-longer policy may provide some advantages in terms of the cumulative output loss, with a moderate increase in cumulative inflation.

Table 4. Interest rate paths for higher-for-longer and linear Taylor rule scenarios. Notes: ψ_p = 2 (dovish reaction) and ψ_p = 4 (hawkish reaction). A correctly chosen RE model is assumed. R_{max} represents a ceiling for the monetary policy rate (set at 3.5% and 4%), which is kept until the interest rate implied by the conventional Taylor rule is lower than these levels.

Scenario	Terminal Monetary Policy Rate	Duration HFL Period	Cumulative Inflation	Cumulative output Loss
Dovish reaction (ψ_p = 2)				
TR	3.73	-	10.90	3.23
HFL, R_{max} = 3.5	3.50	3	11.05	3.03
Hawkish reaction (ψ_p = 4)				
TR	5.51	-	9.68	4.84
HFL R_max = 4	4.00	4	10.19	3.97
HFL R_max = 3.5	3.50	5	10.30	3.49

5. Conclusions

This paper analyses how different sources of uncertainty faced by central banks shape the macroeconomic outcomes in the Smets–Wouters-type (see Smets and Wouters, 2003; Smets and Wouters, 2007) DSGE model. It first compares the optimal policy response given different forms of expectation formation, namely backward-looking expectations with elements of learning and RE. Second, it considers that the central bank may use incorrect expectation formation models to define a future interest rate path and then discusses what this means for inflation and the output. Third, it analyzes different forms of potential non-linearity in the conduct of monetary policy.

Overall, the analysis suggests that an aggressive monetary policy is preferable and more successful in mitigating the high inflation rate for both rational and hybrid expectation formation mechanisms. In the presence of uncertainty regarding the expectation formation mechanism, the hawkish response to high inflation provides lower expected losses for small weights assigned to the output loss.

Overall, a hawkish monetary policy is more robust to uncertainty regarding the expectation formation and whether the correct or incorrect model is used by the central bank with the main mandate of price stability, i.e., a lower weighted cumulative output loss in the loss function. The underestimation of inflation's persistence and a delay in implementing an aggressive response to inflation result in much higher terminal rates and cumulative losses in output. These results are obtained under the assumption of hybrid expectations only for one endogenous variable, namely for inflation. The application of hybrid expectations also to wages does not change the results considerably.

Future developments of the modelling approach considered in this paper could explore the incorporation of a non-linear policy reaction function by including quadratic and cubic terms of inflation deviation from the target. Adding these terms to the standard Taylor rule would introduce asymmetry to the reaction function and allow for a quite aggressive central bank response when inflation is significantly higher than its target.

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Abbreviations

The following abbreviations are used in this manuscript:

CB	Central Bank
DSGE	Dynamic Stochastic General Expectations Model
HFL	Higher-For-Longer
RE	Rational Expectations
HE	Hybrid Expectations
ZLB	Zero Lower Bound

Appendix A. Results of Bayesian Estimation

Table A1. Results from Metropolis–Hastings (parameters). HPD inf is the posterior density 5% interval; HPD sup is the posterior density 95% interval. The notation is the same as in Smets and Wouters (2003).

	Prior			Posterior			
	Dist.	**Mean**	**Stdev.**	**Mean**	**Stdev.**	**HPD Inf**	**HPD Sup**
Productivity shock, ρ_a	beta	0.500	0.2000	0.903	0.0283	0.8745	0.9312
Risk premium shock, ρ_b	beta	0.500	0.2000	0.327	0.0346	0.2921	0.3613
Spending shock, ρ_g	beta	0.500	0.2000	0.966	0.0043	0.9621	0.9708
Risk premium shock, ρ_i	beta	0.500	0.2000	0.923	0.0055	0.9174	0.9284
Monetary policy shock, ρ_r	beta	0.500	0.2000	0.500	0.0101	0.4899	0.5101
Price mark-up shock, ρ_p	beta	0.500	0.2000	0.873	0.0372	0.8360	0.9104
Wage mark-up shock, ρ_w	beta	0.500	0.2000	0.830	0.0681	0.7618	0.8979
MA term price mark-up, μ_p	beta	0.500	0.2000	0.607	0.0855	0.5213	0.6923
MA term wage mark-up, μ_w	beta	0.500	0.2000	0.351	0.0896	0.2617	0.4409
Investment cost, φ	norm	4.000	1.5000	5.108	0.9148	4.1935	6.0232
Risk aversion, σ_c	norm	1.500	0.3750	1.312	0.0556	1.2567	1.3679
External habit degree, λ	beta	0.700	0.1000	0.848	0.0051	0.8424	0.8526
Calvo parameter wages, ξ_w	beta	0.500	0.1000	0.673	0.0030	0.6701	0.6760
Calvo parameter prices, ξ_p	beta	0.500	0.1000	0.684	0.0138	0.6705	0.6980
Frisch elasticity, σ_l	norm	2.000	0.7500	0.784	0.3849	0.3988	1.1685
Indexation to past wages, ι_w	beta	0.500	0.1500	0.262	0.0098	0.2524	0.2720
Indexation to past prices, ι_p	beta	0.500	0.1500	0.167	0.0303	0.1368	0.1975
Capacity utilisation cost, ψ	beta	0.500	0.1500	0.579	0.1726	0.4063	0.7516
Fixed cost share, ϕ_p	norm	1.250	0.1250	1.323	0.1135	1.2091	1.4361
Steady state hours, $\bar{l}$	norm	0.000	2.0000	0.436	0.0347	0.4016	0.4710
Tech spending corr, ρ_{ga}	norm	0.500	0.2500	0.312	0.0401	0.2721	0.3522
Capital share, α	norm	0.300	0.0500	0.290	0.0120	0.2782	0.3021
Employment hours eq, ξ_e	beta	0.500	0.2800	0.807	0.0090	0.7978	0.8158

Table A2. Results from Metropolis–Hastings (standard deviation of structural shocks). The notation is the same as in Smets and Wouters (2003).

	Prior			Posterior			
	Dist.	**Mean**	**Stdev.**	**Mean**	**Stdev.**	**HPD Inf**	**HPD Sup**
Productivity shock, σ^a	invg	0.100	3.0000	0.624	0.0434	0.5810	0.6677
Risk premium shock, σ^b	invg	0.100	3.0000	0.129	0.0184	0.1104	0.1472
Spending shock, σ^g	invg	0.100	3.0000	0.265	0.0030	0.2620	0.2680
Investment shock, σ^i	invg	0.100	3.0000	0.222	0.0355	0.1868	0.2579
Monetary policy shock, σ^m	invg	0.100	3.0000	0.336	0.0062	0.3295	0.3419
Price mark-up shock, σ^p	invg	0.100	3.0000	0.087	0.0018	0.0849	0.0884
Wage mark-up shock, σ^w	invg	0.100	3.0000	0.073	0.0173	0.0554	0.0900

Appendix B. Simulation Results of HE for $\alpha = \delta = 0.5$

Figure A1 shows the path of the interest rate and inflation for various degrees of backward-looking behaviour—RE ($\alpha = \delta = 0$), HE ($\alpha = \delta = 0.8$), and HE ($\alpha = \delta = 0.5$).

Overall, for moderate backward-looking behaviour ($\alpha = \delta = 0.5$), the path of the interest rate and inflation lies between RE and HE ($\alpha = \delta = 0.8$). However, the inflation rate for HE ($\alpha = \delta = 0.8$) is lower than that for HE ($\alpha = \delta = 0.5$) due to the higher interest rate.

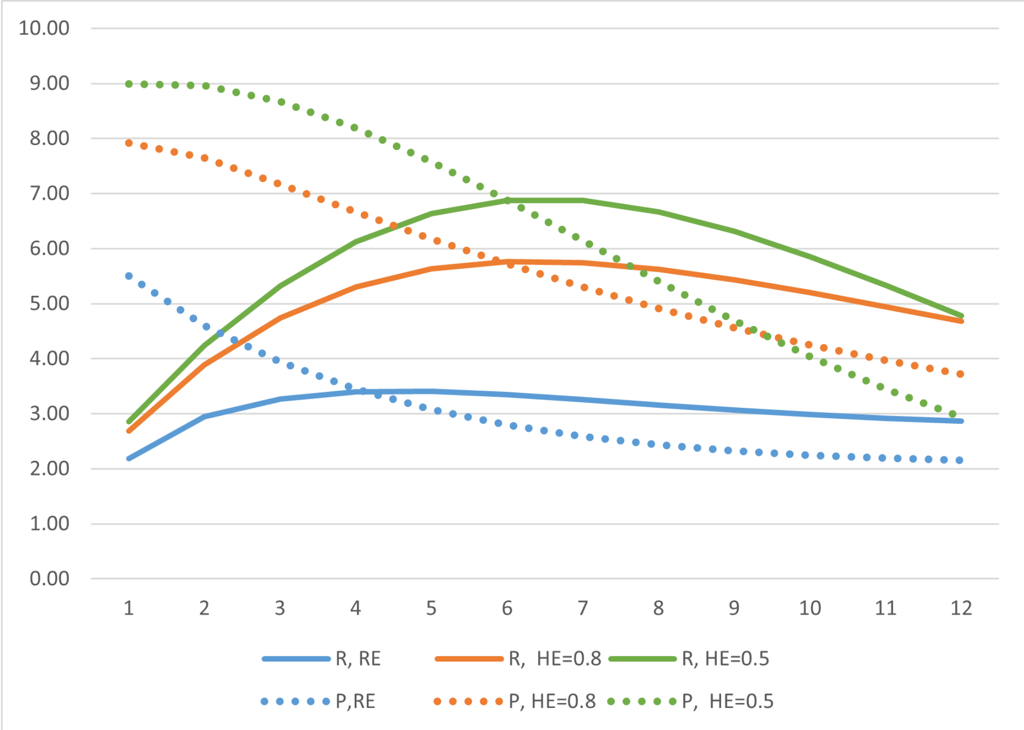

Figure A1. Interest rate and inflation under different degrees of backward-looking behaviour.

Appendix C

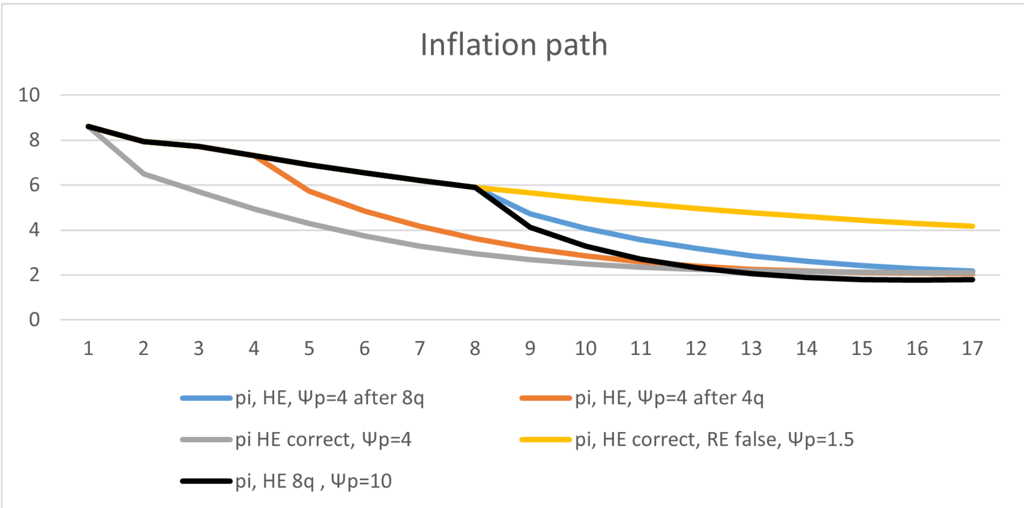

Figure A2. The inflation rate path for a dovish central bank that has used the incorrect RE model for 4 or 8 quarters and then switched to the correct HE model and become hawkish.

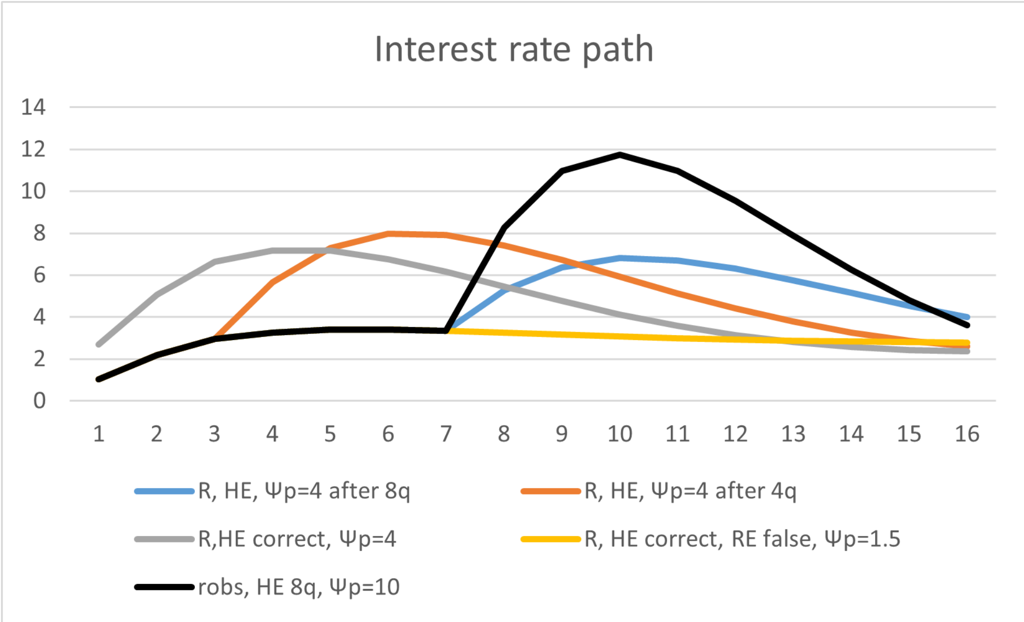

Figure A3. The interest rate path for a dovish central bank that has used the incorrect RE model for 4 or 8 quarters and then switched to the correct HE model and become hawkish.

Appendix D. Methodology and Algorithms Used in Implementation of Scenario Simulation

The initial conditions for the dynamics of the macrovariables are obtained by employing the Kalman filter provided by Dynara's calibrated smoother Adjemian et al. (2024). Estimated mean values are used as calibrated parameters. In further simulations, the initial conditions are defined in Dynara's block histval.

In Section 4.2, we examine scenarios in which the central bank uses an incorrect model to determine the future path of the nominal interest rate. To simulate this scenario, two Dynare files are run in order. For example, in a scenario where the correct model is HE but the central bank uses the incorrect RE model, the first file simulates the incorrect RE model, forecasts the future interest rate path, and saves it. The second file then simulates the behaviour of households and firms, allowing them to compute conditional forecasts based on the constrained interest rate path generated in the simulation of the first file.

In Section 4.5, to simulate switching the model and the central bank reaction function for Case 3 (4), we first forecast endogenous variables for the RE model for four (eight) quarters. Saving the values of the state variables at the fourth quarter, next, we use them as initial values to run the HE model for 12 quarters by employing the command *histval* in another Dynare file. Case 5 is similar to Case 4 except that we run the HE model in the second stage using the Taylor rule with coefficient $\psi_p = 10$. This value of the coefficient is chosen to provide an inflation rate close to 2% after 12 quarters after the beginning of the simulation (i.e., the start of running the RE model), avoiding inflation undershooting.

In Section 4.6, to simulate the higher-for-longer policy, we use Dynare's perfect foresight solver with the following non-linear specification of the Taylor rule:

$$R_t = \max\left(\bar{R}_i, \rho R_{t-1} + (1-\rho)\left(r^* + \pi^* + \psi_p\left(\pi_t^{(4)} - \pi^*\right) + \psi_y y_t^{gap}\right)\right) \quad \text{(A1)}$$

where $\bar{R}_i = 3.5$ and 4 are the possible terminal rates of the central bank. The initial conditions, defined by the command *histval* , are the same as for the other simulations.

Notes

[1] Our modelling framework may be regarded as that of model coefficient uncertainty, because, for the zero coefficients related to backward-looking expectations and learning, the model corresponds to RE.

[2] Under such specifications of the policy reaction function, the model becomes non-linear, and, for the benchmark, ψ_p = 1.5, as well for the HE models, a numerical solution cannot be found; For technical details of the scenarios' implementation, see Appendix D

[3] Since the difference in cumulative inflation between different policies is already quite small, we do not consider the difference in the inflation levels.

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