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Article

Implementing Dirac Approach to Quantum Mechanics in a Hungarian Secondary School

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Abstract: Quantum mechanics is included in many curricula across countries because of its cultural value and technological application. In the last decades, two-state approaches to quantum mechanics became popular because of the age of quantum computers. This article presents an experiment with 24 Hungarian high school students on teaching/learning quantum mechanics according to Dirac's approach to concepts and basic formalism developed in the context of light polarization. Tutorials, pre/post-tests, and oral interviews are the main monitoring tools used to collect data on the students' learning path. From the qualitative and quantitative data analysis, learning progressions emerged in the phenomenology exploration and on the probabilistic nature of single quantum measurement. The students' conceptions of quantum state are enriched, confirming the importance to focus educational approaches on fundamental topics. For one section of students, the complex relationship between quantum state and property remained problematic, but the students' interpretations of a quantum state can be categorized. Two lines of reasoning emerged regarding the impossibility to attribute a trajectory to a quantum system, one more orthodox and one that seeks to avoid the probabilistic nature of the quantum world.

Keywords: physics education; polarization; quantum mechanics; secondary school

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1. Introduction

Quantum physics has been introduced in the curricula of all European countries in the last 10 years [1,2] but with several approaches since there are many different formulations of quantum mechanics (QM) [3]. The first one is the historical framework or reconstruction of crucial experiments (light emission and interaction process, photoelectric and Compton effects, etc.) proposing a descriptive path toward the so-called old theory of quanta [4,5]. The physics education research developed other approaches based on wave formalism [6–8] or based on the statistical interpretation of QM and the role of measurement [9,10]. In this paper, we focus on a conceptual approach to the quantum fundaments and basic vectorial formalism according to the Dirac polarization approach to QM [11–15]. This approach was devised a couple of decades ago, tried and tested extensively in the Italian context. It fits, as a pioneering approach, into a large literature of innovative proposals on the teaching of QM in secondary schools focusing on the phenomenological context of photon polarization states. Nevertheless, in the near past, other physics education research is also performed using single photon experiments in other contexts or methods [16–20]. The role of all secondary school approaches to QM is documented by extensive literature. The methods change according to the educational purposes [21,22], but all of them agree with the need to introduce QM in secondary schools for its cultural value and technological applications [23,24].

In the 2019/20 school year, a teaching/learning experiment based on vector formalism was implemented in Hungary, following the approach developed by the Physics Education Research Unit, Udine, Italy. It was chosen because it is a coherent path proposal on fundamental concepts of QM [11–15,25], according to the Dirac approach to QM [26], based on an adaptive inquiry-based methodology [27,28] by means of simple experiments on light polarization [29,30] and ideal experiments in a computer-based environment [31,32]. The aim of the proposal is to understand some of the fundamental concepts of QM and its basic formalism, offering awareness of its conceptual meaning in two-dimensional real vector space [11] based on the two-state feature of light polarization by simplifying mathematics [12]. In the context of the Physics Education Ph.D. The program carried out at Eötvös Loránd University, Budapest, and in collaboration with the Physics Education Research Unit of the University of Udine, a research-based curriculum was developed and integrated into secondary school education in Hungary, where QM is traditionally discussed in a brief qualitative and narrative way [33]. This article presents the experimentation implemented with Hungarian students and documents how students acquire the basic concepts of QM and what difficulties emerged. It aims to contribute to the literature on students' learning processes for acquiring QM concepts, and presents students' difficulties and gives proposals to overcome them [34–37]. It also provides evidence of the impact of the materials on teaching QM developed by the research unit of the University of Udine in a context different from the Italian one that had not been tested up to now [12,25,38], furthermore the paper also assists other single photon-based approaches [16–20] because some consequences (e.g., categorization of students' thoughts about quantum state represented by polarization state vector restricted to real numbers) are general.

In the next section, we present the research questions and our plan introducing to the examination tools and contribution to the literature. After that, in Sections 3 and 4, we show the research method and instruments and we also sketch the context of the teaching/learning process by presenting the main topics, worksheets, and learning environment. In Section 5, the reader can gain insight into the structure of the learning path with quotations of specific examples, tasks, and available tutorials developed by the Research Unit of Udine. In Section 6, we analyze the results of the multiple-choice questions highlighting the remained problematic aspects and learning progression. In Section 7, we investigate the concept of physical state, in particular in QM, which is a specific indicator of the quantum mechanical way of thinking. We observed that the concept of state is enriched and can be categorized, which is also relevant to other single photon approaches. We present all the answers of students about the quantum state in the Appendix A by categorizing. In Section 8, we asked students about the material and in Section 9 some extra tasks are proposed to extend the tutorials used as research tools.

2. Research Questions

The main research questions with research-based implementation are the following.

- RQ1 What is the impact of the chosen approach on the students' learning process, focusing on discussion of the fundamental concepts of QM?
- RQ2 How did students understand the main aspects of QM?
- RQ3a. How can we identify the problematic aspects faced by students?
- RQ3b. What are the perspectives and difficulties that emerged, in particular on the quantum state?
- RQ4 How do the learning outcomes confirm the previous studies regarding this approach to QM by implementing it in Hungary?

To answer the research questions, a research plan was organized according to the rationale of the validated Dirac polarization approach to QM [11–15], by means of simple real experiments at the macroscopic level and ideal experiments for single photon interacting with polarizers and birefringent crystals, using specific software (JQM) [31,32]. The validated tutorials [39,40] use selected main concepts, investigate the arguments of students, and produce the learning path. Each tutorial focuses on the addressed concepts: mutual exclusive properties, incompatible properties, measurement characteristics and probabilistic nature of single events, quantum state and its formal description with an abstract vector (limited to two dimensions using only real numbers, familiar to the students by assigning it to the direction of polarization of light) and quantum entanglement. The conceptual discussion produced by each tutorial is well described by the tutorial itself and one of the studied aspects is to understand these QM concepts. Specific questions were posed to the students to answer RQ1. For the study of the learning gain of each concept (RQ2) and main difficulties (RQ3), a detailed questionnaire that focused on the reasoning was used. We selected two very different types of students to study the impact of the chosen path in different cultural and formative contexts. To gain evidence of the global vision of the addressed concepts, interviews were carried out with the whole group of students. The multi-instrument qualitative analysis of the same fundamental concepts of QM was oriented to gain evidence on RQ2 and RQ3. We decided to dedicate particular attention to the concepts of the quantum state because the literature shows it to be a special indicator of the conceptual change from the classical to QM way of thinking [9,10,34,35,41,42]. Furthermore, we investigated whether there are differences between Hungarian and Italian results concerning these concepts and RQ4, for which previous research in Italian schools can be used [12,25,38]. This paper contributes to the literature because the Dirac approach to QM was firstly implemented in a Hungarian public high school helping to confirm previous studies and assist in generalization.

3. Research Instruments and Methods

The learning path was carried out with the tutorials developed by the Research Group of Udine [39]. The tutorials were adapted and translated into the Hungarian context. Each topic is shown in Table 1, referring to the Italian version available online [40]. The main difference was that the linear operators and the hypothesis of coexisting properties worksheets were skipped due to the pandemic. The tutorials activate an inquiry-based learning environment during classroom activities, monitoring the dialogues and arguments of students at the same time.

Table 1. Summary of topics and number of questions for each worksheet. The used worksheets [42] are FEN 1, Con 1–7, Form1.

Worksheet Code	Topic	Number of Questions
FEN1	Phenomenological exploration of polarization,	10
Con1	Malus law and its probabilistic interpretation assuming ideal polarizers	4
Con2	Epilogue on photon-polarizer interaction and probabilistic interpretation	2
Con3	Mutually exclusive properties	4
Con4	Formulating hypotheses on superposition	2
Con5	Incompatible properties and distinction with mutually exclusive properties	4
Con6	Non-epistemic uncertainty	2
Con7	Quantum particles and trajectories	6
Form1	From concepts to formalism: vector representation of quantum state	10

Due to COVID-19 pandemic restrictions, the last 3 h were carried out by distance modality. Therefore, we made an electronic version of the last tutorial, so all the answers were accessible. The students filled in the same pre/post-test (the first five were taken over from [38]), which included 7 multiple choice questions and 2 open answer questions. In the following, only the pre/post-test is analyzed in detail.

An initial analysis was carried out of the pre/post-test questions, by evaluating each answer as correct or incorrect. The frequencies of correct answers in the pre-test and post-test were evaluated, the distributions of correct answers were constructed and then compared. To assess whether the distributions of correct answers in the pre-test and post-test were statistically different, a *t*-test was used.

A qualitative analysis of answers was made, concerning the explanation of choices. We focused on students' thoughts regarding QM basic concepts, the different ways of thinking (classical, quantum, hidden variable, according to the profiles defined in [12,25,38]) and searched the QM aspects that proved problematic for students.

Students' solutions on worksheets and their dialogues during work group activities (audio recorded and monitored by free handwritten notes) were also checked and investigated. In this paper, we report only a few hints to support pre/post-test data, leaving the complete presentation of the tutorial analysis to future work.

We evaluated the frequencies of students' choices regarding the multiple-choice questions, analyzed the worksheets, and made notes from student dialogues to better interpret the results. The two open-ended questions were also analyzed and categorized. The categories are defined by the answers of students and by an ex-ante operative definition of classes of answers. Some of the answers are previously defined in research, but there were new types of answers, too, so we formulated new categories according to general qualitative research criteria [43], using those synthesized by Erikson [44].

To get feedback from students and to have some indication of the outcome of the experimentation in the medium term, audio-recorded interviews were made with five students one month after the course, using mirror questions (questions posed by students, re-proposed to them using their own terms) with regard to the answers, according with the Rogersian method [45,46]. The transcribed interviews will be reported and discussed after pre/post-test data as case studies.

4. Context and Sample

This was the first implementation of QM using Dirac polarization approach in Hungary. This pilot project involved two groups of students from the public school Czuczor Gergely Benedictine Secondary School, Győr, Hungary.

The first one (Group A) consisted of students from a class specialized in physics (five people, approximately 18 years old). This means that they had four physics classes per week instead of two last year, and they had already learnt some QM, which is required by the Hungarian system (the wave-particle duality of light and electron, de Broglie relation, Heisenberg's uncertainty relation, and a brief interpretation of the atomic structure with an analogy to vibrating strings (as an electron in a one-dimensional box or potential well)). The learning activity was carried out in a Study Group after school.

The second group (Group B) of students was less interested in physics, as they had a humanities study-orientation (19 people, approximately 17 years old). Most of them were demotivated in studying physics; in addition, their background lacked knowledge of basic physics, but the challenge of learning something up-to-date and extra-curricular attracted them to join this special experience. The learning activity with this group went on within the framework of normal physics educations. Group B had no previous knowledge about QM.

5. The Structure of the Path

The syllabus was the same in both groups, as shown in Table 2.

Table 2. The path schema and structure of the learning material. Due to the COVID-19 pandemic, the 11–14 steps were completed within the framework of online teaching.

Hours	Activities	Contents
1	Pre-test (individual work)	(See Table 1 for tutorial.)
2–3	The students experimented with polarizers, and they explored the phenomenon of light polarization. They worked in groups (2–3 students) and used the worksheet for exploration, and thereafter, the argumentation and explanation happened with the whole class together. Questions were like: *"What happen if we put a third polarizer between two orthogonal polarizers?"*	The main goal is to distinguish the polarization property of light from intensity of light. Using tutorial FEN1.

Table 2. *Cont.*

Hours	Activities	Contents
4	Quantitative experiment on the Malus law (the whole class together): $I_T = T \cdot I_0 \cdot \cos^2\theta$, where I_T is the intensity of transmitted light, T coefficient means polarizers are not ideal, I_0 is the intensity of incident light and θ shows the relative position of polarizers.	Explore and discuss the Malus law.
5–6	Interpreting and practising the Malus law, students made quantitative tasks and explored the meaning of the different factors of Malus law (work groups of 2–3 students or the whole class). Light consists of photons, so the Malus law can be expressed in terms of the photon number, too (monochromatic light): $N_T = T \cdot N_0 \cdot \cos^2\theta$, where N_T is the number of transmitted photons and N_0 that of incident photons. Typical question: *"What information does Malus law: $I_T/I_0 = N_T/N_0 = \cos^2\theta$ provide about the transmission of each photon?"*	Ideal and real polarizers. From macro to micro level by the hypothesis. The probabilistic meaning of Malus law with single photon thought experiment. Using tutorials Con1–2.
7–8	Polarization properties of single photon. Every photon has a well-defined polarization property determined by a well-defined measurement. The property can be indicated by symbols (iconographic representation): * is for horizontal, Δ is for vertical and $\diamondsuit$ is for 45° polarized photons. Discovering that properties * and Δ are mutually exclusive properties. Interpretative hypothesis for $\diamondsuit$ polarized photons and discussion of its meaning: the $\diamondsuit$ polarized photons are not a statistical mixture of * and Δ (workgroup of 2–3 students and discussion with the whole class). Discovering incompatible properties (* and $\diamondsuit$) and understanding the uncertainty principle (whole class).	Polarization properties. Interpretive hypothesis:$[\diamondsuit\diamondsuit\diamondsuit\diamondsuit] \neq [\Delta\Delta + **]$. Mutually exclusive properties. Incompatible properties. Uncertainty principle. Using tutorials Con3–6.
9–10	Phenomenology with birefringent crystals, and nonlocality (workgroup of 2–3 students or the whole class). The calcite crystal splits the light into two mutually exclusive polarized beams. There is a close relationship between polarization property and trajectory. Because of the probabilistic interpretation, we cannot assign trajectory to photons.	Face with birefringent crystals and exploring the nonlocality of photons. Using tutorial Con7.
11	The statistical interpretation of Malus law (the whole class in online teaching). Every photon has a probabilistic nature, so an ensemble of photons shows statistical feature.	Analysis of ideal simulated experiments of interaction of photons with polarizers, using the applet JQM [1] [31,32].
12–13	Assign vectors to the polarizers. The polarization of photons is uniquely given by polarizers, so a vector can be assigned to the photons. Concept of quantum state as a vector. Polarization as a state transition. The superposition principle and generalization (individual work in digital work schedule): $\boldsymbol{u} = \Psi_1\boldsymbol{h} + \Psi_2\boldsymbol{v}$ every state can be expressed as a linear combination of bases. So not only measurable states are possible states, but also their linear combinations too.	Quantum state (via polarization), superposition principle. Using tutorial Form1 in online version.
14	Post-Test (individual work)	

[1] The software JQM is available at http://www.fisica.uniud.it/URDF/secif/mec_q/percorso/avv_11.htm (accessed on 1 September 2022). The final version is protected by a password. Please contact the author of the applet lorenzo.santi@uniud.it for password and instructions to enable Java simulations.

6. Data from Multiple Choice Items of Pre/Post-Test on the Essential Features of QM

Table 3 shows items 1–7 of the questionnaire and summarizes the number of correct answers chosen by the students in the pre/post-test. The systematic increase in the frequency of correct answers (except for question 3 in Group A) highlighted in Figure 1 provides a positive overall picture regarding the impact of the implementation of the didactic proposal in the contexts involved in this research, although the differences between pre and post-test are actually significant only for Group A (t = 2.23, $p < 0.05$).

Table 3. Multiple choice items Q1–Q7 of the pre-post-test: questions and options from which students had to choose the one they considered most appropriate for each question. The frequencies of pre/post-tests for groups A and B are reported in the last four columns. The answers in bold are considered more correct, or more coherent with a completely quantum point of view.

Q1: Measuring a physical observable, which aspect among the following ones characterizes in a peculiar way quantum mechanics in respect of classic mechanics?

Answer options	Pre(A)	Post(A)	Pre(B)	Post(B)
(A) Under some conditions, discrete values of the measured observable are obtained	0	0	1	0
(B) Results of measurements are predictable only in probabilistic terms	**2**	**2**	**10**	**13**
(C) In general, systems initially prepared in the same state evolve in a different way when subjected to a process of measure	**1**	**2**	**5**	**6**
(D) The interaction with the measurement apparatus produces a perturbation on the system	1	1	2	0
(E) The result of a measurement is affected by an unavoidable uncertainty	1	0	1	0
No answer	0	0	0	0

Q2: Consider the following probabilistic forecasts:
(K) The heads outcome in launching a coin has $\frac{1}{2}$ probability to be realized;
(J) A photon with vertical polarization has $\frac{1}{2}$ probability to pass through a polarizer at 45°.

Answer options	Pre(A)	Post(A)	Pre(B)	Post(B)
(A) In the K case, we do not know initial conditions precisely enough, in the J case initial conditions are known, but the phenomenon itself has a probabilistic nature.	**2**	**4**	**8**	**11**
(B) In both cases, we do not know initial conditions precisely enough.	3	1	1	0
(C) In the K case, we do not know initial conditions precisely; in the J case, we do not know with enough precision how the interaction photon-polarizer happens.	0	0	10	8
No answer	0	0	0	0

Q3: Which of the following statements better outlines the meaning of uncertainty relations?

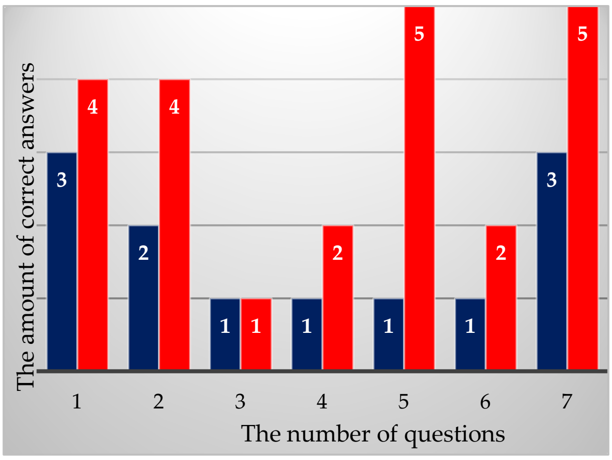
Answer options	Pre(A)	Post(A)	Pre(B)	Post(B)
(A) There are properties of the same system, which cannot be simultaneously determined with arbitrary precision	**1**	**1**	**4**	**6**
(B) It is not possible to measure with arbitrary precision a physical observable	1	1	3	5
(C) It is not possible to make the uncertainty measurement arbitrarily small	0	0	1	0
(D) It is never possible, not even in principle, to predict measurements precisely	0	0	0	1
(E) It is not possible to measure with arbitrary precision position and momentum of a particle.	3	3	11	7
No answer	0	0	0	0

Q4: In classic mechanics, it is always possible to attribute a trajectory to a particle. What statement can be made as far as a quantum particle is concerned (choose only one option)?

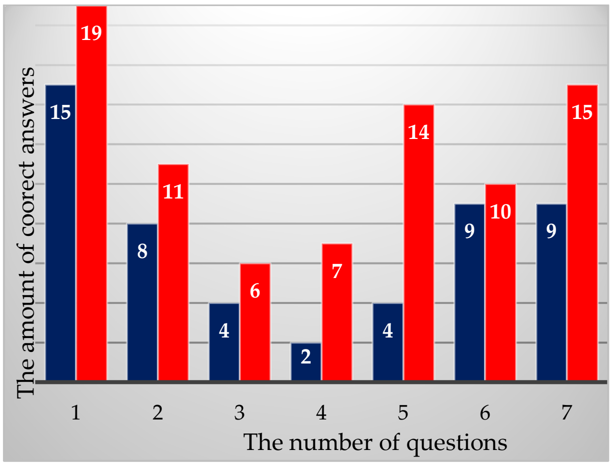
Answer options	Pre(A)	Post(A)	Pre(B)	Post(B)
(A) It is possible to attribute a trajectory, but it is not possible to determine with arbitrary precision all the information needed to determine it with arbitrary precision	3	0	7	3
(B) It is possible to attribute a trajectory, but it is not experimentally accessible	1	1	5	5
(C) It is possible to attribute a trajectory only when a position measurement is performed	0	0	4	2
(D) It is impossible to attribute a trajectory to a particle due to perturbations arising from measurements	0	2	1	2
(E) It is not possible, not even in principle, to associate a trajectory to a particle	**1**	**2**	**2**	**7**
No answer	0	0	0	0

Table 3. *Cont.*

Q5: *Suppose that a beam of light polarized at 45° is split into two beams by a calcite crystal: a beam of light horizontally and vertically polarized. If somehow these two beams are recombined with an inverted calcite crystal, what kind of polarization will the resulting light beam have?*				
Answer options	**Pre(A)**	**Post(A)**	**Pre(B)**	**Post(B)**
(A) The combined light beam consists of semi-horizontal and semi-vertically polarized photons.	2	0	1	1
(B) The combined light beam consists of 45° polarized photons.	**1**	**5**	**4**	**14**
(C) The combined light beam will not be polarized.	0	0	5	4
(D) Photons are polarized both vertically and horizontally.	2	0	8	0
No answer	0	0	1	0
Q6: *Choose the correct statement for the relation between classical physics and quantum mechanics.*				
Answer options	**Pre(A)**	**Post(A)**	**Pre(B)**	**Post(B)**
(A) Classical physics and quantum mechanics operate by very different laws, the two are incompatible.	3	3	4	6
(B) Quantum mechanics includes classical physics as a boundary.	**1**	**2**	**9**	**10**
(C) Classical physics can be applied in the quantum world, but not vice versa.	1	0	5	2
(D) The description of quantum mechanics and classical physics is equivalent.	0	0	1	1
No answer	0	0	0	0
Q7: *A non-polarized light beam passes through two polarizers in succession. The first polarizer halves the intensity of the incoming light and then reduces the intensity of the resulting beam again by half the second polarizer. In the end, the brightness is reduced to $\frac{1}{4}$ of the original. Which statement is correct?*				
Answer options	Pre(A)	Post(A)	Pre(B)	Post(B)
(A) Each polarizer halves the energy of each photon as it passes through.	2	0	2	1
(B) These polarizers are such that they absorb half of the photons at all positions.	0	0	5	1
(C) The position of the polarizers is such that exactly half of the photons pass through them.	0	0	2	2
(D) The position of the polarizers is such that each photon passes through them with $\frac{1}{2}$ probability.	**3**	**5**	**9**	**15**
No answer	0	0	1	0



(**a**)



(**b**)

Figure 1. Summary of pre-test (in blue) and post-test (in red) for Group A in graph (**a**) and for Group B in graph (**b**).

A separate discussion on each of the pre/post-test questions is more appropriate to better appreciate the aspects on which the best learning outcomes have been obtained, as well as the main changes in students' conceptions.

The outcome of *Question 1* shows that the answers focus on two options, (B) and (C). It shows the students' awareness of the probabilistic and stochastic nature of quantum measurements. It is worth recollecting this when dealing with the situation when the class was faced with the problem of interpreting the single photon cases in polarization. Let us recall a specific task from the worksheet, Con1, regarding what happens when only one photon interacts with the polarizer. In this task, the Malus law predicts that only $\frac{1}{2}$ photon goes through a polarizer in conflict with the undividable nature of photons. The first answer by the majority of students was related to the difficulty of interpreting the behavior of a single undividable photon. Half of the students predicted that the photon would certainly be absorbed, while the other half predicted a certain transmission. So, the teacher said: if we imagine light as consisting of photons, the polarizer will interact mostly with one photon at a time, but this interaction happens often and quickly. Therefore, if we say for every photon that it does not go through, then no photons will go through, which is in contrast with our own experiences with polarizers. There was silence in the room for a while, but then one of the students said, "It should be probabilistic." Student answers evidenced how JQM simulation to help students in the probabilistic interpretation of measurements and of photon-polarizer interactions. Almost all students in the sample have achieved an adequate mastery of the meaning of quantum measurement, consolidated or gained through experimentation and in particular by the use of JQM [31,32].

The outcomes of *Question 2* show the same that had emerged in previous studies [12,25,38]: the students are divided between an orthodox quantum vision and a hidden variable point of view (students understood the importance of probabilistic description, but as it is connected to the lack of knowledge about the system and state, students thought the probabilistic behavior was the consequence of the unknowable/inaccessible part of the system, and the probabilistic nature could be avoided if we could have enough knowledge).

The outcome of *Question 3* requires attention. The students automatically thought of the standard Heisenberg's uncertainty relation ($\Delta x \Delta p \geq \hbar/2$). It could be caused by prior quantum knowledge in Group A, but as we do not have specific data on this, we will explore the question of Heisenberg's principle by means of a specific interview in another study.

Question 4 is about the possibility to attribute a trajectory to a quantum particle: about 1/3 of the sample agreed with the orthodox vision (an impossibility in principle to attribute a trajectory). About 1/4 agreed with a hidden-variable perspective (the trajectory exists, but it is not accessible) and another 1/4 agreed with the idea that it is impossible to detect a particle without an irreversible perturbation. These results confirmed the tendencies of some students to develop lines of thinking attempting to preserve the concept of the trajectory (and a deterministic point of view) also in the quantum way of thinking (see also [12,25,38]). It shows how difficult it is to accept nonlocality and incompatibility. One of the interviewed students was asked why he chose option D ("it is impossible to attribute a trajectory to a particle due to perturbations arising from measurements"), to which he replied, "If we observed the photons between the two calcite crystals, we could already be certain of their trajectory. Therefore, the effects of the measurement do not allow for the concept of trajectory to be used." The argument evidenced a partial analysis, forgetting the role of measurement in quantum processes. The need for an in-depth discussion of measurement in QM emerged, and understanding and using formalism for this problem could also be helpful, as other school projects with a different context show [47,48].

Question 6 concerns the recognition that the Malus law appears as the mean of transmitted photons: the classical law is recovered within the limit of large photon numbers. From the answers, it may be worthwhile discussing this in more detail in the future. It is useful to understand the correspondence principle, to see that classical physics emerges as a limiting case of QM. It also helps to establish a correct scientific view and to understand that QM is not in conflict with the macroscopic world. In our path, this point was treated only in the specific case of the Malus law.

Questions 5 and 7 treated simple observations. The conclusion is that most students can interpret the phenomenological part correctly. The outcome of Q5 showed that students are confident with the phenomenology of the interaction between photons and birefringent crystals. The result cannot be taken for granted if we consider the difficulties of many students in understanding this phenomenology, as pointed out in previous research [25,38]. Outcomes of question Q7 also confirm the confidence with the analysis of phenomenology in terms of the probability of transmission of single photons. Students successfully interpreted the decrease of brightness in polarization as probability instead of energy or light intensity.

7. Answers to Open-Ended Questions Q8–Q9

Question 8 was open-ended in the pre/post-test: "Write in your own words what the concept of physical state means in classical physics and in quantum mechanics". In the pre-test, only four students gave their answers, which can be grouped under two categories:

- In QM, state and properties are indeterminate (2/4—A1—"In quantum mechanics, there is no accurate position, there are only probabilities"; A3—"In quantum mechanics, we cannot define the state exactly").
- Each system has some physical properties (2/4 B7—"Every material has physical properties (color, smell) and we know their form, too"; B1—"Physical state: every material has physical properties, and we are able to make predictions"

The post-test answers categories are shown in Table 4 (A1–A5 students of Group A; B1–B12 students of group B), defined operatively with examples of student answers in the post-test, focusing only on the way they define the quantum state (not the classical one). Interested readers can find the complete answers in the Appendix A.

Table 4. The categories of post-test answers.

Code	Category Description
C1	The state is identified with its formal representative entity: a vector (2/17); a probability (3/17). A4—"*There is some data in classical physics that describes the object. There is a vector in quantum mechanics from which we can calculate probabilities*". B3—"*In classical physics, the physical state is what can be measured, and an accurate quantity can be assigned from data and measurements, but in quantum mechanics we can only work with probability*". B4—"*In classical physics, the physical state of an arbitrary body can be described* e.g., *with a vector of motion. In quantum mechanics, this is a bit more complicated: although we can characterize the state of a photon, we can only give it according to a probability*".
C2	In classical mechanics (CM), the properties of a system before and after a measurement are the same, in QM the measurement allows us to attribute properties to the systems, i.e., only after carrying out a measurement or only at the time of the measurement can properties be attributed to the system (3/17). B6—"*The state is different in classical physics and quantum mechanics, namely, we never really get accurate results with the latter, we can just deduce what happens with a photon after it goes through a polarizer, but in classical physics our measurements to define an object's position are accurate (e.g., in motion)*". B10—"*In classical physics, a set of properties can be determined precisely, while in quantum mechanics it can only be observed at a studied moment*". B11—"*Classical: some real state, Quantum: the existence of something given by measurements*".
C3	In CM if you repeat a measurement, you will get the same results, in QM you will get uncertain results (3/17). A3—"*In classical physics, the state that we find by measurement is almost always the same. In quantum mechanics, however, there is always something else, very rarely the same value*". B1—"*In classical physics, measurements yield the same property over and over again. In contrast, in quantum mechanics measurements can provide different results*" B12—"*In classical physics, the physical state of something is a constant property, whether it is during observation or measurement. However, in quantum mechanics, these two are different from each other, because we can get different results out of our observations and measurements (inversion of two calcite crystals, observation of light and examination yield different result)*".
C4	State and property are determined in CM, and indeterminate in QM (2/17). A1—"*Classically, the place and position of a given object can be determined exactly, but not so in the quantum world*". B5—"*Physical state is a measurable factor in classical physics, but in quantum mechanics we can only estimate it*".
C5	In CM, the state is identified by position and velocity, in QM by polarization (and position) (3/17). A3—"*In classical physics, the physical state of an object can be described by its spatial position and velocity. In quantum mechanics, the physical state of a particle can be described by its spatial position and polarization property*". B7- "*In classical physics, it is possible to determine the location of an object and what forces act on it. In quantum mechanics, the physical state of an object can be characterized by its polarization property*".
NA	No answer (1/17).

We observed from the reported examples that the concept of physical state is not well defined in the students' minds, not only in classical physics but also in QM, which is not a surprise. The relevant result is the conceptual progression (however partial) on the concept of the state that can be noted in almost all cases. Only four students answered in the pre-test and only one person in the post-test eluded the question. All the answers in the post-test are much richer than the few answers in the pre-test; only student A1 showed the same conception about the quantum state in the pre/post-tests. The other three students who had initially answered the question in the pre-test highlighted different conceptions in the post-test, including student B1 whose conception remained under the sway of the concept of property.

What the literature on learning processes indicates to us is confirmed here, namely, that we must privilege the founding concepts (as the state) for a conceptual approach that favors learning [34,41]. Making students reflect on the concept of state in QM seems to help them develop a more elaborate (if not more precise) idea of what is meant by the state of a physical system and what implications are connected to this concept. For instance, if we know the state of the system, we can predict its time evolution, or we can predict the outcomes of a measurement.

Moreover, most of the students (11/17 were included in the categories C1-C2-C3) improved as regards quantum state:

- The formal representation is linked to the possibility of calculating the probabilities of measurements outcomes (cat. C1),
- The measurements results are usually uncertain (cat. C3),
- We can only assign properties to a system after performing a measurement of the property (cat. C2).

The latter aspect is very subtle and crucial to the distinction between classical physics and QM. A collective reflection in the class groups would facilitate sharing these partial visions. It would allow students to reach a more complete understanding of the concept of the quantum state, and it would also help to profoundly modify the conceptions of students who autonomously build concepts, such as in categories C4 and C5. Unfortunately, due to the difficulties triggered by the COVID pandemic, this was not possible in our experimentation.

Going into more detail, we can observe that the answers included in category C1 highlight a formal approach to the concept of state, which can also be recognized in other research where some students tend to identify a physical concept with a formula or a formal structure that can define this concept or in which it is involved [39,47–49].

It is obvious that the competence in linking concepts with their formal representations is a fundamental aspect of physics and in particular of QM, which can also provide operational and problem-solving skills. On the other hand, it is also important to help students understand the conceptual role of formalism, not just as a tool for making numerical predictions.

In relation to the answers of category C2 we can still remark the attitude of these students who emphasize the realism of the concept of state in CM (the properties of systems pre-exist the measurement), as opposed to the concept of state in QM in which it is possible to attribute the state of the system with properties only at the time of measurement, but not before it has been performed. As we mentioned, it is a crucial aspect of the distinction between classical physics and QM.

The answers included in category C3 focus on the uncertainty that characterizes the results of measurement in QM. This is a good starting point to introduce students to the concept of quantum state and measurement in QM, though it does not clarify the peculiarity of implications. Indeed, it should be noted that any measurement in CM and QM has its own probabilistic nature. At secondary school level CM, the outcomes fluctuate around the "real" value of the measured quantity due to the random accidents that can stochastically influence the outcome of the measurement. In QM, instead, the stochastic evolution of quantum systems is linked to the particularistic interior property of nature, and even if systems are prepared in the same initial state (under the same initial conditions) the outcome can be dramatically different as far as measurements are concerned.

The answers in category C4 highlight the idea that the state of a system in QM cannot be known accurately, confusing its abstract mathematical meaning with the knowledge of state. The concept of state requires awareness, because of a new way of thinking compared to classical physics and the formal role of mathematics in QM. For that reason, there is an indicator of how deep the fundamental concepts of QM take root in students' mind. In fact, this idea is often presented when students did not acquire the formalism of QM, i.e., the formal aspects are omitted, and the reasoning still proceeds at classical physics level. However, the hypothesis of hidden variable emphasizes the same argument, i.e., the stochastic evolution of a system is linked to the hidden variable (position in the de Broglie-Bohm theory), which is inaccessible to our knowledge, even if its value is actually defined. This line of reasoning has been observed in previous works [12,25,38,47]. Furthermore, classical physics, too, has the potential for failing to understand the concept of state.

The answers found in category C5 are an evident effect of the focus of the proposed educational path on the context of polarization (which is a peculiar property of a quantum system but can also be interpreted within a classical framework). Obviously, students need to consider some other examples of the quantum state to reach a global vision of that concept. On the other hand, the student's answer B7 is included in this category, which shows that his conception of the quantum state is focused on the concept of properties in classical kinematics. It seems quite evident that to modify this student's conception, renegotiation, and generalization of the concept of state in classical physics are required.

Finally, we can observe that categories C4 and C5 could underlie an unclear distinction between the concepts of state and property, which, as mentioned earlier, is a fundamental value in QM but not in CM, because in CM we tend to identify the state of a system with the set of values of the physical quantities that describe the system itself. The Hungarian students involved had not encountered these concepts and distinctions before, therefore the answers given by them highlight an evolutionary process in these concepts, which unfortunately was interrupted by the interruption of classroom schooling during the world pandemic (lessons 11–13).

To better frame this point, we interviewed two students (audio-recorded), the analysis of whose answers was not appropriate (students' answers NA, B6). We asked them what they thought regarding quantum states. The first student replied: "The physical state is the direction in which the photon is rotated", eliciting the following comment from the second student: "This determines the direction of the photon. I mean, we cannot define a concrete direction, but it does define a direction. We have different states".

The identification of the "direction of the photon" [its polarization] with the state of the photon appeared in the student's answers to the tutorial questions (digital work schedule), too: "We may represent the state of the photon transmitted by F1 with a vector $u//U$. Is this association sufficient to reproduce the results of the experiment (Malus law)? Explain". One of the students answered: "Yes, because in Malus law it depends on the angle of inclination of the polarizer and the polarization of the incident light, i.e., the angle of inclination of photons, which is defined by u".

The confusion between direction and state can come from the wave nature of light and from a failure to grasp the concept of physical property. Consistently with previous results [12], documenting difficulties in producing an awareness of the differences between state and properties in the case of polarizations, the iconographic representation can play an important role in distinguishing properties and states, because the icons used for properties are very different from a segment or an arrow (vector). Research also observed that students used icons to shape their interpretative hypotheses, without using mathematical tools, which are not mastered by everyone. The awareness of the new way of thinking requires a comparison between the classical and QM approaches to the concept of property, measurement, and state in relation to the basic formal description. It can also help to understand the concepts, as emerged in previous studies [12].

8. Students' General Opinion about the Curriculum

Feeling comfortable is an important aspect of any learning process. This is not only good for improving general well-being, as also it helps memorize concepts. The students (Group B) were asked at an interview (audio-recorded) to evaluate the course after its completion (they were encouraged to be critical). The question was: "What do you think of the curriculum as a whole?" The opinions were:

"I'll be honest I've hated physics since class 9. Now, I liked it a lot". (His grades were rather moderate in classes 9 and 10). He also said:

> "Experiments in previous courses were more complicated. Anyway, they went so fast that I didn't even understand them right away or I wasn't interested in them, or I just didn't understand. Here, instead, there was more time to learn, it didn't happen so fast. Everything happened bit by bit, and everyone saw the same thing. It was good that we digested the experiments for hours".
>
> Another one said:
>
> "Will there be a course with a similar theme next year? I think it would be great".

(Regretfully, there is no physics for them in the next semester, but we are planning an extra activity devoted to the subject.) One more quote:

> "I liked this approach; it was unique. [. . .] There's always something new here, we always add something, we always approach things differently, not monotonously".

The favorite part of the curriculum was the experiments and tasks carried out with calcite crystals. This interest was observed not only in the classes but in the interviews as well:

> "I'm interested in minerals and that's how I remember them". The knowledge of this student is limited, and yet he was able to keep in mind all the experiments during the course.
>
> "I've never been a fan of physics experiments. Chemistry experiments have always been much more spectacular, but to be honest, I liked them now. When we were after the illustration of principles, and not after the show, that was good. By the way, the calcite crystal was pretty spectacular".

A considerable portion of these responses was given by students with a humanities interest. Because of this, we talked about various quantum mechanical curiosities after the curriculum, and some novels and theatre pieces were also interpreted in this spirit.

The course nicely illustrates indeterminism, as a quantum mechanical principle, with experiments and interpretations, in contrast to the normal Hungarian curriculum where the probabilistic nature of phenomena is mentioned in just two sentences [33]. The following conversation took place during the online teaching period:

- Would you like to talk about these, outside classes?
- Yes, and I suggest that we involve others, such as . . . , he is also trained in philosophy and . . . , I think they may also be interested in it.

It is worth noting that the course was enjoyable also for the teacher, because of the students' engagement and the variability of the questions received. These results mostly show that active engagement is a way to keep up the interest of non-physics-oriented students, an aspect relevant for high schools.

9. Reflection in a Context of Implementation and Suggestions from Interviews

Some additional tasks would be beneficial to consolidate concepts and help generalizations:

- To provide more examples for the state transition $p(\boldsymbol{u} \rightarrow \boldsymbol{w})$ that are not assigned to vertical, horizontal, or 45° degrees directions. Before generalization, there were only these polarization states mentioned, and thus, by seeing the letter u, many students immediately associated it with the 45° state vector ("if $\boldsymbol{v}$ = vertical, $\boldsymbol{h}$ = horizontal then $\boldsymbol{u}$ must be polarized 45° direction").

- The notation $\Psi_1 \boldsymbol{h} + \Psi_2 \boldsymbol{v}$ appears to be too general; there were no exercises with specific numbers. Giving specific numerical examples would be useful: "what is the probability of a photon in the state represented by $0.55\boldsymbol{h} + 0.84\boldsymbol{v}$ passing through a vertical polaroid?"
- Rethinking some previous tasks about trajectory would also be helpful after becoming acquainted with the superposition principle at the end of the curriculum. Earlier, students discovered that if we place another inverted calcite behind a birefringent calcite crystal, it unites the beams, and the polarization of the transmitted photons will be the same as that of the incident beam. However, if we theoretically make a measurement between the two calcites, the outcome will change, so we can help students distinguish the property from the state. The schematic diagram illustrating the problem is reproduced in Figure 2.

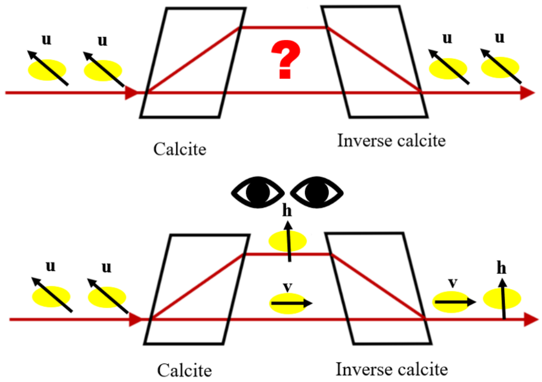


Figure 2. This thought experiment illustrates the lack of trajectory in a previous task. Let us consider $\boldsymbol{u} = \Psi_1 \boldsymbol{h} + \Psi_2 \boldsymbol{v}$. If we make a measurement between the two crystals, the outcome will change.

10. Conclusions

This paper documents experimentation on teaching/learning QM developed in the Physics Education Ph.D. The program carried out at Eötvös Loránd University, Budapest, in collaboration with the Physics Education Research Unit of the University of Udine. The conceptual research-based approach founded on the Dirac approach to QM developed by the Udine group was implemented in two Hungarian school groups involving 24 students (5 from a class specializing in physics and 19 from a humanities class), using tutorials and pre/post-tests to monitor the learning process of students. After two months, interviews were conducted with 5 students. The outcomes discussed in the paper provide answers to the research questions.

The overall positive outcome of the activity is briefly found in the systematic increase in responses consistent with quantum interpretations given by students between pre- and post-tests (see Figure 1). This research is an important test of the exportability of the teaching proposal designed by the research unit of the University of Udine and the accompanying tutorials [11,14,41,50].

The fact that the improvement is statistically significant only for students of the class specializing in physics is an outcome that had not emerged in the previous experiments of the proposal carried out in Italian classes. This could indicate that the effectiveness of the didactic action requires the achievement of a threshold. The limitation of this conclusion is that it is based on a small number of students and is therefore of limited validity. Further studies will be needed to confirm it, understand its actual consistency, and identify where we should place the threshold and the implications for learning/teaching.

Focusing on the discussion of the theoretical fundamental of QM (RQ1a), the interviews show that students enjoyed the lessons, and the curriculum also aroused the interest of students oriented to humanities studies, as were those of our Group B. The students themselves asked for an interdisciplinary approach, so the course not only taught the most important principles and basic formalism of QM but also had cultural value. Results from data analysis evidence that the impact was in terms of motivations, engagement, and multi-perspective activated interest. The interviews show that students' favorite parts were the interaction of photons with calcite crystals and the issue of nonlocality. This confirms what emerged from the experiments conducted in Italy [12,25,38], highlighting in a particular way the cultural value of dealing with the cardinal concepts of theory in high school, also in terms of guidance towards science and the cultural education of citizens. In the future, it might be useful to enlarge these parts when implementing the proposal, to emphasize its role in building a modern vision of the nature of physics as a discipline (RQ4).

Analyzing the pre/post-tests, an overall positive outcome can be observed as regards mastering the addressed concepts by students (RQ2a) (see Figure 1). This is important as a confirmation of the results obtained in the Italian schools in other national contexts [12] (RQ4). There is a learning gain on the addressed concepts and students mastered phenomenology with greater confidence (RQ2a), and they understood the interaction of photons (as quantum objects) with polarizers and birefringent crystals, with an awareness of the probabilistic nature of Malus law and the probabilistic and stochastic nature of quantum measurements. Moreover, concerning the concept of state (both in classical and quantum physics) and the superposition principle, we obtain evidence of improvements and new ways of thinking in comparison with the pre-test stage. We confirmed what the literature on learning processes indicates to us, namely, that we must prioritize the founding concepts (such as the state) for a conceptual approach that favors learning. Making students reflect on the concept of state in QM seems to help them attain a more elaborate (if not more precise) idea of what the state of a physical system means and what implications are connected to this concept, for example with regard to the possibility of predicting its time evolution or making forecasts on the results of measurement for the system.

At the same time, as we can see from an analysis of open questions in the questionnaire, interviews, and tutorial analysis, there were still some difficulties in grasping the concepts of the state as unrelated to that set of values of physical quantities (RQ3b). In more detail, we identified five ways to describe the quantum state by students, differentiating it from the classical concept of state, which was only partially identified in the previous research conducted with Italian students [47,48] (RQ4). Firstly, the state is identified with its formal representative entity, as a vector or a probability. Secondly, the concept of state is defined by specifying in which way a measurement affects the properties of a system (the properties remain the same before and after measurements, but the measurement attributes new properties to a quantum system). The third difficulty refers to the non-probabilistic results in classical physics and the uncertain results in quantum measurements. In this case, students' conceptions show that the quantum state cannot be theoretically accurate. An analogous conception is related to the fourth way of identifying the state, focused on the impossibility of exactly establishing the properties of a quantum system. The last way identifies the direction of polarization property and quantum state (RQ3b).

We have to pay attention to a more detailed and extensive treatment of the superposition principle and its formalism by activating a collective reflection in the class groups. That would facilitate sharing visions and reaching a more complete picture about the concept of the quantum state, in addition to helping modify students' conceptions. To help students improve their understanding of the distinction between state and quantum properties (RQ3b), we can also reinforce the use of appropriate iconographic representations and pay more attention to the visualization of the tasks. The iconic representation of the polarization properties with symbols (*, Δ, ◊) favors the recognition that they concern another sphere (another space) than that of the states represented instead by vectors. This also has an advantage, namely, students like to use icons to construct their interpretative hypotheses, without using mathematical tools they are yet to acquire.

In this experiment, an important aspect that emerged is how the analysis of different phenomenological contexts could help students acquire a more general vision of the concept of state detached from the phenomenology of polarization, which, as we saw, some students failed to perceive as an example of the quantum world, but rather risked identifying with it. Another difficulty evidenced by data analysis is the impossibility to attribute a trajectory to a quantum system, which had been confirmed in previous studies, too [12,25,38] (RQ4).

In summary, an important result is that the analysis and discussion of concepts such as state, property, and measurement in both classical physics and QM plays a central role in shaping the quantum mechanical way of thinking and understanding formalism. Moreover, as discussed in the previous paragraph, an important drawback of the work presented here regards the indications that emerged. The first is that too many students focused only on emblematic cases (45° polarization), which, while allowing them to grasp the concepts, thanks to their simplicity, expose them to the risk that they will veil them from the generality of what has been dealt with. On the other hand, it is important to support the proposal with exercises that allow students to better appropriate formalism operationally and conceptually. The last hints concern how to correlate the simple formalism implemented with the impossibility of associating a trajectory to a quantum system, thereby gaining a better understanding of the quantum view of phenomena.

Learning the fundamentals of quantum mechanics nowadays is of paramount importance not only for a future physicist but also for computer scientists, and engineers, who, for example, want to deal with quantum computing and cryptography. Experts from other fields may also have great interest in the studies on learning quantum concepts that are becoming increasingly relevant in frontier fields such as quantum biology and nanotechnology. We believe that efforts in this direction might be useful in the public education system of other countries, too, and we hope that recently popular photon-based quantum cryptography and quantum computations can also benefit from this paper based on Dirac's polarization approach.

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Informed Consent Statement: Participating students voluntarily applied, by replying to a free call describing the activity, launched by the "Czuczor Gergely" secondary school in Győr (Hungary).

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Appendix A

Table A1. Answers to Q8 Question.

#	Cat.	Pre	Post
A4	C1	No answer	*"There is some data in classical physics that describes the object. There is a vector in quantum mechanics from which we can calculate probabilities".*
B3	C1	No answer	*"In classical physics, the physical state is what can be measured, and an accurate quantity can be assigned from data and measurements, but in quantum mechanics we can only work with probability".*
B4	C1	No answer	*"In classical physics, the physical state of an arbitrary body can be described* e.g., *with a vector of motion. In quantum mechanics, this is a bit more complicated: although we can characterize the state of a photon, we can only give it according to a probability".*
B8	C1	No answer	*"In classical physics, it characterizes the system at a given point in time, while in quantum mechanics it gives information about the polarization of photon".*
B9	C1	No answer	*"Quantum: some vector, which transforms to another vector in a measurement".*
B6	C2	No answer	*"The state is different in classical physics and quantum mechanics, namely, we never really get accurate results, we can just deduce what happens with a photon after it goes through a polarizer, but in classical physics our measurements to define an object's position are accurate (e.g in motion)".*
B10	C2	No answer	*"In classical physics a set of properties can be determined precisely, while in quantum mechanics it can only be observed at a studied moment".*
B11	C2	No answer	*"Classical: some real state, Quantum: the existence of something given by measurements"*
A3	C3	*"In quantum mechanics we cannot define the state exactly"*	*"In classical physics, the state that we find by measurement is almost always the same. In quantum mechanics, however, there is always something else, very rarely the same value".*
B1	C3	*"Physical state: every material has physical properties, and we are able to make predictions"*	*"In classical physics, measurements yield the same property over and over again. In contrast, in quantum mechanics measurements can provide different results".*
B12	C3	No answer	*"In classical physics, the physical state of something is a constant property, whether it is during observation or measurement. However, in quantum mechanics, these two are different from each other, because we can get different results out of our observations and measurements (inversion of two calcite crystals, observation of light and examination yield different result)".*
A1	C4	*"In quantum mechanics, there is no accurate position, there are only probabilities".*	*"Classically, the place and position of a given object can be determined exactly, but not so in the quantum world".*
B5	C4	No answer	*"Physical state is a measurable factor in classical physics, but in quantum mechanics we can only estimate it".*
A2	C5	No answer	*"In classical physics, the physical state of an object can be described by its spatial position and velocity. In quantum mechanics, the physical state of a particle can be described by its spatial position and polarization property".*
B2	C5	No answer	*"In classical physics, the physical state refers to a property, or fact, that is absolutely certain, and we can easily determine the physical state even with the naked eye. In quantum mechanics, the physical state can be determined with probability,* e.g., *polarization, because it is not possible to simply determine the physical state with absolute certainty".*
B7	C5	*"Every material has physical properties (colour, smell) and we know their form, too"*	*"In classical physics, it is possible to determine the location of an object and what forces act on it. In quantum mechanics, the physical state of an object can be characterized by its polarization property".*

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