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How Does Simulation Contribute to Prospective Mathematics Teachers' Learning Experiences and Results?

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Abstract: In this research, the aim was to evaluate a simulation-based learning environment in the context of conditional probability. The study group consisted of 44 prospective mathematics teachers of the Probability and Statistics Teaching course. The data were collected through three probability problems, a survey form for the simulation-based learning environment, and observations. The research was conducted within the scope of the Probability and Statistics Teaching course. In the lessons, conducted in a simulation-based learning environment with distance education, the prospective teachers were asked to solve the questions asked and send the solutions using smartphones. The different ways of thinking that emerged are put forward by the researcher. Then, simulations developed by the researcher were used for the problems, and the prospective teachers were asked to make observations and take notes on important issues. In the last stage, there was a class discussion about the related problems. After the simulation-based learning activities, the prospective teachers were asked to evaluate the learning environment. The data obtained were evaluated qualitatively, and the prospective teachers' ways of thinking about problems, changing thoughts with the use of simulations, and their views on the learning environment are presented as direct quotations. The research findings revealed that the designed learning environment offers unique opportunities for prospective teachers to think about situations with mathematics content and to experience different methods of teaching and learning mathematics. The learning environment provided prospective teachers with the opportunity to understand and evaluate the contribution of simulations to problem-solving and the opportunities they provide for discussion, reflection, and collaboration in a meaningful context. It was concluded that simulations could become a powerful tool and an effective learning environment for learners.

Keywords: mathematics education; probability teaching; simulation; prospective teachers

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1. Introduction

Digitalization has progressed rapidly in the last two decades, and in parallel, the supply of educational software has increased steadily. In terms of education, there are two purposes for using this software. These are to improve the quality of teaching and learning and to improve students' digital literacy [1]. Teachers need to combine their field knowledge, pedagogical knowledge, and technological knowledge.

In today's data-driven world, probability and statistics affect all aspects of public life in every conceivable way. Concepts such as data, chance, probability, randomness, and risk are the concepts of probability and statistics that are encountered every day. In the information age, individuals use the internet, TV, newspapers, etc. every day. They come face-to-face with a lot of data in environments and have to make decisions in situations of uncertainty. Probability and statistics are included in the curriculum in most countries because of their benefits in daily life, their roles in other fields of study, and their contribution to the logical inquiry process. It is seen that in the last two decades, more emphasis has been placed on probability and statistics in curricula. In addition, the number of studies on this subject is increasing day-by-day [2]. However, probability and statistics are a top subject that both teachers have difficulties in teaching and students in learning [3–6]. Hawkins (1990) emphasized that probability education cannot only be reduced to conceptual structures and problem-solving tools, but also to logical inferences and the necessity of establishing correct intuitions in students [7]. Fischbein (1997) stated that students tend to believe that an effect will always produce the same result [8]. Such conceptual errors in the process of learning probability concepts can affect important personal decisions [9].

Probability problems are difficult. Even individuals trained in the fundamentals of probability struggle to solve probability problems correctly. In addition to intuition or personal experience in the classroom, there is empirical evidence to support these claims. With the importance of probability in curricula and the increase in technological equipment in schools, it is recommended that teachers use simulations to enable students to experience repeated attempts at an event [10] because students need experiments to understand the theoretical foundations of probability. These experiments are opportunities that develop their probabilistic intuition, help them build a solid understanding of probability, and motivate them [9]. Studies on probability teaching have suggested the use of computers as a way to understand abstract or difficult concepts and increase students' abilities [11,12]. There are different perspectives on how to best teach probability so that probabilities can be interpreted by everyone in various situations [13]. These opinions are based on different interpretations of the possibility. People think about probability in at least three different ways (classical, frequency, and subjective), and these insights can manifest themselves in the probability teaching and learning process. If an experiment has n simple outcomes, each outcome has an equal probability of occurring; this method assigns 1/n probability to each outcome. That is, the number of ways a particular event can occur is divided by the number of all possible ways. This method is called the classical approach. If probabilities are based on experiments or past data, this method is called frequency approximation. The subjective approach is defined as the degree of belief that an event has occurred. Judgment is used as the basis for assigning probabilities. What is different here is that the use of the subjective approach is often limited to experiments that cannot be replicated. Each of these perspectives has its advantages [10]. For students to develop a meaningful understanding of probability, it is important to be aware of these different perspectives and their interrelationships, and to discover which ones they can benefit from and how in different contexts.

It is inevitable to use different software packages as tools in this period when it is necessary for new teaching technologies to enter learning environments. Software is developed day by day, and some problem situations that cannot be obtained in the real word have become observable easily and quickly. One of them is simulation. Simulation is a teaching method in which students can change their parameters and make experiments one-to-one [4,14] Batanero et al. (2005) and Koparan (2019) emphasize the use of simulations in computer lessons in schools to help students solve simple probability problems that are not possible to solve using physical experiments [10,15]. Simulations are computer applications that allow students the opportunity to observe and interact with real-world experiences [12]. Simulations provide the opportunity to strengthen the understanding of statistical ideas [16] and to support the learning process of students while working on luck experiments [17]. Simulations provide opportunities to strengthen the understanding of statistical ideas [16] and to support learners' learning processes while working on chance experiments [10,17]. Students can build their knowledge using simulation activities [18]. Additionally, simulations encourage active learning and participation in the learning process. Simulations can be used as a tool to improve conceptual understanding and problem-solving. Thus, students can take a more active role in their learning by searching for various alternatives to answer and solve their questions [19]. Instead of closed-ended problems, presenting simulation and design activities where the solution is not open can help students to develop some of the skills required in lifelong learning. Simulation, when combined with the use of technology, is the most appropriate strategy for providing a better focus on probability concepts and reducing time-consuming calculations [9]. Koparan and

Taylan Koparan (2019) stated that simulations are very productive tools for understanding stochastic processes, especially because they can contribute to the development of ideas about randomness and variation [15]. Batanero and Godino (2002) point out that by comparing the results obtained by each student, the variability in small samples should be emphasized, and situations should be created to observe individual results' unpredictability in experiments that produce random results [20]. Students can evaluate by looking at the sum of the results of the whole class, and then, compare the results for large samples with the help of computers so that they can realize the convergence phenomenon (Law of Large Numbers) as well as the relationship between experimental probability and theoretical probability. In this way, the use of computer resources constitutes an important tool for increasing the number of samples for random experimentation in the classroom.

1.1. Simulation as a Tool

A simulation is a form of experiential learning. Students interact with a model prepared by the teacher. The teacher controls the model parameters and uses them to achieve the desired learning outcomes [2]. A simulation is a teaching scenario. Students experience the reality of the scenario and make sense of it. The ambiguous or open-ended nature of a simulation encourages students to reflect on the implications of a scenario. Simulations help students understand concepts in a topic through experimental practice. Simulations are activities that encourage the use of motivating, critical, and evaluative thinking that students of all ages enjoy. Learning using simulation is a form of learning that is very suitable for constructivism principles. As a pedagogical tool in the classroom, simulations can engage students and encourage learning. In this way, it can fill the gap between knowing and doing. The main advantage of simulation in education is that it can be presented as a useful summative and formative assessment tool. As a teaching tool, simulations are a useful part of active learning techniques and can help students combine course content with real-life scenarios. The disadvantages are that it takes time to prepare, the software is costly, and its assessment is more complex than some traditional teaching methods.

It is very important to create a suitable model to perform a simulation [21]. Fischbein (1977) defined a model as a simplified version of the original, a tool that allows easy and complete control of the variables. One of the primary roles of a model is to facilitate the interpretation of certain facts. The other is to help solve problems according to the original facts [8]. Fischbein (1977) identifies some of the other characteristics of good models: A model is a heuristic if it makes it possible to pose problems within the model and to find solutions without leaving the model [8]. A model is a generative model if it can represent an unlimited number of different states using a limited number of items and rules. A model is coherent if its elements are interpreted in the same way for every different situation, and there are naturally no contradictions in its operation and progression. A model is well-constructed if it has an internal structure coherent with the original structure. This structure is provided by the properties and relationships of the elements of the model and is not imposed externally. Intuitive and well-structured features for models by Fischbein (1977) provide some insight to how one might improve the power of simulation in problem-solving [8]. In this study, the dynamic statistics software TinkerPlots was used to create the simulation. TinkerPlots is a software package developed by Konold and Miller (2004) for students of all levels to learn statistics and probability [22]. The core objects of a simulation in TinkerPlots software form a series of essays that demonstrate certain attributes. There are several approaches to the use of simulation in the learning environment; in this study, a simulation-based learning approach was adopted.

1.2. The Use of Simulation in Distance Education

It has been determined that distance education from primary school to university has great potential, especially during the pandemic period. It has also been found that well-designed interactive media tools such as simulations and virtual environments can offer learners interesting ways to bring in content. In this research, the aim is to bring the two together. Academic and lifelong success requires not only an accumulation of facts and conceptual insights, but also attitudes, tendencies, and values that are compatible with science. Learning from this perspective is about building the knowledge, skills, beliefs, and attitudes that form the identity of a person who is the producer of scientific knowledge. If students develop their identities as competent performers in science and acquire knowledge, skills, and beliefs aligned with those valued by the scientific community, they will undertake these practices outside of formal school contexts. Therefore, in parallel with the use of new digital media, a current trend in education is to accept the emergence of new learning experiences such as simulations, and to try to understand their implications in how we think, act, and learn [2,23,24].

1.3. Theoretical Framework

The enormous amount of knowledge accumulated in recent years, along with the large number of studies carried out in various disciplines, has led to an overburden of knowledge, most of which remains in the field of theory, and thus, has limited contribution to novices [25]. This has motivated educators to develop new teaching methods in which all knowledge is studied but various learning processes are used to create a tangible link between theory and practice [26]. The simulations have been developed as an educational tool for professionals to efficiently apply and deal with complex and challenging situations that they face daily in their field. Considering the existence of different modelling and simulation approaches, the need to create a common language that allows researchers, educators, and trainers to share their developments about simulation has emerged. The simulation-based learning (SBL) cycle adopted in this research is presented in Figure 1.

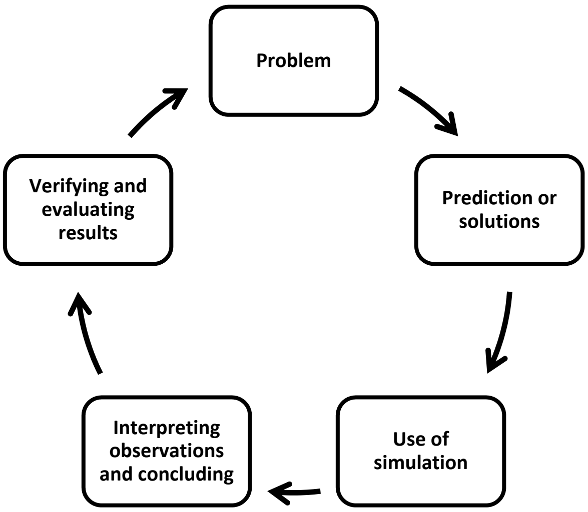


Figure 1. Simulation-based learning cycle.

The simulation cycle takes about 25–30 min, and each cycle consists of several stages that are repeated sequentially. Below is an explanation of the stages that make up a cycle.

1. Problem: The cycle starts with the presentation of a problem situation to the participants. Care is taken to ensure that the problem situation is from daily life and can be modeled.

2. Prediction or solutions: After the problem situation is read by the participants, the participants are given enough time to present a prediction or solution to the problem.

3. Simulation use: At this stage, simulations prepared by the researcher are used for the problems and the participants are asked to observe the simulation environment.

4. Interpreting observations and concluding: At this stage, participants are asked to examine the observation results (What happened here? Why did this happen?) and conclude.

5. Verification and evaluation of the results: In the last stage, participants are asked to present theoretical explanations regarding the results they obtained from the observations and evaluate the simulation-based learning process. The aim is to enable participants to develop their reflection skills on the problems posed, as well as to provide feedback on the learning environment and process (Which communication skills helped to advance the discussion and solve the problem, and which prevented progress?). Finally, the researcher is informed about the issues that emerged in the discussion.

It is thought that it would be appropriate to consider the loop formed in the context of this study as a case study. To summarize, the purpose of this study is to clarify the advantages of the SBL environment and improve its use. For this reason, the research questions focus on prospective teachers' SBL experiences. Probability knowledge is essential in making rational decisions, and more attention needs to be paid to both the students' learning of probability and the preparation of teachers in this field. Current teacher training tends to focus more on theory and concepts and less on their practical application. This is one of the main reasons why pre-service teachers are unprepared when they start working, whereas skills and experiences can be developed effectively in environments similar to real-life settings. However, most of the current studies suggest that this needs to be built up through ongoing and further research. Although there are important studies based on the literature, experimental and evidence-based studies are needed for the development and widespread use of simulation in education. Hawkins (1996) stated that using technology effectively in terms of how students learn and how best to teach them is the same as planning non-technology teaching [27].

This study aimed to examine the role of prospective teachers' reasoning in the context of probability problems and the use of simulation as a tool in solving probability problems. For this purpose, we asked: How does simulation contribute to higher education students' learning experiences and results? In this context, the learning experiences realized by the prospective teachers in the SBL process, and the insights they gained as a result of the SBL experiences according to the perspective of the prospective teachers, were focused on. In addition to these objectives, we asked: What should the content of a research simulation-based course be? How should this content be organized to be a useful tool when solving problems? This study sheds light on the questions in an instructional atmosphere.

1.4. Research Questions

In this study, to determine the influence of the learning environment created for teaching probability on the learning experiences and results of prospective teachers, answers were sought to the following questions:

(1) How does the SBL environment affect prospective teachers' thinking about probability?

(2) What are the opinions of prospective teachers on the use of simulation in probability teaching?

2. Methods

In this study, the case study method was adopted. Case studies, the aim is to examine and reflect on the special case of a certain phenomenon in depth [28]. In this type of research, the environment, individuals, and processes in which the research is conducted are investigated using a holistic approach and the relationships and interactions between them are focused on. The use of more than one data collection technique in case studies allows researchers to reach rich and mutually supportive data diversity [29]. In this study, the simulation-based learning environment is considered a special case in teaching probability with simulation. In this designed learning environment, we tried to determine the way of thinking about probability problems of prospective teachers. Qualitative research can provide in-depth and rich information about participants' views and personal understandings, but because research is typically based on a small non-random samples, the findings cannot easily be generalized beyond its participants [30].

2.1. Working Group

This research was carried out in a university in the Black Sea Region of Turkey in the Spring Semester of the 2020–2021 Academic Year. The study group consisted of 44 mathematics teacher candidates, 31 women (70.5%) and 13 men (29.5%), who took the Probability Statistics Teaching course. A purposeful sampling method was used in the study. In purposeful sampling, the researcher specifies the characteristics of the population of interest and finds individuals with these characteristics, and selects a sample that he thinks will provide the best understanding of the subject studied (Fraenkel and Wallen, 2006). Among the most suitable sample groups for the use of simulation are prospective teachers who are directly exposed to SBL environments. The Probability Statistics Teaching course is a compulsory and theoretical course held for 3 h a week, included for the first time in the curriculum in 2020–2021. Prospective teachers took this course for the first time. In the previous year, they took the probability course, which was conducted without simulations for 2 h per week.

2.2. Data Collection Tools

According to Yin (2009), a reliable case study should be based on at least two data sources [31]. Therefore, the present study is based on two data collection tools. The first is the data collected through the problems (Table 1) the researcher presented to the participants. These data were collected by smartphones in distance education and analyzed one-by-one by transferring them to the computer.

Table 1. Some examples of problems used in the learning environment.

	Examples of Problems
1	What is the probability that two of the children are girls and two are boys in a family of 4 children?
2	One of the two children of a father is a boy. What is the probability that his other child is also a boy?
3	There are three cards in a hat. One of the cards is red (RR) on both sides, the other is white (WW) on both sides, and the last card is coloured red on one side and white (RW) on the other side. A random card is chosen from the hat and placed on the table. It seems that the visible side of the card is red. What is the probability that the bottom side of the card is also red?

The second is the view form used to reveal the participants' views on the SBL environment at the end of the lesson. A total of 21 items from the opinion form are of the 5-point Likert type, ranging from "completely disagree" to "agree". Two items are open-ended questions and five items are two-answer (yes–no) questions. With this opinion form, the aim was to determine the prospective teachers' thoughts on simulation, not to develop a scale. Participants were asked to reflect on their views, experiences, and insights they gained, both as participants and observers, regarding the lessons in which the SBL cycle was used.

To prevent bias, the participants were unaware of the objectives of the research. The learning environment used in this study was also used in the lessons held throughout the semester (Figure 1). The SBL cycle was designed to be a fixed component of the lessons. Each week, prospective teachers experienced different problems related to probability and statistics, observed simulations in problem-solving, and participated in self- and peer-feedback. Each course can be considered an independent workshop. The opinion form was prepared as a "Google form" and the questionnaire link was sent to the participants who took the course, and they were asked to fill it. For the participants to express their opinions about the SBL environment freely, their identity information was not requested; only gender information was requested.

2.3. Data Analysis

The data presented are the written responses of the prospective teachers to the questions. This method was adopted in distance education to enable all prospective teachers to participate in the activities comfortably. However, prospective teachers were allowed to express their thoughts during the course. The courses were recorded. The recordings were monitored repeatedly during the analysis of the data. An answer was sought to the question of what kinds of new opportunities simulation offers in solving probability problems that are not present in the paper-and-pencil process. This question has been one of the most important focal points of the research throughout the activities. For this purpose, it examined what prospective teachers can and cannot do in the learning environment. The first predictions and solutions to the problem using paper and pencil, different experiments conducted by changing the simulation variables, the interpretation and conclusions of the observations, and reflections from the verification and evaluation stages of the results were presented as evidence with direct quotes and screenshots. The researcher researched the use of technology in probability statistics teaching, its integration into lessons, and its effects. In this study, unlike others, the simulation activities were not limited to a few problems or a few weeks but covered a period of 14 weeks. The sections presented in this article are part of every lesson. This was observed repeatedly by the researcher. In addition, some sections recorded by the researcher were examined by a field expert and four teachers, and opinions were received on whether the simulation contributed to the solution of the related problems. Teachers and experts stated that simulations created by different researchers for similar questions would provide similar advantages. The types of actions expected from the teacher candidates during the activities can be defined as academic and social actions. These actions are presented in Table 2.

Table 2. Types of Actions Expected from prospective teachers.

Academic Goals		Social Goals
Reasoning (R)	Generalizing (GE)	Attending events (AE)
Hypothesis (H)	Determining model problem fit (DMPF)	Freely sharing thoughts (FST)
Guessing (GU)	Performing a few and many experiments (PE)	Using mathematics as a communication tool (UMCT)
Testing (T)	Interpreting the test outputs (ITO)	Valuing the use of technology (VUT)
Observing (O)	Learn from mistakes (LFM)	Peer learning (PL)
Explaining (E)	Calculating experimental probability (CEP)	
Making inferences (MI)	Calculating theoretical probability (CTP)	
Using the language of probability (LP)	Bridging experimental and theoretical probability (BETP)	
All possible situations (APS)	Desired situations in all situations (DS)	

Social actions were observed at every stage of the research (class participation, problem-solving, participation in class discussions, and participation in the questionnaire). Evidence that academic actions were provided is included in the findings.

2.4. Application Process

SBL activities were carried out within the scope of the Probability Statistics Teaching course, which was conducted for 14 weeks, for 3 h a week. The lessons were carried out with distance education due to COVID-19 and all the lessons were video recorded. The researcher was also in charge of the course and played a role in identifying and presenting the problems, using simulations related to the problems, and encouraging prospective teachers to solve the problems and explain the solutions. After the problems were presented, the teacher candidates were given enough time to solve them and they were asked to send photos of the solutions via a smartphone instant messaging application (WhatsApp). Thus, the aim was to better understand prospective teachers' approaches to problems and their thinking styles. Then, experiments were conducted using simulations for the problems, and the prospective teachers were asked to observe the outputs. They were asked to record important results related to their observations and to conduct experimental probability calculations. Finally, considering the experimental results, they were asked to rethink the solution to the problem and calculate the theoretical probability. The activities ended with class discussions. Each of the activities lasted 25–30 min. At the end of the activities, they were asked to watch the video recordings and evaluate the SBL. In the simulation-based activities, the whole process, and especially the classroom discussions, were observed by the researcher by watching them again from the video recordings. The trainer started the activities by presenting the problems to the prospective teachers, and closely followed the activities of the prospective teachers (their solutions to the problems, their inferences for simulation observations, and their discussions on theoretical probability). Throughout the lesson, the trainer had to guide the prospective teachers according to the SBL cycle.

3. Results

The results are presented to answer the research questions. For the first question, the SBL cycle adopted in the study was scrutinized to reveal how simulation affects the learning experiences and the results of prospective teachers. For the second question, the findings, obtained from Google Forms to determine the opinions of the prospective teachers about the use of simulation and the SBL environment, are presented as frequencies and percentages.

3.1. The Findings Obtained from the First Research Problem

3.1.1. Problem 1

What is the probability that two of the children are girls and two are boys in a family of 4 children?

Predictions or Solutions to the Problem

The predictions or solutions to the problem are presented in Table 3.

Table 3. Predictions or solutions to the 1st problem.

Response	Some Extracts of Ways of Thinking	f (%)
1/16	*PT3: The probability that 2 out of 4 children are boys and 2 are girls* $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{16}$ *PT18: The probability that 2 of them are men is* $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$ *The probability that 2 of them are girls is* $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$ *We multiply them and conclude* $\frac{1}{4}\cdot\frac{1}{4}=\frac{1}{16}$	30(68%)
1/5	*PT1: BBBB, BBBG, BBGG, BGGG, and GGGG are all states. We want one, probability 1/5.* *PT15: There may be cases of 4G, 4B, 3G1B, 2G2B, and 1G3B. There is only one situation we want. Probability 1/5.*	9(20%)
3/8	*PT27: BBGG* $\frac{\frac{4!}{2!.2!}}{2.2.2.2}=\frac{6}{16}=\frac{3}{8}$ *PT39: There is 1 case in the form of BBBB, 4 cases in the form of BBBB, 6 cases in the form of BBGG, 4 in the form of BGGG, and 1 case in the form of GGGG. There are a total of 16 situations. The state we want is six, with probability 6/16 = 3/8.*	3(7%)
1/4	*PT43: There are 4 situations we want. These are BBGG, BGBG, GBGB, GGBB. All cases are 16. The probability is 1/4.* *PT17: The desired situation is 2 girls and 2 boys. There are 4 different situations in this way. GGBB, 2.1.2.1 = 4. In all cases, 2.2.2.2 = 16. Probability 4/16 = 1/4*	2(5%)

In Table 3, it was observed that 30 (68%) prospective teachers gave the answer of 1/16, and they did not take into account the fact that the children were older or younger in the problem. It was seen that only two (5%) prospective teachers took into account all their situations and gave the answer of 3/8. When the solutions of the participants who gave answers other than 3/8 were examined, it was determined that the situations that could arise or the sample space were evaluated incorrectly.

Use of Simulation

The model created for the experimental solution of the problem is shown in Figure 2. B represents the boy, G represents the girl, Draw indicates that the random selection will be made four times, and Repeat 1000 indicates the number of trials.

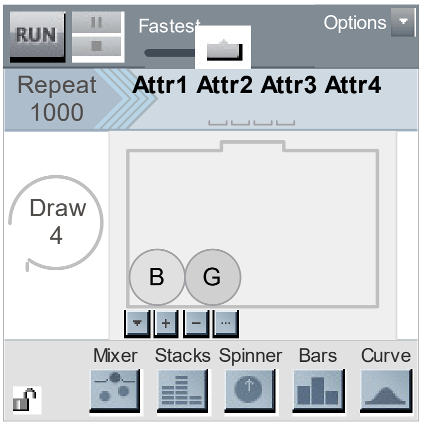
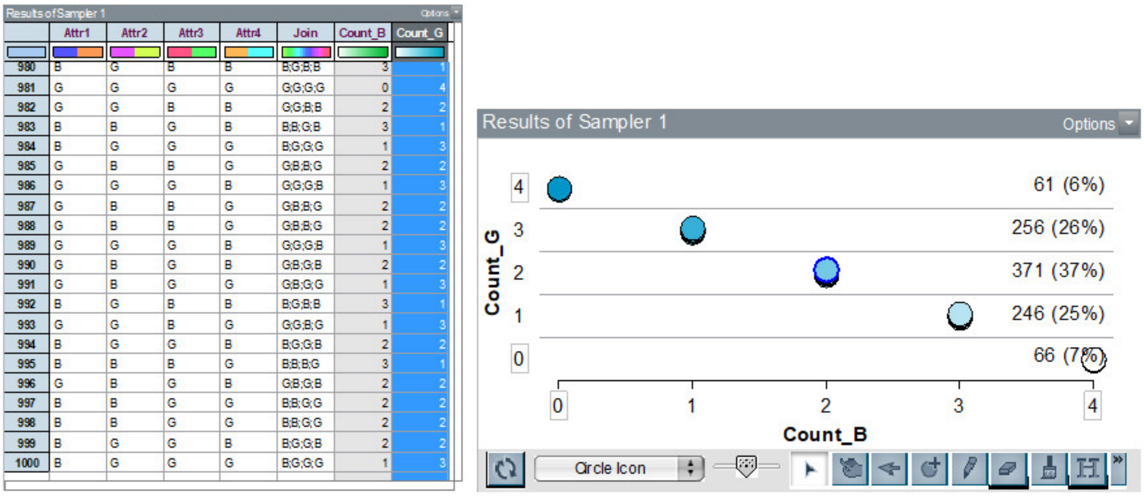


Figure 2. A simulation model was created for the 1st problem.

In the simulation created for the problem (Figure 2), the number of trials was taken as 10, 100, and 1000; the resulting results were presented with graphics containing frequencies and percentages, and the participants were asked to make observations. Figure 3 shows the results for 1000 trials.


Results of Sampler 1

	Attr1	Attr2	Attr3	Attr4	Join	Count_B	Count_G
980	B	G	B	B	B;G;B;B	3	1
981	G	G	G	G	G;G;G;G	0	4
982	G	G	B	B	G;G;B;B	2	2
983	B	B	G	B	B;B;G;B	3	1
984	B	G	G	G	B;G;G;G	1	3
985	G	B	B	G	G;B;B;G	2	2
986	G	G	G	B	G;G;G;B	1	3
987	G	B	B	G	G;B;B;G	2	2
988	G	B	B	G	G;B;B;G	2	2
989	G	G	G	B	G;G;G;B	1	3
990	G	B	G	B	G;B;G;B	2	2
991	G	B	G	G	G;B;G;G	1	3
992	B	G	B	B	B;G;B;B	3	1
993	G	G	B	G	G;G;B;G	1	3
994	B	G	G	B	B;G;G;B	2	2
995	B	B	B	G	B;B;B;G	3	1
996	G	B	G	B	G;B;G;B	2	2
997	B	B	G	G	B;B;G;G	2	2
998	B	B	G	G	B;B;G;G	2	2
999	B	G	G	B	B;G;G;B	2	2
1000	B	G	G	G	B;G;G;G	1	3

Figure 3. The outputs of the simulation model for 1000 trials.

Interpreting Observations and Concluding

PT18: *According to the results of 100 trials, 2 boys 2 girls 31, 39, 36, 44, 37, 36, 37.39, 35, 37, 43, 41. The probability is between 31% and 41%.* [O-R-E-ITO-CEP-GU]

PT7: *Should we increase the number of experiments? If we increase the number of attempts, we can find a more precise range. (This on-demand simulation is used for 1000 trials)* [T-R-PE-E-MI]

PT26: *According to the results of 1000 trials, the number of 2 boys and 2 girls is 382, 371, 356, 369, 350, and 392. According to these, the probability value can be between 35% and 40%.* [O-R-E-ITO-CEP-GU]

PT4: *Yes, but this is experimental probability. The theoretical probability is a single value, while the experimental probability can take on different values in a certain range.* [BETP]

PT4: *There are 4B, 3B1G, 2B2G, 1B3G, and 4G states in the simulation. These are all occasions.* [APS]

PT33: *The situation with 2 boys and 2 girls is more*. [ITO-MI-R-E]

PT39: *According to the simulation results, the numbers 1B3G and 3B1G are close to each other. Also, the 4B and 4G numbers are close to each other*. [ITO-MI-R-E]

Verifying and Evaluating Results

PT25: *I noticed this in the simulation. There are cases when children are born before or after. The genders can be ordered differently. Our fault is that we neglect some situations.* [OE-ITO-R-E-LFM]

PT41: *2 boys and 2 girls can be in different ways. We can think of these as repetitive permutations. The number of different arrays of these* $\frac{4!}{2!.2!} = 6$.

Since there will be 2 different gender situations for a child, there are 2.2.2.2 = 16 different situations for 4 children. From here the probability is $\frac{\frac{4!}{2!.2!}}{2.2.2.2} = \frac{6}{16} = \frac{3}{8}$. [APS-R-CTP-GE]

PT20: *I understood properly now. There are cases 4G, 3G1B, 2G2B, 1G3B, and 4B. We can express them in combinations such as* $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4}$.

In other words, there is 1 case in the form of GGGG, 4 cases in the form of GGGB, 6 cases in the form of GGBB, 4 cases in the form of GBBB, and 1 case in the form of BBBB. There are a total of 16 situations. The state we want is six, probability 6/16 = 3/8 = 0.375. [PE-APS-R-ITO-CTP-GE]

3.1.2. Problem 2

One of the two children of a father is a boy. What is the probability that his other child is also a boy?

Predictions or Solutions to the Problem

The predictions or solutions for to problem are presented in Table 4.

Table 4. Predictions or solutions to problem 2.

Response	Some Extracts of Ways of Thinking	Frequency Percentage
1/2	*PT7: The sex of the child is independent of the other. It can be a boy or a girl. Therefore, the probability of being a boy is 50%.*	33(75%)
	PT11: The gender of the children is always 50% girls and 50% boys for each child.	
	PT32: It could be a girl, or it could be a boy. There are two sexes. So there are two situations. Probability 1/2.	
	PT40: %50. Because it doesn't matter what the gender of the other child is. It is already known.	
	PT17: When a child is born, either a boy or a girl is born. In other words, the probability of a boy or girl being born is 50%. Even if ten boys are born, there is a 50% chance that the eleventh child will be born a boy or a girl.	
	PT12: All situations BG and BB. The desired state is BB. The probability is 1/2.	
	PT9: XX (Female) and XY (Male) from here XX (Girl), XY (Male), XX (Girl), XY (Male).	
1/3	*PT10: All occasions for two kids BB, BG, GB, GG. Since someone is a boy, there can be no GG status. There are three states in total (BB, BG, GB) and there is a desired state (BB). The probability is 1/3.*	5(11%)
	PT33: All situations are situations where there is at least one male. $B = \{BG, GB, BB\}$*, the desired state is* $A = \{BB\}$*. The intersection of these two sets is* $A \cap B = \{BB\}$*. According to the conditional probability formula* $P(A\|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{3}$.	
1/4	*PT27: There are a total of four cases GG, BB, BG, and GB. The desired state is BB. So the probability is 1/4.*	4(9%)
1/5	*PT3: Both are men (BB),* $\binom{2}{2} = 1$*The first is a boy and the other is a girl (BG),* $\binom{2}{1} = 2$*. The first is a girl and the other is a boy (GB),* $\binom{2}{1} = 2$*. If we divide the desired situation into all situations, we find the probability 1/5.*	2(5%)

As can be seen from Table 4, it was seen that 33 (75%) prospective teachers gave the answer 1/2 for this problem. The majority of the prospective teachers did not realize that the problem was a conditional probability problem. Some of the teacher candidates, such as S12, who answered 1/2, interpreted the fact that one of their children was a boy and that the first of their children was a boy. It was seen that two prospective teachers used the information they learned in the biology lesson to support the 1/2 answer. (Table 4.). It was observed that only five (11%) prospective teachers gave an answer of 1/3 with conditional probability. In the answers, all situations were taken into consideration or a conditional probability formula was seen. Four (9%) prospective teachers gave the answer 1/4. When they reached this result, it was determined that they included the situation that they were both girls in the solution. Two (5%) prospective teachers reached the answer 1/5. It was observed that these students included combinations in their solutions.

Use of Simulation

The model created for the experimental solution of the problem is shown in Figure 4. B represents the boy, G represents the girl, Draw shows that the random selection will be made two times, and Repeat 1000 indicates the number of trials.

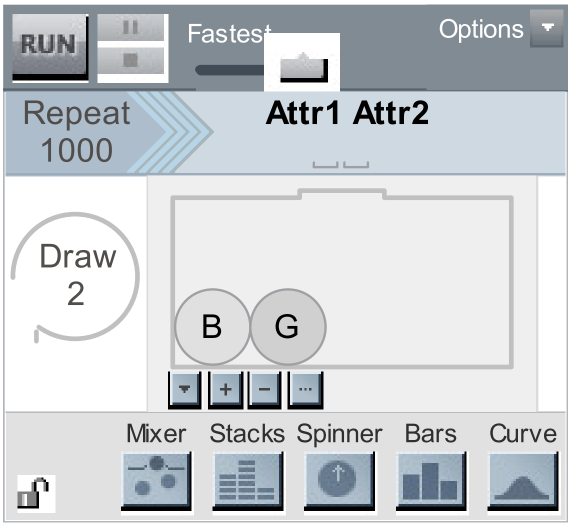
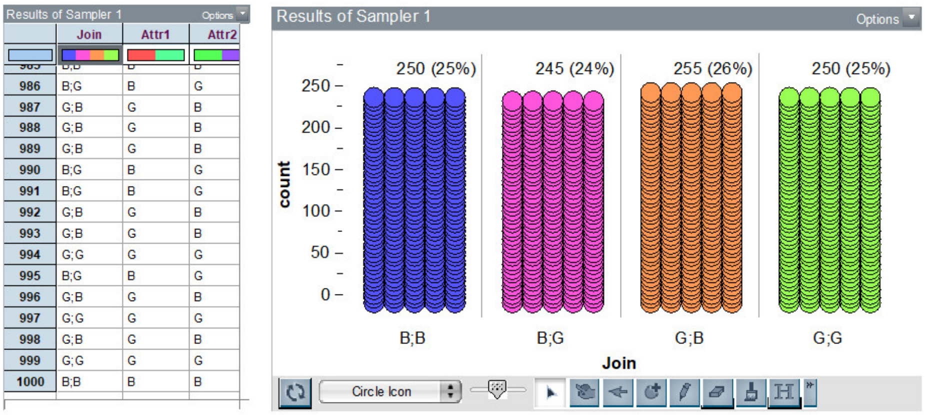


Figure 4. A simulation model was created for the 2nd problem.

In the simulation created for the problem (Figure 4), the number of trials was taken as 10, 100, and 1000; the resulting results were presented with graphics containing frequencies and percentages and the participants were asked to make observations. Figure 5 shows the results for 1000 trials.


Results of Sampler 1

	Join	Attr1	Attr2
986	B;G	B	G
987	G;B	G	B
988	G;B	G	B
989	G;B	G	B
990	B;G	B	G
991	B;G	B	G
992	G;B	G	B
993	G;B	G	B
994	G;G	G	G
995	B;G	B	G
996	G;B	G	B
997	G;G	G	G
998	G;B	G	B
999	G;G	G	G
1000	B;B	B	B

Figure 5. Outputs from the simulation model for 1000 trials.

Interpreting Observations and Concluding

PT8: There was a balanced distribution in the simulation. All of them are equally likely to happen. So 25%. [O-R-E-ITO-MI-CEP]

PT15: We know a boy is a boy. So we will not consider the situation where they are both girls. [O-R-ITO-MI]

PT28: Yes, I agree. There are 3 situations where one is a boy. [O-R-E-ITO-MI]

PT16: What is required of us is the situation in which both of them are men. There is one situation like this. [O-R-E-ITO-MI]

Verifying and Evaluating Results

PT36: The probability is 1/3. Because there are 3 situations. There is 1 situation requested from us. [O-R-E-ITO-MI-CTP-GE]

PT20: I also thought it was 1/2 at first. I saw the situations one by one in the simulation. The children of a family of two children can be both boys (BB), the older boy (BG), the older girl (GB), or both girls (GG). In the question, it is stated that one of the children is male, that is, the only possible situations are BB, BG, and GB. Since only one of them is the other boy, the probability is, or 33%. [O-R-E-ITO-MI-CTP-LFM-GE]

3.1.3. Problem 3

There are three cards in a hat. One of the cards is red (RR) on both sides, the other is white (WW) on both sides, and the last card is colored red on one side and white (RW) on the other side. A random card is chosen from the hat and placed on the table. It seems that the visible side of the card is red. What is the probability that the bottom side of the card is also red?

Predictions or Solutions to the Problem

The predictions or solutions to the problem are presented in Table 5.

Table 5. Predictions or solutions to problem 3.

Response	Some Extracts of Ways of Thinking	Frequency Percentage
1/2	*PT12: It becomes 1/2. Because the card with a red side on one side is either the 1st card or the 3rd card. So there are 2 cards. The card with red on both sides is only the 1st card. So the probability is 1/2.*	28(64%)
	PT23: All states are RR and RW. There are two situations. The desired state is RR, that is, a situation. The probability is 1/2.	
	PT10: It is obvious that the WW card is not drawn, and then the card on the table is either RR or RW. The probability of both must be the same; since we pulled randomly.	
1/3	*PT41: Since the cards are RR, WW, and RW, a total of in the 6 faces. In the first case we want, probability 2/6 = 1/3.*	7(16%)
	PT18: There are 6 faces in total. Since we see a red, there are 2 reds left. Since all faces are 6, the probability is 2/6 = 1/3.	
	PT32: Out of three cards, there is one card with red on both sides. Therefore, the probability is 1/3.	
	PT11: RR, WW, RW. Red one hundred three. We see one of his faces. So two red faces remain. Probability 2/3. But we have to multiply this by 1/2. Because there are two red cards. There is a card that is red on both sides. As a result,(2/3).(1/2) = 1/3 is found.	
	PT5: Using the combination, I found the probability as $\frac{\binom{2}{1}}{\binom{3}{1}}\cdot\frac{\binom{1}{1}}{\binom{2}{1}} = \frac{1}{3}$.	

Table 5. *Cont.*

Response	Some Extracts of Ways of Thinking	Frequency Percentage
1/6	*PT16: Three sides of the cards are red and three sides are white. The probability of being red is 3/6, one of the three cards with two red sides, so the probability is 1/3. If these probabilities are multiplied, the result is 3/6. 1/3 = 1/6.*	6(14%)
	PT38: First of all, the probability of choosing one of the three cards is 1/3. There are 2 cards with red on them (RR, RW). The probability of choosing one with red on both sides is 1/2. If we multiply, the result is 1/3. 1/2 = 1/6.	
	PT15: RR, WW, RW cards, 6 states are obtained in the form of RW, RB, RR, RW, RW, WW. Only one of these situations (RR) is required. The probability is 1/6.	
1/4	*PT13: All faces are shaped like KKBBKB. The probability that the randomly selected face is red is 1/2. The probability that the back of the red card is also red* $\frac{1}{2}$*. Probability 1/2.1/2 = 1/4.*	1(2%)
3/4	*PT21: These three cards have 3 red and 3 white faces. We should not take into account the card with white on both sides. So there are a total of 4 faces. Since what we want is red, the probability is 3/4.*	1(2%)
2/3	*PT28: We neglect the card that is white on both sides. The remaining two cards have 3 red and 1 white face. Since we see a red face, we have 2 red faces and 1 white face. So the probability of being red is 2/3.*	1(2%)

As can be seen from Table 5, it was seen that 28 (64%) prospective teachers gave the answer 1/2 for this problem. The vast majority of prospective teachers were not focused on the faces of the cards. It was observed that some focused on the cards.

Use of Simulation

The model created for the experimental solution of the problem is shown in Figure 6. 1, 2, and 3 represent cards in Figure 6. Draw shows that the random selection will be made two times, and Repeat 1000 indicates the number of trials.

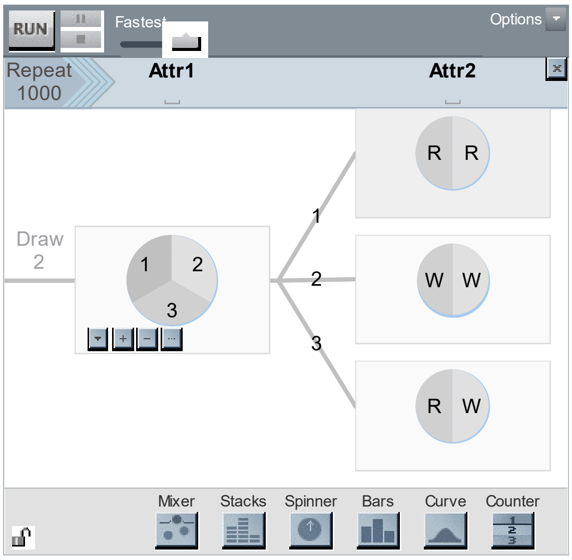
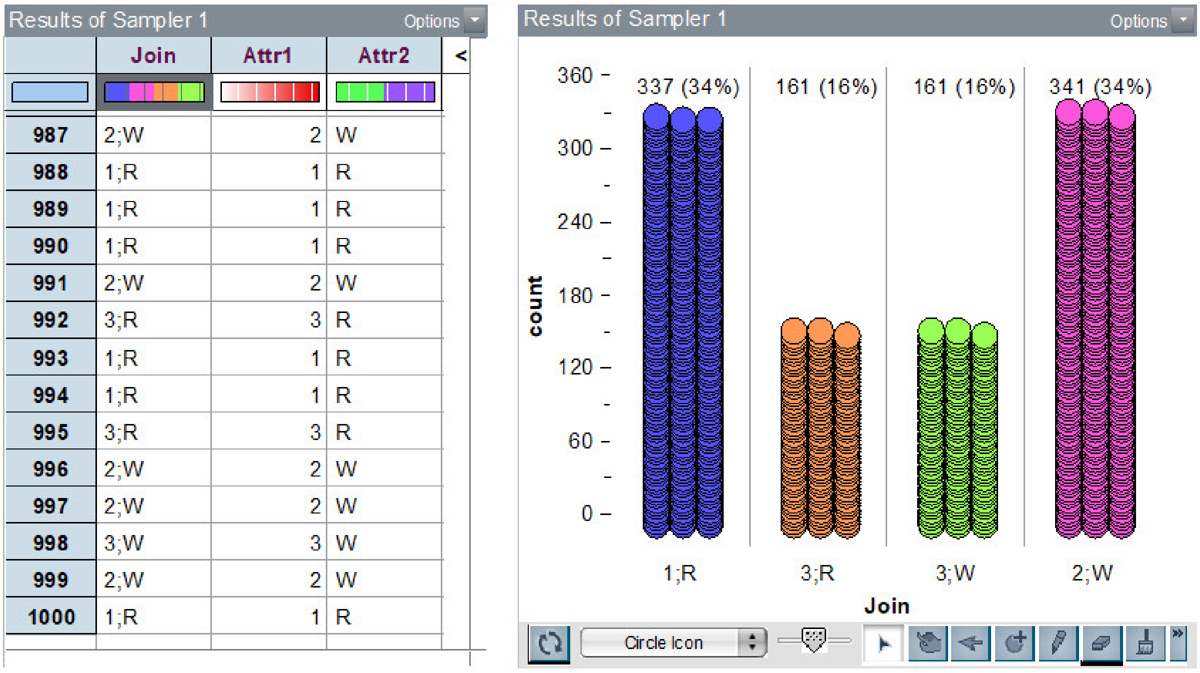


Figure 6. A simulation model was created for the 3rd problem.

In the simulation created for the problem, the number of trials was taken as 10, 100, and 1000; the resulting results were presented with graphics containing frequencies and percentages and the participants were asked to make observations. Figure 7 shows the results for 1000 trials.



Results of Sampler 1

	Join	Attr1	Attr2
987	2;W	2	W
988	1;R	1	R
989	1;R	1	R
990	1;R	1	R
991	2;W	2	W
992	3;R	3	R
993	1;R	1	R
994	1;R	1	R
995	3;R	3	R
996	2;W	2	W
997	2;W	2	W
998	3;W	3	W
999	2;W	2	W
1000	1;R	1	R

Figure 7. Outputs from the simulation model for 1000 trials.

Interpreting Observations and Concluding

R: Is the model suitable for the problem? (They saw the stage of building the model)

PT24: Yes, cards and percentages suit the problem. [DMPF]

R: What do you observe according to the results of 1000 trials?

PT35: The probability of getting red from card 1 and white from card 2 is equal. The probability of getting red from the 3rd card and white from the 3rd card is also equal. [O-R-E–ITO-MI]

PT8: There is a limitation in the problem. We must neglect the white ones (3W, 2W). All cases are cases 1 and 2. The desired situation is situation 1. [O-R-E-ITO-APS-DS-LFM]

PT7: According to the simulation result, the probability that the other side of the card is also red is 34/50 (68%). [O-R-E-ITO-CEP]

Verifying and Evaluating Results

PT39: The face we see in front of us may be one of the two red faces of the RR or is the only red face of the RW. So, there are two cases where the other face is red while there is a case where it is white. So the probability sought becomes ⅔. [O-R-E-ITO-CTP-GE]

As can be seen from the simulation-based learning cycle stages, the prospective teachers had the opportunity to see all the situations with the simulation. With the simulation, they were able to change variables such as the number of experiments, the number of coins, and the number of dice. They were able to graphically see the data produced by the simulation and the relationships between them. They became more willing to discuss and explain what they saw. Meanwhile, they used their mathematical skills such as estimating, making assumptions, testing, observing, analyzing, making inferences, and generalizing.

3.2. The Results Obtained from the Second Research Problem

The results obtained from the opinions of the prospective teachers about the use of the simulation are presented in Table 6.

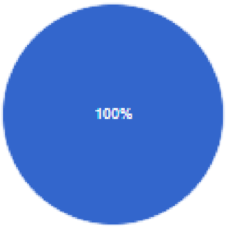
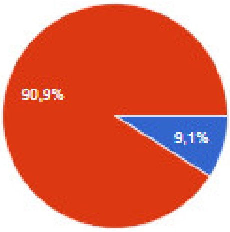
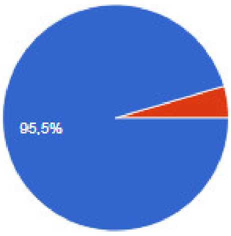
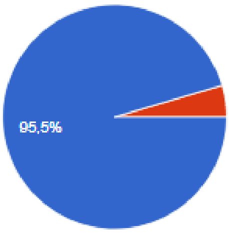
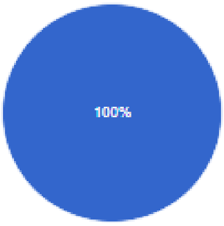
Table 6. Prospective teachers' views on the simulation.

	Items	1	2	3	4	5
1	Simulation offers different representations of data				12 (27.3%)	32 (72.7%)
2	Simulation allows variables to be changed			3 (6.8%)	12 (27.3%)	29 (65.9%)
3	Simulation increases communication skills			10 (22.7%)	14 (31.8%)	20 (45.5%)
4	Simulation provides visual learning			4 (9.1%)	12 (27.3%)	28 (63.6%)
5	Simulation makes observation easy			2 (4.5%)	9 (20.5%)	33 (75%)
6	Simulation can be used as a problem-solving tool			2 (4.5%)	13 (29.5%)	29 (65.9%)
7	The simulation requires little effort for long trials			4 (9.1%)	5 (11.4%)	35 (79.5%)
8	Using simulation in probability statistics teaching is a waste of time.	25 (56.9%)	11 (25%)	3 (6.8%)	3 (6.8%)	2 (4.5%)
9	The use of simulation in teaching is interesting			2 (4.5%)	17 (38.6%)	25 (56.8%)
10	The use of simulation is unnecessary in probability statistics teaching	20 (45.5%)	15 (34.1%)	5 (11.4%)	2 (4.5%)	2 (4.5%)
11	Simulation offers a different learning experience			2 (4.5%)	11 (25%)	32 (70.5%)
12	Simulation allows us to see unpredictable processes			4 (9.1%)	10 (22.7%)	30 (68.2%)
13	The simulation provides the opportunity to test assumptions			2 (4.5%)	9 (20.5%)	33 (75%)
14	Simulation supports classroom discussion		2 (4.5%)	9 (20.5%)	10 (22.7%)	23 (52.3%)
15	Simulation bridges experimental probability and theoretical probability	1 (2.3%)	1 (2.3%)	1 (2.3%)	9 (20.5%)	32 (72.7%)
16	Simulation improves generalization skills		1 (2.3%)	2 (4.5%)	14 (31.8%)	27 (61.4%)
17	Simulation enables guesswork and prediction		1 (2.3%)	5 (11.4%)	11 (25%)	23 (52.3%)
18	The simulation produces results compatible with real-word	1 (2.3%)	1 (2.3%)	2 (4.5%)	11 (25%)	29 (65.9%)
19	A simulation is a tool for revealing and correcting misconceptions		1 (2.3%)	5 (11.4%)	12 (27.3%)	26 (59.1%)
20	Simulation makes learning easier with feedback		1 (2.3%)	2 (4.5%)	13 (29.5%)	28 (63.6%)
21	Simulation requires preparation	2 (4.5%)	3 (6.8%)	10 (22.7%)	9 (20.5%)	20 (45.5%)

The distributions in Table 6 reveal that the majority of the prospective teachers saw simulation as an effective tool and found the features of the simulation useful.

Finally, the data obtained from the opinions of the teacher candidates about the SBL environment are presented in Table 7.

Table 7. Participants' views about the SBL environment.

I gained new techniques and ideas in the SBL environment	Difficult to understand in the SBL environment	SBL contributed to my better understanding of probability and statistics	SBL is very useful for probability and statistics teaching	I would like to use simulation when I am a math teacher
				

From Table 7, all of the prospective teachers acquired new techniques and ideas in the learning environment, 40 (90.9%) of the 44 prospective teachers did not agree that learning in a simulation-based learning environment was difficult, 42 (95.5%) understood probability and statistics better, 42 (95.5%) stated that they found simulation very useful for teaching probability and statistics, and all of them wanted to use simulation when they became mathematics teachers.

4. Discussion

In this study, it was determined that most of the prospective mathematics teachers could not form the sample space in the conditional probability questions in the paper-and-pencil process. Similarly, Demirci, Özkaya, and Konyalıoğlu [32] concluded that prospective teachers could create a sample space and calculate probability in non-conditional probability questions, but they could not create a sample space in conditional probability problems [31].

In this study, some prospective teachers approached the problems intuitively and reached the right solution. However, this was quite uncommon. Although it is expected that probabilistic problems be approached intuitively, it is just as important to integrate these intuitions with the right operations. Kazak [33] stated that it would be possible to approach this situation intuitively since the subject of probability is closely related to daily life situations, but these intuitions would be insufficient in situations that require complex reasoning and cause misconceptions [33]. Some researchers stated that sufficient training and support are needed to develop both teachers 'and students' intuition in probability teaching [2,7,15,24]. In this context, it was seen that simulations provided different opportunities for prospective teachers in the process of problem-solving by enabling them to visualize invisible processes (sample space and the frequencies and percentages of desired situations). In the created learning environment, prospective teachers were able to form hypotheses, distinguish various scientific processes, change the variables in the mathematical model, and conduct experiments, and they were able to make predictions, inferences, and verifications by observing the experimental results. In addition, the trainees found that the skills learned in a virtual environment could easily be transferred to a real-world scenario through simulation. The use of simulation contributed to the prospective teachers' ability to bridge between probability knowledge and problem situations and to reconstructing their mathematical knowledge. Koparan (2016) stated that the visuality required for probability problems in traditional environments cannot be provided and that alternative learning environments are needed. However, when moving from experiments to theoretical probability, it became easier for prospective teachers who participated in a combination of simulations and classroom discussions to understand the relationship between experimental and theoretical probability.

In this study, both pen-and-paper and simulation-based learning processes were employed. Demirci, Özkaya, and Konyalıoğlu [32] stated that teacher candidates should create learning environments where they can face their mistakes [31]. Konold et al. (2007) stated that students should be given opportunities to predict according to their previous knowledge and intuitions, review their first predictions based on the data, develop another possible theory in light of new knowledge, and test its validity [16]. It is not possible to collect enough data during the course hours with concrete experiments. For this reason, the role of simulations in the later stages of the activities developed by Konold et al. is quite big. Batanero et al. (2005) stated that for students to develop a meaningful understanding of probability, they should accept different interpretations and explore the connections between them and different contexts in which one of them could be useful [10].

While the simulations' ability to generate data quickly made it possible to run thousands of trials, calculate results, and calculate relative frequencies, alone, they did not lead to satisfactory results in problem-solving. However, it can be said that these problems have a facilitating effect on the participants in seeing their determined relationships. It was observed that prospective teachers had difficulties finding the correct answer, especially in conditional probability problems. This may be due to false intuitions. For one of these false intuitions, it was determined that they did not take into account that the information given in the problem reduced the number of possible situations. However, the learners' performance was found to improve when the problems were presented in a natural frequency format. Some insights that teacher candidates gained during the activities may affect this performance increase. In this study, it was found that simulations easily generate repetitive long-term events, encouraging participants' reasoning with insights that were different from insights about the probability that would emerge from more limited experimental trials. It was observed that simulations provide a rich scientific context for prospective teachers to practice and learn through inductive learning. Inductive reasoning has the characteristics of constructivist methods in which learners construct their reality instead of obeying the instructions given to them. Simulations have become a positive active learning tool that connects prospective teachers' skills, knowledge, and competencies with their attitudes and motivations toward learning. Their familiarity with technology kept their motivation alive. Feedback was the source of the reflective discussion that accompanied the simulation. As in a previous study focusing on reflection [34], the combination of systematic observation and mindful follow-up dialogue is critical to the transition from reflection to transformational learning. SBL helps to promote reflection in learning [35,36]. The prospective teachers reported that they learned from the simulation experiences and reflective observations. This situation provided an opportunity for a prospective teacher to learn from the experiences of other prospective teachers in order to learn what works and what does not. SBL being an integral part of the lesson, and its regular use, allowed trainees to analyze and learn from their peers' experiences, which contributed to the advancement of learning. The reflective discussions accompanying the simulation facilitated this learning process.

One of the findings of this study is the advantage of SBL in broadening the views of prospective teachers about the simulated situation. The use of SBL in teacher education is relatively new [2,24,37–41]. and there are still known limitations associated with SBL [42–44]. However, Cruz and Patterson (2005) claimed that SBL processes can help improve teaching techniques [45]. This view is supported by the findings of the present research, which emphasizes aspects of peer learning that are specific to SBL and are known to benefit learning processes in general, and teacher education in particular [46]. The current findings also show that prospective teachers can improve their use of the reviewed teaching techniques and, perhaps more importantly, recognize the importance of improving them.

5. Conclusions and Suggestions

In this research, the aim was to fill the gap created by the lack of studies on teaching probability theory using simulations. In the study, conducted within the scope of a case study, it was concluded that simulations are an important tool that supports intuition and learning in teacher education processes. It was determined that the prospective teachers found the designed learning environment interesting and useful, experienced a different teaching experience, and also wanted to use simulation when they became mathematics teachers.

The simulations provided a context for combining the rich content of probability theory with the stunning technological achievements of the digital age and allowed learners to reconsider classical problems through contemporary discourse. Sampling procedures in probability teaching can be created with the help of dynamic statistics software such as TinkerPlots. The probability of generating sequences of random experiments using software and the operations that can be carried out with them can be seen as a useful toolbox for generating models of random situations.

The presented work provides an overview of the use of simulations in probability teaching, from which implications for development, application, and research are derived. However, some limitations need to be taken into account, such as the number of participants and the method chosen. Various implications can be drawn from the results of this research for classroom practices and future research. Teachers need to develop competencies in creating simulations, integrating them into their teaching, and thinking about their use in their lessons. For this reason, training for creating simulations can be given to instructors. More research is recommended to understand the effect of simulations on mathematical knowledge so that they can be applied to other subject areas. Additional studies at different levels are recommended for SBL. Qualitative approaches such as classroom practices and observation studies, as in this particular study, can provide new insights into the selection and use of simulations in mathematics education.

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Informed Consent Statement: Voluntary participation form was obtained from all participants who participated in the study.

Data Availability Statement: Data is not publicly available due to privacy or ethical restrictions on participants.

Conflicts of Interest: The author declare no conflict of interest.

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