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Teachers' Beliefs and Their Influence on Math Instructions for Gifted English Learners

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Abstract: The dynamic interplay between teachers' beliefs and practices significantly impact the quality of instruction and the trajectory of talent development in young students. This case study explores the beliefs and practices of two elementary teachers instructing gifted ELs in mathematics. The constant comparison method was used to analyze data collected from classroom observations, semi-structured interviews, and field notes. Three factors were found to affect the (in)consistency between teachers' expressed beliefs and observed practices: compatibility among core and peripheral beliefs, knowledge about evidence-based practices, and classroom management skills. Students exhibit higher levels of participation, communication, and engagement in critical thinking skills when their teacher embraces constructive perspectives in teaching mathematics, demonstrates pedagogical expertise, and employs a proactive classroom management approach. Conversely, students encounter restricted opportunities to independently construct their own understanding of mathematics when their teacher holds maladaptive beliefs about teaching mathematics, has limited knowledge of evidence-based practices, and has an authoritarian classroom management style. These findings underscore the need for a new approach to professional development (PD) that encourages teachers to critically examine the connection between their beliefs and instructional practices and their impact on the student's mathematical talent development.

Keywords: English learners; elementary math; giftedness; teaching practices; teachers' beliefs; perceptions

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1. Introduction

The U.S. has seen disappointing results from policy initiatives aimed at the inclusion of English learners (ELs) in gifted education [1–5]. In contrast to the 60% growth in EL enrollment nationwide, reaching as high as 200% in some states [6], the number of ELs in gifted programs stagnated at a mere 2% [7]. So, how did a country that prides itself on being a "nation built by immigrants" fail to capitalize on the diverse talents of its youngest citizens? This study contends that the inadequate scrutiny of the interplay between teachers' beliefs and instructional practices, as related to gifted ELs, plays a central role in this shortcoming. To foster an environment of equity, one that empowers ELs to flourish in gifted education programs, it is imperative for us to gain a robust understanding of how teachers' instructional practices are shaped by their beliefs regarding gifted EL students, talent development, and the teaching of mathematics. So far, endeavors toward this objective have been hindered by the extremely low enrollment of ELs in gifted programs. We are unlikely to gain a comprehensive understanding of how to effectively teach ELs in gifted classes when these students are either absent entirely or grossly underrepresented in these programs. This dearth of research evidence was underscored by Mun and her colleagues [8] in their systematic review of literature on ELs in gifted education, in which only seven (7) empirical studies on effective instructions were identified.

This present case study stands out for its distinctiveness, as it delves into the beliefs and actions of two teachers who were tasked with instructing classes exclusively composed of gifted ELs in mathematics. The insights gained from this study concerning teachers' beliefs about gifted English learners (GELs) and the consequential impact on their instructional practices will significantly contribute to the advancement of teacher training. By illuminating the intricate relationship between teachers' beliefs, instructional approaches, and quality of teaching, this study will inform the development of targeted interventions aimed at enhancing the teacher's capacity to effectively teach diverse learners. Promising avenues for such interventions include professional development and peer mentorship programs, which offer valuable opportunities for teachers to engage in reflective practices. Through these initiatives, teachers are encouraged to critically evaluate their own beliefs and instructional practices and to identify areas where they might deviate from evidence-based recommendations. By actively participating in a continuous cycle of self-reflection and professional learning, teachers gain the power to serve as catalysts in fostering the talent development of every student.

This study is informed by a situated-sociocultural perspective on mathematics teaching and learning. We posit that learning is meaning-making, "a process by which people interpret situations, events, objects or discourses, in light of their previous knowledge and experience" [9] (p. 106). The critical role of meaning-making, with particular attention to the shift away from students' mastery of discrete elements of content towards the development of reasoning, communication, and problem-solving skills, is a central tenet of the effective teaching practices identified by the National Council of Teachers of Mathematics [10,11]. For GELs, learning is optimized in classrooms that offer them abundant and diverse opportunities to engage in cognitively demanding tasks that encourage risk-taking, sense-making, and reinterpretation of knowledge within compatible social contexts [12]. There is substantial evidence [13,14] that with appropriate curricular and instructional support, ELs can participate, contribute, and succeed in math in spite of and because of their language diversity. ELs have the capacity to develop their proficiency in both languages as they participate in communicative and meaningful tasks [15], and they can bring new perspectives and resources to the classroom that can benefit their peers [16]. This strength-based perspective of Els positions these students as strong candidates for gifted and talented services.

1.1. Teacher's Role in Gifted EL's Talent Development

The role of teachers in facilitating ELs' access to rigorous learning opportunities is crucial in these students' development of mathematical talent [17–19]. Effective mathematics instruction for ELs necessitates the teacher's deep understanding of the linguistic and cultural backgrounds of his/her students, as well as their unique learning needs [20,21]. Teachers who implement pedagogical approaches that promote inquiry-based learning, nurture critical and creative thinking skills, and provide support for language-rich mathematical discussions have been found to significantly enhance ELs' mathematical comprehension and proficiency [18,20]. Teachers who hold positive beliefs about EL's potential for academic growth are more likely to set challenging goals and provide the necessary support to help ELs reach their full potential [22]. Considering the substantial body of evidence indicating that ELs can achieve remarkable levels of academic success when supported by teachers who possess asset-based beliefs and a toolkit of culturally responsive teaching strategies [23–25], we must reject any notion that cast ELs as less-than-capable students. They should be viewed as multi-competent [26] learners who can draw from their cultural and linguistic knowledge as they discover and employ multiple ways of meaning-making.

1.2. Relationship between Teachers' Beliefs and Practices

The scholarship on the role of teachers' beliefs in teaching and learning spans decades and can trace its roots to the first edition of the Handbook of Research on Teaching [27]. It has grown considerably since then and has taken many directions. The complexity of teachers' beliefs manifests in both the range of beliefs and the intricate manner in which these beliefs are structured and applied. Beliefs are organized within a complex, interconnected, and multidimensional system [28] and are held with varying degrees of certitude [29] by the individual, subject to change with time and experience [30], and can coexist with conflicting beliefs [31,32]. The strong elements of subjectivity and fluidity that undergird the construct of teachers' beliefs affect how beliefs are used as contextual filters through which teachers interpret their experiences, shape their interactions with students, and enact classroom practices [33,34]. Hence, it is not surprising when discrepancies arise between what teachers think they should do (beliefs) and what they actually do (observed practices).

1.3. Relationship between Teachers' Beliefs and Instructional Practices

Teachers' beliefs play a crucial role in shaping instructional practices, particularly in the domain of mathematics and gifted education [35–37]. These beliefs influence the choices teachers make regarding curriculum, instructional strategies, and classroom interactions. Understanding teachers' beliefs about math and giftedness and their impact on instructional practices is essential for providing appropriate educational opportunities that foster the talent development of gifted students. While the connection between teachers' beliefs and their practice may seem self-evident, there are many times when a teacher's expressed beliefs are incongruent with his or her actual classroom practice [38,39].

Teachers' beliefs about gifted students can vary widely, influencing their perceptions of giftedness and the instructional strategies they employ in the classroom. Unfortunately, certain teachers may have a fixed mindset, believing that giftedness is innate and unchangeable. This mindset may lead to limited expectations for their gifted students' academic growth and a lack of differentiation in instruction. In contrast, teachers with a growth mindset view giftedness as a malleable trait that can be developed through effort and effective instruction. These teachers are more likely to provide challenging and engaging learning opportunities for gifted students [40]. However, there are times when teachers profess growth mindset beliefs but do not translate them into effective instructions in the classroom [41]. This disconnect is not exclusive to general education teachers who may not have received adequate professional training in research-based practices. It can even be found in teachers who were trained to teach gifted students. Tofel-Grehl and Callahan [36] found that while teachers in specialized STEM high schools ranked inquiry-based learning as a priority, the observed instruction consisted of lectures with a high proportion of work correction and homework practice. This incongruency between beliefs and practices can be so stark that even their students are acutely aware of it. In interviews with the researchers, students readily expressed their frustration about the dissociation between their teachers' words and actions, "they [the teachers] say we do inquiry, but all we do is what we are told. It's kinda lame sometimes" [36] (p. 48).

While many experts consider mathematics-related beliefs to be a significant, or perhaps the most influential [29] predictor of teacher behavior, the relationship between teachers' beliefs and practice is not unequivocal nor linear. Teachers' beliefs about the nature of mathematics can vary along a continuum from a procedural-oriented, deductive view to an inquiry-driven, constructive view [42]. The constructive perspective supports a learner-focused model of teaching that prioritizes individual sense-making and supports the establishment of a student-centered environment [43]. The deductive view aligns with a teacher-directed transmission approach that focuses on students following rules and replicating procedures rather than constructing knowledge [44]. However, there are many times in which teachers who endorse constructivist views about teaching and learning do not necessarily implement classroom practices that reflect those beliefs [45–47].

1.4. Factors That Affect the Relationship between Teachers' Beliefs and Practice

Why do teachers often fail to align their actions with their stated intentions? Although the question appears straightforward, the answer is a multifaceted issue with intricate layers. We will discuss four reasons for this misalignment and their implications in regard to student learning. First is the vital role that teacher preparation programs play in shaping teachers' beliefs and instructional practices. Research suggests that these programs may not adequately prepare teachers to bridge the gap between theory and practice [48]. Teachers may enter the classroom with strong beliefs about student-centered, inquiry-based instruction but lack the necessary pedagogical skills and strategies to implement them effectively. The absence of explicit training and support in translating beliefs into actionable instructional practices can contribute to the misalignment between what teachers believe is effective and what they actually teach in the classroom. The second factor is the various constraints and pressures under which teachers operate within our complex educational system. Time limits, standardized testing requirements, and curriculum mandates can limit teachers' ability to implement their preferred instructional approaches [49]. In such cases, teachers may feel compelled to prioritize coverage of content over student-centered, inquiry-based instruction. Additionally, external demands from non-teaching duties can negatively affect how much teachers spend in the classroom and teaching quality. For example, one (1) out of four (4) teachers loses at least 30% of his/her time through disruptions caused by disciplinary issues or administrative tasks [50]. The third factor is the lack of resources and support. Teachers require adequate resources and support to implement their instructional beliefs effectively. However, limited access to instructional materials, technology, professional development opportunities, and collaboration with colleagues can hinder the ability of teachers to align their practices with their beliefs [51]. Without the necessary resources and support, teachers may struggle to implement student-centered, differentiated instruction or lack the confidence to experiment with new strategies. As a result, their instructional practices may deviate from their beliefs. Lastly, the instructional practices of teachers are influenced by their own prior experiences as learners and habits. These ingrained habits and beliefs can be resistant to change, even when teachers hold progressive beliefs about effective instruction [48]. For example, a teacher who was primarily exposed to traditional, teacher-centered instruction during their own schooling may unconsciously default to similar practices despite recognizing the benefits of student-centered approaches. Overcoming deeply ingrained habits and beliefs requires deliberate reflection, ongoing professional development, and support from instructional leaders.

1.5. Need for Study

There is a plethora of research aimed at providing insight into the complexity of teachers' beliefs–practice relationship [52–60]. However, the beliefs–practice relationship within the context of teaching gifted English learners (ELs) remains unexplored. In Lucas, Villegas, and Martin's review on this topic [61], they were only able to locate five studies [62–66] that examined whether and in what ways teachers' beliefs about ELs relate to instructional practices. None of those studies investigated the direction and strength of the association between teachers' beliefs and practice as it pertains to gifted students or mathematics. This scarcity of research is in dissonance with the rapidly changing landscape of education in the United States and elsewhere in the world. As a result of migration and globalization, ELs are the fastest-growing student group, and two-thirds of these students are in grades K-5 [67]. Although teachers play an enormous role in the math talent development of young students, the connection between teachers' beliefs and practices remains underexamined, and even less is known about how beliefs inform the pedagogical choices of teachers in support of particular groups of underserved students, such as gifted ELs. This lack of understanding is part of the reason why ELs are continuously underrepresented in STEM fields in schools and in the workplace [68]. The purposes of the current study are to address limitations in previous research; examine teachers' beliefs and their teaching practices with gifted ELs; and explore how teacher and classroom characteristics affect the correlation between teachers' beliefs and teacher practices. The current study has the potential to provide insights into teacher preparation and professional development for teachers of culturally and linguistically diverse students. With more information about the interaction between teaching beliefs and practices, teacher educators will be able to develop strategies to support effective teacher behaviors and target and remediate undesirable ones.

1.6. Research Questions

The specific questions guiding this case study are: (a) What are elementary school teachers' beliefs about teaching math to gifted ELs? (b) How do elementary school teachers teach math to gifted ELs? (c) How consistent are teachers' beliefs and instructional practices? To answer these questions, we explore the experience of two second-grade teachers to probe the dynamics of the beliefs–practice relationship and its effect on the quality of teaching and learning in a math class of gifted English learners.

2. Materials and Methods

A qualitative case study [69] was used to examine teachers' beliefs about gifted English learners in mathematics and the way in which these beliefs are translated into classroom practices. Case studies offer an in-depth and holistic exploration of individual teachers within their unique classroom contexts [70], allowing researchers to capture the complexity and nuances of teachers' instructional decision-making. By conducting interviews and engaging in extensive observation, we, as researchers, deeply immersed ourselves in the natural setting of the classroom. This allowed us to directly witness, analyze, and evaluate teaching quality within the context of the instructional activity, teacher–student interaction, and learning objectives. This immersive approach enabled us to uncover the dynamics that underlie teachers' beliefs, the conditions under which beliefs are translated into practice, and the factors that either facilitate or hinder the relationship between teachers' expressed beliefs and their observed practices.

2.1. Context of the Study

This case study is part of a larger investigation of the teaching and learning behaviors in elementary math classes of GELs from underprivileged communities in a large urban school district located in the northeast of the United States [71,72]. Students were identified as mathematically gifted based on teacher observations of their mathematical skills and motivation to learn. This identification system is based on the position that access to gifted and talented programs should be expanded to include students with exceptional talent and/or who express a high level of interest in mathematics [73–75]. The students attended enrichment math programs 3 times a week after school for approximately 40 min per session for 6 months. At the beginning and end of the year, a 12-question math test was administered to assess students' knowledge in geometry, measurement, number sense, and algebraic reasoning. The test comprised a combination of single-answer questions and open-ended questions. Regularly scheduled classroom observations were carried out by the authors and graduate assistants in the eight participating schools. For the present study, Ms. A and Mr. B, two teachers from one of these schools, were selected for fine-grained analyses of the teaching and learning processes enacted in their classrooms.

2.2. Participants and Setting

We employed purposeful, criterion-based sampling for this study [76]. The selected school has a large, culturally, linguistically, and economically diverse student population and is located in a multicultural community. The school was chosen because its students and the community from which they come are representative of those who are often overlooked in scholarly discourses about talent development. By intentionally situating our study in this school, we take the position, as advocated by NCTM [75], that mathematical talent is evenly distributed across geographic, demographic, and economic boundaries. The selected elementary school serves 1702 students from preschool to fifth grade; 747 of these students are identified as ELs. The school's minority student enrollment is 100%, and 89% are economically disadvantaged students. In 2022, 38% of students scored at or above the proficient level for math, and 36% scored at or above that level for reading on the state assessment. The math proficiency rate for third-grade ELs was 15%, 8% for 4th, and 5% for 5th. There are 140 full-time teachers, 36 (26%) of whom are in their first or second year of the profession [77].

The two teachers selected for participation are representative of the two predominant types of teachers of ELs: (a) generalists trained as broad-field elementary school teachers and (b) specialists with a degree or certification in teaching English as a second language or bilingual education. Mr. B—a generalist—is the type of teacher that most ELs will encounter, as the number of teachers who are trained to work with language-minority students has not kept pace with the rapid growth of ELs in the school system. It is estimated that more than 60% of teachers have ELs in their classrooms, but only 10% of these teachers have completed sufficient coursework [78]—like Ms. A—to support these students. More information on the school and teachers can be found in Tables 1 and 2, respectively.

Table 1. School demographic information.

Students		**Number**	**Percentage**
Grade Level	Student K-5	1702	
	Grade 2	312	18%
Ethnicity	White	8	<1%
	Hispanic	1623	95%
	Black	1	<1%
	Asian/Pacific Islander	69	4.0%
	Other	1	<1%
Eligible for Free and Reduced Lunch		1521	89%
English Learners		970	57%
Gender	Female	852	50%
	Male	850	50%
Teachers with 3 or More Years of Experience		124	96%

Table 2. Teachers' demographic information.

	Ms. A	**Mr. B**
Teaching position	Dual language teacher	General and special education teacher
Age	Early 50s	Mid 40s
Gender	Female	Male
Education	BA: Political ScienceMaster: Education	BA: Business and MediaMaster: Education
Ethnicity	Hispanic	Caucasian
Number of years of teaching	17	5
Number of years of teaching in high-need schools	17	5
Number of years of teaching gifted students	3	2

2.3. Data Collection

The triangulation process [79] for these multiple cases relied on data collected from (1) classroom observations during the after-school math enrichment program, (2) a semi-structured interview about teachers' perceptions of and beliefs about mathematics and teaching math to gifted ELs, and (3) field notes from the interviews and observations.

***Teacher Interviews*:** Both teachers were interviewed twice in the six-month period during which they taught the gifted ELs in an after-school math enrichment program. The semi-structured interviews were approximately an hour long and conducted by the authors of this study. The questions focused on these teachers' perceptions of and beliefs about the characteristics of gifted ELs (How would you describe gifted English language learners (Els) in your class?), effective strategies in teaching mathematics (What are methods or strategies that you find to be effective and ineffective that you would change or remove?), and how to support gifted ELs in math (How would you describe a teacher's role is in supporting students? What can you do to help students to overcome challenges?). The interviews were audio-recorded and transcribed verbatim.

***Classroom Observations*:** Each teacher was observed for 24 after-school class sessions, about 40 min each, during a six-month period. The non-participant observations were conducted by the authors and graduate assistants. Teacher behaviors were observed across diverse settings and activities within the classroom (e.g., whole-class instruction, large-group activity, small-group work, free play, cleanup time, and transition). The aim of conducting observations across different settings is to comprehensively capture the variations in teacher practices that may be influenced by the specific characteristics of the immediate environments and activities. Extensive training was conducted prior to commencing observations to enhance reliability and validity among observers. The training was based on both videotaped classroom interactions and live observation in classrooms not in the sample. The verbal exchanges in the observation were captured by audio recording and were transcribed verbatim. The observer placed the recording device in a position that could best capture the discourse between the teachers and the students. The observer also positioned herself in the classroom where she could view and document the non-verbal interactions that were taking place between teachers and students and among students themselves. These observational notes were used in conjunction with the audio transcription to create a comprehensive observation document of the classroom.

***Field Notes*:** Field notes were immediately completed after each observation to enhance data and provide a rich context for analysis [80]. The field notes were used by the observers to create a condensed account of the class session, fill in details that were not able to be recorded on the spot, and provide reflections on the events that occurred. The field notes were used in conjunction with the data from the interviews and observations to help us make sense of the context in which the teacher–student interactions were taking place, gain insights into the observed teaching and learning processes, and generate questions about behaviors that are noteworthy for future investigation. The creation and analysis of field notes allowed the authors to engage in reflection about the study's framework and questions [81,82] and track our analytical thinking from the outset of the data-collection period and into an analysis period.

2.4. Data Analyses

The constant comparison method [83] was used to search for the meaning of every piece of information. First, the interviews, classroom observations, and field notes were thoroughly examined individually. This was followed by an initial round of open coding grounded in the framework of teaching behaviors [84]. After identifying the open codes from each case, we used cross-comparison [69] to coalesce and array the evidence across the two cases to identify the central themes relevant to teaching gifted ELs in mathematics. Looking between and within themes for each teacher, we developed an instructional profile for each teacher, characterized by their observed practices and explanations for specific actions. Finally, the data were categorized, restructured, and presented in narrative form [85–87].

Qualitative research acknowledges the role of the researcher as an instrument in shaping the results of the study [88,89]. As part of the process of identifying patterns in teachers' beliefs and instructional practice, the data analysis was deliberately interpretive. The interpretive framework is used to make assertions and comparisons regarding teachers' beliefs and practices based on the standards of practice established by the National Council of Teachers of Mathematics [10]. The expressed beliefs of the teachers and observed practices that align with the perspective of math as sense-making [10] were categorized as "constructive" or "student-centered." Beliefs and practices that reflect the math-as-procedures [10] mindset were categorized as "deductive" or "teacher-directed." Peer debriefing, triangulated sources, and thick descriptions of the data were used to add to the credibility and dependability of the findings [90]. The process of interpretive analysis involved extensive discussion between the authors of this study and members of the research team. Codes and themes were iteratively refined during periodic group meetings.

3. Results

The presentation of findings includes the comparison and contrast of the two teachers' expressed beliefs, observed practice, and the (in)consistency between beliefs and practice, followed by a discussion of the factors that influence the beliefs–practice relationship. Students' names are presented as pseudonyms.

3.1. Case Study: Ms. A

Ms. A believed that learning mathematics is a process of exploration. During interviews, Ms. A used the term "research" several times to describe mathematical learning: "I give them [students] the tools so that they can deepen their research. How to push it [learning] forward, how to question when they're researching." This perception of learning math as "doing" math is strongly correlated with Ms. A's selection of instructional activities for her students. In a lesson about measurement, several stations with cups and containers in various sizes were set up around the classroom, with three (3) to four (4) students assigned to work cooperatively at each station (Figure A1).

In the following excerpt, Ms. A reviews the students' results from the previous day's activity, in which students compared how much water each type of cup (1 cup, 1/2 cup, and 1/4 cup) could hold.

> **Ms. A:** So, what did you learn from our experiment on Thursday? That was really interesting? Let us start with Leandro.
>
> **Student 1 (Leandro):** It was interesting that I found two half cups are one cup.
>
> **Ms. A:** So, there are two halves in one, in one whole cup . . . isn't that interesting? Love your observations. How about you, Isabell?
>
> **Student 2 (Isabell):** Four (4) fourths made a whole cup.
>
> **Ms. A:** How many fourths make a whole cup? Four. It took four (4) of these (points to the 1/4 cups) to make one full cup. Very interesting.

Ms. A purposefully elicited responses from multiple students, creating an environment where students were encouraged to share their observations, imbuing the activity with individual significance for each student. After students had developed an understanding that a "cup" is an ambiguous term that can be used to refer to several different types of measuring instruments, Ms. A challenged the students further with this question, "When you say this container holds four (4) cups of water, which cup? Which cup?" In the subsequent activity, Ms. A played an active role as a facilitator as students began to grapple with the idea that measurements can differ depending on the size of the measuring unit. Throughout this solution-finding process, Ms. A introduced tools to help students develop their problem-solving and reasoning skills. These tools extended beyond simple physical objects such as measuring cups, encompassing a wider array of elements, including graphic organizers. One such tool, the data chart (Figure A2), was distributed to students to help them accurately document their predictions and observations during the experiment. Tools such as these play a crucial role in fostering the development of essential critical thinking skills, such as data gathering, analysis, and presentation.

Ms. A believed that problem-solving extends beyond the task of finding the correct answer. When asked about how she judges the success of a student activity, Ms. A indicated that she looks for instances where "more conversation is happening between the students on [math] ideas". Ms. A was rarely observed asking students to produce a singular answer to a question. She was often found **to ask open-ended questions** such as "What was the most surprising discovery that you made while we were doing the water experiment"?, or "What did you discover that was interesting"? Ms. A was quite deliberate about the types of questions she asked. She explained the value of using open-ended questions to engage students in the activities: "open-ended questions make it [challenging problem] accessible . . . then usually you'll see these kids more involved, and more peers involved too." These statements showed that Ms. A was aware of the effect of purposeful questioning and deployed them accordingly to guide students through inquiry-based learning. These questions prompted students to reflect on their experience, evaluate possible solutions, and plan the next step of action. For Ms. A, the development of these critical thinking skills involved in the problem-solving process are more important than finding a pre-designated answer.

Ms. A believed that students learn through classroom talk. Ms. A asserted that talk is the pivotal element in mathematical learning. She stated, "The more they talk, the more they engage, the more English learners engage with content matter, the more they learn". Ms. A believes that student talk, either between peers or with the teacher, is a key indicator of effective learning taking place. And if a teacher hears "lots of mm-hmm", according to Ms. A, that is a sign that students are disengaged. Ms. A's high valuation of talk is reflected in her practice when she repeatedly prompted her students to openly share their ideas, defend the ones that they agree with, and critique the ones they do not. "Do you agree or disagree with [student's name] just said?" is a common question that Ms. A posed to her students during whole-class and group activities. Ms. A believed that students develop a deeper level of understanding of math through talk, as she explained the following: "If you can explain something . . . it is deeper, and it is also more internalized". Ms. A is also keenly aware of the obstacles that prevent students from participating in classroom discourse. She explained, "If children are afraid or they're shy and they don't want to engage, they're not going to learn. If you have the best math program, but your children are afraid because they don't know the language, and nobody makes them feel welcoming in their life".

Ms. A demonstrated a deep understanding of the significance of fostering a safe and inclusive environment for her students. She took deliberate measures to empower them to take initial steps towards open communication, often reminding them they were free to express their thoughts without reservation, "You can say anything you want". Moreover, she consistently emphasized the importance of mutual respect, asking students to "listen to everyone's ideas" when they are engaged in agree/disagree discourse. In a lesson on the measurement of area, Ms. A took further steps to demonstrate to her students that their voice mattered when she recorded every student's contribution to a whole-class discussion (Figure A3). She validated the students' ideas when she announced to the class, "Everybody makes different predictions . . . I am writing your ideas on the board".

3.2. Case Study: Mr. B

Mr. B believed that students should learn how to apply their knowledge to solve problems. When asked about expectations and learning objectives in mathematics, he said, "You need to get them to understand the steps of the problem and what is in the problem. So, they can . . . understand and how to apply it". Even though Mr. B used the term "problem" here, it had a very different connotation from the type of open-ended problems posed in Ms. A's class. Mr. B perceived math "problems" as an exercise in efficiency, to be solved quickly, and allocating time for students to explore their own sense-making could be a potential distraction. This attitude was reflected in Mr. B's implementation of an instructional activity in a lesson about measuring length. Students were each given a one-foot ruler and were asked to measure the length of their shoes. Most of the students were observed to be able to use the ruler appropriately. Mr. B asked students, "How many inches are your shoe?" and received an array of answers. Mr. B often repeated the students' responses, such as "OK, 9." or "Daniel is 6". He did not pose follow-up questions to ask students how they used the ruler, what they understood about the markings on the ruler, or more critical questions, such as why a one-foot ruler is an appropriate tool to measure the length of a shoe. Mr. B's behavior in this event indicated that he prioritized procedural application rather than conceptual knowledge acquisition in student learning.

Mr. B believed that the role of the teachers is to facilitate student learning. When asked about strategies to engage students in mathematics, Mr. B responded, "I am a facilitator ... [I] walk around the classroom ... make sure they [the students] have the right things". This emphasis on the right or correct answer portrayed Mr. B's perception of mathematics as a deductive process. Hence, Mr. B adopted a close-ended instructional approach that herds his students towards one singular solution rather than allowing students the time and space to make sense of the problem at hand. While Mr. B may use phrases such as "I am like a partner" or "I help guide them" to convey his intention of sharing space and fostering collaboration with his students, his actions exhibited a dictatorial approach. This became evident in the teacher–student interactions during the measurement activity. As each student reported his or her shoe length, Mr. B recorded the measurements and constructed a graph on the projector (Figure A4).

Then, Mr. B instructed the students to mimic his actions, stating, "I want you to do the same thing". As Mr. B walked around the classroom to monitor students' progress, he repeatedly pointed to the project to remind students, "Look at the numbers and put them on the line plot". The rest of the lesson proceeded without solicitation of any student contributions.

Mr. B believed that language is a useful teaching tool. In his interview, Mr. B emphasized the importance of language and communication skills as an integral aspect of teaching and learning math. Mr. B stated that he found classroom discussions are an effective way to teach problem-solving strategies, stating, "Discussions help because you can sort of get them to understand the steps of the problem and what is in the problem. So, they [students] can both understand the words and how to apply them". Mr. B also stated that "My goal is, of course, for them to understand what they are reading and understand the process of how to solve the problem". Although Mr. B's responses may have suggested a commitment to creating a language-rich environment, the actual classroom interactions between the teacher and students did not reflect the same discourse-focused approach. Below is an excerpt from an observation of a lesson on regrouping in subtraction. The lesson began with Mr. B instructing the students to open their workbooks to a specific word problem.

> **Mr. B:** Let's look at number three. Marcel jumped 39 cm high. Jamal jumped 48 cm high. How much higher did Jamal jump than Marcel? Okay. So, how much higher, what does that mean I have to do?
>
> **Students (multiple students answering in chorus):** Subtract.
>
> **Mr. B:** Minus, subtraction. Okay. Jamal is 48 minus 39, Okay?
>
> **Student 1:** I know the answer from minus.
>
> **Mr. B:** This is what you should do. Put them on top of each other.
>
> **Student 2:** A number up here?
>
> **Mr. B:** 48. Good.
>
> **Student 3:** 39, where?
>
> **Mr. B:** It says here in your book, 48 minus 39. Like this (writes on the board as $\begin{array}{r} 48 \\ -39 \\ \hline ? \end{array}$).

In this short exchange, we identified two instances in which Mr. B failed to seize valuable moments to utilize language and communication to develop the students' math and literacy skills. Mr. B could have asked the students to read the question and used this opportunity to evaluate their language proficiency. Mr. B could have queried Student 1 on the potential answer when the student volunteered a response. Instead, the student was ignored, and Mr. B proceeded to provide overly simplistic instructions on setting up the subtraction problem in column form. The students, either as a whole class or as individuals, were excluded from the discourse space in the room.

Mr. B believed that students could benefit from scaffolding and differentiated instruction. When asked about how he helps students who may be struggling in mathematics, Mr. B responded, "I think scaffolding is pretty important because a lot of the times, some math problems, require many steps. So, when you scaffold, you can break a problem apart into easier things, and you can focus on the one thing, get your information first, understand that. Then move on to the next step". Although Mr. B's responses may have suggested an orientation towards differentiated instruction, the actual classroom interactions between the teacher and students did not reflect the same student-focused approach. When he noticed a student struggling with subtraction involving regrouping, he failed to provide the intended personalized assistance. Instead, Mr. B made a general announcement to the class, "Chris has trouble, so let's show her". He then proceeded to demonstrate the computation procedure on the board for all students. He explicitly instructed the class, "We regroup and borrow one from four, change that to three, either to an 18 minus nine ... OK, write it". These actions contradicted Mr. B's professed belief in individualized instruction and revealed a reliance on teacher-directed, explicit teaching methods. This whole-class, teacher-directed instructional practice left very little, if any, room for students to demonstrate their own individual mastery of the content, even though Mr. B professed that he evaluates the success of his lessons by contemplating, "What could I have done to maybe support them [students] and for them to understand more or maybe to connect to it [math] more". This type of reflexive teaching was not documented in the observed practices. By not querying the student on the answer or where and how s/he made the mistake, Mr. B missed the opportunity to gain insight into the student's thinking process. Without taking any measures to establish a foundational knowledge of the students' capabilities, Mr. B was ill-equipped to differentiate his instruction and effectively support his students in overcoming challenges.

3.3. Consistency between Teachers' Beliefs and Practices

By comparing teachers' expressed beliefs and observed instructions related to critical thinking and problem-solving skills, we determined that there is a high level of consistency between Ms. A's beliefs and practices, as demonstrated by the following indicators:

- Allowing students to develop mathematical reasoning skills through experimentation.
- Utilizing critical thinking strategies to encourage students to evaluate their ideas on problem solving.
- Ask open-ended questions to prompt students to explore different ideas.

We also determined that Mr. B showed a high level of inconsistency in the following indicators:

- The role of the teacher as a facilitator in student-centered activities.
- Engage students in learning and applying language and communication skills in mathematical reasoning.
- Differentiate instructions to provide students with multiple methods to interact with the content.

4. Discussion

Teachers' beliefs about mathematics can range from viewing mathematics as a static, deductive application of facts and formulas to a dynamic domain of knowledge based on constructive sense-making and pattern-seeking. These beliefs are often seen as direct precursors to behavior [28]. However, empirical evidence with respect to the degree of alignment between the mathematical beliefs of teachers and their practices has been inconclusive. Like in prior studies [45,47,53], we found that teachers can espouse constructive beliefs about mathematics but do not exhibit evidence of them in their teaching, or the practices were implemented in an ineffective manner. The incongruency between beliefs and practice suggests that the construct of teachers' beliefs is not held in isolation but exists as a part of a multi-layered ecological model. Within this larger framework, the enactment of beliefs into practice can be influenced by various internal and external factors. We have identified three of these factors as possible explanations for the variation in the teachers' beliefs–practice relationship.

Factors That Affect Teachers' Beliefs–Practice Relationship

Compatibility between Beliefs. Although both Ms. A and Mr. B implemented similar measurement activities, the depth of mathematical reasoning and level of engagement by the students displayed in Ms. A's class far surpassed Mr. B's. Although Mr. B perceived himself as a facilitator and may have planned to conduct the shoe measurement activity as a student-centered learning experience, his actions portrayed his beliefs about mathematics as a deductive process. The conflicts between Mr. B's beliefs negatively affected the implementation of the measurement activity, which failed to promote the students' critical thinking skills. Instead, speed and replication were prized over the development of mathematical reasoning for the students. For Ms. A, her beliefs about learning math through exploration aligned with her goal for the water measurement activity, both of which emphasized the students' development of reflection, evaluation, and planning skills. The contrasting outcomes between Ms. A's and Mr. B's implementation of measurement activities underscored the critical impact of the compatibility or conflict between teachers' beliefs on their practice. When teachers possess a congruent belief system that supports student-centered and inquiry-based learning, the potential for enhancing the students' critical thinking and conceptual understanding becomes more pronounced. Conversely, when there is a misalignment, such as in the case of Mr. B, the quality of teaching and learning suffers, even if the teacher has good intentions.

Where did Mr. B's transmission-oriented belief about math come from? Additionally, how is he not conscious of it? The most likely genesis of Mr. B's beliefs is his personal schooling experiences. New teachers may be novices to the profession, but they already possess strong beliefs about teaching and learning [28], shaped by their decade-long experience as students [28]. Multiple studies [91–93] have found the persistence and transfer of teachers' beliefs about math, formed as students, into their current teaching practice. Mr. B, as a student, could have been heavily influenced by the traditional, teacher-directed approaches to mathematics that are prevalent in our schools. Mr. B, as a teacher, did not relinquish this deductive perspective on mathematics upon entering the profession. His transmission-oriented beliefs about mathematics became apparent in his description of a typical lesson structure: "Might be five, 10 min of me [teaching] . . . another 40 min, half hour [for students] to do work". This type of "I do, you watch" approach, followed by student replicating the algorithm modeled by the teacher, is emblematic of teacher-directed instruction. The research-informed, constructive belief about mathematics advocated by the teacher education program did not supplant but was superimposed on top of his pre-existing traditionalist one. Consequently, we observe a constant tug-of-war within Mr. B's beliefs, both in his words and actions, resulting in an overall less-than-satisfactory teaching quality.

The conflict within Mr. B's belief system highlights the inherent tension between his deeply rooted core belief in mathematics as procedure-driven and his less-firmly-held peripheral belief in students exercising autonomy in constructing meaning. While the tension between these conflicting beliefs may be readily apparent to an external observer, the beholder of these beliefs may not be aware of this juxtaposition [37,41]. Hence, uncovering conflicts within an individual's belief system is a crucial initial step for a teacher in modifying maladaptive beliefs [94]. In a case study of a second-grade teacher, Wood, Cobb, and Yackel [95] observed profound changes in both the teachers' beliefs and teaching methods after she engaged in intensive reflective teaching practices for a year. Her transition from traditional approaches to prioritization of the construction of mathematical meaning by students was precipitated by an iterative process of analysis and evaluation of daily video recordings of her lessons. This type of lengthy, transparent reflective teaching would be particularly crucial for teachers like Mr. B, as it enables them to pause and assess the impact of the interplay between their beliefs and instructional practices on their students. This introspective approach allows them to gain valuable insights from their experiences and break free from the entrenched teacher-directed mindset they experienced themselves as students.

Knowledge of Evidence-based Practices. An additional factor that contributed to the difference in teaching quality between Ms. A and Mr. B was their knowledge of evidence-based practices. Ms. A stated in the interview that she was familiar with the eight practice standards, eight Effective Teaching Practices identified in NCTM's *Principles to Actions: Ensuring Mathematical Success for All* [10]. Mr. B responded that he was not familiar with NCTM as an organization or their professional recommendations. This lack of knowledge of research-based guidelines points to a weakness in teacher education on math content and pedagogy [96–99]. This under-preparation is evident in Mr. B's interview response when he stated, "I can't remember if I have heard of NCTM, I don't know about them". While Mr. B expressed beliefs that align with the student-centered approach endorsed by NCTM [10], we struggled to find instances where students were afforded voice or choice over their learning, two defining characteristics of student-centered learning. Instead, Mr. B was observed to give explicit directions on every part of the measurement activity. Students were often instructed to "look at the board" and copy down the teacher's answer. This discrepancy between words and actions stems from Mr. B's interpretation of what qualifies as student-centered learning. During his interview, Mr. B frequently used the term "student-centered" when discussing situations that involved hands-on learning. This suggests that he may have misconstrued the meaning of student-centered teaching, mistakenly associating it with any activity involving concrete or visual materials in some way, regardless of the quality of the instructional delivery.

Without a foundational knowledge of evidence-based practices, Mr. B could not accurately assess whether his understanding and implementation of student-centered instruction aligned with the recommended practices and their true essence. So, while a few objectives of student-centered learning were partially fulfilled in Mr. B's class, the opportunities for students to exercise their own agency were severely constrained. Rather than encouraging students to take the initiative to record the data about shoe sizes and create their own graphs, Mr. B monopolized the task by providing a pre-made graph for students to copy. This discrepancy between Mr. B's beliefs and instructional practices highlighted the need for teachers to develop an authentic understanding of student-centered learning and the importance of empowering students in learning mathematics. We cannot assume that this enlightenment will naturally occur over time. Despite Mr. B being less experienced compared to Ms. A, his five years of teaching should not be considered short by any means. In the context of large urban schools, where the average number of years a teacher teaches is 14 and where 36% of teachers have between 3 and 9 years of experience [100], Mr. B's 5 years can be seen as a substantial duration. Yet, Mr. B did not report attending any math-specific professional development in recent years. Good teaching, or more specifically good math teaching, requires teachers to develop their own problem-solving, critical thinking, and reasoning skills. It is unrealistic to rely on time spent in the profession as a guarantor of effective teaching. Engaging in rigorous professional development focused on math content and pedagogy is vital for teachers to refine and enhance their teaching skills.

Classroom Management. The classroom management styles of Ms. A and Mr. B had a notable impact on the translation of their instructional beliefs into instructional practice. Ms. A nurtured a collective sense of responsibility among her students. She proactively communicated her expectations to the students in advance, ensuring clarity and understanding. For instance, when it was time for students to write and reflect on the water-measurement activity, Ms. A addressed the class by saying, "You may continue working quietly as you answer the questions. I will use this time to start the cleanup while you write in your journals". In this manner, she effectively conveyed to the students the expectation of independent and quiet work. At the same time, she demonstrated her willingness to share the responsibility of tidying up. Ms. A's classroom management style, which prioritizes student investment and participation in creating a conducive learning environment, aligned seamlessly with her student-centered approach to content instruction. This alignment can also be observed in Mr. B's classroom management style and instructional approach, albeit at the opposite end of the continuum. Mr. B adopted an authoritarian approach to maintain or at least attempt to have strict control over his classroom. During the shoe-measurement activity, when he noticed off-task behavior from a student, Mr. B expressed his displeasure, "Put the ruler away. You still haven't even copied. I have already provided you with the answers. I don't understand why we need to discuss it further". Then, Mr. B threatened to relocate the student to sit alone if he did not resume work. While such disciplinary actions often elicited immediate compliance, over time, the students in question would revert to their previous off-task behaviors, such as ceasing their work, engaging in chitchat, or displaying restlessness through fidgeting. The students' off-task behaviors are symptomatic of their limited sense of ownership in a classroom that frames mathematical learning within a limited, deductive perspective and where compliance becomes the easiest route to reach a predetermined solution.

Ms. A's proactive and Mr. B's reactive approaches to classroom management are representative of their distinct responses to curriculum implementation. Ms. A interpreted the curricular expectation for her gifted ELs as providing opportunities for them to "express their unique ideas ... as each of them think so differently". Ms. A placed great value on the exploration of ideas as it encourages students to "think outside the box". "All ideas must be tried", Ms. A stated in her interview, because each student's response can "come to you with a different meaning". Meaning-making takes time [101–104], and time is generously given in Ms. A's class as she and her students spend time talking, writing, and experimenting with different problem-solving strategies. Challenging instructions and meaningful learning opportunities require time to implement. When time is not given, as in Mr. B's classroom, the teaching quality suffers, and the learning opportunities shrink. Mr. B preferred to adhere strictly to the prescribed curriculum because "The standards are already embedded into the curriculum and the lessons themselves". He expressed reluctance to deviate from the curriculum as it contained detailed information about which standards should be addressed and how to teach them. This preference for a prescribed curriculum is common among new teachers who often feel more comfortable following established plans in the nascent stage of their career [105]. While it is normal and expected for novice teachers to accept curriculum guidance, excessive reliance on any particular curriculum, no matter how well-designed, hampers the development of expertise necessary to handle the uncertainties of teaching [106,107]. When teachers feel ill-equipped to respond effectively to classroom uncertainties, such as disruptions or student disengagement, they often resort to authoritarian approaches to regain control. This negative consequence is evident in Mr. B's observed practices. He allowed his instructions to be confined by the parameters set by the curriculum, implementing authoritarian measures to enforce strict adherence to the predetermined order and sequence of the lesson plans outlined in the curriculum.

5. Conclusions

The sweeping reforms in mathematics that gained momentum in the United States during the 1980s and have since resonated globally have firmly established constructivist principles as the bedrock of mathematics education. The reforms operate on the assumption that teachers who embrace inquiry-based, student-centered approaches are more adept at implementing such practices in the classroom, thereby enhancing the overall quality of teaching. However, this study presents compelling evidence that challenges this very premise. Both Ms. A and Mr. B expressed beliefs that are aligned with the constructivist perspective of mathematics that emphasizes meaning-making and problem solving. However, only Ms. A exhibited a strong and positive correlation in her beliefs–practice relationship. The incongruency between beliefs and practice, as seen in Mr. B, has been noted in other studies that have found similar inconsistencies [37,42,92,108,109]. This misalignment among individual teachers perpetuates the prevalence of a transmissive, teacher-directed approach to teaching mathematics on a global scale [110]. The teacher's dominant presence within math classes undermines the envisioned ideal of co-constructing knowledge advocated by proponents of math reforms.

When reviewing the findings of this study, it is tempting to attribute the disparity in the teaching quality between Ms. A and Mr. B to the relatively short duration of Mr. B's 5-year tenure in the teaching profession, especially when contrasted with the extensive 17 years of experience held by Ms. A. One could even argue that Ms. A was naturally endowed with the ability to create a more inclusive and engaging classroom environment, given that she shares the same ethnicity (Latinx) and home language (Spanish) as most of her students. While we acknowledge that cultural background and years of experience can positively impact teaching quality, it is unproductive to focus solely on immutable attributes such as race and time. These factors cannot be altered to help Mr. B or any other teacher to improve their practice. Let us turn our attention to actionable measures that we can undertake to support teachers to effectively nurture the mathematical talents of young gifted English learners.

6. Implications

6.1. The Need for a New Approach to Professional Development

In the cases of both Ms. A and Mr. B, their beliefs about teaching mathematics exerted a significant influence on their instructional practices. However, the power of teachers' beliefs is often overlooked in the design of most professional development (PD) programs. Instead, professional developers often make the assumption that teachers share the same beliefs as they do. Consequently, a typical PD tends to focus primarily on aspects such as lesson planning, instruction, and assessment, neglecting the critical role of teachers' beliefs in shaping effective teaching practices. According to Hill's [111] observations of local and regional math PD sessions, teachers assumed passive roles, listening or watching as professional developers explained concepts and practices. Although teachers engaged in math activities during most sessions, they were primarily applying the strategies illustrated by the professional developers. This direct transplantation of practices without providing teachers with the opportunity to critically examine their purpose or align them with their personal beliefs can lead to a cognitive disconnect. As a result, teachers may struggle to fully invest in and adopt the demonstrated strategies, regardless of their recommendations or research evidence, into their instructional repertoire. Consequently, it is not surprising that the evidence on the effectiveness of math professional development is mixed and, at times, disheartening. Research has indicated that many PD programs, even those incorporating elements associated with rigorous standards and high quality, did not enhance teacher knowledge or student achievement [112–115]. For those programs that did yield positive effects, the impact often diminished within months to a year [116], and in some instances, within days [117]. The criticism against the typical one-shot professional development model is fierce [111,118–120].

6.2. Tackling Conflicts between Beliefs and Practice

Building upon the insights gained from our study and recognizing the factors that had contributed to the limited success of the traditional PD model, an effective PD program for teachers of gifted ELs should incorporate several key elements. First, the program should be intensive, spanning several months to a year and allowing for sustained engagement and growth. Second, the focus should be on mathematics, emphasizing content-specific knowledge and strategies. Third, the program should address teacher planning and instruction, providing practical guidance and support in implementing evidence-based practices. Additionally, the program should be aligned with the mathematical practices outlined in the NCTM standards, ensuring that instructional approaches are grounded in research and best practices. PD program facilitators should create opportunities for teachers to observe and analyze the application of student-centered instruction in teaching advanced mathematics to gifted English learners. Teachers should be encouraged to identify aspects of the intended curriculum materials and instructions that may not align with their own beliefs or seem unfeasible within their classroom contexts. By engaging in this examination of “intended” and “implemented”, teachers can develop a deeper understanding of the recommended practices, the underlying rationale of the curriculum objectives, and how their current understanding aligns or diverges from the intended goals. Explicitly recognizing the disparities between the desired instructional approach and their current teaching practices enables teachers to take the necessary steps to address maladaptive beliefs and ineffective strategies. This type of reflective analysis is instrumental in bridging the gap between what should be practiced and what is currently being practiced in the classroom, facilitating the growth and improvement of instructional practices.

6.3. Adapting the Beliefs–Practice Relationship in Response to Contextual Factors

Furthermore, it is essential to provide teachers with exposure to a diverse range of methods for implementing evidence-based practices in the classroom. By redesigning professional development in this manner, we can enhance teachers’ pedagogical knowledge and strengthen their ability to create optimal and engaging learning experiences for their students. This new approach to professional development begins with the recognition that there is no one-size-fits-all recipe for translating constructive beliefs about teaching mathematics or enacting student-centered instruction. Instead, teachers should be encouraged and supported in utilizing their knowledge and expertise to tailor instruction based on the specific context of their schools and students while still ensuring alignment with evidence-based recommendations.

This approach acknowledges that teachers are the experts in their classrooms, with valuable insights into their students’ strengths, challenges, and learning styles. It respects their autonomy and encourages them to take ownership of their professional growth. By exploring various methods of implementing student-centered instruction, teachers have the opportunity to select and adapt approaches that align with their personal beliefs and teaching style. This empowers teachers to create an instructional style that feels authentic and genuine to them rather than imposing a prescribed set of instructions that may not resonate with their personality or experience. When teachers have the freedom to choose and adapt instructional methods that align with their values and teaching philosophy, they are more likely to effectively implement student-centered practices. This flexible approach promotes a stronger connection between the teacher’s intentions and the actual teaching practices in the classroom. By embracing flexibility and demonstrating respect for teachers’ expertise and decision-making, this new approach to PD fosters a culture of ongoing growth and development, ultimately benefiting both teachers and students.

7. Contribution

This study makes a valuable contribution to the field of gifted education and talent development by examining not only the beliefs held by teachers regarding teaching mathematics to English learners but also how those beliefs are manifested in their instructional practices. Our findings highlighted several factors that either facilitate or hinder the translation of teachers’ student-centered beliefs into practice. These facilitating factors include having compatible and constructive beliefs about teaching mathematics, possessing pedagogical knowledge, and utilizing effective classroom-management strategies. Conversely, hindering factors encompass holding conflicting beliefs (both constructive and transmissive), lacking knowledge about evidence-based practices, and adopting authoritarian classroom management approaches. Now that we have identified some of the obstacles that hinder the translation of teachers’ constructive beliefs into student-centered instruction, the next step for future research is to evaluate the effectiveness of professional development programs designed to remove these obstacles or mitigate the adverse effects of maladaptive teachers' beliefs and practices in regard to students' math talent development.

8. Limitations

Despite the interviewer's efforts to remain neutral and objective, the nuanced nature of human communication may inadvertently reveal aspects of their beliefs or biases. Even without explicit expressions of beliefs, subtle indications or unintentional signals from the interviewer can potentially reveal their underlying stance. This interplay between the interviewer and the teacher can impact the overall tone and content of the interview, potentially influencing the teacher's responses. We also need to acknowledge that our assumptions about "teacher-directed" and "student-centered" behaviors may not always be valid. Simple categorization of teacher behavior may not be possible. For example, explicit direction-giving by teachers may sometimes precede collaborative group work, and close-ended questions may precede scaffolded support. In these cases, what may be interpreted as teacher-directed actions were used as part of student-centered instruction.

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Appendix A

Figure A1. Water-measurement activity station.

4. How much water do you think it can hold? Write down your **Prediction**!
5. Use the cup and pour water into Container. How much water can it actually hold? Write down your **Measurement**!

Container	**Prediction**	**Measurement**
A		
B		
C		

Figure A2. Water measurement data chart.

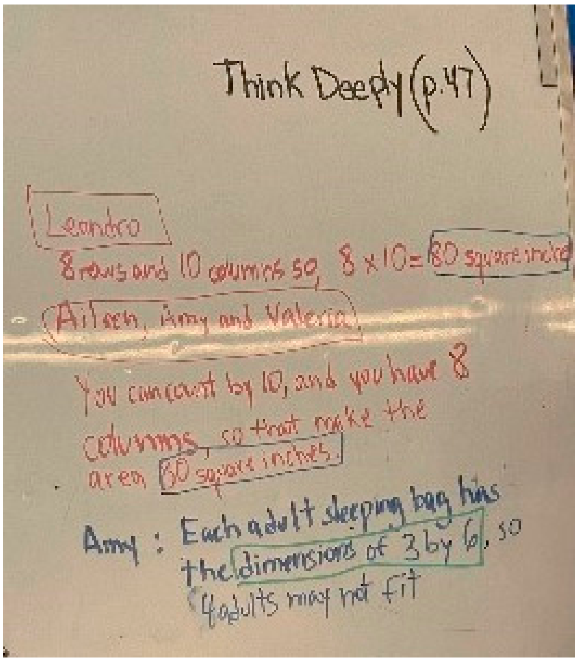

Figure A3. Students' ideas on how to measure area.

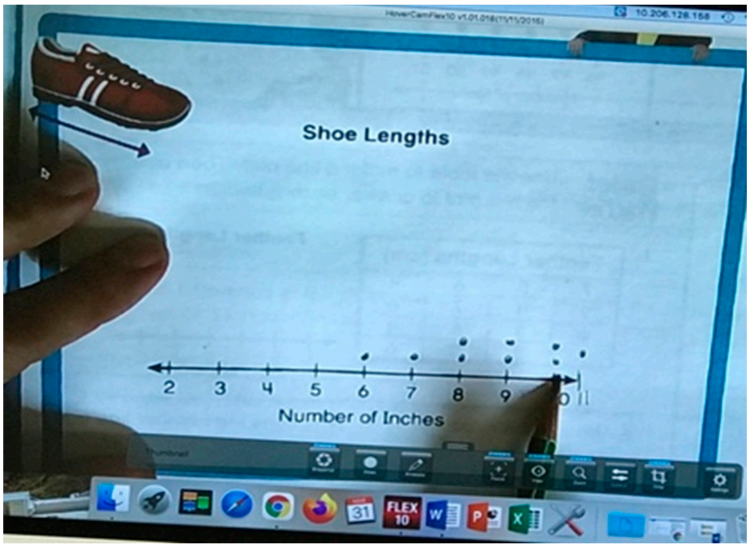

Figure A4. Mr. B's graph, as seen on the projector.

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