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Rough Draft Math as an Evolving Practice: Incremental Changes in Mathematics Teachers' Thinking and Instruction

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Abstract: This paper presents exploratory findings suggesting that mathematics teachers can implement Rough Draft Math (RDM) by making small, incremental changes that align with their current practices and local contexts, including curriculum materials, with minimal support. Following a conference presentation and/or reading a book about pedagogy, teachers reported shifts in their thinking that facilitated their interest in enacting RDM and small changes they made to their teaching. The flexibility of RDM, as a general concept rather than a set of prescribed practices, allowed teachers to incorporate RDM to meet their own teaching goals. We propose that this adaptability enables teachers to incorporate RDM into their classrooms incrementally, reflecting their existing objectives for their students.

Keywords: mathematics teaching; engagement; motivation; teaching practice

1. Introduction

A persistent challenge for mathematics teaching is motivating and engaging students to participate actively in their learning. Teachers have reported a need to put forth additional efforts to increase students' motivation to learn since the COVID-19 pandemic, e.g., [1]. However, rather than locating this problem within students, we view students' engagement and motivation as a dilemma for teaching. It is important to identify teaching practices that encourage more students to participate in mathematics class and that support students with learning mathematics content.

One persistent reason why students shut down in mathematics classrooms is that, in many cases, they have experienced that being good in mathematics means that they must obtain correct answers quickly [2]. Mathematics educators have challenged this perspective. Researchers have been advocating for learning environments that engage students in productive struggle [3] and perseverance [4] in the face of worthy challenges.

An approach to teaching that appears to have potential in engaging students in mathematics learning is Rough Draft Math [RDM] [5,6]. "Rough draft thinking happens when students share their unfinished, in-progress ideas and remain open to revising those ideas" [5] (p. 3). Teaching that incorporates RDM involves treating all students' work and thinking as a rough draft and explicitly incorporates opportunities for students to revise their work and thinking. There is not one specific way to teach with RDM. Rather, RDM is a generative concept that teachers can enact in ways that fit into their contexts. If teachers treat students' thinking as a potentially revisable draft and incorporate revising into their teaching, they are teaching with RDM.

We conjecture that RDM can promote students' engagement in learning mathematics. If all draft ideas are welcome, students may feel safer to participate, particularly if strengths in students' drafts are recognized (see [7]). When drafted ideas are treated as valuable, and then workshopped to build on those strengths in order to revise them, it is possible that students will be less likely to avoid taking intellectual risks and not as likely to avoid

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participating to save face (lower performance avoidance goals) [8]. Additionally, honoring the strengths in students' draft thinking can support students with seeing themselves as mathematically competent, which can potentially increase their self-efficacy [9]. When students have opportunities to revise their thinking regularly in mathematics classrooms, students may be able to develop a growth mindset [10] such that they recognize that their efforts to persevere can lead to greater learning. This process could lead to students developing higher mastery goals [11] or participating to learn rather than to appear smart. Additionally, if students experience that they can learn from their peers' draft thinking and if students experience that their peers can learn from their drafts, then students may build a stronger sense of community in the classroom as they come to value learning from each other (stronger sense of belonging in mathematics) [12].

Our claim in this paper is that Rough Draft Math can be viewed as an incremental change to support students' engagement in learning mathematics that, with minimal support, can be understood by teachers and feasibly taken up in the contexts of teachers' lived constraints. First, we will elaborate on the construct of RDM and argue that it can be an incremental change in teaching mathematics that could lead to supporting students' engagement. Then, we will describe forms of relatively minimal support for teachers (e.g., conference presentations, reading a book alone or in a book club).

Next, in the rest of the paper, we describe results from a series of studies. Study 1 describes shifts in teachers' thinking after attending a conference presentation. Study 2 summarizes variations in teachers' enactment of RDM after reading a book. Study 3 is a case of teachers' curricular noticing, using RDM as a frame for noticing curriculum materials, which is also an example of incremental change that is possible after reading a book. Study 4 illustrates a case of a teacher who enacted RDM because it aligned with her current motivations for teaching, which helps to explain why RDM could be an incremental change. This set of studies, taken together, provides insight into the ways that teachers can incorporate RDM in their teaching without extensive professional support.

1.1. Rough Draft Math as an Incremental Change in Teaching

RDM can be viewed as an incremental change in teaching. Teachers could enact RDM without buying new curriculum materials or without revamping their pedagogical approach. For instance, some teachers teach mathematics by first, directly demonstrating how to solve a problem, giving students opportunities for guided practice, and then, giving students time for independent practice. With this direct instruction approach [13,14], teachers can invite students to share their draft ideas or document their first thinking, and they can ask students to revise a problem that they have initially attempted after guided practice. Alternatively, other teachers may teach mathematics by launching a task that students can begin solving with their prior knowledge and develop new knowledge through making progress on that task. With this teaching through problem-solving approach [15,16], teachers can treat students' initial attempts to solve the problem as drafts, ask students to compare and contrast their solution strategies, and then, invite students to revise their first attempts. In any teaching approach, teachers are likely to close the lesson by consolidating ways of thinking about mathematics during a lesson closure, either promoting connections across strategies or emphasizing the important ideas from the lesson. A lesson closure could be an opportunity to invite students to reflect upon how their thinking changed, or how they revised their thinking, during the lesson. As we detail below, drafting and revising can be woven into how teachers already approach teaching mathematics.

1.1.1. Tag Students' Talk as Rough Drafts

One example of an incremental change associated with RDM is explicitly tagging students' talk about their thinking as "rough drafts" [6]. This involves telling students that it is okay to share their thinking in rough draft form or telling students that in-progress or incomplete thinking is valuable to share with the class. This tag, "share your rough draft thinking", has the potential to elicit more students' thinking. If students hear that rough

drafts are welcome, they may feel less pressure to be correct and more welcome to share whatever they are currently thinking.

Tagging students' talk as a rough draft is an example of an incremental change because teachers are likely already working on strategies to elicit their students' thinking; inviting students to share their rough drafts is a quick move to integrate. Students often adopt the language, too, saying, "I don't know, this is just my rough draft thinking, but. . ." and then go on to explain their thinking. In this way, the tag of "rough draft" can be used as a face-saving caveat so that students share their thinking when they are not sure about whether or not it is correct.

1.1.2. Document How Thinking Changed

Additionally, another incremental change related to RDM is explicitly asking students to document how they revised their thinking at a moment in a lesson, perhaps after collaborating with peers or at the end of a whole class discussion. Teachers often ask their students to share their strategies with one another. This might be through a turn and talk [17] or small group activity. Discussion of students' strategies also can occur during a whole class discussion. Regarding whole class discussions, teachers are encouraged to intentionally select students' solutions to be discussed in class, and sequence them for the discussion in ways that lead to mathematical connections [18].

After the interactions (in partners, small groups, or whole class discussions), teachers could ask students to write down any new ideas they had after listening to others' thinking. This allows students to record any revisions they made to their thinking while listening to peers or their teacher and promotes reflection. Documenting revisions normalizes that we are constantly changing our thinking, promotes listening to one another, and allows students to record what they learned. Asking students to document how they revised their thinking during a class discussion or small group discussion is a minimal change that can support students to learn more from interacting in mathematics class.

1.1.3. Ask Students to Reflect on How Their Thinking Changed over Time

Another example of an incremental change in the spirit of RDM is incorporating opportunities for students to reflect on how they revised their thinking across a lesson, unit, marking period, or semester. At the end of any class discussion or as an exit ticket at the end of a lesson, teachers could promote metacognitive reflection [19] by asking students to write down how they revised or changed their thinking. This sort of reflection could be a part of an end of unit assessment or an end of marking period assessment. Incorporating brief moments of reflection is a small change that helps students become more aware of their learning.

1.2. Minimal Supports Provided to Teachers to Enact Rough Draft Math

One way to think about incremental changes in teaching is whether the change can occur with relatively minimal support. For instance, teachers have learned about RDM by attending a single presentation at a conference, such as a keynote talk or a breakout session. When the first author facilitates these single-session professional learning opportunities, she engages the participants in thinking about the value of drafting and revising in mathematics classrooms, and she facilitates opportunities for the participants to draft and revise their thinking as they do mathematics together. A single conference presentation could be viewed as a relatively minimal support because it is a short period of time to learn about an idea, particularly if no systematic follow up is in place to support teachers' learning beyond that session.

Another form of minimal support for professional learning is reading a book. Teachers might read a professional book alone or in a study group of other teachers, without an external facilitator or the time and resource commitments of a professional development leader. If teachers adjust their teaching after reading a book, this is a relatively low cost way to support teachers. The costs include the price of a book, the time spent reading and,

potentially, time spent on reflecting and discussing the book. If teachers want to learn about RDM, they could read *Rough Draft Math: Revising to Learn* [5].

Certainly, more extensive support could support teachers' learning and uptake of RDM. For example, RDM has been promoted in school districts through multi-session professional learning opportunities, both remotely and in person. However, it is worth considering the potential impacts of more minimal levels of support for RDM. Below, we will explore the impacts of two minimal supports: (a) Study 1 illustrates the impacts on teachers' understandings of mathematics teaching with RDM after attending a single professional learning presentation and (b) the results from Studies 2–4 illustrate the impacts on teachers' enactments of mathematics teaching with RDM after reading a book.

2. Methods

To illustrate incremental changes in teachers' thinking and practice, as informed by RDM, we present findings from four studies. To understand incremental changes in teachers' thinking after a relatively minimal intervention of a single conference presentation, we present an analysis of participants' reflections after the presentation (Study 1). To understand incremental changes in teaching practices after a relatively minimal intervention of reading a book, we present three studies. We summarize findings from Study 2, which has already been published [20]; these findings describe variations in how teachers enacted rough drafting and revising. Study 3 illustrates a case of how a teacher noticed opportunities to engage their students in drafting and revising in their curriculum materials. Study 4 describes a case of how a teacher saw RDM as helping her achieve her goals for mathematics teaching. Studies 2–4 demonstrate the feasibility of incorporating RDM into current teaching practice by making relatively small changes.

For all studies, the sampling of participants was conducted by soliciting volunteers. Thus, our process for recruiting participants for all studies was not random, but we used convenience sampling practices. For Study 1, participants attended conference sessions about RDM voluntarily, and it was their choice whether or not to complete the Google Form at the end of the session. Participants in Study 2 and 4 were recruited by soliciting volunteers from those who participated in book studies (referred to the first author by leaders of those book studies). Participants for Study 3 were recruited via social media (e.g., Twitter/X).

Data analyses for all studies were conducted using an inductive approach [21] to conduct a thematic analysis [22] of the Google Form responses and interviews. For the Google Forms in Study 1, we sought to identify trends in the sample. For the interviews in Study 2, 3, and 4, we used the teaching practices in the book about RDM [5] as a starting point to identify themes that were described by the participants. We selected quotations to present in this paper that were representative of the sample.

3. Results

3.1. Study 1: Teachers' Shifts in Thinking About Mathematics Teaching with RDM After Attending One Professional Learning Presentation

This study addressed the following research question: *After attending a conference presentation about RDM, what do participants report about how their thinking about teaching changed?* At the end of conference presentations for teachers about RDM, to model inviting learners to reflect on how their thinking changed, the first author asked participants to respond to a reflection prompt on a Google Form. The following sentence starter was used as a quick way to prompt reflection: "I used to think... and now I think..." (This prompt is from the Visible Thinking project from Harvard's Project Zero [23].) Typically, approximately 25–33% of attendees at these presentations completed these exit questions on the Google Forms.

To understand ways that teachers' thinking shifted incrementally after experiencing a conference presentation about RDM, we analyzed 262 participants' responses to this prompt from seven different single professional learning sessions conducted by the first

author. These presentations took place at events from February 2023–February 2024 in six states and the District of Columbia. Attendees were classroom teachers and mathematics teacher leaders (e.g., coaches, curriculum supervisors). We identified common themes across responses. These themes represent incremental changes in teachers' thinking that could occur from one single professional learning session. Figure 1 summarizes the themes for Study 1.

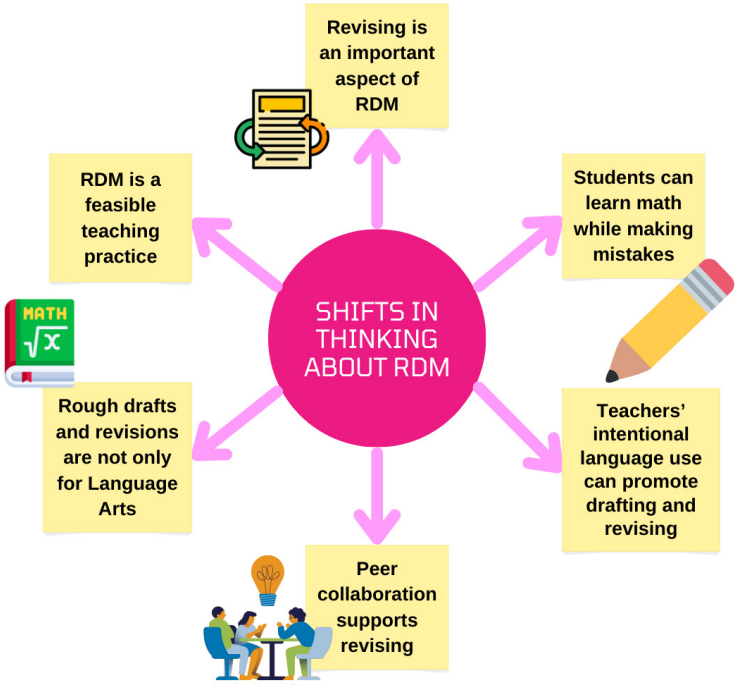

Figure 1. Changes in thinking after attending a conference presentation about Rough Draft Math. Note: Attendees responded to the following prompt: "I used to think. . . and now I think. . .".

3.1.1. Revising Is an Important Aspect of RDM

Teachers reported developing greater appreciation for the value of revising in mathematics (27.1% of responses). A reflection that demonstrated this incremental change in thinking was the following,

> I used to think more about the importance of the 'rough draft' portion and less about the benefits of revision. The activity we did to write and revise our story about the graph reminded me of the benefits of revising to clarify, make connections more clear, and think of our audience.

Some teachers may be initially attracted to RDM because they want to welcome students' drafts to invite more students to participate and share their in-progress thinking. However, the role of revising is essential for moving students' drafts forward. During a professional learning session, teachers can begin to recognize the importance of revising in mathematics.

Additionally, teachers could become more open to what should or could be revised when learning mathematics. A teacher wrote, "I used to think that revising and editing was more limited to students correcting their work, but now I think it is really about learning other perspectives, looking at things differently than we do initially". Revising certainly can involve analyzing errors and correcting mistakes, but it can be so much more. We can revise our thinking by expanding our points of view and developing alternative solution strategies or ways of viewing a situation. We can revise by making new connections between strategies. We can create a new representation to illustrate a mathematical relationship. We can revise a justification to make it more precise, concise, or more illuminating, even if it is already correct. Shifting understandings about what it

means to revise is another incremental change in teachers' thinking that can occur during a single professional learning session.

3.1.2. RDM Could Be a Feasible Teaching Practice

One shift that teachers reported is that, after experiencing multiple moments of drafting and revising in a single PL session, they could see that this approach to teaching is feasible to enact, even in their own professional contexts. This theme was evident in 18.7% of the responses. A teacher wrote, "I used to think that this would be difficult to bring into the classroom but it makes sense on how it would be so useful and not so difficult to incorporate into a class". One teacher responded, "I can change tasks already created to implement this strategy". Another teacher wrote that they used to think "this concept was more time consuming to implement and now I think there are quick ways to incorporate this as a daily practice and shift in classroom practice". If teachers can understand how RDM fits into their current teaching practices, they may be more likely to put it into practice. It is promising that some teachers became more aware of the feasibility of RDM from one professional learning session

3.1.3. Rough Drafts and Revisions Are Not Only for Language Arts

Teachers recognized that they could leverage practices from literacy, rough drafting and revising, and use them to support mathematics learning (13.4% of the sample). A teacher wrote that they used to think "revision was primarily for ELAR [English Language Arts and Reading]" and now they think that "revision happens everyday and is an essential component of problem solving, learning, and thinking". Revision can be viewed as any changes in how we think; so, really, revising is learning. Teachers shifted to see how revising can occur for any academic content, including math, even after one single professional learning session.

3.1.4. Peer Collaboration Supports Revising

After attending a single professional learning session, some teachers reported that they see collaborative learning as a process where students could gain inspiration from their peers about how they could revise their thinking (12.6% of the sample). This teacher wrote about shifting from viewing revising as an individual activity to a collaborative one:

> I used to think about revision as looking at only your work and making it better. Now I see the power of looking at another's work, understanding their strategy, and then revising my work using their method. Sometimes it isn't 'comfortable', but it has the power to open a new avenue of thinking or solidify that the method I'm using is the one that works best for me.

This teacher recognized that comparing and contrasting one's own work with another person's work and trying to understand another person's thinking can provide ideas for revising or improving one's own thinking. Additionally, a teacher wrote, "I used to think that rough draft thinking just meant having students fix mistakes. I now think it's about an initial thought followed by learning from others and then expanding your initial thinking based on what you learned". This response captures the idea that revising can involve expanding your perspective on what strategies are valid for a problem, and this kind of expansion can occur through collaboration. If teachers revised their thinking after collaborating with a colleague on a task during a professional learning session, they may begin to see the value of peer collaboration among students to support revising to learn mathematics.

3.1.5. Intentional Language Use Can Promote Drafting and Revising

Teachers (10.7% of the sample) noticed that they could make small changes in their language use that would support students' engagement in rough drafting and revising. One teacher wrote, "By just adding the words/question 'what's your rough draft thinking on this?' creates a safer space for students to express themselves". Another teacher wrote,

"I used to ask, 'is anyone brave enough to share something they did wrong?' Now I plan on asking, 'Can anyone share something they revised in their thinking?'" Teachers reported that they became aware of small shifts in their language that they could use with students. After one professional learning session, some teachers recognized that such intentional uses of language could help students feel safe to share their thinking.

3.1.6. Students Can Learn Mathematics While Making Mistakes

Some teachers (8.8% of the sample) reported coming to value what students can learn from being incorrect. One teacher wrote that they used to think "I had to focus on just the right answer. However, I now think of the power of just working on the answer. I love the feeling of a rough draft because you know you can fix it later". This teacher appeared to shift away from thinking that their job was getting all students to correct answers. This reflection suggests that they shifted to recognize that students are learning as they work toward the correct answer. Another teacher wrote, "I used to think teachers were meant to discourage mistakes and now I think that mistakes are simply a path to accomplishment". If teachers can make this kind of incremental shift from one professional learning session, it is exciting that teachers could shift to be less likely to view students' thinking through a deficit lens if they make mistakes while engaging in mathematics, and instead they can come to appreciate that students are learning through the process of working toward a correct answer.

3.1.7. Conclusions from Study 1

Overall, we found that this prompt ("I used to think. . . and now I think. . .") has been a helpful tool for capturing incremental changes in teachers' thinking. From this analysis, we are more aware of possible incremental changes in teachers' thinking from a single opportunity to learn about RDM. Naming and labeling small changes in teachers' thinking provides insight about the kinds of learning that might take place when teachers attend one professional learning session.

The two most prominent themes in these responses were that teachers came to appreciate the role of revising in RDM and the feasibility of RDM. Although the role of drafting matters in RDM, revising is the unique contribution of RDM. Without RDM, many mathematics teachers already attempt to create safe spaces for students to participate. Although framing the activity as sharing one's "rough draft thinking" does contribute to creating that safe space, incorporating revising is something that teachers may not otherwise do in mathematics classrooms without being introduced to the idea of RDM. Additionally, if teachers can recognize feasible ways to incorporate RDM after one conference presentation, that is a powerful finding.

3.2. Study 2: Variations in Enactments of RDM After Reading

Another way of investigating how a minimal intervention could prompt an incremental change in practice is by exploring small changes teachers made in their teaching after reading a book. After all, when a book is published, authors are curious about what readers take away from it. Facilitators of book studies of *Rough Draft Math: Revising to Learn* [5] reached out to the first author. They asked for reflection questions to support the book studies, invited the first author to attend one of the book study meetings, or generally informed the first author that the book studies were taking place. The first author then asked facilitators for names of book study participants who might be willing to share how they enacted RDM after reading.

In a previously published study [20], we documented variations of teachers' self-reported enactments of RDM, and we provide a short summary of the results from this paper below. Our research questions were: *When teachers described enacting rough draft math, which teaching practices were salient and feasible? Among these salient and feasible enactments of rough draft math, how did teachers' descriptions vary and in what ways could these variations be viewed as potentially having productive and/or powerful impacts?* Participants in the study were

recruited in two ways: invited to participate from a list of names of book club members provided by facilitators or invited to participate via social media if teachers read the book on their own.

These findings of feasible and salient incremental changes in teaching were identified from self-reports from 32 teachers in eight states in the U.S.A. Teachers were interviewed remotely. Prior to the interview, teachers sent a digital artifact that represented how they integrated RDM into their teaching practices. Artifacts included student work samples, Desmos activities or other student-facing tasks, and short video clips or photographs of their students interacting and problem solving. During the interviews, teachers were asked to describe how their artifact was an example of RDM, what RDM meant to them, why they enacted this approach to teaching, and how much RDM was a part of their regular mathematics teaching practices. Findings were first identified by classifying teachers' practices by categories that aligned with the book's contents (i.e., building and sustaining a culture in support of RDM, task selection and enactment, revising practices, incorporating reflection on changes in thinking), and then, these themes became refined through further passes through the data.

There were two enactments that were the most salient and feasible for teachers in this sample: (a) incorporating explicit revising opportunities into mathematics lessons and (b) selecting and implementing mathematics tasks purposefully to invite rough drafts and revising. Salient enactments were those described repeatedly in the interviews with extensive detail. We considered enactments that were described by a high percentage of teachers in the sample to be the most feasible for these teachers. These salient and feasible enactments can be viewed as incremental changes because they are relatively small changes teachers were able to make to their teaching after reading a book. Below, we briefly summarize variations in these enactments, as more detailed findings were published previously [20].

3.2.1. Variations in Revising

Teachers described that incorporating explicit opportunities for students to revise their thinking was one incremental change that they made to their teaching after reading. However, teachers engaged their students in revising in different ways. One variation was whether or not teachers incorporated structured or unstructured opportunities for revision into their lessons. An example of a structured revision opportunity was when a teacher would ask the students to draw a line across or down a piece of paper (or fold the paper) and told the students that they would have opportunities to make more than one attempt to solve a problem or write an explanation. An unstructured revision opportunity was when students were told that they could look back at their work at any point and make changes to their work using a different color of a writing instrument, but they were not necessarily given directions at a particular moment to write a new revised solution.

Another variation in enacting revising was related to assessments. Some teachers invited students to revise by making corrections to their tests or quizzes, which involved revising their work. Other teachers invited students to instead revise their thinking by incorporating self-assessments. Students wrote reflections on how their thinking grew and changed by looking across their work over time and drew upon their previous work as artifacts to justify how their thinking grew and changed. This self-assessment approach sometimes looked like a portfolio assessment as the end of a unit.

3.2.2. Variations in Selecting and Implementing Tasks to Promote Drafts and Revisions

Another incremental change that teachers reported enacting after reading was their approach for selecting and implementing tasks. Some teachers described selecting tasks from their textbook and modifying the tasks to invite more reasoning and sense making. An example was implementing a task by showing students only part of a prompt, which included removing the question that students were expected to address in the task. Then, they asked students what they noticed and wondered. This process invites students to

share drafts of what they are beginning to notice about the task. Another example was changing the prompt of the problem to invite multiple solution strategies, which would invite revisions after students compared and contrasted their strategies. Revising, then, is a process of expanding students' repertoires of strategies.

Additionally, teachers described implementing instructional routines in a way that more intentionally centered rough drafts and revising. For example, one possible routine that a teacher could implement is a dot talk. This is a routine where students share how they chunked or segmented dots in an image in order to count them. (The focus here is not on the total dots, but on sharing different ways of seeing structure in the image.) One intentional language change was naming all possible ways of initially chunking the image as "first thinking" or a "draft". Also, an intentional implementation shift often involved asking students to share how they revised their thinking after hearing other ways of chunking the image. They would ask students to turn and talk and reflect. They might invite students to complete a sentence starter like, "A way of chunking the image that I didn't think about at first was. . ." The purpose of the dot talk routine is, indeed, to support students with seeing structure in a variety of ways, and the concept of RDM supported teachers with enacting this routine in ways that made this purpose more explicit to students.

In these ways, teachers found ideas in the book *Rough Draft Math: Revising to Learn* [5] to be salient and feasible, which suggests that incremental changes are possible for teachers to enact after reading a book. It is promising that mathematics teachers reported enacting revising and enacting their tasks in ways that invited drafting and revising after reading, and without extensive coaching or extended time in professional learning sessions. Additional examples and greater details about these variations in enacting RDM can be found in [20].

3.3. Study 3: Incorporating RDM While Using Curriculum Materials

Teachers reported that it was feasible to enact RDM in their teaching, as indicated above in their reflections after a conference presentation ("I used to think. . . and now I think. . ."). We interpreted this to mean that some teachers saw it possible to enact RDM using their own textbooks or curriculum materials. To seek to understand more about using RDM with their own tasks, we conducted a study to investigate what teachers noticed in their curriculum materials regarding opportunities to enact RDM. We addressed this research question: *When teachers view their curriculum materials using rough drafts and revising as a lens, what do teachers report about opportunities to engage students in rough drafting and revising while enacting their curriculum materials?*

Research on teachers' noticing suggests that what teachers notice is shaped by the frame that they use while noticing. Teachers do not simply observe; they make sense out of what they notice, and their frame for noticing is their lens for interpreting what they notice [24]. Louie and colleagues [25] explored teachers' frames for noticing in terms of the degree to which teachers' frames emphasized deficit perspectives on students' thinking and learning. If teachers hold an interpretative frame of mathematics learning as absorbing a fixed body of knowledge, then they might attend to accuracy and correctness in students' thinking, interpret mathematical work in relation to whether it is correct or incorrect, and respond to students' thinking by correcting errors or praising correctness. Alternatively, according to Louie and colleagues [25], if teachers view mathematics learning through a frame of creatively exploring ideas, then they may attend to students' diverse ways of making sense of mathematics, interpret students' work as sensible and a valuable resource for their classmates' learning, and respond by giving students opportunities to develop their own ideas and leverage those ideas to advance their learning. This alternative approach is an anti-deficit frame because students' thinking is assumed to be viable and to have strengths worth building upon.

3.3.1. Curricular Noticing

We were interested in how teachers engaged in curricular noticing for enacting RDM. Curricular noticing is a concept informed by research on teachers' noticing of students'

thinking [26,27]. Skills for curricular noticing are ways that a teacher makes sense of the opportunities provided in mathematics curriculum materials [28]. An assumption underlying curricular noticing is that all teachers participate with curriculum materials [29] as they read, adapt, and evaluate their materials to plan and enact lessons. Curricular noticing skills are attending, interpreting, and responding to curriculum materials.

Curricular attending "describes the skills involved in viewing information within curriculum material to inform the teaching and learning of mathematics" [28] (p. 525). For the purposes of our study, we considered what teachers attended to in curriculum materials. In other words, *what* in their curriculum materials did teachers notice when asked how they saw opportunities to engage students in drafting and revising?

Curricular interpreting "refers to the skills used by teachers to make sense of that to which they attended" [28] (p. 536). In our study, this meant that we were interested in how teachers interpreted what they attended to in the materials regarding opportunities to incorporate drafting and revising into mathematics teaching. Essentially, *how* could teachers use what they noticed (through their attention) to invite students to share their rough draft thinking or revise their thinking?

Curricular responding "describes the skills involved in making curricular decisions based on the interpretation of curricular materials" [28] (p. 526). Responding to materials is a process of enactment. To investigate curricular responding, researchers could ask teachers to describe how they would enact the materials in practice.

3.3.2. A Case of Curricular Noticing with RDM as a Frame

Below, we provide evidence from a curricular noticing study. We intentionally asked teachers to use RDM as a frame to notice their materials. We assumed that asking teachers about the opportunities that they see in their curriculum materials to enact drafting and revising would mean that teachers would use RDM as a frame for attending and interpreting their materials.

We conducted 12 interviews with mathematics teachers and teacher leaders (e.g., coaches who support mathematics teachers with curriculum materials) to investigate what they noticed regarding opportunities to enact RDM while using their curriculum materials. All interviewees were solicited via social media (Twitter/X), with the caveat that participants had to be familiar with RDM. All participating teachers reported that they had read *Rough Draft Math: Revising to Learn*. For these interviews, which were conducted remotely, we asked participants to choose a single lesson of their choice and to send us lesson artifacts from their curriculum materials, including the teachers' guide and student facing materials for that lesson. We asked the teachers about the opportunities they saw in that lesson specifically, and in the materials more generally, to invite students to engage in drafting and revising.

This analysis is important for considering the ways in which RDM could be viewed as an incremental change in teaching. If teachers can use their curriculum materials to enact RDM, then RDM is feasible to enact in the teachers' current contexts. Below, we report what one teacher attended to in his materials and how he interpreted what he noticed as potentially affording an enactment of RDM. We focus here on what he saw as possible in the materials as they were written, not on modifications to materials to enact RDM.

Mr. Louis Johnson (pseudonym) shared about his work implementing *Illustrative Math* in a middle school classroom. He self-identified as a Hispanic man. He has been both a teacher leader and a classroom teacher in the Mid-Atlantic region of the United States. He reported having 11 years of classroom teaching experience, and 24 years in all working in the field of education. He said that he learned about RDM through reading the book and attending a presentation by the first author. We selected this teacher as the case for this paper because his interview represented some common themes across the interviews.

He found *Illustrative Mathematics* [*IM*] to align well with enacting RDM. He said the following when asked about enacting RDM with these curriculum materials:

> So, the way that I think about it is that there are opportunities for students throughout a lesson to revise some of their initial thinking. And that can happen

in multiple ways. That can happen from teacher feedback. That can happen from student-to-student feedback. So that's what I think of with rough draft math—the idea of revising student thinking or student work within a certain space and time.

He focused on the role of revising in RDM when reflecting on his curriculum materials more than the role of drafting. We summarized what Mr. Johnson noticed in his curriculum materials in Table 1 and described his noticings in more detail below.

Table 1. Mr. Johnson's curricular noticing using rough drafts and revising as a frame.

Features of Curriculum Materials	Work of Teaching	Connections to Rough Drafting and Revising
Curriculum materials as source of rich tasks	Select and enact tasks that promote reasoning and sense making that align with the central lesson goal.	If the task asks students to generate a representation (or a strategy or a justification), then, there can be opportunities for students to draft and revise.
Curriculum materials' recommendations for student collaboration	Foster collaboration from students so they can learn from one another and develop productive dispositions.	Intentionally encourage revising by asking students to come to a consensus.
Curriculum materials as source of instructional routines	Opens access for students for opportunities to engage in reasoning.	Revising is often built into an instructional routine, or the routine can be enacted more intentionally to incorporate revising.
Structure of lessons in curriculum materials	Provide students with collaborative problem-solving experiences. Then make connections explicit to support achievement of the learning goal of the lesson.	Identified that the lesson synthesis invited students to compare and contrast their thinking and resolve disagreements, which is a process of revising.

3.3.3. Role of the Task

When teachers noticed rigorous tasks in their lessons, they saw RDM as an incremental change that could be helpful to engage students in solving them. For example, Mr. Johnson reported that he selected a particular lesson to reflect upon its potential for RDM because of the tasks in the lesson: students were asked to create tape diagrams to represent story problems. His rationale for selecting this lesson was as follows: "If students are cre-ating something, that's a space where they are able to revise something". He interpreted tasks in this lesson to be amenable to drafting and revising, because students were asked to create a representation, and then they could compare and contrast their representations to better represent the story. Teachers in these interviews regularly mentioned that RDM could support their students to persevere while solving challenging tasks, such as these tasks involving creating tape diagrams.

3.3.4. Intentionally Enact Group Work by Explicitly Encouraging Drafts and Revisions

Teachers in these interviews described how RDM provided them with insights about small changes they could make to support students in learning through collaborating. Mr. Johnson noticed an activity in *IM* that provided specific instructions for collaboration (see Figure 2):

Mr. Johnson interpreted the potential to enact this activity with language that could intentionally connect with RDM:

> I would have a little bit more intentionality of each person doing their own draft. There are three stories. And so, maybe what I would do is, if you're in a group of three, assign one person; you do the rough draft for number one, number two, number three. Then you get the feedback from the others that they agree or disagree with the way that you drew it. And then, you would come back together. So, each group would have a consensus.

Getting feedback from peers and obtaining a consensus as a group could lead to students revising their drafts. Mr. Johnson noticed the potential to respond to his curriculum

materials by making small changes in how he talked with students about how to engage; he could use the activity as written, otherwise, to enact RDM.

Let's use tape diagrams to make sense of different kinds of stories.

In this activity, you will work in groups of 2–3.

You will take turns speaking and listening to your group members as you analyze how tape diagrams represent different situations.

You and your group will have two things to do with the diagram: explain why it represents the story and also figure out any unknown values in the story.

We will end this activity with a whole-class discussion.

Figure 2. Instructions for collaboration on Mr. Johnson's math activity. Note: This activity is from Illustrative Mathematics.

3.3.5. Routines Written into Curriculum Materials Promote RDM

Some of these curriculum materials incorporated instructional routines, such as mathematics language routines [30], directly into lessons. Mr. Johnson observed that *IM* contained routines throughout its units. He reported that, if teachers implement lessons as written, including these routines, students may be engaging in RDM without even realizing it. He attended to a routine called Stronger and Clearer Each Time [31]. He described this routine as follows: "...where someone writes a response, they have a partner read it, they get feedback from the partner, they revise their response, they give it to a second partner, that second partner reads it and gives feedback again". He interpreted that the routine has drafting and revising through collaboration built into it. He said, "When that routine comes up, and it comes up fairly regularly in the lessons... you're essentially implementing a version of Rough Draft Math when doing that routine". If teachers do not skip over these routines, they are engaging students in RDM even if they do not know that they are doing so. With an awareness of RDM, teachers may even respond to the materials by enacting the routine using the language of RDM to incorporate greater intentionality (e.g., "now you will write your first draft... after you read your partner's draft, revise your first draft...").

3.3.6. Lesson Structure Aligned with RDM

Curriculum materials may have lesson structures that promote drafting and revising. Mr. Johnson reported that the *IM* lessons tended to have an opening routine, collaborative learning experiences with challenging tasks, and a synthesis discussion. Mr. Johnson noticed the directions for the lesson synthesis, shown in Figure 3.

Mr. Johnson interpreted these question as supporting students with revising their thinking:

> So, how are they alike or different to me helped unpack some of the thinking and potential revisement. It's the thinking. How are they like or different? Then, do you have any disagreements? That, to me, is a space where, some of these rough drafts can be; that's where the revision can happen. And, so, how are your disagreements resolved? Could be a space where I shared, you know, or Mandy

> shared her thinking about my diagram, and I realized that her thinking was right. And, so, that led me to want to alter it or change it or something like that.

The synthesis prompt elicited students to compare and contrast their thinking, and through these comparisons, they could have new ideas about how to revise. Resolving disagreements also supported revising.

Our analysis of these interviews is ongoing; for the purposes of this paper, we only reported this single case. Our findings illustrate not only other ways that curriculum materials, as written, could align with RDM, but also, teachers reported small modifications to promote RDM that teachers made to materials. Across the interviews, we have heard how these teachers viewed RDM as an incremental enhancement to achieve their goals for students' learning while implementing their curriculum materials.

Let's discuss our diagrams:

How are the group diagrams alike and different?

Did you have any disagreements in your groups? If so, how were they re-

solved?

Figure 3. Instructions for lesson synthesis used by Mr. Johnson. Note: These instructions are from Illustrative Mathematics.

3.4. Study 4: A Teacher's Motivations to Enact RDM

We have a conjecture about why teachers can integrate RDM into their thinking and instructional practices with minor supports: RDM can be woven into teachers' current practices when RDM aligns with teachers' current motivations for mathematics teaching or their goals for what they are trying to achieve with their students. In other words, teachers are drawn to enact RDM if it fits into what they are already attempting to accomplish in their classrooms.

The concept of Professional Working Theory [PWT] [32] refers to understandings that develop when teachers reflect upon and interweave their professional knowledge, practical experiences, and ethical and moral principles or beliefs. As teachers learn about new ideas (professional knowledge), they consider how those ideas might fit into the rest of their knowledge, experiences, and beliefs. They may integrate the new ideas into their PWTs or they may reject them.

Integrating new knowledge into a PWT can be viewed as weaving. Jónsdóttir and Gísladóttir [33] are teacher educators who conducted a self-study of how they supported teachers with the development of their professional working theories. As teachers weave what they learn from professional learning opportunities (e.g., conference presentations, reading a professional book), they craft coherence in their larger PWT about their teaching practice [34] through their efforts to continuously improve as professionals. In Jónsdóttir and Gísladóttir's study [33], one teacher, Hanna, wrote a metaphor of weaving a tapestry:

> My professional working theory—who I am in my work and what I want to stand for—consists of many influences from different sources. These threads of influence weave together into the tapestry of my professional working theory. Each thread is important but individually fragile. When woven together with the others, each thread is strengthened, can bear more strain, and progresses towards its fullest potential. (p. 154)

Previous research suggests that teachers are more willing to take up a practice if they view these practices as being able to be implemented immediately [35]. Our work suggests that some mathematics teachers view RDM as a practice that advances what they are already trying to achieve and as a practice they are able to integrate into their current teaching practice through incremental changes.

A Case of How RDM Can Be Woven into a Teacher's Motivations

To illustrate how RDM fits into a teachers' current motivations, we present reflections from a sixth-grade teacher in the Pacific Northwest. Ms. Alderman self-identified as a white woman. At the time of the study, she had been a mathematics teacher for eight years. She read the book about RDM and had participated in a multi-session professional development experience in her school district led by the first author. In an interview, Ms. Alderman reported enacting both revising and implementing math tasks to invite drafts and revisions with salience. (For more on her enactment of RDM, see [20]. She was a participant in that study, but we did not analyze teachers' motivations for that publication.)

Ms. Alderman shared how RDM helped her achieve goals that she held for her students, such as supporting their mathematics learning and growing students' positive dispositions toward learning mathematics. For Ms. Alderman, RDM helped her focus students on the process of doing mathematics and de-emphasized a focus on getting a correct answer quickly. As she enacted RDM, she observed, "...it was like, immediately, the pressure of being correct, the pressure of having the right answer was, like—we didn't have to worry about that anymore". If she could reduce the pressure of being correct quickly for her students, they were more likely to take intellectual risks. Ms. Alderman found RDM to be particularly useful with engaging her students after coming back to face-to-face learning after the period of remote instruction during the initial phase of the COVID-19 pandemic.

> I wanted students talking. They weren't talking to each other and they weren't talking about math, and it was really hard to get them talking. There was a lot of fear, and I think more than just the typical, like, math anxiety, there was a lot of, like, we've been behind a computer for a year and how do I do this? It was shocking [after introducing RDM]... it was just, immediately, kids started owning their right [to share what made sense to them]. Like, 'well, I'm gonna share my thinking, and it's not done, but that's okay'.

Regarding her work with RDM, Ms. Alderman said,

> I don't think I could teach math another way now. <laugh> I really don't think I could... hearing students just so excited. And they're like, oh, can we come back to this activity tomorrow? Or could we, can we look at this more? Could we, could we do this more? Or students are like, can I take this home and think about this more? <laugh> like, well, yeah, I didn't assign homework, but yes, you can definitely think about it more...

She reported that enacting RDM supported students with developing a desire to continue learning and persevering. She wanted students to engage in actively making sense of mathematics through discourse, to build a classroom culture that focuses on understanding the process of doing mathematics, and to support her students' confidence. She found that RDM helped her work toward these goals. Overall, we conjecture, based on our interviews with teachers, that teachers who are drawn to enact RDM see it as a teaching practice that helps them achieve what they are already striving to enact.

4. Discussion

The results of Studies 2–4 provide evidence that RDM can be enacted with incremental changes to a teacher's practice and, in the cases of these participants, with relatively minimal support. Additionally, the results of Study 1 show that even attending a single conference talk can provide opportunities for teachers to shift their thinking about their teaching. We are hopeful that readers will potentially be inspired to make incremental

changes to their own teaching (or their thinking about their teaching) after reading about what was possible among these participants.

Across these studies, it appears that revising was possible for teachers to integrate into their mathematics teaching practice; they found revising to be valuable, and revising could be conducted in a variety of ways in a mathematics classroom. Teachers considered ways to invite students to revise assessments. They enacted tasks in ways that gave students more than one attempt at solving them, and they enacted instructional routines and collaborative work in a manner that made opportunities to draft and revise explicit to their students. Drafting and revising also could help teachers achieve some of their goals, such as engaging more students in discourse so that they persevere to make sense of mathematics.

Results across these analyses demonstrate the potential for RDM to be a feasible incremental change for mathematics teachers to enact in their current contexts. However, we acknowledge that our participants are those who volunteered to reflect upon RDM. Above, when we asked participants in a conference session to share how their thinking changed, not all participants at conference sessions completed the Google Forms at the end of the sessions. On average, about one third of attendees at a conference session respond to the invitation to complete a Google Form and share how their thinking changed. We conjecture that participants whose thinking was not impacted or participants who did not enjoy the presentation also did not complete the Google Forms. This means we are not aware of how or why the sessions did not impact some teachers' thinking.

In the interviews we conducted about teachers' enactments of RDM after participating in book studies, we spoke with teachers who were interested in sharing their thinking about RDM with us. We did not systematically investigate the thinking of all teachers who participated in the book studies. This means that we did not learn about how the book did not impact some teachers' practice.

Similarly, we solicited volunteer participants for our study about enacting RDM with curriculum materials. Teachers who participated in interviews were interested in talking about how they saw opportunities for enacting RDM while using their current textbooks. This means we did not learn from teachers who found it more challenging to integrate RDM with their curriculum materials.

Although we do not yet have a strong understanding about teachers' resistance to enacting RDM, we also have not faced challenges with finding participants for our research on RDM. Teachers and teacher leaders around the country have been willing and interested to share how they enact RDM. Findings from those who are willing to enact RDM provide existence proofs of what is possible. The ease with which we have been able to solicit participants around the country suggests that there are teachers who are invested in this instructional approach. We hope that what we have learned from these participants will allow us to support more teachers with enacting RDM.

Additional Supports Needed to Enact RDM

In this article, we have shared a range of ways that teachers reported being able to enact RDM with minimal levels of support, which suggests that RDM is a feasible incremental change in mathematics teaching. However, it is possible that RDM could be enacted even more powerfully with more intensive support. Mathematics teacher educators could build on the momentum of incremental changes by providing additional opportunities for teachers to grow their practice in enacting RDM.

As an example, we conjecture that for students to feel safe to share their draft thinking, teachers must recognize and highlight strengths in students' drafts. This involves believing that students' thinking makes sense to them and is viable for making progress in their learning. Teachers can monitor which students participate to assess if teachers appear to be operating out of implicit biases [36]. With support, teachers can learn to recognize strengths in students' work [37,38]. Engaging in identifying implicit biases and learning to see strengths in students' thinking is an intensive, but important, endeavor that goes beyond incremental changes in teaching.

5. Conclusions

In this paper, we have shared exploratory findings to illustrate that mathematics teachers can enact RDM by making incremental changes that align with their current teaching practices and local contextual expectations, including curriculum materials, with relatively minimal support. After experiencing a conference presentation, teachers could articulate small changes in their thinking that support an interest in enacting RDM. After reading *Rough Draft Math: Revising to Learn* [5], teachers could take up and enact practices from the book, often while using their current curriculum materials. We conjecture that this is possible, in part, due to RDM being a general concept, not a set of prescribed practices, that teachers can enact in ways that make sense to them. We also conjecture that teachers were able to make incremental changes to enact RDM if they saw RDM as supporting the goals that they have for their students.

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