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Developing a Constructive Conceptual Framework of a Pre-Service Mathematics Teachers' Content Knowledge Instrument on Space and Shape

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Abstract: Space and shape is one of the geometry topics that should be mastered by students and require proper teachers' Mathematics Content Knowledge (MCK) for teaching to avoid misconception. This study aimed at developing a constructive conceptual framework as an instrument to examine mathematics pre-service teachers' MCK on space and shape contents and describing their profile on this topic. The present study used mixed methods, in which the obtained data were analyzed both quantitatively using Exploratory Factor Analysis (EFA) and qualitatively described in nature. The developed MCK instrument was administered to 21 senior Indonesian mathematics pre-service teachers who were in their third year of study which and by a purposive sampling technique. The results showed that the instrument had very good 10 final items with a consistent reliability coefficient of 0.67 and resulted in four factor components, namely, figural representation, area and circumference of object, relationship between properties of objects, and figural reasoning. Of the four factors, figural representation and reasoning factors had mostly been the challenges for Indonesian mathematics pre-service teachers. On the contrary, they performed better in the area and circumference of objects and the relationships between properties of objects. The findings lead to redesigning the curriculum for mathematics pre-service teachers' learning to accommodate all their challenges.

Keywords: Mathematics Content Knowledge (MCK); mathematics pre-service teachers; space and shape; MCK instrument; Exploratory Factor Analysis

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Citation: Ekawati, R.; M.; Rosyidi, A.H.; Prawoto, B.P.; Prahmana, R.C.I.; Lin, F.-L. Developing a Constructive Conceptual Framework of a Pre-Service Mathematics Teachers' Content Knowledge Instrument on Space and Shape. *Mathematics* **2022**, *10*, 137. https://doi.org/10.3390/math10010137

Academic Editors: Michael Gregory Voskoglou and Jay Jahangiri

Received: 17 November 2021
Accepted: 26 December 2021
Published: 4 January 2022

1. Introduction

Teachers' Mathematics Content Knowledge (MCK) is one of the essential things for effective teaching; therefore, it is important to be researched. Gearhart and Saxe [1] described teachers who have knowledge about the subject and flexible pedagogical knowledge as perfect teachers. In addition, Berliner [2] considered those who are more flexible in teaching as the expert ones. Recently, investigations on knowledge scheme for teaching as well as the MCK are the familiar approach used in much research, i.e., Programme of International Student Assessment (PISA) and Teacher Education and Development Study in Mathematics (TEDS-M). In addition, numerous studies on the assessment of MCK have been conducted in several projects, such as COACTIV [3] and MT21 [4].

The notion of content knowledge is initially described by Shulman [5]. In the field of mathematics, Ma [6] considered content knowledge as understanding of the breadth and depth of relevant topics in mathematics, including an awareness of the interconnections. There are five content standards in mathematics: numbers and operations, algebra, geometry, measurement, and data analysis and probability [7]. Geometry is one of the mathematics topics that requires students to have the ability of analyzing, reasoning, and visualizing because it forms students' concrete-to-abstract thinking [8]. One of the basic concepts in geometry is space and shape, a topic that offers opportunities to practice logical, critical, creative, and systematical reasoning [9]. The space and shape topic encompasses a wide range of phenomena that are encountered everywhere in our visual and physical world, such as patterns, properties of objects, positions and orientations, representations of objects, decoding and encoding of visual information, and navigation and dynamic interaction with real shapes as well as with representation [10]. In short, geometry becomes the essential foundation to mastering the space and shape topics. In terms of geometry teaching, Fauzan [11] described that there are several problems in geometry instruction in Indonesia. The approach to teaching geometry is very theoretical, while many abstract concepts and formulas are introduced without paying much attention to logical reasoning and understanding [11].

The range of content used in national mathematics curricula is designed to supply students with knowledge and skills on mathematics phenomena. In Indonesian curriculum, there are three mandatory education levels, namely elementary school, junior high school, and senior high school. At each level, there are some geometry topics referring to the space and shape content that students need to learn, including but not limited to two-dimensional figures, three-dimensional figures, the relationships of angles, the relationships of lines, the Pythagorean theorem, geometry transformation, congruence and similarity, vectors, and trigonometry [12]. Mastering space and shape requires students to have the ability to think visually and spatially. Anwar and Juandi [13] described visual thinking as the ability to interpret and understand information in the form of images, graphics, or other related forms. Meanwhile, the ability to visualize, interpret, relate, and define object movements is defined as spatial thinking [14]. On the other hand, students need to have a spatial reasoning ability that consists of a set of cognitive processes by which the mental representations for spatial objects, relationships, and transformations are constructed and manipulated. Before delivering knowledge to their students, teachers must master the knowledge first to avoid any misconceptions.

The instruments used for measuring teachers' knowledge have been developed for some mathematics content in which some of them concern on a specific content such as ratio and proportion [15]. To the best of the researchers' knowledge, there is still a limit on the number of studies on developing the instruments for assessing teachers' knowledge in their specific knowledge on geometry. Geometry plays significant roles in primary, secondary, and higher mathematics curricula in many countries. It further serves as essential foundation for space and shape, but the category extends beyond traditional geometry in content, meaning, and method, drawing on elements of other mathematical areas such as spatial visualization, measurement, and algebra. Previous literatures and statistics addressed that the space and shape content problem is both complex and difficult for students to solve. This is reflected on the mass failure in mathematics examination on geometry and the trend of students' performance decreasing [16]. The cause of such a phenomenon could be related to teachers' MCK on Geometry or Space and Shape. For instance, Kambilombilo and Sakala [17] reported teachers' inadequate content knowledge and some misconceptions in teaching secondary school geometry in Africa. Another study revealed that mathematics pre-service teachers already addressed some learning components, such as their experience in learning Geometry and their confidence to teach it [18]. However, there are still few studies focusing on mathematics pre-service teachers' knowledge on geometry in Indonesia. To have more insight on the pre-service teachers' MCK along with their mathematics teaching practices, developing a qualified instrument for measuring teachers' MCK on space and shape is necessary. The present study aimed at developing a Mathematics Content Knowledge instrument for examining Indonesian mathematics pre-service teachers using a PISA and TEDS-M framework. The study intended to develop a constructive conceptual framework for measuring mathematics pre-service teachers' MCK in the space and shape topic. Further, to put forward the reliable measure, the study started with developing an initial instrument framework and items and continued with pretesting and undertaking detailed MCK item analysis.

Space and Shape Content

Van de Walle [7] mentioned shapes and properties as one of the geometries subject's contents applied to all school levels. The study of shapes and properties includes the properties of shapes in two dimensions and in three dimensions is called space and shape. Moreover, it also includes the relationships between the properties. Not quite different from PISA, space and shape is a content knowledge that covers patterns, properties of objects, positions and orientations, representations of objects, decoding and encoding of visual information, and navigation and dynamic interaction with real shapes as well as with representations [10]. Regarding the Indonesian curriculum on mathematics education, the space and shape topic is learned by students from elementary to senior high school. The study of space and shape in general includes the study of two-dimensional figures (e.g., properties of object, area and perimeter, and lines of symmetry) and three-dimensional figures (e.g., volume, surface, nets, and distance) [12].

2. Research Methods

2.1. Research Design

The present study used explanatory sequential mixed methods. This method conducts quantitative research to analyzes the results at first and then builds the explanation of the results in more detail with qualitative research. The quantitative research explored the relationships between variables and connected those relationships to the theories [19]. This part explained the Explanatory Factor Analysis of MCK items in the form of a correlation analysis for each item to the total score. This also used Exploratory Factor Analysis and factor loading for each variable to find the pattern. The factor scores and loadings differed from the patterns of pre-service teachers' responses to MCK items. After analyzing the results using quantitative research, we continued by explanation using qualitative research, as Frances et al. [20] described that qualitative research was an approach that enabled researchers to explore detailed social and organizational characteristics and individual behaviors along with their meanings. The qualitative research method was necessary to be undertaken to describe and explore more deeply about pre-service students' performances of MCK items in each factor loadings.

2.2. Sample

The participants were Indonesian mathematics pre-service teachers who were in their third year of study at State University in Java, Indonesia. They had already learned several concepts encompassing much professional/mathematics content, pedagogical theories, and teaching skills. The study delivered 15 MCK items on space and shape content based on PISA and TEDS-M framework to the participants. There were 21 Indonesian mathematics pre-service teachers determined using a purposive sampling technique.

2.3. Test Items Developments and Validation

This study aimed at proposing a conceptual framework for an instrument to examine pre-service teachers' MCK on space and shape content and examine their profile. First, the study developed a framework for assessing pre-service teachers' knowledge on space and shape content based on the PISA and TEDS-M framework. The item design involved two phases, namely defining the conceptions of item components and developing and validating the test instrument.

2.3.1. Phase 1: Defining Conceptions of Item Components

The framework of the MCK items was influenced by some existing theories related to space and shape and elaborated with previous frameworks developed by PISA, TIMSS, and

TEDS-M. Van de Walle [7] stated that the initial MCK of space and shape included a study of the properties and the relationship between the properties. Fujita et al. [14] stated that the core MCK of space and shape referred to the skills of coding information, manipulating and constructing nets, constructing 2D drawings of 3D shapes, decoding information, and interpreting geometrical properties. In this study, content knowledge on space and shape was developed according to van de Walle [7] and Fujita et al. [14] and PISA's framework. The content knowledge on space and shape was the underlying element of 15 initial MCK items. Referring to OECD [10] and IAEEA [21], there were three important variables on the corresponding problems of MCK items, such as context situations, sub domain, and item format.

The first variable was context situations. There were four types, including personal, occupational, societal, and scientific [10]. Among these four context situations, scientific is the main context used in the framework of MCK items. According to OECD [10], the variable sub domain of PISA included (a) patterns, (b) properties of objects, (c) positions and orientations, (d) representations of objects, (e) decoding and encoding of visual information, and (f) navigation and dynamic interaction with real shapes as well as with representations. Regarding the variable sub domain of PISA, this study focused on four sub domains, namely pattern, properties of objects, positions and orientations, and the representation of objects. In solving the pattern problems, students might use a mathematical reasoning with respect to geometrical pattern. On the other hand, students might use mathematical reasoning to solve representation of object problems. In addition, mathematical reasoning (e.g., relationship between properties of geometrical figures) could be used in solving properties of objects and positions and orientations problems.

The MCK items were generated in four different forms: (1) multiple choice (MC) items to raise problems with correct answer and distracters [22], (2) complex multiple choice (CMC) items with true–false statements—which were designed to test mathematics pre-service teachers to justify statements regarding the properties of geometrical figures and pattern, (3) open constructed response (OCR) items developed to test how pre-service teachers had possibly a different way to sketch geometrical patterns and cube nets and analysed the properties of geometrical figures and areas of sections, and (4) close constructed response (CCR) items designed to asses properties of two and three-dimensional figures. Table 1 presents the overview of fifteen MCK items instrument in more detail.

Table 1. Overview of 15 MCK items.

Code	Overview	Problem	Sub Domain	Item Format
MCK 1	The shape of a geometrical pattern	Drawing an object based on a given geometrical pattern	Pattern	OCR
MCK 2	Properties of a quadrilateral	Justifying statements related to properties of a quadrilateral	Properties of objects	MC
MCK 3	Relationship of quadrilaterals	Generalizing the relationship among quadrilaterals by representing it in a Venn diagram	Properties of objects	OCR
MCK 4	Properties of a quadrilateral	Justifying statements related to the properties of quadrilaterals	Properties of objects	CMC
MCK 5	Properties of a quadrilateral	Identifying the special case of each quadrilateral	Properties of objects	CCR

Table 1. *Cont.*

Code	Overview	Problem	Sub Domain	Item Format
MCK 6	Visuospatial skills	Drawing a cube net completed with a vertex appropriate with the direction of a cube cutting on its edges	Positions and orientations	OCR
MCK 7	Circumference of geometrical figures	Determining the circumference of two-dimensional figures	Positions and orientations	MC
MCK 8	Area of shaded region of geometrical figures	Analysing the area of the shaded region of two-dimensional figures	Positions and orientations	MC
MCK 9	Relationship of properties of geometrical figures	Determining the least number of sides needed to measure to calculate the area of geometrical figures	Representation of object	CCP
MCK 10	Relationship of properties of geometrical figures	Determining the side to be measured to calculate the area of geometrical figures	Representation of object	OCP
MCK 11	Volume of three-dimensional figures	Analysing the volume of three-dimensional figures	Representation of object	CCP
MCK 12	The shape of geometrical patterns	Justifying statements related to the given patterns	Pattern	CMC
MCK 13	Volume of three-dimensional figures	Determining how many cubes would be needed to make a geometrical figure	Pattern	CCP
MCK 14	Lines of symmetry of geometrical figures	Determining lines of symmetry of two-dimensional figures	Properties of objects	CCP
MCK 15	Area of a section	Comparing the areas of three sections	Positions and orientations	OCP

2.3.2. Phase 2: Developing and Validating the Test Instrument

Before the MCK items being administered, there were some stages of revision. The study started with considering the theoretical frameworks of PISA and TEDS-M for space and shape content for developing the instrument and continuing to discuss the first draft with four mathematics education experts from Indonesia and Taiwan, and considered this as face validation. The experts were experienced for 20–40 years in mathematics education. There were fifteen MCK items developed. The instruments were initially developed in English, so it was needed to require a valid translation to make sure that the questionnaire would properly measure the MCK on space and shape content of Indonesian mathematics pre-service teachers.

The first stage was adaptation, in which the instruments were translated independently from English to Bahasa Indonesia by a mathematics educator. The method used to translate the instrument was back-translation. Back-translation was a process to re-translate the translation text by someone who did not see the original text [23]. The second stage was integrating the translations into one common translation to check the validity and consistency of each item's translation by an English language expert. Table 1 shows the overview of MCK items on space and shape. The overview refers to the Indonesian mathematics curriculum on the topic of geometry in elementary to senior high schools.

2.4. *Coding Scheme Score*

The comprehensive coding scheme for all MCK items described in Table 1 was developed and discussed with expert mathematics educators. MCK items with one solution were coded as "correct = score 1" and "incorrect = score 0". The responses were coded more than once to ensure the coding consistency. If some answers were coded doubtfully, the researchers discussed with the coders and consulted the experts. The MCK with 15 items with its form of scoring level rubric was adapted from Ekawati et al. [15]. There were four categories of scoring levels, such as incorrect or blank with score 0, partially correct with score 0.500 or 0.700, correct with score 1, and blank with score 0. More descriptions about the problem and explanation of scoring rubric are depicted in Figure 1 and Table 2. We conducted a pilot study with two pre-service teachers to check the readiness of given problems. The work duration is two hours for two knowledge domains (MCK and Mathematics Pedagogical Content Knowledge).

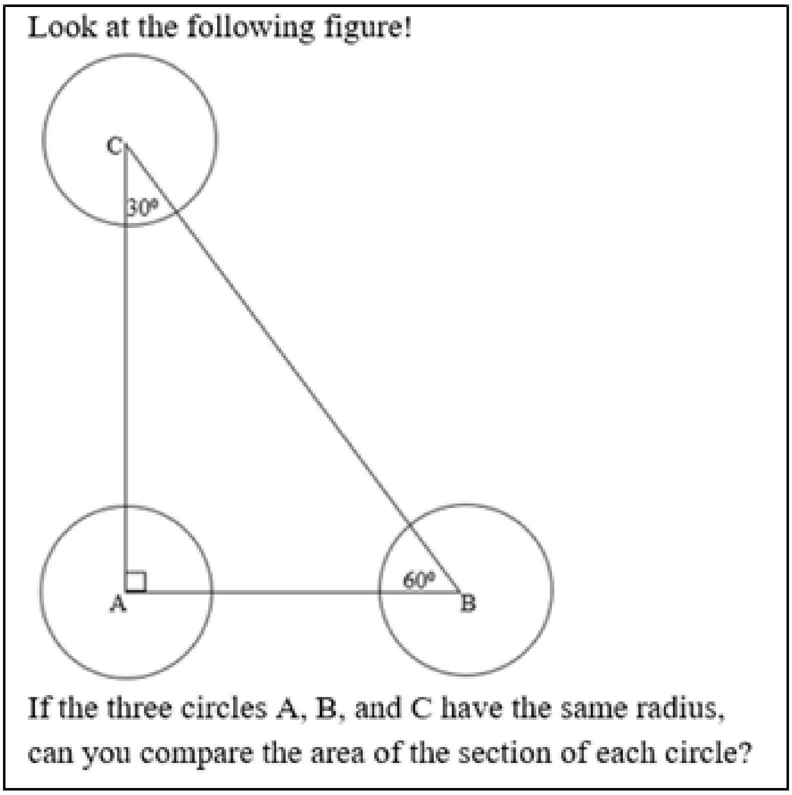

Figure 1. MCK 15 Items.

The mathematics pre-service teachers' responses to the MCK items were mainly discussed in this study by applying both Exploratory Factor Analysis and cluster analysis. Item analysis conducted before the exploratory factor analysis aimed at "investigating the performance of items considered individually either in relation to some external criterion or in relation to the remaining items on the test" [24]. The study explored correlation analysis for each item to the total score, and then deleted the items with correlations less than 0.300. This corresponded to Ebel and Frisbie [25], who explained that based on the discrimination index, very good items have a correlation above 0.400, whereas items with a reasonably good but possibly subject-to-improvement correlation ranges from 0.300 to 0.390. The items with a correlation from 0.200 to 0.290 were marginal and needed some revision, while items were considered poor and needing major revision with a correlation below 0.190. Afterward, exploring factor analysis and factor loading were undertaken for each variable to look for the pattern on the MCK data. Furthermore, the relationship between each variable that showed the overall pattern was clearly observed by conducted exploratory factor analysis.

Table 2. Scoring rubric for MCK 15 items.

Code	Responses	Score
60	Incorrect (writing the wrong comparison)	0
61	Incorrect (said that the area of the three sections is comparable without explaining the reason)	0
70	Partially correct (explaining how to compare the area of the three sections without comparing the area of the three sections)	0.5
71	Partially correct (calculating the area of the three sections without comparing each area)	0.5
72	Partially correct (explaining how to compare the area of the three sections without calculating the area, but stating the comparison)	0.7
80	Correct (explaining how to compare the area of the three sections and comparing the area of the three sections)	1
81	Correct (calculating the area of each section and comparing each area)	1
99	Blank	0

2.5. Exploratory Factor Analysis and Cluster Analysis

3. Results

3.1. Descriptive Statistics

The reliability result of the MCK items on space and shape was 0.673, considered as very good items with a consistent reliability coefficient [25]. Based on the three methods in item analysis with the criteria explained above, MCK items 2, 4, 5, 8, and 13 were deleted because of their correlation to the total score, and the factor loading values of these items were less than 0.300. Moreover, MCK 8 and 13, with a correlation below 0.190, were considered poor and needing major revision. MCK 2, 4, and 5, with correlations from 0.200 to 0.290, were marginal and needed some revisions. However, it was difficult to find the pattern and the factor analysis within the MCK items. The means and standard deviations of the MCK items (after deleting MCK 2, 4, 5, 8, and 13) were obtained from the factor analysis process specifically (see Table 3). Table 3 portrays that the maximum standard deviations of the MCK items were between 0.350 and 0.490, which indicated that the dispersion of data points tended to be close to the mean.

Table 3. Descriptive statistics of MCK item scores (except 2, 4, 5, 8, and 13).

	Number of Samples	Mean	Std. Deviation
MCK 1	21	0.543	0.494
MCK 3	21	0.357	0.478
MCK 6	21	0.357	0.478
MCK 7	21	0.763	0.436
MCK 9	21	0.714	0.463
MCK 10	21	0.667	0.483
MCK 11	21	0.467	0.478
MCK 12	21	0.286	0.463
MCK 14	21	0.657	0.478
MCK 15	21	0.576	0.353
Valid N (listwise)	21		

3.2. Exploratory Factors Analysis

In this study, the dimensionality of the framework of MCK items was explored in more detail. By using exploratory principal component factor analysis with the Oblimin with Kaiser Normalization rotation method, the responses of MCK items were submitted. This method enabled maximizing the number of factors with the maximum number of statements, so the factors do not remain completely independent, which showed by the correlation of each item to a total score of more than 0.300 [26]. Using different rotation methods can form distinct factors from the statements, as Mets and Torokoff [26] found that Varimax with Kaiser Normalization using many statements creates different factor loadings which could not clearly differentiated to the criterions. Variances of the factor on the communalities of items were from 0.466 to 0.958, which could be considered high communality. The procedure identified four underlying factors in MCK items with an eigenvalue greater than 1, as shown in the scree plot (see Figure 2) and rotated structure matrix (see Table 4). Then, the value of Kaiser–Meyer–Olkin's measure of sampling adequacy (KMO) of MCK items was 0.542, which could be interpreted as an appropriate number. Coakes and Steed [27] suggested that the KMO measure should exceed 0.6, so that factoring could be processed. Bartlett's test of the sphericity value of MCK items also showed its significance (73.736) with p = 0.004, that the factor analysis was considered appropriate [27].

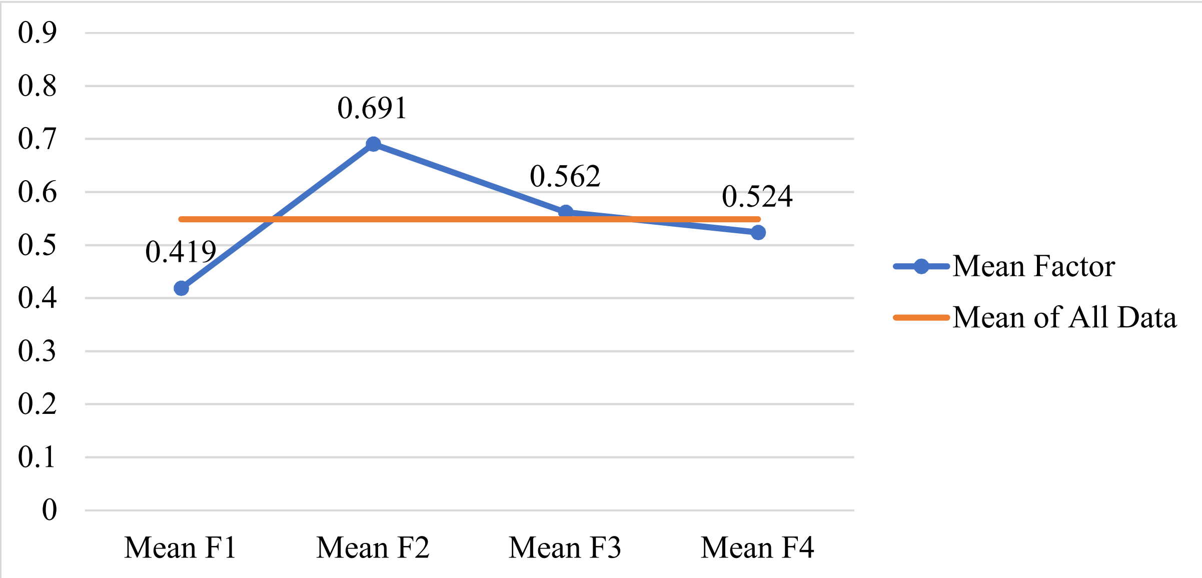

Figure 2. Mean score of items in each factor.

In accordance with Table 4, the 10 MCK items were distributed over four factors. MCK items 1, 3, and 6, which loaded highly on factor 1 (F1), were interpreted as figural representation. F1 pertained to drawing figural representations within different situations, including on pattern, Venn diagram, and cube net with its edges. The knowledge used in F1 items was considered the incorporation of teachers' ability on spatial representation, as Clements and Battista [28] stated that making a drawing was an act of representation, not a perception. Toptaş [29] claimed that the inadequacy of mental tools for spatial representation reflected on inaccurate drawing. Therefore, the present study labeled this factor as figural representation.

The two items (e.g., MCK 9 and MCK 10) that loaded on factor 2 (F2) had common characteristics interpreted as the area and circumference of objects. F2 pertained to connect the relationship between area, circumference, and sides of two-dimensional figures from the given design of a house. Pre-service teachers were asked to determine the least number of sides needed to measure to calculate the area of house and which sides they needed to be mentioned.

Besides those three factors above, factor 4 (F4) was likewise identified and interpreted as figural reasoning, in which MCK 7, 12, and 15 loaded highly. This factor pertained to property-based spatial analytic reasoning decoded by Fujita et al. [14] that property-based spatial analytic reasoning referred to interpreting the structural elements of shapes and decomposing objects into their parts using geometric properties for reasoning and decision-making. Other researchers also proposed property-based spatial analytic reasoning as decomposing objects into their parts using geometric properties to specify how the parts or shapes were related and, using these relationships, operated on the parts [30]. In this factor, one item was about to determine the perimeter from various two-dimensional figures, and two other items were about reasoning ability to justify statements in patterns and area of sections. In the end, the 10 MCK items on these four factors about shape and space were applied in our study to explore the content knowledge of Indonesian pre-service teachers.

Table 4. Rotated structure matrix of principal component factor analysis.

Descriptive	Factor			
	1	2	3	4
Drawing object based on a given geometrical pattern (MCK 1)	0.856			
Generalizing the relationship among quadrilaterals by representing it in a Venn diagram (MCK 3)	0.858			
Drawing a cube net completed with vertex appropriate with the direction of cube cutting on its edges (MCK 6)	0.899			
Determining the least number of sides needed to measure to calculate the area of geometrical figure (MCK 9)		0.959		
Determining the side to be measured to calculate the area of geometrical figure (MCK 10)		0.975		
Analysing the volume of three-dimensional figures (MCK 11)			0.859	
Determining lines of symmetry of two-dimensional figures (MCK 14)			0.859	
Determining the circumference of two-dimensional figures (MCK 7)				0.620
Justifying statements related to the given patterns (MCK 12)				0.652
Comparing the areas of three sections (MCK 15)				0.766

In accordance with Figure 2, F1 had the lowest mean score of 0.419, indicating that most of pre-service teachers found some difficulties in solving F1 problems. Besides, most of them solved problems quite well among other factors, a with mean score of 0.691. Furthermore, F3 with a mean score of 0.562 and F4 with a mean score of 0.524 approached the overall mean score of all MCK items. Most pre-service teachers found some difficulties in solving F1 problems, where only 42% of pre-service teachers could solve F1 problems. In the problem of finding the pattern of a geometrical figure (MCK 1), pre-service teachers usually asked to find the last pattern, but they must find the third pattern that was unusual. Data from Figure 2 showed that 69% of pre-service teachers could solve F2 problems. Here, the pre-service teachers were asked to determine the least number of sides needed to measure to calculate the area of objects and which sides they needed to be mentioned. The F3 mean score was close to the overall mean score of all MCK items (see Figure 2), which showed that 56% of pre-service teachers could solve F3 problems. Furthermore, F4 had mean score below the overall mean score of all MCK items (see Figure 2), which showed that 52% of pre-service teachers could solve F4 problems.

4. Discussion

Students' mathematical knowledge and skills were affected by their teachers' mathematical knowledge and skills [31]. Therefore, it was also important to assess the teachers' mathematical knowledge. Pre-service teachers who were prepared to become teachers in the future were expected to have a good Mathematics Content Knowledge (MCK). In response to this, by designing items to assess pre-service teachers' MCK on space and shape, the present study aimed to reveal the factors to measure pre-service teachers' MCK for an assessment instrument.

There were four important factors to measure pre-service teachers' MCK on space and shape based on statistical analysis results that indicated spatial reasoning within different sub domains as the underlying elements. It was essential because spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world [32]. The importance of spatial reasoning on students' performance [33] and how students used spatial reasoning skills to solve geometric problems [14] were strongly documented in existing studies. Spatial reasoning skills are needed to be proficient both for students and teachers [34–36]. Thus, pre-service teachers prepared for being a teacher in the future as facilitators for their students' spatial reasoning must master spatial reasoning. In this study, factor 1 (F1/figural representation) could be observed as items for measuring the ability to draw figural representations within different situations including on patterns, Venn diagrams, and cube nets with its edges. Factor 2 (F2/area and circumference of objects) could be observed as items for measuring the ability to determine the least number of sides needed to measure to calculate the area of objects and which sides they needed to mention. Factor 3 (F3/relationship of properties of objects) could be observed as items for measuring an understanding of spatial orientation-to-operating relationships of properties of two- and three-dimensional figures. Finally, factor 4 (F4/figural reasoning) could be observed as items for measuring the understanding of property-based spatial analytic reasoning to decompose objects into their parts using geometric properties for reasoning and decision making. These four factors were included in the MCK instrument for measuring Indonesian pre-service teachers' content knowledge of space and shape.

Overall, the MCK performance of pre-service teachers in F1 (Figural representation) and F4 (Figural reasoning) were below the overall mean score. F1 showed the lowest mean score value while F2 (area and circumference of objects) showed the highest score. Furthermore, regarding the difficulties of pre-service teachers in solving F1 problems, the possible reasons for such phenomenon were due to their limitation in visual thinking ability or related to the topics. The problem was about finding the pattern of the Koch snowflake, which was unfamiliar for the pre-service teachers. Visual thinking was the ability to process and create interpretations, uses, and ideas about images [13,37]. The ability of thinking visually was required in understanding concepts and solving mathematical problems, especially in geometrical problems [13]. Bad visual thinking ability was also one of the reasons why pre-service teachers had difficulty in solving problem MCK 6, about drawing cube nets based on the given instruction in cutting the side. Drawing cube nets was not a difficult problem. It was well known by all the pre-service teachers. However, it looked difficult because of the instruction about where and which side should be cut. Some pre-service teachers properly drew and labelled the nets, and some of them drew the right type of the nets but wrongly labelled it. In addition, there were also those who just drew cube nets without following the instructions, so they got the wrong answer. Furthermore, another problem given in F1 was MCK 3, about making a Venn diagram of the rectangular family. The key to this problem was understanding the concept and characteristics of the rectangular family, which is a basic concept that must be mastered by all pre-service teachers. Most pre-service teachers did not recognize the special form of each shape and drew an incorrect Venn diagram for the rectangular family. Meanwhile, F2 problems were the easiest factor for all pre-service teachers.

Referring to MCK in F2 as MCK 9 and MCK 10, pre-service mathematics teachers should have the knowledge about length conservation, where the length of an object did not change under a variety transformation [38]. Regarding the concept of length conservation and considering that MCK 10 was an open constructed response (OCR), there were some alternative answers of MCK 10 (see Figure 3). Pre-service teachers made different signs in the picture on MCK 10 to show there are six sides that measured. Some used "√", "x" and number 1–6 as signs for the six sides. Although F2 was an easy problem, there were some pre-service teachers who could not handle it. They did not determine some parts of the sides needed to be measured that required their ability to connect the relationship between the areas, perimeter, and sides of two-dimensional figures. Furthermore, they also determined all sides without looking carefully at the question that asked them to determine the least number of sides needed to calculate the areas of objects.

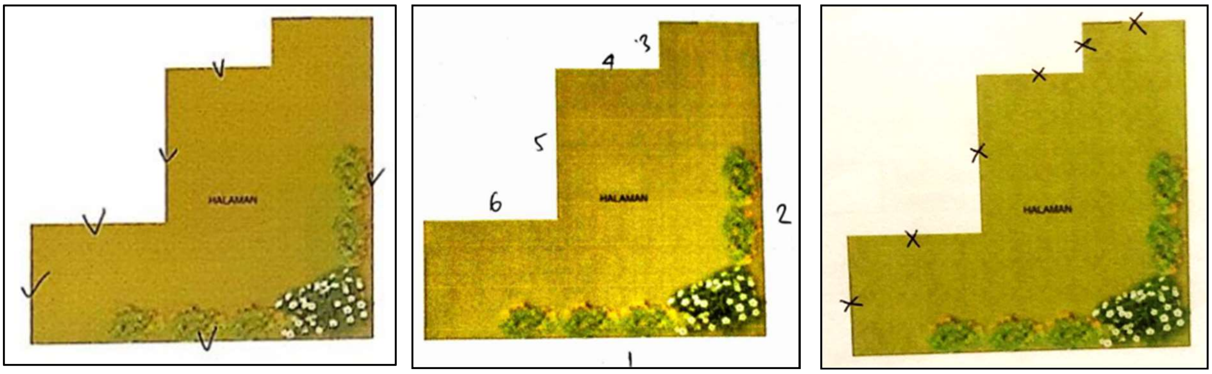

Figure 3. Alternative answers of pre-service teachers on MCK 10 (F2).

Understanding of spatial orientation-to-operating relationship of properties of two- and three-dimensional figures were measured on F3. MCK 11 asked pre-service teachers to analyze the volume of three-dimensional figures (e.g., four prisms with same heights and different base areas) (see Figure 4).

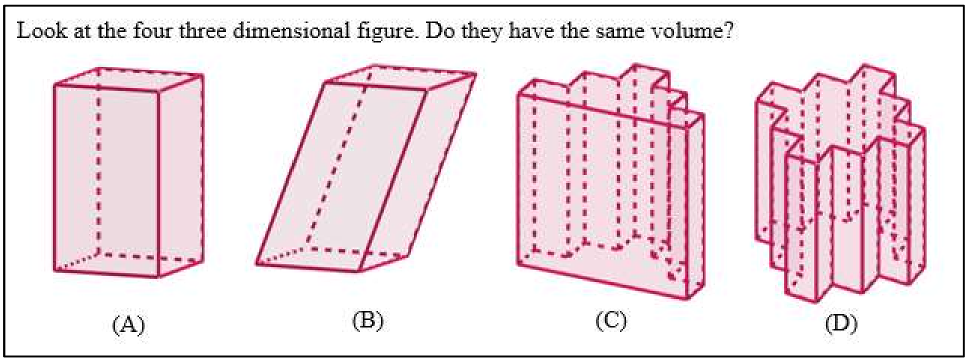

Figure 4. MCK 11: (**A**–**C**) are prism with equal volume and (**D**) prism is different volume

Half of pre-service teachers solved the problems well and explained the reasoning why the prism had the same volume or not. They explained that prism A, B, and C had the same volume because prism B had the same base area, but the figure was moved aside, and part of base area of prism C could be moved to form the same base area with prism A. In the other hands, prism D did not have the same volume as the others' because the base area could not form the same even though it could move around it. Hence, half of pre-service teachers still could not understand the relationships between the positions of objects in three-dimensional figure with respect to one's own position [28]. For overcoming this, pre-service teachers needed to understand the concept deeply so that in the future they did not make misconceptions in their teaching classes. Moreover, MCK 14 asked pre-service teachers to determine lines of symmetry of two-dimensional figures. There were hexagons with six lines of symmetry, circles with infinity lines of symmetry, pentagons with five lines of symmetry, and parallelograms with no lines of symmetry. The minority of pre-service students had wrong answers on determining lines of symmetry of hexagons and parallelograms, mentioning lines of symmetry from each edge only, without considering if it folded into same area or not. It showed that their understanding of spatial orientation-to-operating relationships of properties of two-dimensional figures still needed to be increased.

F4 consisted of three items, namely MCK 7, 12, and 15. The highest mean score among F4 problems was MCK 7, of which pre-service teachers were asked to determine the perimeter of two-dimensional figures. To solve it, they should know how to use figural reasoning for decision-making on which sides needed to be calculated so that the perimeter was sixty. The wrong answer (25%) showed that they did not recognize that there were some overlapping sides that needed to be calculated twice or three times. It showed that they could not use property-based spatial analytic reasoning well on geometric properties to specify how the parts or shapes were related and operated on the parts for decision-making [30]. Then, the rest of them could make the right decision on which perimeter of figure A was sixty and the others were not. In other hands, MCK 15 asked pre-service teachers to compare the area of three sections (in a triangle). Half of pre-service teachers (55%) solved the problems well and explained how to compute the comparison of the area of three sections in detail. There were some kinds of alternative answers explained in Table 2.

Pre-service teachers explained how to compare the areas of the three sections and comparing the areas of the three sections or just calculating them, meaning that they could use their figural reasoning to compute the relationships between the positions of objects in two-dimensional figure with respect to one's own position [28] especially between the sections of a circle, the angle of its sections, and the radius of a circle. Hence, there were still some pre-service teachers who did not explain the reason or calculated it wrongly. Moreover, MCK 12 had the lowest mean score among the F4 problems, in which pre-service teachers were asked to justify statements related to the given patterns. They should analyze different statements and made decisions that tiles would go in any position that matched the pattern. Statements 2 and 6 were the right answers where statement 2 " If m + n is even, then use tile A, otherwise use tile B." and for statement 6 "If m and n are both odd or both even, then use tile A, otherwise use tile B.". Only 28% of pre-service teachers solved it correctly, and the rest had difficulties to analyze it. Most of them chose statement 1 (10%), statement 3 (36%), statement 4 (21%), and statement 5 (15%). Statement 3, "(3) If m $\times$ n is odd, then use tile A, otherwise use tile B.", was the most chosen one, but it could not be suitable for any positions of tile because tile A also used when m and n were even that were contradicted with the statement. Statement 4, "(4) If m $\times$ n is even, then use tile A, otherwise use tile B.", was the second most chosen one, but it contradicted with the statement that when m $\times$ n was even, one should use tile B. Regarding this result, it showed that pre-service teachers still had difficulties in figural reasoning, especially in judging whether statements were true or not. Because F4 problems had the lowest mean score compared to the other factors, pre-service teachers' figural reasoning still needs to be increased. To overcome this, pre-service teachers need to understand property-based spatial analytic reasoning to decompose objects (e.g., two and three-dimensional figures) into their parts using geometric properties for reasoning and decision-making so that in the future they do not make misconceptions in their teaching classes.

5. Research Implication

The MCK instrument with these four factors (e.g., F1, F2, F3, and F4) could be used to explore teachers' MCK related to space and shape. Based on the research, there is a potential implication on using the conceptual framework as the basis for MCK instruments on space and shape to measure pre-service Mathematics teachers' MCK in each level (e.g., elementary, junior high school, and senior high school). As described by Schmidt et al. [4], there is a strong relationship between teachers' knowledge and students' achievement. In this study, the profile of pre-service Mathematics teachers' MCK on space and shape were described, and there are several factors that needs to be improved. Llinares [39] explained that from the mathematics teacher education perspective, the design for the teaching programs will be based on both teachers' knowledge and instructional quality. With regards to the information of pre-service teachers' knowledge, mathematics teachers' educators can redesign learning activities on geometry/space and shape as well as school mathematics to improve pre-service mathematics teachers' knowledge using several strategies. For example, Gambini and Lenart [40] emphasized design activities for teaching non-Euclidean geometry with several activities such as using manipulative activities, analyses of an example, group activities, and general discussion.

6. Conclusions

This study proposes conceptual framework for instrument to examine pre-service teachers' Mathematics Content Knowledge (MCK) on space and shape content. The characteristic of the framework such as sub-domain, context situations, and item format of problems are elaborated. There are four factors of MCK of Space and Shape resulted from Exploratory Factor Analysis, which can be used as framework for data analysis and further study to develop MCK instrument. These will lead to understanding pre-service teachers' MCK performance on space and shape.

The four factors of MCK instrument consist of figural representation, area and circumference of objects, the relationships between properties of objects, and figural reasoning. With respect to those four factor components, Indonesian pre-service teachers can apply their knowledge in the area and circumference of objects and the relationships between properties of objects. However, they face challenges in figural representation and figural reasoning. Furthermore, a comprehensive overview of mathematics pre-service teachers' knowledge on space and shape is essentially needed in future studies to develop instruments for assessing mathematics pedagogical content knowledge (MPCK).

Author Contributions: Conceptualization, R.E.; methodology, R.E. and F.-L.L.; validation, R.E., F.-L.L., M. and A.H.R.; formal analysis, A.H.R., B.P.P. and R.C.I.P.; investigation, R.E. and R.C.I.P.; data curation, R.E., M. and B.P.P.; writing—original draft preparation, R.E.; writing—review and editing, R.E. and R.C.I.P.; visualization, R.E. and A.H.R.; supervision, F.-L.L. and R.C.I.P. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors are grateful to Universitas Negeri Surabaya, Universitas Ahmad Dahlan, and National Taiwan Normal University for giving us a research collaboration opportunity and Kai-Lin Yang for valuable suggestions.

Conflicts of Interest: The authors declare no conflict of interest.

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