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Quantized Fault-Tolerant Control for Descriptor Systems with Intermittent Actuator Faults, Randomly Occurring Sensor Non-Linearity, and Missing Data

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Abstract: This paper examines the fault-tolerant control problem for discrete-time descriptor systems that are susceptible to intermittent actuator failures, nonlinear sensor data, and probability-based missing data. The discrete-time non-homogeneous Markov chain was adopted to describe the stochastic behavior of actuator faults. Moreover, Bernoulli-distributed stochastic variables with known conditional probabilities were employed to describe the practical features of random sensor non-linearity and missing data. In this study, the output signals were quantized and a dynamic output feedback controller was synthesized such that the closed-loop system was stochastically admissible and satisfied the strictly $(\mathbf{Q},\mathbf{S},\mathbf{R})$-$\gamma$-dissipative performance index. The theoretical developments are illustrated through numerical simulations of an infinite machine bus.

Keywords: intermittent actuator faults; Markov process; probabilistic missing data; randomly occurring sensor non-linearity; output feedback control; dissipativity

MSC: 93C55

1. Introduction, Notations, and Outline

In this section, we provide the literature review and the notations and acronyms used in the present document, as well as the objectives and outline of this work.

1.1. Bibliographical Review

Singular systems, well known as descriptor systems, appear to be mathematical models that are able to depict the relationship between static and dynamic equations that simultaneously describe the behavior of different components in a system. Numerous applications of singular systems have been explored, including mechanical systems, robotics, chemical processes, and economical systems [1,2]. For digital control purposes, discrete-time singular systems have received great attention, with many publications in causality, asymptotic stability, and some prescribed performances [3–7].

Most dynamic systems, including actuators and sensors, are susceptible to unexpected faults or failures; in fact, many reasons may be responsible for a system's instability and performance degradation. For specific reasons, one can cite sensors, actuators' ages, sudden changes in working conditions, and internal components being corroded, among others, which could cause significant damage to the systems. Fault-tolerant control and fault diagnosis are crucial approaches for dynamic systems that seek to design satisfactory controllers to maintain the critical functionality of a system within admissible levels when suffering from faults or failures. Different elegant results have been developed to address the issue of reliable control for various classes of systems. Kavarian et al. develop a method for designing fault-tolerant controllers for power systems subject to random changes and actuator failures in [8]. The fault-tolerant control (FTC) method for wind-diesel hybrid systems with time-varying bounded sensor faults was proposed in [9]. In [10], the reliable observer-based control problem for discrete-time Takagi-Sugeno fuzzy systems with time-varying delays and stochastic actuator faults was formulated from the input-output approach. We also report on some results relating to fault-tolerant control for singular systems. For systems with actuator and/or sensor faults, sliding mode control was used [11]. The reliable control problem for nonlinear singularly perturbed systems with random actuator failures is discussed in [12]. Besides, as is mentioned in [13], the faults can be classified as permanent faults or intermittent faults. The first class of faults refers to faults that appear and are permanent and the last class of faults corresponds to the case where the faults occur occasionally and are limited in time. By virtue of its randomness, and intermittence, an intermittent fault needs a stochastic model to describe it [14,15]. In the meantime, Markovian jump systems have been proverbially regarded as a significant model for describing many systems with random structure changes [16–18]. Recently, the authors in [14] employed the Markov chain as a mathematical model to characterize the intermittent fault where the Markov chain was assumed to be homogeneous, with the probability of failure being independent of time. The hypothesis was, however, too restrictive in practice since the failure rate of any component usually depends on multiple aspects, for instance, its age and the extent of its solicitation. Thus, time-varying transition probabilities are more convenient, and the investigation of the non-homogeneous Markov process is more attractive, which is what motivated this study [19–21]. On the other hand, for most complex systems, the states are, however, not usually available for measurement and only a piece of partial information is accessible from the system outputs. A static/dynamic output feedback control design is often considered for such complex systems. In recent research, the dynamic output feedback control problem was also considered for singular systems [22–24]. Nevertheless, it should be pointed out that the output signals were generally taken from the sensors, which work practically under severe environments with aggressive conditions.Thus, it was necessary to pay close attention to the control problems for engineering systems with sensor non-linearities [4,11,25,26]. Moreover, it is understood that in engineering systems that employ digital channels for signal transmission, signal quantization becomes indispensable, especially when the bandwidth and energy are limited. Nevertheless, it may impact the system's performance when signals are quantized. Thus, it is not surprising that researchers have recently investigated the problems of control and filtering using various quantization approaches [27,28]. For instance, in [29] the authors studied quantized non-stationary filtering for networked Markov switching repeated scalar non-linear systems. The work in [30], examined the quantification H_∞ control problem for non-linear stochastic network systems accompanied by probabilistic missing data. In [31], the authors studied the problem of event-triggered admissibilization for discrete-time singular Markovian jump network systems with delay and output quantizations. To the extent of our knowledge, few research efforts have been made on singular systems, and this established the second motivation for the present work.

1.2. Objective and Outline

This paper reveals the following significant contributions of our work: (1) the system under examination exhibited intermittent actuator faults, represented by a non-homogeneous Markov chain, and the data from different sensors might be missing and affected by stochastic nonlinearities; (2) sufficient conditions were established such that the closed-loop system was stochastically admissible under the strictly $(\mathbf{Q}, \mathbf{S}, \mathbf{R})$-$\gamma$-dissipative index; (3) a feasible control strategy was formulated for the considered control problem using the decoupling matrix procedure; and (4) simulation results of a physical plant to demonstrate the effectiveness of the control scheme was presented.

Table 1 displays the notations and acronyms that are used in this paper.

Table 1. Notations and acronyms used in the present document.

Symbol	Acronym/Notation
$\mathbb{N}$	the set of positive integer numbers
$\mathbb{R}$	the set of real numbers
$\mathcal{X} \in \mathbb{R}^n$	n-dimensional Euclidean space
$\boldsymbol{X} \in \mathbb{R}^{n\times m}$	$n \times m$ real matrix
$\boldsymbol{X} > 0$	real symmetric positive definite matrix $\boldsymbol{X}$
$\lVert\boldsymbol{X}\rVert$	norm of the matrix $\boldsymbol{X}$
$\boldsymbol{X}^\top$	transpose of the matrix $\boldsymbol{X}$
$\mathrm{sym}(\boldsymbol{X})$	$\boldsymbol{X} + \boldsymbol{X}^\top$
$\lambda()$	eigenvalue of a matrix
$\mathbb{E}$	mathematical expectation
$*$	term that is induced by symmetry
r_k	discrete-time Markov process
LMI	linear matrix inequalities
MJS	Markovian jump system
FTC	fault-tolerant control

2. Preliminaries and Problem Statements

In this section, we introduce some preliminaries, which facilitate the comprehension of our proposal, and state the problem under study.

2.1. The Model

Consider a class of discrete-time descriptor systems described by the following state-space equations:

$$\begin{cases} Ex(k+1) = A(k)x(k) + Bu_F(k) + B_1 w(k) \\ z(k) = C_1 x(k) + D_1 w(k) \\ y(k) = g(C_2 x(k)) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u_F(k) \in \mathbb{R}^m$, and $w(k) \in \mathbb{R}^w$ represent, respectively, the state vector, control input vector, and disturbance input vector, which lies in the square additive space $L_2[0,\infty)$. $z(k) \in \mathbb{R}^p$ is the controlled output vector, and $y(k) \in \mathbb{R}^{n_y}$ is the measurement output. The system is defined by the matrices E, B, B_1, C_2, C_1, and D_1, which are assumed to be known, real, and constant with appropriate dimensions. The uncertain matrix $A(k)$ is defined as $A(k) = A + \Delta A(k)$, where A is a constant matrix, and $\Delta A(k)$ represents the parametric uncertainties.

2.2. Assumption

Throughout this paper, the following are assumed:

A1 $E \in \mathbb{R}^{n\times n}$ is a singular matrix such that $\mathrm{rank}(E) = q < n$.

A2 Matrix ΔA is defined as $\Delta A = H\Delta(k)F$, where matrices H and F are known and constant with appropriate dimensions, and matrix $\Delta(k)$ is an unknown matrix verifying $\Delta^T(k)\Delta(k) \leq I$.

A3 Due to actuator perturbation, we assume that there may exist an intermittent fault in the actuator, which is described as

$$u_F(k) = \Theta_{r(k)} u(k) \tag{2}$$

where matrix $\Theta_{r(k)}$ defines the actuator fault. For the sake of notation, in each $r_k = i \in \mathbb{N}$, the related matrices or vectors to r_k are denoted using the index i.
Matrix Θ_i is defined as $\Theta_i = \text{diag}\left(\bar{\theta}_{1i}, \bar{\theta}_{2i}, \cdots, \bar{\theta}_{mi}\right)$, where the degradation levels of the actuator, $\theta_{si},\ s = 1, 2, \cdots m$, are defined in $\mathbb{N}$ with a probability matrix $\Pi(k) = \pi_{ij}(k),\ (i, j \in \mathbb{N})$. $\pi_{ij}(k)$ defines the transition probability such that $\pi_{ij}(k) = Pr(r(k+1) = j|r(k) = i)$, $\pi_{ij}(k) \geq 0$ and $\sum_{j=1}^{N} \pi_{ij}(k) = 1$ for each i. A Markov chain that exhibits time-dependent transition probabilities is known as a non-homogeneous Markov chain. Transition matrix $\Pi(k)$ is assumed to have the following structure:

$$\Pi(k) = \sum_{l=1}^{M} \alpha_l(k) \Pi^l \tag{3}$$

where $0 \leq \alpha_l(k) \leq 1$ and $\sum_{l=1}^{M} \alpha_l(k) = 1$.
Accordingly, the time-varying transition probability matrix $\Pi(k)$ evolves on a polytope defined by its vertices $\Pi^l, l = 1, \cdots, M$, as well as referring to the polytopic time-varying transition matrix.

A4 Sensor outputs are sent over an unreliable network, where random non-linearities may affect the sensors. Here, we assume that the sensor output is as follows:

$$\hat{y}(k) = \theta(k) C_2 x(k) + (1 - \theta(k)) \varphi(C_2 x(k)) \tag{4}$$

where $\varphi(C_2 x(k)$ is a non-linear function, which can be defined as

$$\varphi(C_2 x(k)) = L_1 C_2 x(k) + \phi(C_2 x(k))$$

where $\phi(C_2 x(k)) \in [L_1,\ L_2]$ is a nonlinear continuous function satisfying the sector condition [32,33]

$$\phi^T(C_2 x(k))[\phi(C_2 x(k)) - L\phi(C_2 x(k))] \leq 0, \tag{5}$$

where diagonal matrices L_1 and L_2 are known and verify $0 \leq L_1 < L_2$ and $L = L_2 - L_1$.

A5 Additionally, this study attempts to develop a controller using the quantization of sensor output. Based on the logarithmic quantizer, the following model can be used to define sensor output:

$$q(\hat{y}(k)) = \begin{bmatrix} q_1(\hat{y}_1(k)) & q_2(\hat{y}_2(k)) & \cdots & q_{n_y}(\hat{y}_{n_y}(k)) \end{bmatrix} \tag{6}$$

where $\hat{y}_s(k)$ is the $'s'$th component of $\hat{y}(k)$.
To define the logarithmic quantizer, we propose the following set of quantization levels:

$$\mathcal{U} = \{u^l, u_i^l = \rho^l u_0,\ l = 0, \pm 1, \pm 2, \cdots\} \cup \{0\},\ u_0 > 0 \tag{7}$$

where ρ^l is the quantization density verifying $0 < \rho^l < 1$.
Specifically, the corresponding logarithmic quantizer $q(\nu)$ is defined as follows:

$$q(\nu) = \begin{cases} u^l & \text{if } \dfrac{1}{1+\delta}\rho^l u_0 < \nu \leq \dfrac{1}{1-\delta}\rho^l u_0 \\ 0 & \text{if } \nu = 0 \\ -q(-\nu) & \text{if } \nu < 0 \end{cases} \tag{8}$$

in which ν defines the input of the quantizer, and $\delta = \frac{1-\rho}{1+\rho}$.

The quantization error of a logarithmic quantizer is $\Delta_i = q_i(\nu_i) - \nu_i$, where $\Delta_i \in [-\delta, \delta]$. Then, we have

$$q(\hat{y}(k)) = (I + \Delta q(k))\hat{y}(k)$$

where

$$\Delta q(k) = diag\Big(\Delta_1, \Delta_2, \cdots \Delta_{n_y}\Big) \tag{9}$$

A6 The measured output suffers from both signal losses and quantization arriving to the controller, thus, the output is given by

$$\begin{aligned} y(k) &= \beta(k)q(\hat{y}(k)) = \beta(k)(I + \Delta q(k))\Big(\theta(k)C_2x(k) + (1-\theta(k))\varphi(C_2x(k))\Big) \\ &= g((C_2x(k))) \end{aligned} \tag{10}$$

Stochastic parameters $\beta(k)$ and $\theta(k)$ in (4) and (10) are governed by the Bernoulli distribution so that we have the following:

$$Pr(\tau(k) = 1) = \tau, \; Pr(\tau(k) = 0) = 1 - \tau$$

where $\tau(k) = \{\beta(k), \theta(k)\}$, $\tau = \{\beta, \theta\}$, and $0 \leq \beta, \theta \leq 1$ are known constants.

Let $\rho_1(k) = \beta(k) - \beta$ and $\rho_2(k) = \beta(k)\theta(k) - \beta\theta$. It can be seen that $\mathbb{E}\{\rho_l(k)\} = 0$, $(l = 1, 2)$.

The purpose of this study was to design a mode-dependent full-order dynamic output controller of the following format:

$$\begin{cases} E\hat{x}(k+1) = \hat{A}_i\hat{x}(k) + \hat{B}_iy(k) \\ \quad\;\; u(k) = \hat{C}_i\hat{x}(k) \end{cases} \tag{11}$$

where $\hat{A}_i$, $\hat{B}_i$, and $\hat{C}_i$ are the designed controller gains, and $\hat{x}(k) \in \mathbb{R}^n$ identifies the controller state. As a result of combining (1) and (11) we obtain the dynamics of a closed-loop system as follows:

$$\begin{cases} \bar{E}\bar{x}(k+1) = (\bar{A}_i + \Delta\bar{A})\bar{x}(k) + \bar{B}_1w(k) + \bar{A}_{\phi i}\phi(C_2x(k)) + \rho_1(k)\Upsilon_1(k) + \rho_2(k)\Upsilon_2(k) \\ \quad\;\; z(k) = \bar{C}_1\bar{x}(k) + \bar{D}_1w(k) \end{cases} \tag{12}$$

where $\bar{x}^T(k) = \begin{bmatrix} x^T(k) & \hat{x}^T(k) \end{bmatrix}^T$, $\bar{E} = \text{diag}(E, E)$, $\Delta\bar{A}(k) = \bar{H}\Delta(k)\bar{F}$, and

$$\bar{A}_i = \bar{\mathbb{A}}_i + \mathbb{H}_i\Delta Q(k)\mathbb{F}, \; \bar{A}_{\phi i} = \bar{\mathbb{A}}_{\phi i} + \mathbb{H}_i\Delta Q(k)\mathbb{I}, \; \Delta Q(k) = \text{diag}(\Delta q(k), \Delta q(k))$$

$$\bar{\mathbb{A}}_i = \begin{bmatrix} A & B\Theta_i\hat{}C_i \\ \beta\theta\hat{B}_iC_2 + \beta(1-\theta)\hat{B}_iL_1C_2 & \hat{A}_i \end{bmatrix}, \; \bar{\mathbb{A}}_{\phi i} = \begin{bmatrix} 0 \\ \beta(1-\theta)\hat{B}_i \end{bmatrix}$$

$$\bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \bar{C}_1 = \begin{bmatrix} C_1 & 0 \end{bmatrix}, \; \bar{D}_1 = D_1, \bar{C}_2 = \begin{bmatrix} C_2 & 0 \end{bmatrix},$$

$$\bar{H} = \begin{bmatrix} H \\ 0 \end{bmatrix}, \; \bar{F} = \begin{bmatrix} F & 0 \end{bmatrix}, \; \mathbb{H}_i = \begin{bmatrix} 0 & 0 \\ \beta\theta\hat{B}_i & \beta(1-\theta)\hat{B}_i \end{bmatrix}$$

$$\mathbb{F} = \begin{bmatrix} C_2 & 0 \\ L_1C_2 & 0 \end{bmatrix}, \; \mathbb{I} = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$\Upsilon_1(k) = \begin{bmatrix} 0 \\ \hat{B}_i\bar{\Delta}qL_1C_2x(k) + \hat{B}_i\bar{\Delta}q\phi(C_2x(k)) \end{bmatrix}, \; \bar{\Delta}q(k) = I + \Delta q(k)$$

$$\Upsilon_2(k) = \begin{bmatrix} 0 \\ \hat{B}_i\bar{\Delta}qC_2x(k) - \hat{B}_i\bar{\Delta}qL_1C_2x(k) - \hat{B}_i\bar{\Delta}q\phi(C_2x(k)) \end{bmatrix}$$

Remark 1.

1. *As proposed in [34,35], expression (3) provides a non-homogeneous Markovian chain characterized by a time-varying transition probability, which may be described by a polytope with time-varying parameters. This case of Markovian chain degenerates the piecewise homogeneous and homogeneous cases.*
2. *The Markov process is used here to model the actuator fault; hence, the precise transition probabilities were difficult to ascertain in practice. To circumvent this problem, time-varying transition probabilities were introduced in a convex polytopic set modeled by a non-homogeneous Markovian chain.*
3. *From a practical point of view, the transition probability matrix can be be obtained using the method suggested in [36].*

Before doing so, we recall, for nominal singular Markovian jump system (13), the subsequent definitions as follows:

$$Ex(k+1) = A_i x(k) \tag{13}$$

Definition 1 ([37])**.**

1. *For each $i \in \mathbb{N}$, if $\det(zE - A_i)$ is not identically zero, then pair (E, A_i) is said to be regular;*
2. *For each $i \in \mathbb{N}$, if $deg\Big(det(zE - A_i)\Big) = rank(E)$, then pair (E, A_i) is said to be causal;*
3. *If for any initial state (r_0, x_0), $\mathbb{E}\Big\{\sum_{k=0}^{\infty} \|x(k))\|^2 | r_0, x_0\Big\} < \infty$ is verified, then system (1) is stochastically stable;*
4. *If system (1) is regular, causal, and stochastically stable, then it is said to be stochastically admissible.*

Throughout this study, we use a quadratic supply rate defined as

$$J_{zw}(k) = z^T(k)\mathbf{Q}z(k) + 2z^T(k)\mathbf{S}w(k) + w^T(k)\mathbf{R}w(k)$$

where matrices $\mathbf{Q}$, $\mathbf{S}$, $\mathbf{R}$ are real, and $\mathbf{Q} = \mathbf{Q}^T$, $\mathbf{R} = \mathbf{R}^T$. We suppose that $\mathbf{Q} \leq 0$ and $-\mathbf{Q} = \mathbf{Q}_-^T \mathbf{Q}_-$.

Definition 2 ([38])**.** *System (1) is strictly $(\mathbf{Q}, \mathbf{S}, \mathbf{R})$-$\gamma$ dissipative for a given scalar γ, if the following inequality holds under zero initial condition:*

$$\sum_{s=0}^{\infty} J_{zw}(s) > \gamma \sum_{s=0}^{\infty} w^T(s) w(s) \tag{14}$$

Remark 2. *The dissipativity criterion defined above unifies the H_∞ performance and positive realness by an appropriate choice of different parameters. Actually, inequality (14) is equivalent to an H_∞ performance index for $\gamma > 0$ when $\mathbf{Q} = -I$, $\mathbf{S} = 0$, and $\mathbf{R} = (\gamma^2 + \gamma)I$. The criterion corresponds to strict passivity or strictly positive realness if $\mathbf{Q} = 0$, $\mathbf{S} = I$, and $\mathbf{R} = 0$.*

The following lemmas are introduced to help in the controller design process:

Lemma 1 ([39])**.** *Let $Q = Q^T$ and M and N be given matrices. For any matrix $F(k)$ satisfying $F^T(k)F(k) \leq I$, $Q + MF(k)N + N^T F^T(k) M^T < 0$ holds, if and only $Q + \varepsilon M M^T + \varepsilon^{-1} N^T N < 0$ for any scalar $\varepsilon > 0$.*

Lemma 2. *If there exists a scalar α, matrices Q, N, M, U, and T satisfying*

$$\begin{bmatrix} Q & M & \alpha N \\ * & -\alpha\,\mathrm{sym}(U)+T & 0 \\ * & * & -T \end{bmatrix} < 0 \tag{15}$$

then,

$$Q+\mathrm{sym}(MU^{-1}N^T)<0 \tag{16}$$

Proof. By checking the congruence transformation to (15) by $[I,\ N(U)^{-T},\ N(U)^{-T}]^T$, condition (16) holds. □

3. Admissibility and Dissipativity Analysis

This section shows the results on the stochastic mean square admissibility with dissipativity performance of the closed-loop system.

Theorem 1. *Given the scalars $\gamma > 0$, $\delta_1, \cdots, \delta_{n_y}$, and matrices $\mathbf{Q}$, $\mathbf{S}$, and $\mathbf{R}$, if there exist matrices $P_i^l > 0$, S_i, W_i, $\mathbb{V}_{1i}$, $\mathbb{V}_{2i}$, and positive scalars ε_{0i}, ε_{1i}, ε_{2i} such that*

$$\begin{bmatrix} \Xi_i^{lq}(\bar{\mathbb{A}}_i,\bar{\mathbb{A}}_{\phi i}) & \mathbf{Y}_{1i}\Lambda & \mathbf{Y}_2 W_i \\ * & -W_i & 0 \\ * & * & -W_i \end{bmatrix} < 0 \tag{17}$$

where

$$\Xi_i^{lq}(\bar{\mathbb{A}}_i,\bar{\mathbb{A}}_{\phi i}) = \begin{bmatrix} \Xi_{11i}^{lq} & \Xi_{12i} & \Xi_{13i} & \Xi_{14i} & \bar{C}_{1i}^T\mathbf{Q}_- & \mathbb{V}_{1i}\bar{H} \\ * & \Xi_{22i}^{lq} & \mathbb{V}_{2i}\bar{\mathbb{A}}_{\phi i} & \mathbb{V}_{2i}\bar{B}_{1i} & 0 & \mathbb{V}_{2i}\bar{H} \\ * & * & -2\varepsilon_{1i}I & 0 & 0 & 0 \\ * & * & * & -\Xi_{44i} & \bar{D}_{1i}^T\mathbf{Q}_- & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\varepsilon_{0i}I \end{bmatrix} \tag{18}$$

$$\begin{aligned}
&\mathbf{Y}_{1i} = \mathrm{col}\left\{\mathbb{V}_{1i}\bar{\mathbb{H}}_i,\ \mathbb{V}_{2i}\bar{\mathbb{H}}_i,\ 0,\ 0,\ 0,\ 0\right\},\ \mathbf{Y}_2 = \mathrm{col}\left\{\bar{\mathbb{F}}^T,\ 0,\ \bar{\mathbb{I}}^T,\ 0,\ 0,\ 0\right\} \\
&\Xi_{11i}^{lq} = \hat{\Xi}_{11i}^{lq} + \varepsilon_{0i}\bar{F}^T\bar{F},\ \hat{\Xi}_{11i}^{lq} = -\bar{E}^T(P_i^l - X_i^{lq})\bar{E} + sym(\mathbb{V}_{1i}(\bar{\mathbb{A}}_i-\bar{E})) \\
&\Xi_{12i} = (X_i^{lq}\bar{E})^T + S_i^T\bar{R} - \mathbb{V}_{1i} + (\bar{\mathbb{A}}_i-\bar{E})^T\mathbb{V}_{2i}^T,\ \Xi_{22i}^{lq} = -sym(\mathbb{V}_{2i}) + X_i^{lq} \\
&\Xi_{13i} = \mathbb{V}_{1i}\bar{\mathbb{A}}_{\phi i} + \varepsilon_{1i}LC_2,\ \Xi_{14i} = \mathbb{V}_{1i}\bar{B}_{1i} - \bar{C}_{1i}^T\mathbf{S} \\
&\Xi_{44i} = (\mathbf{R}-\gamma I) - \mathrm{sym}(\bar{D}_{1i}^T\mathbf{S}),\ \Lambda = \mathrm{diag}(\delta_1,\delta_2,\cdots,\delta_{n_y}) \\
&X_i^{lq} = \sum_{j=1}^{N}\sum_{l=1}^{M}\sum_{q=1}^{M}\alpha_l(k)\eta_q(k)\pi_{ij}^l P_j^q,\ \alpha_l(k+1)=\eta_q(k)
\end{aligned} \tag{19}$$

then, closed-loop system (12) is stochastically mean-square admissible with a strictly $(\mathbf{Q},\mathbf{S},\mathbf{R})$-$\gamma$-dissipative performance. $\bar{R}$ is a full column rank matrix such that $\bar{R}\bar{E}=0$ and $\mathrm{rank}(\bar{R}) = 2n-2r$.

Proof. Performing the Schur complement to (17), we get

$$\Xi_i^{lq}(\bar{\mathbb{A}}_i,\bar{\mathbb{A}}_{\phi i}) + \mathbf{Y}_{1i}\Lambda W_i^{-1}\Lambda\mathbf{Y}_{1i}^T + \mathbf{Y}_2 W_i \mathbf{Y}_2^T < 0 \tag{20}$$

On the other hand, from the following inequality

$$\begin{aligned}0 &\le (\mathbf{Y}_{1i}\Delta Q(k)W_i - \mathbf{Y}_2)W_i^{-1}(\mathbf{Y}_{1i}\Delta Q(k)W_i - \mathbf{Y}_2)^T \\ &= \mathbf{Y}_{1i}\Delta Q(k)W_i\Delta Q(k)\mathbf{Y}_{1i}^T + \mathbf{Y}_2 W_i^{-1}\mathbf{Y}_2^T - \mathrm{sym}(\mathbf{Y}_1\Delta Q(k)\mathbf{Y}_2^T) \\ &\le \mathbf{Y}_{1i}\Lambda W_i \Lambda \mathbf{Y}_{1i}^T + \mathbf{Y}_2 W_i^{-1}\mathbf{Y}_2^T - \mathrm{sym}(\mathbf{Y}_1\Delta Q(k)\mathbf{Y}_2^T)\end{aligned}$$

we get

$$\mathrm{sym}(\mathbf{Y}_1\Delta Q(k)\mathbf{Y}_2^T) \le \mathbf{Y}_{1i}\Lambda W_i\Lambda\mathbf{Y}_{1i}^T + \mathbf{Y}_2 W_i^{-1}\mathbf{Y}_2^T \tag{21}$$

Considering (9) with $\Delta_s \in [-\delta_s,\ \delta_s],\ s = 1, 2\cdots, n_y$, results in

$$\Xi_i^{lq}(\bar{\mathbb{A}}_i, \bar{\mathbb{A}}_{\phi i}) + \mathbf{Y}_{1i}W_i\mathbf{Y}_{1i}^T + \mathbf{Y}_1\Delta Q(k)\mathbf{Y}_2^T < 0 \tag{22}$$

Using (21), it leads from (22) to

$$\Xi_i^{lq}(\bar{\mathbb{A}}_i, \bar{\mathbb{A}}_{\phi i}) + \mathrm{sym}(\mathbf{Y}_{1i}\Delta Q(k)\mathbf{Y}_2^T) = \Xi_i^{lq}(\bar{A}_i, \bar{A}_{\phi i}) < 0 \tag{23}$$

Assume $\Delta\bar{A} = 0$. We will firstly demonstrate that system (12) is admissible. From (23), we have

$$\Psi_i^l = \begin{bmatrix} -\bar{E}^T(P_i^l - X_i^{lq})\bar{E} + sym(\mathbb{V}_{1i}(\bar{A}_i - \bar{E})) & (X_i^{lq}\bar{E})^T + S_i^T\bar{R} - \mathbb{V}_{1i} + (\bar{A}_i - \bar{E})^T W_i^T \\ * & -sym(\mathbb{V}_{2i}) + X_i^{lq} \end{bmatrix} < 0 \tag{24}$$

Following this, we apply the congruence transformation to (24) by $\left[I, (\bar{A}_i - \bar{E})^T\right]^T$, and we get

$$\bar{A}_i^T X_i^{lq}\bar{A}_i - \bar{E}^T P_i^l \bar{E} + \mathrm{sym}\,(S_i^T\bar{R}\bar{A}_i) < 0 \tag{25}$$

For matrix $\bar{E}$, there exist two non-singular matrices $\hat{M}$ and $\hat{N}$ such that $\hat{E} = \hat{M}\bar{E}\hat{N} = \begin{bmatrix} I_{2r} & 0 \\ 0 & 0\end{bmatrix}$.

Define

$$\begin{aligned} &\hat{A}_i = \hat{M}\bar{A}_i\hat{N} = \begin{bmatrix}\hat{A}_{11i} & \hat{A}_{12i} \\ \hat{A}_{21i} & \hat{A}_{22i}\end{bmatrix}, && \hat{R} = \hat{M}\bar{R}\hat{M}^{-1} = \begin{bmatrix}\hat{R}_{11} & \hat{R}_{12} \\ \hat{R}_{21} & \hat{R}_{22}\end{bmatrix} \\ &\hat{S}_i = \hat{M}^{-T}S_i\hat{N} = \begin{bmatrix}\hat{S}_{11i} & \hat{S}_{12i} \\ \hat{S}_{21i} & \hat{S}_{22i}\end{bmatrix}, && \hat{P}_i^l = \hat{M}^{-T}P_i^l\hat{M}^{-1} = \begin{bmatrix}\hat{P}_{11i}^l & \hat{P}_{12i}^l \\ * & \hat{P}_{22i}^l\end{bmatrix} \\ &\hat{X}_i^{lq} = \hat{M}^{-T}X_i^{lq}\hat{M}^{-1} = \begin{bmatrix}\hat{X}_{11i}^{lq} & \hat{X}_{12i}^{lq} \\ * & \hat{X}_{22i}^{lq}\end{bmatrix} \end{aligned} \tag{26}$$

Using the fact that $\bar{R}\bar{E} = 0$, it can be seen that $\hat{R}\hat{E} = 0$, $\hat{R}_{11} = 0$ and $\hat{R}_{21} = 0$.

Performing the congruence transformation to (25) by $\hat{N}^T$ and $\hat{N}$, respectively, the following inequality holds using (26):

$$\begin{bmatrix} \star & \star \\ \star & sym(\hat{S}_{12i}^T\hat{R}_{12i}\hat{A}_{22i} + \hat{S}_{22i}^T\hat{R}_{22i}\hat{A}_{22i} + \hat{A}_{12i}^T\hat{X}_{12i}^{lq}\hat{A}_{22i}) + \hat{A}_{12i}^T\hat{X}_{12i}^{lq}\hat{A}_{12i} + \hat{A}_{22i}^T\hat{X}_{22i}^{lq}\hat{A}_{22i} \end{bmatrix} < 0 \tag{27}$$

where $\star$ represents the non concerned elements of the matrix. Thus, from (27), it can be verified that $sym(\hat{S}_{12i}^T\hat{R}_{12i}\hat{A}_{22i} + \hat{S}_{22i}^T\hat{R}_{22i}\hat{A}_{22i} + \hat{A}_{12i}^T\hat{X}_{12i}^{lq}\hat{A}_{22i}) < 0$, which implies $\hat{A}_{22i}$ is nonsingular. By Definition 1, it can be concluded that pair $(\bar{E}, \bar{A}_i)$ is regular and casual.

Our next step is to demonstrate that system (12) is stochastically stable. This can be accomplished by selecting a Lyapunov function which is defined as follows:

$$V(k) = \bar{x}^T(k)\bar{E}^T\Big(\sum_{l=1}^{M}\alpha_l(k)P_{r_k}^l\Big)\bar{E}\bar{x}(k) \tag{28}$$

Define $\Delta V(k)$ as the forward difference of $V(k)$. Therefore, along the trajectory of system (12), we can calculate

$$\begin{aligned}\mathbb{E}\{\Delta V(k)\} &= \mathbb{E}\Big\{V(k+1) - V(k)|x(k), r_k = i\Big\}\\ &= \bar{x}^T(k+1)\bar{E}^T\Big(\sum_{j=1}^{N}\sum_{l=1}^{M}\sum_{l=1}^{M}\alpha_l(k)\alpha_l(k+1)\pi_{ij}^l P_j^l\Big)\bar{E}\bar{x}(k+1)\\ &\quad - \bar{x}^T(k)\bar{E}^T\Big(\sum_{l=1}^{M}\alpha_l(k)P_i^l\Big)\bar{E}\bar{x}(k)\end{aligned} \tag{29}$$

Note that $\sum_{l=1}^{M}\alpha_l(k+1)P_j^l = \sum_{q=1}^{M}\eta_q(k)P_j^q$, where $0 \le \eta_q(k) \le 1$, and $\sum_{q=1}^{M}\eta_q(k) = 1$. Thus, we know that

$$\begin{aligned}\mathbb{E}\{\Delta V(k)\} &= \bar{x}^T(k+1)\bar{E}\Big(\sum_{j=1}^{N}\sum_{l=1}^{M}\sum_{q=1}^{M}\alpha_l(k)\eta_q(k)\pi_{ij}^l P_j^q\Big)\bar{E}\bar{x}(k+1)\\ &\quad - \bar{x}^T(k)\bar{E}^T\Big(\sum_{l=1}^{M}\alpha_l(k)P_i^l\Big)\bar{E}\bar{x}(k)\\ &= \bar{x}^T(k+1)\bar{E}^T X_i^{lq}\bar{E}\bar{x}(k+1) - \bar{x}^T(k)\bar{E}^T\Big(\sum_{l=1}^{M}\alpha_l(k)P_i^l\Big)\bar{E}\bar{x}(k)\end{aligned} \tag{30}$$

Let $\bar{x}_s(k) = \bar{x}(k+1) - \bar{x}(k)$. Equation (29) is equivalent to

$$\mathbb{E}\{\Delta V(k)\} = \bar{x}_s^T(k)\bar{E}X_i^{lq}\bar{E}^T\bar{x}_s(k) - \bar{x}^T(k)\bar{E}^T\Big(\sum_{l=1}^{M}\alpha_l(k)P_i^l - X_i^{lq}\Big)\bar{E}\bar{x}(k) + 2\bar{x}_s^T(k)\bar{E}X_i^{lq}\bar{E}^T\bar{x}(k) \tag{31}$$

Additionally, using the fact that $\bar{R}\bar{E} = 0$, we have

$$2\bar{x}^T(k)S_i^T\bar{R}\bar{E}\bar{x}_s(k) = 0 \tag{32}$$

Moreover, with appropriates matrices $\mathbb{V}_{1i}^T$ and $\mathbb{V}_{2i}^T$ it can be established from (12) that

$$\begin{aligned}&2\mathbb{E}\Big\{\zeta^T(k)\begin{bmatrix}\mathbb{V}_{1i}^T & \mathbb{V}_{2i}^T & 0\end{bmatrix}^T[-\bar{E}\bar{x}_s(k) + (\bar{A}_i - \bar{E})\bar{x}(k) + \bar{A}_{\phi i}\phi(C_2x(k)) + \rho_1(k)\Upsilon_1(k) + \rho_2(k)\Upsilon_2(k)]\Big\}\\ &= 2\zeta^T(k)\begin{bmatrix}\mathbb{V}_{1i}^T & \mathbb{V}_{2i}^T & 0\end{bmatrix}^T\begin{bmatrix}(\bar{A}_i - \bar{E}) & -I & \bar{A}_{\phi i}\end{bmatrix}\zeta(k) = 0\end{aligned} \tag{33}$$

where $\zeta(k) = \mathrm{col}\Big\{\bar{x}(k),\ \bar{E}\bar{x}_s(k),\ \phi(C_2x(k))\Big\}$. According to (5), one has

$$-2\varepsilon_{1i}\Big\{\phi^T(C_2x(k))\Big(\phi(C_2x(k)) - L\bar{C}_2\bar{x}(k)\Big)\Big\} \ge 0 \tag{34}$$

where ε_{1i} is a positive scalar.

Substituting (32)–(34) into (31) gives us

$$\mathbb{E}\{\Delta V(k)\} \le \zeta^T(k)\bar{\Psi}_i^{lq}\zeta(k) \tag{35}$$

where

$$\bar{\Psi}_i^{lq} = \begin{bmatrix} \hat{\Xi}_{11i}^{lq} & \Xi_{12i} & \Xi_{13i} \\ * & \Xi_{22i}^{lq} & \mathbb{V}_{2i}\bar{A}_{\phi i} \\ * & * & -2\varepsilon_{1i}I \end{bmatrix} \tag{36}$$

From (23), we can deduce that $\bar{\Psi}_i^{lq} < 0$, and (35) provides the following

$$\mathbb{E}\{\Delta V(k)\} \leq \kappa \|\zeta(k)\|^2 \tag{37}$$

where $\kappa < 0$ is the largest eigenvalue of $\bar{\Psi}_i^{lq}$. Thus, from (37) we have

$$\mathbb{E}\Big\{\sum_{l=0}^{\infty} \|\zeta(l)\|^2\Big\} \leq \frac{1}{\kappa}\mathbb{E}\Big\{\sum_{l=0}^{\infty} \Delta V(k)\Big\} \tag{38}$$

$$\mathbb{E}\Big\{\sum_{l=0}^{\infty} \|\zeta(l)\|^2\Big\} \leq -\frac{1}{\kappa}V(0) < \infty \tag{39}$$

Thus, system (12) is stochastically admissible according to Definition 1.

In order to show that system is $(\mathbf{Q}, \mathbf{S}, \mathbf{R})$-$\gamma$ dissipative, we present the following performance index:

$$J_0 = \mathbb{E}\Big\{\sum_{k=0}^{\infty} (J_{zw} - \gamma w^T(k) w(k))\Big\} \tag{40}$$

As with previous steps, the null equation

$$2\psi^T(k)\begin{bmatrix}\mathbb{V}_{1i}^T & \mathbb{V}_{2i}^T & 0 & 0\end{bmatrix}^T \begin{bmatrix}(\bar{A}_i - \bar{E}) & -I & \bar{A}_{\phi i} & \bar{B}_{1i}\end{bmatrix}\psi(k) = 0 \tag{41}$$

can be applied to get

$$\mathbb{E}\{\Delta V(k)\} - J_{zw}(k) + \gamma w^T(k) w(k) = \psi^T(k)(\bar{\Xi}_i^{lq})\psi(k) \tag{42}$$

where $\psi(k) = \mathrm{col}\Big\{\zeta(k),\ w(k)\Big\}$, and

$$\bar{\Xi}_i^{lq} = \begin{bmatrix} \hat{\Xi}_{11i}^{lq} & \Xi_{12i} & \Xi_{13i} & \Xi_{14i} & \bar{C}_{1i}^T\mathbf{Q}_- \\ * & \Xi_{22i}^{lq} & \mathbb{V}_{2i}\bar{\mathbb{A}}_{\phi i} & \mathbb{V}_{2i}\bar{B}_{1i} & 0 \\ * & * & -2\varepsilon_{1i}I & 0 & 0 \\ * & * & * & -\Xi_{44i} & \bar{D}_{1i}^T\mathbf{Q}_- \\ * & * & * & * & -I \end{bmatrix} \tag{43}$$

According to (23), $\bar{\Xi}_i^{lq} < 0$, and under zero initial conditions, we are left with the following equation:

$$J_0 \leq \sum_{k=0}^{\infty} \mathbb{E}\Big\{\Delta V(k) - J_{zw} + \gamma w^T(k) w(k)\Big\} \leq \mathbb{E}\Big\{V(\infty) + \sum_{k=0}^{\infty}(-J_{zw} + \gamma w^T(k) w(k))\Big\} < 0 \tag{44}$$

Since $V(\infty) \geq 0$, it is easy to verify that $\mathbb{E}\Big\{J_{zw} - \gamma w^T(k) w(k)\Big\} > 0$. Therefore, according to Definition 2, system (12) is stochastically admissible and strictly $(\mathbf{Q}, \mathbf{S}, \mathbf{R})$-$\gamma$ dissipative.

Assume that $\Delta\bar{A} \neq 0$. In the same manner as above, we arrive at

$$\bar{\Xi}_i^{lq} + \mathrm{sym}\left(\Gamma_{1i}^T \Delta(k) \Gamma_{2i}\right) < 0 \tag{45}$$

where $\Gamma_{1i} = \begin{bmatrix}(\mathbb{V}_{1i}\bar{H})^T & (\mathbb{V}_{2i}\bar{H})^T & 0 & 0 & 0\end{bmatrix}$ and $\Gamma_{2i} = \begin{bmatrix}\bar{F} & 0 & 0 & 0 & 0\end{bmatrix}$.

Because of $\Delta^T(k)\Delta(k) \leq I$, inequality (22) holds according to Lemma 1. This ends the proof. □

Remark 3. *Theorem 1 establishes the existence of the controller such that the closed-loop system is stochastically admissible and strictly* $(\mathbf{Q}, \mathbf{S}, \mathbf{R})$-$\gamma$ *dissipative. Due to the non-linear nature of the condition in the theorem, it cannot be solved with existing LMI solvers. The next section demonstrates the procedure to design the controller and overcome the BMI terms in* (17).

4. Dissipativity Controller Design

In the sequel, the corresponding controller gains $\hat{A}_i$, $\hat{B}_i$, and $\hat{C}_i$ will be designed based on the following conclusion.

Theorem 2. *System (12) is stochastically admissible and strictly dissipative for a scalar $\gamma > 0$, matrices $\mathbf{Q}$, $\mathbf{S}$, and $\mathbf{R}$, and tuning parameters a_1, a_2, b_1, and b_2, if the matrices $\mathbf{P}_i^l > 0$, $T_i > 0$, V_{1i}, V_{2i}, W_{1i}, W_{2i}, Y_i, W_i, U_i, $\hat{\mathcal{A}}_i$, $\hat{\mathcal{B}}_i$, $\hat{\mathcal{C}}_i$ and scalars $\varepsilon_{0i} > 0$, $\varepsilon_{1i} > 0$, and $\varepsilon_{2i} > 0$ exist such that the following LMI is true:*

$$\left[\begin{array}{ccc|c|c} \Psi_i^{lq}(\mathcal{A}_{1i}, \mathcal{A}_{2i}) & \tilde{\mathbf{Y}}_{1i}\Lambda & \mathbf{Y}_{2i}W_i & \tilde{\mathbf{Y}}_{3i} & \alpha\tilde{\mathbf{Y}}_{4i} \\ * & -W_i & 0 & & \\ * & * & -W_i & & \\ \hline & * & & -\alpha\,\mathrm{sym}(U_i) + T_i & 0 \\ & * & & * & -T_i \end{array}\right] < 0 \tag{46}$$

where

$$\begin{aligned}
\Psi_i^{lq}(\mathcal{A}_{1i}, \mathcal{A}_{2i}) &= \begin{bmatrix} \Psi_{11i}^{lq} & \Psi_{12i} & \Psi_{13i} + \varepsilon_{1i}LC_2 & \Psi_{14i} - \bar{C}_{1i}^T\mathbf{S} & \bar{C}_{1i}^T\mathbf{Q}_- & \Psi_{16i} \\ * & \Psi_{22i}^{lq} & \Psi_{23i} & \Psi_{24i} & 0 & \Psi_{26i} \\ * & * & -2\varepsilon_{1i}I & 0 & 0 & 0 \\ * & * & * & -\Phi_{44i} & \bar{D}_{1i}^T\mathbf{Q}_- & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\varepsilon_{0i}I \end{bmatrix} \\
\tilde{\mathbf{Y}}_{1i} &= \mathrm{col}\left\{\Psi_{17i},\ \Psi_{27i},\ 0,\ 0,\ 0,\ 0\right\}, \\
\tilde{\mathbf{Y}}_{3i} &= \mathrm{col}\left\{\Psi_{19i},\ \Psi_{29i},\ 0,\ 0,\ 0,\ 0,\ 0\right\}, \\
\tilde{\mathbf{Y}}_{4i} &= \mathrm{col}\left\{\begin{bmatrix}0 & \hat{\mathcal{C}}_i\end{bmatrix}^T,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}, \\
\Psi_{11i}^{l} &= -\bar{E}^T(P_i^l - X_i^{lq})\bar{E} + sym(\mathcal{A}_{1i}) + \varepsilon_{0i}\bar{F}^T\bar{F}, \\
\Psi_{12i} &= (X_i^{lq}\bar{E})^T + S_i^T\bar{R} - \mathbb{V}_{1i} + (\mathcal{A}_{2i})^T, \\
\Psi_{22i}^{lq} &= -sym(\mathbb{V}_{2i}) + X_i^{lq}, \\
\mathbb{V}_{ji} &= \begin{bmatrix} V_{ji} & a_jY_i \\ W_{ji} & b_jY_i \end{bmatrix},\ j = 1, 2 \\
\mathcal{A}_{ji} &= \begin{bmatrix} V_{ji}(A-E) + a_j(\beta\theta\hat{\mathcal{B}}_iC_2 + \beta(1-\theta)\hat{\mathcal{B}}_iL_1C_2) & a_j(\hat{\mathcal{A}}_i - Y_iE) + B\Theta_i\hat{\mathcal{C}}_i \\ W_{ji}(A-E) + b_j(\beta\theta\hat{\mathcal{B}}_iC_2 + \beta(1-\theta)\hat{\mathcal{B}}_iL_1C_2) & b_j(\hat{\mathcal{A}}_i - Y_iE) + B\Theta_i\hat{\mathcal{C}}_i \end{bmatrix}, \\
\Psi_{j3i} &= \begin{bmatrix} a_j\beta(1-\theta)\hat{\mathcal{B}}_i \\ b_j\beta(1-\theta)\hat{\mathcal{B}}_i \end{bmatrix} \\
\Psi_{j4i} &= \begin{bmatrix} V_{ji}B_1 \\ W_{ji}B_1 \end{bmatrix},\ \Psi_{j6i} = \begin{bmatrix} V_{ji}H \\ W_{ji}H \end{bmatrix}, \\
\Psi_{j7i} &= \begin{bmatrix} a_j\beta\theta\hat{B}_i & a_j\beta(1-\theta)\hat{\mathcal{B}}_i \\ b_j\beta\theta\hat{B}_i & b_j\beta(1-\theta)\hat{\mathcal{B}}_i \end{bmatrix} \\
\Psi_{j9i} &= \begin{bmatrix} V_{ji}B\Theta_i - B\Theta_iU_i \\ W_{ji}B\Theta_i - B\Theta_iU_i \end{bmatrix}
\end{aligned} \tag{47}$$

Furthermore, the controller gains are given by

$$\hat{A}_i = Y_i^{-1}\hat{\mathcal{A}}_i, \quad \hat{B}_i = Y_i^{-1}\hat{\mathcal{B}}_i, \quad \hat{C}_i = U_i^{-1}\hat{\mathcal{C}}_i \tag{48}$$

Proof. Based on Theorem 2, a feasible solution must satisfy the condition $-\operatorname{sym}(\mathbb{V}_{2i}) < 0$ and $-\operatorname{sym}(U_i) < 0$. It follows that Y_i and U_i are nonsingular, and we get

$$\hat{\mathcal{A}}_i = Y_i\hat{A}_i, \qquad \hat{\mathcal{B}}_i = Y_i\hat{B}_i, \qquad \hat{\mathcal{C}}_i = U_i\hat{C}_i \tag{49}$$

According to Lemma 2, we obtain from (46)

$$\begin{bmatrix} \Psi_i^{lq}(\mathcal{A}_{1i},\mathcal{A}_{2i}) & \tilde{\mathbf{Y}}_{1i}\Lambda & \mathbf{Y}_{2i}W_i \\ * & -W_i & 0 \\ * & * & -W_i \end{bmatrix} + \operatorname{sym}(\tilde{\mathbf{Y}}_{3i}U_i^{-1}\tilde{\mathbf{Y}}_{4i}^T) < 0 \tag{50}$$

Note that $\hat{C}_i = U_i^{-1}\hat{\mathcal{C}}_i$. Thus, it is easy to get

$$\tilde{\mathbf{Y}}_{3i}U_i^{-1}\tilde{\mathbf{Y}}_{4i}^T = \begin{bmatrix} \begin{bmatrix} 0 & V_{1i}B\Theta_i\hat{C}_i - B\Theta_i\hat{\mathcal{C}}_i \\ 0 & W_{1i}B\Theta_i\hat{C}_i - B\Theta_i\hat{\mathcal{C}}_i \end{bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \begin{bmatrix} 0 & V_{2i}B\Theta_i\hat{C}_i - B\Theta_i\hat{\mathcal{C}}_i \\ 0 & W_{2i}B\Theta_i\hat{C}_i - B\Theta_i\hat{\mathcal{C}}_i \end{bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{51}$$

Using (51), inequality (50) is equivalent to

$$\begin{bmatrix} \Psi_i^{lq}(\mathbf{A}_{1i},\mathbf{A}_{2i}) & \tilde{\mathbf{Y}}_{1i}\Lambda & \mathbf{Y}_{2i}W_i \\ * & -W_i & 0 \\ * & * & -W_i \end{bmatrix} < 0 \tag{52}$$

where

$$\mathbf{A}_{1i} = \mathcal{A}_{1i} + \begin{bmatrix} 0 & V_{1i}B\Theta_i\hat{C}_i - B\Theta_i\hat{\mathcal{C}}_i \\ 0 & W_{1i}B\Theta_i\hat{C}_i - B\Theta_i\hat{\mathcal{C}}_i \end{bmatrix} = \mathbb{V}_{1i}(\bar{\mathbb{A}}_i - \bar{E})$$
$$\mathbf{A}_{2i} = \mathcal{A}_{2i} + \begin{bmatrix} 0 & V_{2i}B\Theta_i\hat{C}_i - B\Theta_i\hat{\mathcal{C}}_i \\ 0 & W_{2i}B\Theta_i\hat{C}_i - B\Theta_i\hat{\mathcal{C}}_i \end{bmatrix} = \mathbb{V}_{2i}(\bar{\mathbb{A}}_i - \bar{E})$$
$$\mathbb{V}_{1i} = \begin{bmatrix} V_{1i} & a_1Y_i \\ W_{1i} & b_1Y_i \end{bmatrix}, \mathbb{V}_{2i} = \begin{bmatrix} V_{2i} & a_2Y_i \\ W_{2i} & b_2Y_i \end{bmatrix}$$

Moreover, using the fact that

$$\Psi_{j3i} = \mathbb{V}_{ji}\bar{\mathbb{A}}_{\phi i}, \; \Psi_{j4i} = \mathbb{V}_{ji}\bar{\mathcal{B}}_{1i}, \; \Psi_{j7i} = \mathbb{V}_{ji}\bar{\mathbb{H}}_i, \; j = 1,2$$

it can be verified that (52) is equivalent to (17). Hence, according to Theorem 1, if (46) holds then system (12) is stochastically admissible and strictly dissipative. □

Remark 4. *In comparison with existing results in [23,40,41], the proposed control design scheme's main benefit is its simplicity and lesser conservativeness. In [40], the SVD decomposition technique was applied with a specific structure of auxiliary matrices. Our proposed methodology differed from the one in [23] since it is valid only for systems with measurable states and requires the tuning of* *many scalars. Additionally, contrary to our study [41], some scalars were introduced in the matrix $\mathbb{V}_{1i}$ and $\mathbb{V}_{2i}$ in order to reduce the conservatism of the method.*

5. A Numerical Application

5.1. A Machine Infinite-Bus System

With the help of the example of a machine infinite-bus system displayed in Figure 1, and borrowed from [42], we demonstrate both the efficiency and correctness of the proposed control scheme.

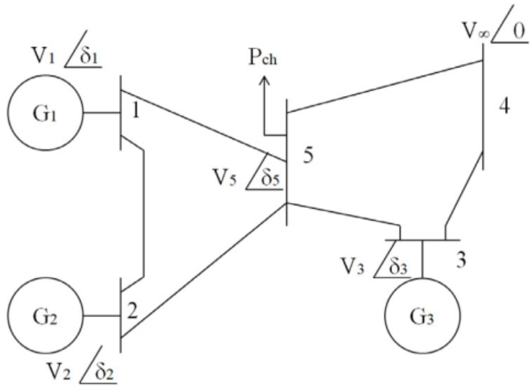

Figure 1. Three machine infinite buses.

From the publication [43], the following model describes the corresponding simulation system:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0.01 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0.01 & 0 \\ -1.0714 & 0.7143 & 0 & 0.9593 & 0 & 0 & 0.3571 \\ 0.3846 & -0.8461 & 0 & 0 & 0.9423 & 0 & 0.4615 \\ 0 & 0 & -0.75 & 0 & 0 & 0.945 & 0.4 \\ 0.005 & 0.012 & 0.008 & 0 & 0 & 0 & -0.035 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.7142 & 0 & 0 \\ 0 & 0.3846 & 0 \\ 0 & 0 & 0.5 \\ 0 & 0 & 0 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.01 \end{bmatrix} \; C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

and $E = \mathrm{diag}(1,1,1,1,1,1,0)$. The other parameters are respectively selected as follows:

$$\begin{bmatrix} H \\ \hline F \\ \hline C_{11} \\ \hline D_1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0.01 & 0.01 & 0.01 & 0.01 \end{bmatrix}^T \\ \hline \begin{bmatrix} 0.1 & -0.1 & 0.1 & 0 & 0 & 0 & -0.1 \end{bmatrix} \\ \hline \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0 & 0 & 0 & 0.1 \end{bmatrix} \\ \hline 0 \end{bmatrix}$$

We suppose that an intermittent actuator fault may occur randomly according to a time-varying Markov chain with fault matrices selected as $\Theta_1 = \mathrm{diag}(1,1,1)$, and $\Theta_2 = \mathrm{diag}(0.5, 0.5, 0.5)$, and a transition probability defined by the following vertices:

$$\Pi^1 = \begin{bmatrix} 0.5 & 0.5 \\ 5/14 & 9/14 \end{bmatrix}, \qquad \Pi^2 = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}, \qquad \Pi^3 = \begin{bmatrix} 0.75 & 0.25 \\ 0.65 & 0.35 \end{bmatrix},$$

Moreover, the following non linear function

$$\varphi(C_2x(k)) = \frac{L_1 + L_2}{2}C_2x(k)) + \frac{L_2 - L_1}{2}\sin(C_2x(k))$$

is borrowed to represent the sensor non-linearity with $L_1 = \mathrm{diag}(0.5,\ 0.5,\ 0.5,\ 0.5)$ and $L_2 = \mathrm{diag}(0.8,\ 0.8,\ 0.8,\ 0.8)$. The parameters of the logarithmic quantizer are selected as $u_0 = 0.1$ and $\rho = 0.9$. Let $\mathbf{Q} = -0.1, \mathbf{S} = 0.1, \mathbf{R} = 10$, $\gamma = 0.1$, $R_0 = \begin{bmatrix} 0_{7,6} & I_{7,1} \end{bmatrix}$, $\bar{R} = \mathrm{diag}(R_0, R_0)$, $a_1 = 1$, $b_1 = 0$, $a_2 = 1$, $b_2 = 0$, $\alpha = 0.1$, $\varepsilon_0 = 1$, and $\varepsilon_1 = 1$.

A viable solution is found using Yalmip's toolbox in conjunction with Mosek's solver at $\beta = 0.9$, $\theta = 0.8$, and

$$\hat{A}_1 = \begin{bmatrix} 0.65823 & 0.0004064 & 0.001535 & 0.013364 & 0.000794 & 0.0061627 & -0.0025474 \\ 0.0047828 & 0.66035 & 0.0028214 & 0.017249 & 0.016817 & 0.0067394 & -0.0020184 \\ 0.0023727 & 0.0023443 & 0.65729 & 0.0074535 & 0.0030585 & 0.0075329 & -0.0015125 \\ -0.001726 & -0.042823 & -0.0083913 & 0.59108 & -0.036362 & -0.023284 & -0.0025816 \\ 0.035663 & 0.0067386 & 0.0093891 & -0.048313 & 0.61179 & -0.011463 & -0.0015296 \\ -0.010618 & -0.017083 & 0.020782 & -0.014012 & -0.011638 & 0.6231 & -0.0011323 \\ 0.003645 & 0.0030258 & 0.00222 & 0.0017253 & -0.0011264 & 0.00022401 & -0.49802 \end{bmatrix}$$

$$\hat{B}_1 = \begin{bmatrix} -0.05209 & 0.013598 & -0.0007033 & 0.047023 \\ 0.015709 & -0.062511 & 0.0060032 & 0.047707 \\ -0.0024841 & 0.0037372 & -0.058534 & 0.032604 \\ 0.19178 & -0.017745 & 0.0011332 & -0.16227 \\ 0.026467 & 0.15574 & -0.0050712 & -0.16976 \\ 0.026843 & -0.0054818 & 0.16765 & -0.1031 \\ -0.0014821 & -0.0058124 & -0.0033069 & 0.010051 \end{bmatrix}$$

$$\hat{C}_1 = \begin{bmatrix} -0.0256 & 0.039964 & 0.019995 & 0.016935 & -0.077525 & -0.038215 & -8.7587e-05 \\ -0.056836 & -0.082333 & -0.013872 & 0.083229 & 0.13775 & 0.0011287 & 9.8431e-05 \\ 0.026176 & 0.023931 & -0.11231 & -0.11 & -0.086408 & 0.14403 & -1.9844e-05 \end{bmatrix}$$

$$\hat{A}_2 = \begin{bmatrix} 0.65804 & 0.00034327 & 0.0013678 & 0.0039296 & 0.0007452 & 0.0013286 & -0.0017139 \\ 0.0027157 & 0.65996 & 0.0017229 & 0.0081953 & 0.0060933 & 0.0016663 & -0.0014004 \\ 0.0015253 & 0.0015426 & 0.65774 & 0.00092097 & 0.00060671 & 0.0040795 & -0.00092355 \\ 0.018616 & -0.021029 & -0.002425 & 0.64437 & -0.0028923 & 4.4378e-05 & -0.0010219 \\ 0.013199 & 0.021075 & -0.0037748 & -0.013745 & 0.64909 & 0.0026206 & -0.00024366 \\ -0.0044677 & -0.0065649 & 0.030394 & 0.0021783 & 0.0034892 & 0.6412 & -0.0002276 \\ 0.0022379 & 0.0022817 & 0.0014652 & 0.00017662 & -0.0012586 & -0.0005046 & -0.49805 \end{bmatrix}$$

$$\hat{B}_2 = \begin{bmatrix} -0.061413 & 0.017146 & 0.0064549 & 0.046044 \\ 0.013098 & -0.063389 & 0.0080104 & 0.048076 \\ 0.0036008 & 0.0070987 & -0.071588 & 0.028373 \\ 0.11256 & -0.022614 & -0.011207 & -0.077291 \\ 0.0031899 & 0.10688 & -0.016069 & -0.096737 \\ -0.002898 & -0.009896 & 0.13612 & -0.055888 \\ -0.0019342 & -0.0064795 & -0.0036871 & 0.011058 \end{bmatrix}$$

$$\hat{C}_2 = \begin{bmatrix} -0.1033 & 0.076039 & 0.016612 & 0.062354 & -0.087665 & -0.032075 & -3.8798e-05 \\ -0.038447 & -0.17818 & 0.020649 & 0.027777 & 0.12359 & -0.042474 & -3.3119e-05 \\ 0.03359 & 0.030564 & -0.21223 & -0.076694 & -0.065088 & 0.19051 & -7.1046e-06 \end{bmatrix}.$$

The control scheme proposed in [41] is not feasible with the above parameters for this system, which indicates the superiority of the new strategy.

5.2. Results and Graphical Plots

As a way of demonstrating that the developed control strategy is effective, let us suppose that $w(k) = 0.01\sin(5k)$, and the initial condition
$x(0) = \begin{bmatrix}0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1\end{bmatrix}$.

Figures 2–7 illustrate the simulation results of the resulting closed-loop system, achieved by applying the designed fault-tolerant controller (11) to the uncertain system (1). Figures 2–5 record the actual and quantized output responses of the system, while Figures 6–9 show, respectively, the control input $u(k)$, the Markov chain mode of the actuator failure, and the Bernoulli distributions. The fault-tolerant dynamic output feedback controller could keep the closed-loop system dynamically stable under actuator faults, external disturbances, model uncertainties, and unmeasured states.

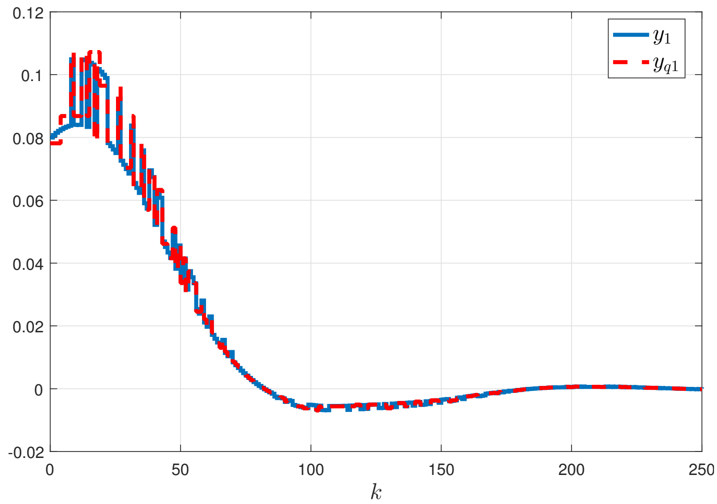

Figure 2. Actual output, y_1, and quantized output, y_{q1}.

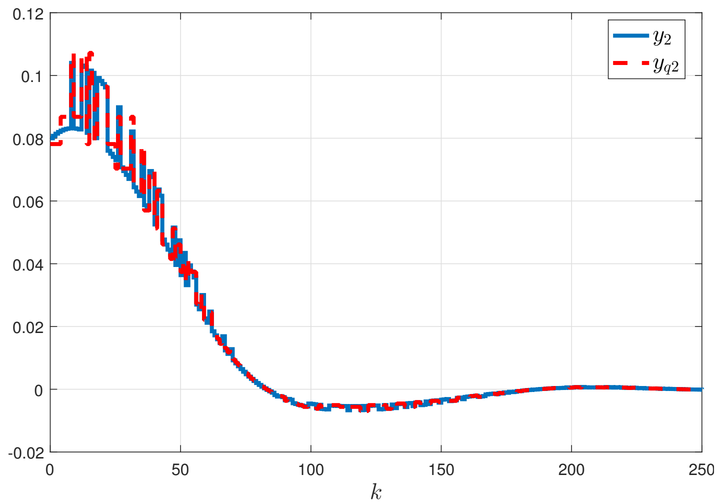

Figure 3. Actual output, y_2, and quantized output, y_{q2}.

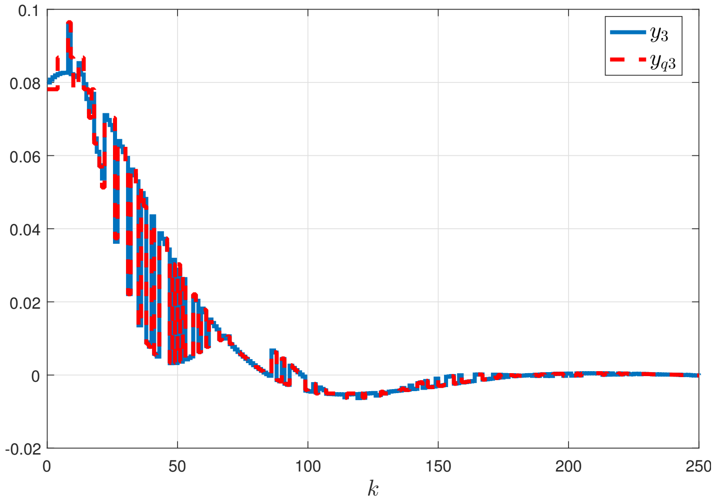

Figure 4. Actual output, y_3, and quantized output, y_{q3}.

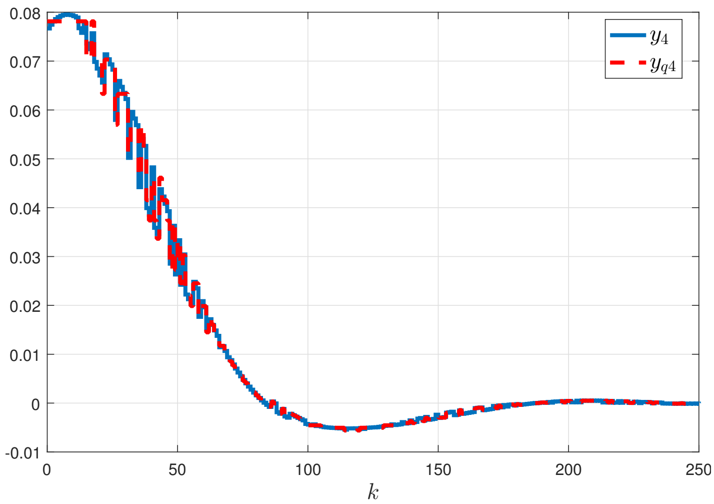

Figure 5. Actual output, y_4, and quantized output, y_{q4}.

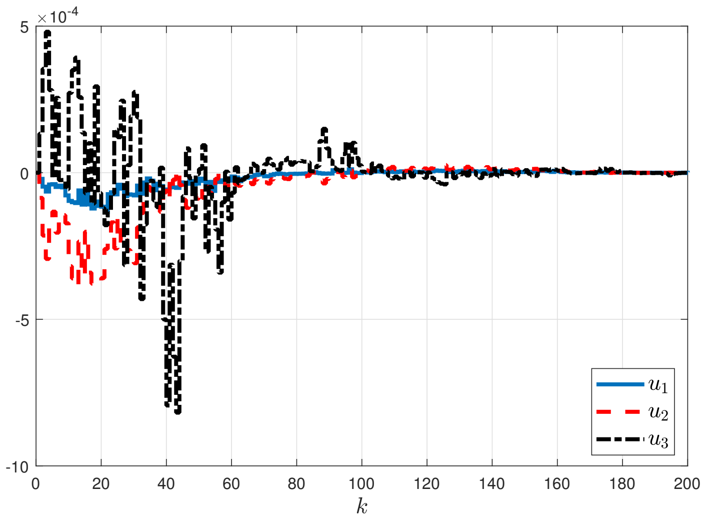

Figure 6. Input trajectories.

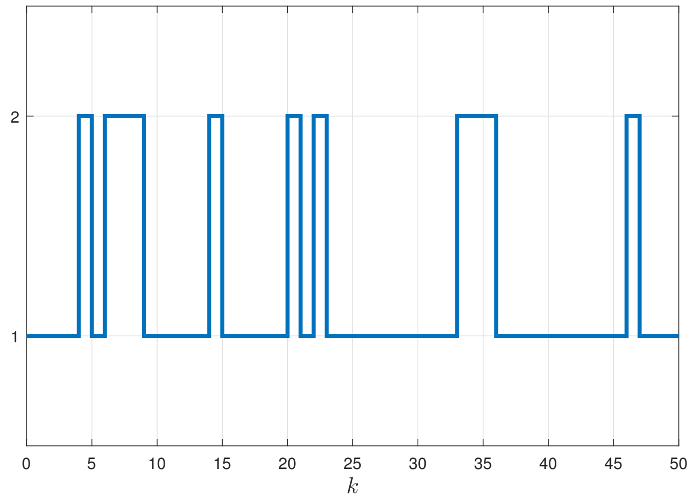

Figure 7. Mode values of Markov chains of an intermittent fault.

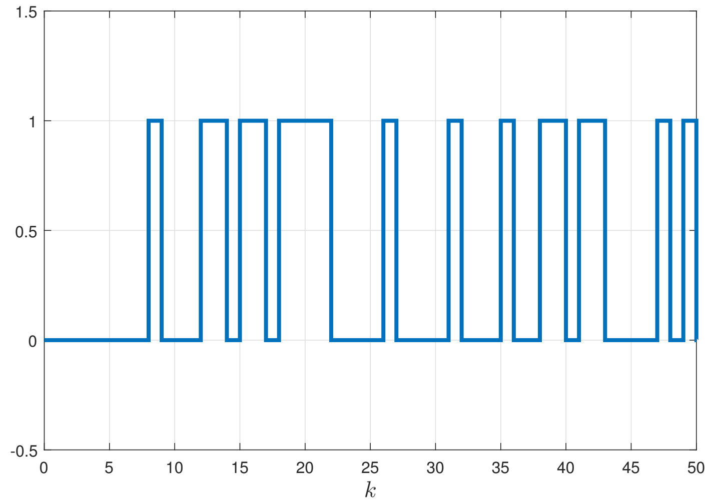

Figure 8. Bernoulli distribution variable $\beta(k)$.

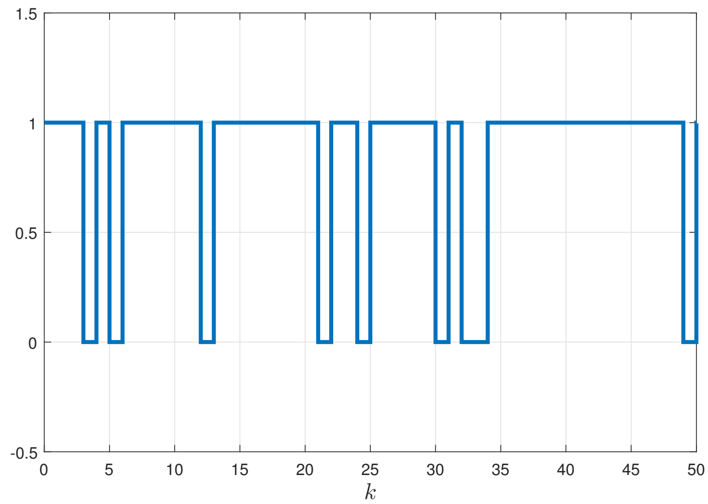

Figure 9. 'Bernoulli distribution variable $\theta(k)$.

5.3. Comparative Explanations

With the fault-tolerant control of discrete-time descriptor systems characterized by intermittent actuator failures and unpredictable sensor non-linearities, this paper has proven effective in overcoming this challenge. In spite of various control issues for discrete-time Markovian jump systems having been explored in the literature [35,41,44], our approach differed in the following ways:

- Ref. [35] describes a discrete-time Markovian jump system with a quantized and resilient state feedback control law. As we considered a more general class of singular systems with partially measured states, our approach was more general. Additionally, random sensor non-linearity and missing data were taken into account.
- Although the reliable control problem for discrete-time descriptor systems, using a dynamic output feedback controller, had been explored in our previous work [41], the present investigation differed with the following points:
  - The intermittent actuator failures were described by a non-homogeneous Markov process with time-varying transition probabilities. Moreover, the randomly occurring sensor non-linearity, suggested in this study, was more general and might include the saturation non-linearity.
  - To handle a networked control system, the output quantization and missing data might be an effective scheme to reduce the storage space and transmission bandwidth [44].
- Between resilient controllers proposed in [35,44] and the reliable controller developed in this study, resilient controllers were employed to precisely handle gain fluctuations, whereas the reliable controller was used to compensate for failures of components in the system, especially actuators and sensors.

6. Conclusions and Future Work

6.1. Concluding Remarks

The fault-tolerant control problem presented in this paper is for discrete-time singular systems with intermittent actuator faults, randomly occurring sensor nonlinearity, and probabilistic missing data. Among the main results, the main findings are as follows: (i) Random sensor non-linearity and random missing data have been studied for singular linear systems, where related impacts have been assessed. (ii) We have discussed the intermittent actuator faults described by a non-homogenous Markov model. (iii) A dynamic output feedback controller with quantized output signals was proposed. The synthesized controller might guarantee the stochastic stability of closed-loop systems with satisfactory dissipative performance. A numerical simulation of the machine infinite bus confirmed the efficiency and potential value of the results obtained.

6.2. Limitations

This study revealed a quantized input model, which was independent of the failure modes. In spite of this, as presented in [29], this model could not be used to describe physical applications due to the absence of some available information of target modes. Accordingly, since information relating to a mode could not be transmitted correctly, the problem of combining mode-independent and mode-dependent systems when dealing with control/filtering problems pertaining to Markovian jump systems is fascinating [44]. This issue could be considered in the future.

6.3. Future Work

Other research areas that should be also pursued in the future include stabilization problems for nonlinear processes [45–47].

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