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Output Tracking Control of Random Nonlinear Time-Varying Systems

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Abstract: This paper is concerned with the output tracking control problem for random nonlinear systems with time-varying powers. A distinct feature of this paper is that we consider time-varying powers and the second-order moment process simultaneously, which is more practical in real applications than the existing results where only one factor is considered. We propose a new design scheme, which ensures that the fourth moment of the tracking error can be adjusted to be arbitrarily small and all the states of the closed-loop system are bounded in probability. Finally, a numerical simulation is given to demonstrate the feasibility of the control idea.

Keywords: random nonlinear systems; time-varying powers; tracking

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1. Introduction

Consider the random nonlinear systems (RNSs) with time-varying powers described by

$$\begin{array}{rcl} \dot{x}_j & = & [x_{j+1}]^{r_j(t)} + f_j(\bar{x}_j) + g_j^T(\bar{x}_j)\zeta(t), j = 1, \cdots, n-1, \\ \dot{x}_n & = & [u]^{r_n(t)} + f_n(\bar{x}_n) + g_n^T(\bar{x}_n)\zeta(t), \\ y & = & x_1, \end{array} \quad (1)$$

where $\overline{x}_j = (x_1, \cdots, x_j)^T \in R^j$, $u \in R$, $y \in R$ are the state, input, and output of the system, respectively. $\overline{x}_j(t_0) = (x_{10}, \cdots, x_{j0})^T$, $t \in [t_0, \infty)$. The time-varying power $r_j(t) : R^+ \to R^+$ is a continuous bounded function complying $1 \le \underline{r} \le r_j(t) \le \bar{r}$ with two positive constants $\underline{r}$ and $\bar{r}$, and we define $[\cdot]^{a(t)} = sign(\cdot)|\cdot|^{a(t)}$ with $a(t)$ as the time-varying continuous function. The functions $f_j : R^j \to R$ and $g_j : R^j \to R$, $j = 1, \cdots, n$, are smooth, vanishing at the origin. $\zeta(t) \in R^m$ is a standard second-order moment process (SOMP) defined on the complete probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathcal{P})$ with a filtration $\mathcal{F}_t$ complying the general requirements. A given $\mathcal{C}^1$ target signal is defined as $y_0(t) \in R$ ($\mathcal{C}^1$ represents a class of functions whose derivatives are continuous).

When the noise $\zeta(t)$ in system (1) is white noise, system (1) is called stochastic nonlinear systems (SNSs), and there are many results on its control design. Reference [1] explores the adaptive output feedback tracking problems for SNSs with the unknown state, and [2] considers the strict-feedback SNSs with unknown parameters in the drift terms or the diffusion terms. State feedback tracking control of SNSs was studied in [3]. Reference [4] investigates the global output feedback stabilization for SNSs. Reference [5] presents mean-nonovershooting tracking control designs for strict-feedback SNSs. Reference [6] solves the prescribed-time mean-square stabilization and inverse optimality control problems for strict-feedback SNSs by developing a new nonscaling backstepping design scheme. Reference [7] solves the finite-time stabilization problem of stochastic low-order nonlinear systems with time-varying orders and stochastic inverse dynamics. In [8], a new observer design for a class of nonlinear systems with unknown, bounded, time-varying delays was presented. In [9], the authors studied the finite time stability of equilibrium points of the Caputo–Katugampola fractional neural networks with time delays and proved its existence and uniqueness.

The results in [1–7] is based on $r_j(t) = 1$. When $r_j(t)$ is greater than 1, system (1) is understood as high power systems. There are also many studies on higher power systems. Reference [10] investigated the finite-time stabilization of output-constrained systems with stochastic inverse dynamics and high-order and low-order nonlinearities. Reference [11] presented an adaptive state-feedback strategy for state-constrained systems. In [12], the authors were concerned with the problem of robust cooperative output tracking. According to the results in [10–12], the orders are required to be constants. However, there are many systems with time-varying powers in practical industrial applications. For example, it is clear that the power of boiler turbine units in [13] is time-varying. In addition, the underactuated mechanical system in [14] is also a time-varying system. The reason is the performance hidden trouble brought by spring hardening. Recently, ref. [15] presented two types of controllers for SNSs with time-varying powers, namely the state feedback controller and optimal controller. Reference [16] studied the adaptive control of systems with time-varying power.

White noise is considered a disturbance in the above results. It is undeniable that white noise has its own unique advantages in theoretical analysis. However, in many engineering systems, SOMP is more reasonable for model disturbances. References [17–19] propose two stability theories for this type of system (RNSs with SOMP). Reference [19] considers the stabilization for RNSs. The trajectory tracking of random Lagrange systems disturbed is studied in [20]. Reference [21] discussed the adaptive tracking control for RNSs. Reference [22] investigated the stability of the nonlinear benchmark system in vibrating environments. Reference [23] focused on cooperative control for multiple benchmark systems. References [19–23] focused on tracking problems. There are also some studies on the stability of random systems. For example, stability in the presence of time delay [24], unified stability criteria [25], global asymptotic stability, and stabilization [26]. Nevertheless, there are currently no published results on tracking the control of higher-order RNSs with time-varying powers.

In this paper, we focus on output tracking control for a class of high-order RNSs with time-varying powers. Compared with the available results, the main contributions of this paper are two-fold:

(1) This paper is the first result on the output tracking topic of high-order RNSs with time-varying powers. To extend the order of the system to the time-varying power domain, a new method is proposed to design the controller to achieve stability analysis. Different from [15]'s method, the time-varying order of the system considered in this paper is not uniform $r(t)$ and we consider different orders, i.e., $r_i(t) \neq r_j(t)$, $i \neq j$.

(2) Unlike the deterministic systems [16], the systems studied in this paper are perturbed by SOMP. In the controller design, how to reasonably separate the SOMP from the nonlinear functions is a challenging problem. This is completely different from the designs with white noise in [1–15].

This paper includes four parts. Section 2 is the control design and analysis. Section 3 illustrates the effectiveness of the control method by a simulation example. Section 4 presents the conclusions.

2. Control Design and Analysis

For system (1), we need the following assumptions.

Assumption 1. *For the target signal $y_0(t) \in R$, we assume that $y_0(t)$ and $\dot{y}_0(t)$ satisfy $|y_0| + |\dot{y}_0| \leq M$, where M is a positive constant.*

Assumption 2. *There exist nonnegative smooth functions $\theta_i(\bar{x}_i)$ and $\phi_i(\bar{x}_i)$, $i = 1, \ldots, n$, such that*

$$|f_i(\bar{x}_i)| \leq \theta_i(\bar{x}_i),$$
$$|g_i(\bar{x}_i)| \leq \phi_i(\bar{x}_i).$$

Assumption 3. *$\zeta(t)$ is the $\mathcal{F}_t-$adapted and piecewise continuous process satisfying $\sup_{t \geq t_0} E|\zeta(t)|^2 < K$, where $K > 0$ is a constant.*

Remark 1. *Assumption 3 shows that the random process $\zeta(t)$ is a second-order moment process. As shown in [19–26], this kind of noise characterizes the physical system more reasonably than white noise.*

The objective of this paper was to design an output tracking controller for system (1), such that the closed-loop system has a unique solution on $[t_0, \infty)$, all states are bounded in probability and the tracking error's 4th moment can be tuned arbitrarily small.

2.1. Controller Design

For system (1), we adopted the coordinate changes

$$\eta_i = x_i - x_i^*, \tag{2}$$

where $x_i^*, i = 2, \cdots, n$, are intermediate controllers, the specific form of which is given in the following section. In particular, $x_1^* = y_0$. Then we have

$$\begin{aligned} \dot{\eta}_i &= [x_{i+1}]^{r_i(t)} + f_i(\bar{x}_i) + g_i^T(\bar{x}_i)\zeta(t) - \sum_{k=1}^{i-1} \frac{\partial x_i^*}{\partial x_k}([x_{k+1}]^{r_k(t)} + f_k(\bar{x}_k) \\ &\quad + g_k^T(\bar{x}_k)\zeta(t)) - \frac{\partial x_i^*}{\partial y_0}\dot{y}_0, \end{aligned} \tag{3}$$

where $x_{n+1} = u$.

Next, we give the design process of the system (1).

Step 1. We first designed x_2^*.

According to (2) and (3), we obtain

$$\eta_1 = x_1 - x_1^* = x_1 - y_0, \tag{4}$$

and

$$\dot{\eta}_1 = [x_2]^{r_1(t)} + f_1 + g_1^T\zeta - \dot{y}_0. \tag{5}$$

Meanwhile, we choose the Lyapunov function $V_1 = \frac{1}{4}\eta_1^4$. From (5) and Assumptions 1 and 2, we have

$$\begin{aligned} \dot{V}_1 &= \eta_1^3\dot{\eta}_1 \\ &= \eta_1^3([x_2]^{r_1(t)} + f_1 + g_1^T\zeta - \dot{y}_0) \\ &\leq \eta_1^3([x_2]^{r_1(t)} + f_1 + g_1^T\zeta + M). \end{aligned} \tag{6}$$

By Assumption 2 and Lemma 2.2 in [27], we obtain

$$\eta_1^3(f_1 + M) \leq \eta_1^3(\theta_1(\bar{x}_1) + M) \leq \beta_{11}(\bar{x}_1)|\eta_1|^{\bar{r}+3} + \epsilon_{11}, \tag{7}$$

where $\beta_{11}(\bar{x}_1) = \frac{3}{\underline{r}+3}(\frac{\bar{r}}{\epsilon_{11}(3+\underline{r})})^{\frac{\bar{r}}{3}}(\theta_1(\bar{x}_1) + M)^{\frac{\bar{r}+3}{3}}$, and ϵ_{11} is a positive constant.

By Assumption 2 and Lemma 2.2 in [27], we have

$$\begin{aligned} \eta_1^3 g_1^T \zeta &\leq \eta_1^3 \phi_1(\bar{x}_1)\zeta \\ &\leq \epsilon_{121}\eta_1^6\phi_1^2(\bar{x}_1) + \tfrac{1}{4\epsilon_{121}}|\zeta|^2 \\ &\leq \beta_{12}(\bar{x}_1)|\eta_1|^{\bar{r}+3} + \tfrac{1}{4\epsilon_{121}}|\zeta|^2 + \epsilon_{122}, \end{aligned} \tag{8}$$

where ϵ_{121} and ϵ_{122} are positive constants, $\beta_{12}(\bar{x}_1) = \frac{3}{\underline{r}+3}\left(\frac{\epsilon_{122}\bar{r}}{(3+\underline{r})}\right)^{\frac{\bar{r}}{3}}\epsilon_{121}^{\frac{\bar{r}+3}{3}}\ \eta_1^{\bar{r}+3}(\phi_1(\bar{x}_1))^{\frac{2\bar{r}+6}{3}}$.

Importing (7) and (8) into (6) can cause

$$\begin{aligned} \dot{V}_1 &\leq \eta_1^3[x_2]^{r_1(t)} + \beta_{11}(\bar{x}_1)|\eta_1|^{\bar{r}+3} + \epsilon_{11} + \beta_{12}(\bar{x}_1)|\eta_1|^{\bar{r}+3} + \frac{1}{4\epsilon_{121}}|\zeta|^2 + \epsilon_{122} \\ &= \eta_1^3([x_2]^{r_1(t)} - [x_2^*]^{r_1(t)}) + \eta_1^3[x_2^*]^{r_1(t)} + \beta_1(\bar{x}_1)|\eta_1|^{\bar{r}+3} + \delta_{11} + \delta_{12}|\zeta|^2, \end{aligned} \tag{9}$$

where $\beta_1(\bar{x}_1) = \beta_{11}(\bar{x}_1) + \beta_{12}(\bar{x}_1)$, $\delta_{11} = \epsilon_{11} + \epsilon_{122}$, $\delta_{12} = \frac{1}{4\epsilon_{121}}$.

So, we choose

$$x_2^* = -\alpha_1(\bar{x}_1)(\eta_1 + [\eta_1]^{\bar{r}}) = -(c_1 + \beta_1(\bar{x}_1))^{\frac{1}{\underline{r}}}(\eta_1 + [\eta_1]^{\bar{r}}), \tag{10}$$

such that

$$\eta_1^3[x_2^*]^{r_1(t)} = -\alpha_1^{r_1(t)}(\bar{x}_1)(\eta_1 + [\eta_1]^{\bar{r}})^{r_1(t)}\eta_1^3, \tag{11}$$

where $c_1 \geq 1$ is a free parameter, $\alpha_1(x_1) \geq 1$ is a smooth function uncorrelated of $r_1(t)$.

By Lemma 2.3 in [27], we have

$$\eta_1^3(\eta_1 + [\eta_1]^{\bar{r}})^{r_1(t)} \geq |\eta_1|^{r_1(t)+3} + |\eta_1|^{\bar{r}r_1(t)+3}. \tag{12}$$

Thus, we have

$$\eta_1^3[x_2^*]^{r_1(t)} \leq -(c_1 + \beta_1(\bar{x}_1))(|\eta_1|^{r_1(t)+3} + |\eta_1|^{\bar{r}r_1(t)+3}), \tag{13}$$

and by Lemma 3 in [16], we obtain

$$|\eta_1|^{\bar{r}+3} \leq |\eta_1|^{r_1(t)+3} + |\eta_1|^{\bar{r}r_1(t)+3}. \tag{14}$$

From (9) and (10), we have

$$\dot{V}_1 \leq -c_1|\eta_1|^{\bar{r}+3} + \eta_1^3([x_2]^{r_1(t)} - [x_2^*]^{r_1(t)}) + \delta_{11} + \delta_{12}|\zeta|^2. \tag{15}$$

Step 2. We then design x_3^*.

From (2), we have

$$\begin{aligned} \dot{\eta}_2 &= [x_3]^{r_2(t)} + \left(f_2(\bar{x}_2) - \frac{\partial x_2^*}{\partial x_1}([x_2]^{r_1(t)} + f_1(\bar{x}_1))\right) - \frac{\partial x_2^*}{\partial y_0}\dot{y}_0 \\ &\quad + g_2^T(\bar{x}_2)\zeta - \frac{\partial x_2^*}{\partial x_1}g_1^T\zeta. \end{aligned} \tag{16}$$

Choosing $V_2 = V_1 + \frac{1}{4}\eta_2^4$, by (15) and (16), we obtain

$$\begin{aligned} \dot{V}_2 \leq\ & -c_1|\eta_1|^{\bar{r}+3} + \eta_1^3([x_2]^{r_1(t)} - [x_2^*]^{r_1(t)}) + \delta_{11} + \delta_{12}|\zeta|^2 \\ & + \eta_2^3\Big([x_3]^{r_2(t)} + \Big(f_2(\bar{x}_2) - \tfrac{\partial x_2^*}{\partial x_1}([x_2]^{r_1(t)} + f_1(\bar{x}_1))\Big) - \tfrac{\partial x_2^*}{\partial y_0}\dot{y}_0 \\ & + g_2^T(\bar{x}_2)\zeta - \tfrac{\partial x_2^*}{\partial x_1}g_1^T\zeta\Big). \end{aligned} \tag{17}$$

According to Lemma 2.1 in [27], we obtain

$$\begin{aligned} \eta_1^3([x_2]^{r_1(t)} - [x_2^*]^{r_1(t)}) &\leq \bar{r}(2^{\bar{r}-2}+2)(|\eta_1|^3|\eta_2|^{r_1(t)} + \alpha_1^{\bar{r}-1}|\eta_1|^{r_1(t)+2}|\eta_2|) \\ &\leq \bar{r}(2^{\bar{r}-2}+2)|\eta_1|^3(|\eta_2|^{\bar{r}} + |\eta_2|) + \bar{r}(2^{\bar{r}-2}+2)\alpha_1^{\bar{r}-1} \\ &\quad \cdot(|\eta_1|^{\bar{r}+2} + |\eta_1|^3)|\eta_2|. \end{aligned} \tag{18}$$

From Lemma 2.2 in [27], we have

$$\begin{aligned} \bar{r}(2^{\bar{r}-2}+2)(\alpha_1^{\bar{r}-1}+1)|\eta_1|^3|\eta_2| &\leq \epsilon_{21} + \beta_{211}(x_1)|\eta_2|^{\bar{r}+3}, \\ \bar{r}(2^{\bar{r}-2}+2)\alpha_1^{\bar{r}-1}|\eta_1|^{\bar{r}+2}|\eta_2| &\leq \tfrac{1}{2}|\eta_1|^{\bar{r}+3} + \beta_{212}(x_1)|\eta_2|^{\bar{r}+3}, \\ \bar{r}(2^{\bar{r}-2}+2)|\eta_1|^3|\eta_2|^{\bar{r}} &\leq \tfrac{1}{2}|\eta_1|^{\bar{r}+3} + \beta_{213}|\eta_2|^{\bar{r}+3}, \end{aligned} \tag{19}$$

where

$$\begin{aligned} \beta_{211}(x_1) &= \frac{1}{\underline{r}+3}\left(\frac{\bar{r}+2}{(\underline{r}+3)\epsilon_{211}}\right)^{\bar{r}+2}\left(\bar{r}(2^{\bar{r}-2}+2)((x_1+1)^2+1)^{\frac{3}{2}}(\alpha_1^{\bar{r}-1}+1)\right)^{\bar{r}+3}, \\ \beta_{212}(x_1) &= \frac{1}{\underline{r}+3}\left(\bar{r}(2^{\bar{r}-2}+2)\alpha_1^{\bar{r}-1}\right)^{\bar{r}+3}\left(\frac{\underline{r}+3}{2\bar{r}+4}\right)^{-(\bar{r}+2)}, \\ \beta_{213} &= \frac{\bar{r}}{\underline{r}+3}\left(\frac{6}{\underline{r}+3}\right)^{\frac{\bar{r}}{3}}\left(\bar{r}(2^{\bar{r}-2}+2)\right)^{\frac{\bar{r}+3}{\underline{r}}}. \end{aligned}$$

Substituting (19) into (18), we have

$$\eta_1^3([x_2]^{r_1(t)} - [x_2^*]^{r_1(t)}) \leq |\eta_1|^{\bar{r}+3} + \beta_{21}(x_1)|\eta_2|^{\bar{r}+3} + \epsilon_{21}, \tag{20}$$

where $\beta_{21}(x_1) = \beta_{211}(x_1) + \beta_{212}(x_1) + \beta_{213} \geq 0$ is uncorrelated of $r_1(t)$, ϵ_{21} is positive constant.

Estimate the sixth term of (17) as

$$\begin{aligned} &\eta_2^3\left(f_2(\bar{x}_2) - \tfrac{\partial x_2^*}{\partial x_1}([x_2]^{r_1(t)} + f_1(\bar{x}_1)) - \tfrac{\partial x_2^*}{\partial y_0}\dot{y}_0\right) \\ \leq\ &\eta_2^3\left[\theta_2(\bar{x}_2) + \tfrac{\partial x_2^*}{\partial x_1}(x_2+1)^{\frac{\bar{r}}{2}} + \theta_1(x_1)) + \tfrac{\partial x_2^*}{\partial y_0}M\right] \\ \leq\ &\beta_{22}(x_1)|\eta_2|^{\bar{r}+3} + \epsilon_{22}, \end{aligned} \tag{21}$$

where

$$\beta_{22}(x_1) = \frac{3}{\bar{r}+3}\left(\frac{\bar{r}}{\epsilon_{22}(\underline{r}+3)}\right)^{\frac{\bar{r}}{3}}\left[\theta_2(\bar{x}_2) + \frac{\partial x_2^*}{\partial x_1}(x_2+1)^{\frac{\bar{r}}{2}} + \theta_1(x_1)) + \frac{\partial x_2^*}{\partial y_0}M\right]^{\frac{\bar{r}+3}{3}},$$

with ϵ_{22} being a positive constant.

For the term in (17) involving the SOMP, we have

$$\begin{aligned} \eta_2^3\left(g_2^T(\bar{x}_2)\zeta - \frac{\partial x_2^*}{\partial x_1}g_1^T(\bar{x}_1)\zeta\right) &\leq \epsilon_{231}\eta_2^6\left(\phi_2(\bar{x}_2) + \frac{\partial x_2^*}{\partial x_1}\phi_1^T(\bar{x}_1)\right)^2 + \frac{1}{4\epsilon_{231}}|\zeta|^2 \\ &\leq \beta_{23}|\eta_2|^{\bar{r}+3} + \epsilon_{232} + \frac{1}{4\epsilon_{231}}|\zeta|^2, \end{aligned} \tag{22}$$

where ϵ_{231}, ϵ_{232} are positive constants, and $\beta_{23}(\bar{x}_2) = \frac{3}{\bar{r}+3}\left(\frac{\bar{r}}{\epsilon_{232}(\underline{r}+3)}\right)^{\frac{\bar{r}}{3}}\left(\epsilon_{231}\eta_2^3\left(\phi_2(\bar{x}_2) + \frac{\partial x_2^*}{\partial x_1}\phi_1(\bar{x}_1)\right)^2\right)^{\frac{\bar{r}+3}{3}}$.

From (17), (21) and (22), we obtain

$$\begin{aligned} \dot{V}_2 \leq\ &-(c_1-1)|\eta_1|^{\bar{r}+3} + \eta_2^3([x_3]^{r_2(t)} - [x_3^*]^{r_2(t)}) + \eta_2^3[x_3^*]^{r_2(t)} + (\beta_{21}(\bar{x}_2) + \beta_{22}(\bar{x}_2) \\ &+\beta_{23}(\bar{x}_2))|\eta_2|^{\bar{r}+3} + (\delta_{11}+\delta_{21}) + (\delta_{12}+\delta_{22})|\zeta|^2 \\ \leq\ &-(c_1-1)|\eta_1|^{\bar{r}+3} + \eta_2^3([x_3]^{r_2(t)} - [x_3^*]^{r_2(t)}) + \eta_2^3[x_3^*]^{r_2(t)} + \beta_2(\bar{x}_2)|\eta_2|^{\bar{r}+3} \\ &+\textstyle\sum_{k=1}^2 \delta_{k1} + \sum_{k=1}^2 \delta_{k2}|\zeta|^2, \end{aligned} \tag{23}$$

where $\delta_{21} = \epsilon_{21} + \epsilon_{22} + \epsilon_{232}$, $\delta_{22} = \frac{1}{4\epsilon_{231}}$.

Constructing the virtual controller x_3^* as

$$x_3^* = -(c_2 + \beta_2(\bar{x}_2))^{\frac{1}{\bar{r}}}(\eta_2 + [\eta_2]^{\bar{r}}) = -\alpha_2(\bar{x}_2)(\eta_2 + [\eta_2]^{\bar{r}}), \tag{24}$$

then we have

$$\eta_3^3[x_3^*]^{r_2(t)} \leq -(c_2 + \beta_2(\bar{x}_2))(|\eta_2|^{r_2(t)+3} + |\eta_2|^{\bar{r}r_2(t)+3}), \tag{25}$$

where $c_2 \geq 1$ is the design parameter, the smooth function $\alpha_2 = -(c_2 + \beta_2(\bar{x}_2))^{\frac{1}{\bar{r}}}$ is irrelevant of $r_2(t)$.

By (23) and (24), we have

$$\dot{V}_2 \leq -(c_1 - 1)|\eta_1|^{\bar{r}+3} - c_2|\eta_2|^{\bar{r}+3} + \eta_2^3([x_3]^{r_2(t)} - [x_3^*]^{r_2(t)}) + \sum_{k=1}^{2}\delta_{k1} + \sum_{k=1}^{2}\delta_{k2}|\zeta|^2. \tag{26}$$

Deductive Step. In this step, we design the virtual control x_{i+1}^*.

Suppose that at step $i-1$, we have a positive function V_{i-1} and a virtual controller x_i^*

$$\begin{aligned} x_i^* &= -(c_{i-1} + \beta_{i-1}(\bar{x}_{i-1}))^{\frac{1}{\bar{r}}}(\eta_{i-1} + [\eta_{i-1}]^{\bar{r}}) \\ &= -\alpha_{i-1}(\bar{x}_{i-1})(\eta_{i-1} + [\eta_{i-1}]^{\bar{r}}), \end{aligned} \tag{27}$$

such that

$$\dot{V}_{i-1} \leq -\sum_{k=1}^{i-1}(c_k - 1)|\eta_k|^{\bar{r}+3} + \eta_{i-1}^3([x_i]^{r_{i-1}(t)} - [x_i^*]^{r_{i-1}(t)}) + \sum_{k=1}^{i-1}\delta_{k1} + \sum_{k=1}^{i-1}\delta_{k2}|\zeta|^2, \tag{28}$$

where $\alpha_{i-1}(\bar{x}_{i-1})$ is a smooth function uncorrelated of $r_i(t)$.

In step i, we select a function

$$V_i = V_{i-1} + \frac{1}{4}\eta_i^4. \tag{29}$$

From (28) and (29), we have

$$\begin{aligned} \dot{V}_i \leq\ & -\textstyle\sum_{k=1}^{i-1}(c_k - 1)|\eta_k|^{\bar{r}+3} + \eta_{i-1}^3([x_i]^{r_{i-1}(t)} - [x_i^*]^{r_{i-1}(t)}) + \sum_{k=1}^{i-1}\delta_{k1} + \sum_{k=1}^{i-1}\delta_{k2}|\zeta|^2 \\ & + \eta_i^3([x_{i+1}]^{r_i(t)} - [x_{i+1}^*]^{r_i(t)}) + \eta_i^3[x_{i+1}^*]^{r_i(t)} + \eta_i^3\Big(f_i - \textstyle\sum_{k=1}^{i-1}\frac{\partial x_i^*}{\partial x_k}([x_{k+1}]^{r_k(t)} + f_k) \\ & - \frac{\partial x_i^*}{\partial y_0}M\Big) + \eta_i^3\Big(g_i^T\zeta - \textstyle\sum_{k=1}^{i-1}\frac{\partial x_i^*}{\partial x_k}g_k^T\zeta\Big). \end{aligned} \tag{30}$$

Similar to the proof process of (18), we have

$$\begin{aligned} \eta_{i-1}^3([x_i]^{r_{i-1}(t)} - [x_i^*]^{r_{i-1}(t)}) \leq\ & \bar{r}(2^{\bar{r}-2} + 2)(|\eta_{i-1}|^3|\eta_i|^{r_{i-1}(t)} + \alpha_{i-1}^{\bar{r}-1}|\eta_{i-1}|^{r_i(t)+2}|\eta_i|) \\ \leq\ & \bar{r}(2^{\bar{r}-2} + 2)|\eta_{i-1}|^3(|\eta_i|^{\bar{r}} + |\eta_i|) + \bar{r}(2^{\bar{r}-2} + 2)\alpha_{i-1}^{\bar{r}-1} \\ & \cdot(|\eta_{i-1}|^{\bar{r}+2} + |\eta_{i-1}|^3)|\eta_i| \\ \leq\ & |\eta_i|^{\bar{r}+3} + \beta_{i1}(\bar{x}_{i-1})|\eta_{i+1}|^{\bar{r}+3} + \epsilon_{i1}, \end{aligned} \tag{31}$$

where $\epsilon_{i1} \geq 0$ is a free constant and $\beta_{i1}(\bar{x}_{i-1})$ is a smooth function, both of them uncorrelated of $r_i(t)$.

With the help of (27), Assumption 1 and Lemma 2.1 in [27], we obtain

$$\begin{aligned} & \eta_i^3(f_i - \textstyle\sum_{k=1}^{i-1}\frac{\partial x_i^*}{\partial x_k}([x_{k+1}]^{r_k(t)} + f_k) - \frac{\partial x_i^*}{\partial y_0}M) \\ \leq\ & \eta_i^3\Big(\theta_i + \textstyle\sum_{k=1}^{i-1}\frac{\partial x_i^*}{\partial x_k}(|x_{k+1}| + |x_{k+1}|^{\bar{r}} + \theta_i(\bar{x}_i)) + \frac{\partial x_i^*}{\partial y_0}M\Big) \\ \leq\ & \beta_{i2}(\bar{x}_i)|\eta_i|^{\bar{r}+3} + \epsilon_{i2}, \end{aligned} \tag{32}$$

where $\beta_{i2}(\bar{x}_i) \geq 0$ are uncorrelated of $r_i(t)$, ϵ_{i1} is a positive constant.

By (3) and (27), Assumption 1 and Lemmas A.2, A.4, we have

$$\begin{aligned}\eta_i^3(g_i\zeta - \sum_{k=1}^{i-1}\frac{\partial x_i^*}{\partial x_k}g_k\zeta) &\leq \eta_i^3\Big(\phi_i(\bar{x}_i) + \sum_{k=1}^{i-1}\frac{\partial x_i^*}{\partial x_k}\phi_k(\bar{x}_i)\Big)\zeta \\ &\leq \beta_{i3}(\bar{x}_i)|\eta_i|^{\bar{r}+3} + \epsilon_{i31} + \epsilon_{i32}|\zeta|^2, \end{aligned} \tag{33}$$

where $\beta_{i3}(\bar{x}_i) \geq 0$ are uncorrelated of $r_i(t)$. ϵ_{i31} and ϵ_{i32} are positive constants.

Substituting (31)–(33) into (30), we have

$$\begin{aligned}\dot{V}_i \leq & -\sum_{k=1}^{i-1}(c_k-1)|\eta_k|^{\bar{r}+3} + \eta_i^3([x_{i+1}]^{r_i(t)} - [x_{i+1}^*]^{r_i(t)}) + \eta_i^3[x_{i+1}^*]^{r_i(t)} + \sum_{k=1}^{i}\delta_{k1} \\ & + \sum_{k=1}^{i}\delta_{k2}|\zeta|^2 + (\beta_{i1}(\bar{x}_i) + \beta_{i2}(\bar{x}_i) + \beta_{i3}(\bar{x}_i))|\eta_i|^{\bar{r}+3}. \end{aligned} \tag{34}$$

The virtual controller

$$x_{i+1}^* = -\alpha_i(\bar{x}_i)(\eta_i + [\eta_i]^{\bar{r}}) = -(c_i + \beta_{i1}(\bar{x}_i) + \beta_{i2}(\bar{x}_i) + \beta_{i3}(\bar{x}_i))^{\frac{1}{\underline{r}}}(\eta_i + [\eta_i]^{\bar{r}}), \tag{35}$$

leads to

$$\dot{V}_i \leq -\sum_{k=1}^{i}(c_k-1)|\eta_k|^{\bar{r}+3} + \eta_i^3([x_{i+1}]^{r_i(t)} - [x_{i+1}^*]^{r_i(t)}) + \sum_{k=1}^{i}\delta_{k1} + \sum_{k=1}^{i}\delta_{k2}|\zeta|^2, \tag{36}$$

where $c_k \geq 1$ is a design parameter and $\alpha_i(\bar{x}_i) \geq 1$ is uncorrelated of $r_i(t)$.

Step n. Finally, we design the controller u. Let

$$V_n = \sum_{k=1}^{n}\frac{1}{4}\eta_k^4. \tag{37}$$

In the case of (37), we have

$$\dot{V}_n \leq -\sum_{k=1}^{n-1}(c_k-1)|\eta_k|^{\bar{r}+3} + \beta_n(\bar{x}_n)\eta_n^{\bar{r}+3} + \eta_n^3[u]^{r_n(t)} + \sum_{k=1}^{n}\delta_{k1} + \sum_{k=1}^{n}\delta_{k2}|\zeta|^2, \tag{38}$$

where $\beta_n(\bar{x}_n) \geq 0$ is a smooth function.

If we design the actual controller as

$$u = -\alpha_n(\bar{x}_n)(\eta_n + [\eta_n]^{\bar{r}}) = -(c_n + \beta_n(\bar{x}_n))^{\frac{1}{\underline{r}}}(\eta_n + [\eta_n]^{\bar{r}}), \tag{39}$$

then we obtain

$$\dot{V}_n \leq -\sum_{k=1}^{n}(c_k-1)|\eta_k|^{\bar{r}+3} + \sum_{k=1}^{n}\delta_{k1} + \sum_{k=1}^{n}\delta_{k2}|\zeta|^2, \tag{40}$$

where $c_n \geq 1$, $\alpha_n(\bar{x}_n) \geq 1$ is uncorrelated of $r_i(t)$.

Remark 2. *The design idea of this paper is completely different from the design idea of [15]. Although the system in [15] also has time-varying power, the system noise considered in this paper is a kind of color noise, which is a completely different white noise from [15]. A new design scheme is proposed in this part.*

Remark 3. *With the effect of time-varying powers $r_i(t)$ in system (1), it is a challenging problem to design a time-independent controller. The time-varying powers make our design much more difficult and essentially different from the constant power cases [10–12]. In our control scheme, we designed the virtual controllers and real controller with the upper bound $\bar{r}$ and lower bound $\underline{r}$ of $r_i(t)$.*

2.2. Stability Analysis

In this part, we present the main results on stability.

Theorem 1. *Consider the high-order RNSs (1), if Assumptions (1)–(3) hold, with the controller (39), we have*

(1) The closed-loop system has a unique solution on $[t_0, \infty)$;
(2) All the states of the closed-loop system are bounded in probability;
(3) The fourth moment of the tracking error can be tuned to be arbitrarily small.

Specifically, for $\forall \varepsilon$ and initial value $x(t_0)$, there is a finite-time $T(x(t_0), \varepsilon)$, such that

$$E|x_1(t) - y_0(t)|^4 < \varepsilon, \forall t > T(x(t_0), \varepsilon).$$

Proof. Let $V = V_n$, for (40), if $r_i(t) = 1$, we have

$$\begin{aligned} \dot{V} &\leq -\sum_{k=1}^{n}(c_k - 1)|\eta_k|^4 + \sum_{k=1}^{n}\delta_{k1} + \sum_{k=1}^{n}\delta_{k2}|\zeta|^2 \\ &\leq -c_0 V + b + a|\zeta|^2, \end{aligned} \tag{41}$$

where $c_0 = \min_{1\leq k\leq n}\{4(c_k - 1)\}$, $b = \sum_{k=1}^{n}\delta_{k1}$, $a = \sum_{k=1}^{n}\delta_{k2}$. □

If $r_i(t) > 1$. By Lemma 2.3 in [27], we have

$$|\eta_k|^4 \leq \tau + \sigma|\eta_k|^{\bar{r}+3}, \tag{42}$$

where $0 < \tau \leq 1$ is a design parameter and $\sigma = \frac{4}{\bar{r}+3}\left(\frac{\bar{r}-1}{\tau(\bar{r}+3)}\right)^{\frac{\bar{r}-1}{4}}$, which yields

$$|\eta_k|^{\bar{r}+3} \geq \sigma^{-1}|\eta_k|^4 - \sigma^{-1}\tau. \tag{43}$$

Substituting (43) into (40) yields

$$\dot{V} \leq -c_0 V + b + a|\zeta|^2, \tag{44}$$

where $c_0 = \min_{1\leq k\leq n}\{4\sigma^{-1}(c_k - 1)\}$, $b = \sum_{k=1}^{n}(c_k - 1)\sigma^{-1}\tau + \sum_{k=1}^{n}\delta_{k1}$, $a = \sum_{k=1}^{n}\delta_{k2}$.

Let $\eta(t) = (\eta_1(t), \cdots, \eta_n(t))^T$, define the first exit time

$$\chi_l = \inf\{t : t \geq t_0, |\eta(t)| \geq l\}, \forall l > 0. \tag{45}$$

Under the concurrence of Assumption 3 and Fubini's theorem

$$\begin{aligned} EV(\eta(t \wedge \chi_l)) &\leq V(\eta(t_0)) + b(t - t_0) + aE\{\int_{t_0}^{t}|\zeta(s)|^2 ds\} \\ &\leq V(\eta(t_0)) + (b + aK)(t - t_0). \end{aligned} \tag{46}$$

From (46) and Lemma 5 in [19], Conclusion (1) is proved.
Next, we present a proof of Conclusion (3).
Let $t_l = \min\{t, \chi_l\} = t \wedge \chi_l$, by (41), we have

$$E(e^{c_0 t_l}V(\eta(t_l)) \leq e^{c_0 t_0}EV(\eta(t_0)) + \frac{b}{c_0}(e^{c_0 t} - e^{c_0 t_0}) + aE\Big\{\int_{t_0}^{t}e^{c_0 s}|\zeta(s)|^2 ds\Big\}. \tag{47}$$

Then, letting $l \to \infty$, by (47), we have

$$e^{c_0 t}E(V(\eta(t)) \leq e^{c_0 t_0}EV(\eta(t_0)) + \frac{b}{c_0}(e^{c_0 t} - e^{c_0 t_0}) + aE\Big\{\int_{t_0}^{t}e^{c_0 s}|\zeta(s)|^2 ds\Big\}. \tag{48}$$

It can be inferred from (48), Assumption 3,

$$e^{c_0 t}E(V(\eta(t)) \quad \leq \quad e^{c_0 t_0}EV(\eta(t_0)) + \frac{b+aK}{c_0}(e^{c_0 t} - e^{c_0 t_0}), \tag{49}$$

or equivalently

$$E(V(\eta(t)) \quad \leq \quad e^{-c_0(t-t_0)}EV(\eta(t_0)) + \frac{b+aK}{c_0}(1 - e^{-c_0(t-t_0)}). \tag{50}$$

Referring to the definition of a and b, it can be obtained that the information of $EV(\eta(t))$ can be adjusted as small as you want. So, noting $\eta = (x_1 - y_0, \eta_2, \cdots, \eta_n)^T$, for $\forall\varepsilon$ and initial value $x(t_0)$, $\exists$ a finite-time $T(x(t_0), \varepsilon)$, the sufficient large L leads to

$$E|x_1 - y_0|^4 < \varepsilon, \forall t > T(x(t_0), \varepsilon). \tag{51}$$

Next, we will prove Conclusion (2). From (50), we obtain

$$EV(\eta(t)) \quad \leq \quad V(\eta(t_0)) + \frac{b+aK}{c_0}. \tag{52}$$

For any constant $h > 0$, note that

$$EV(\eta(t)) \geq \int_{|\eta|>h} V(\eta(t))P(dw) \geq \inf_{|\eta|>h} V(\eta(t))P(|\eta| > h), \tag{53}$$

from which (52), we have

$$P(|\eta| > h) \leq \frac{V(\eta(t_0)) + \frac{b+aK}{c_0}}{\inf_{|\eta|>h} V(\eta(t))}. \tag{54}$$

By (54) and $V(\eta(t))$, we have

$$\lim_{h\to\infty} \sup_{t>t_0} P(|\eta| > h) \leq \lim_{h\to\infty} \sup_{t>t_0} \frac{V_{\eta(t_0)} + \frac{b+aK}{c_0}}{\inf_{|\eta|>h} V(\eta(t))} = 0. \tag{55}$$

By (48), $\eta(t)$ is bounded in probability. This shows that $\eta_i(t), i = 1, \cdots, n$ is bounded in probability. Moreover, considering the $\eta_1 = x_1 - y_0$ and $\eta_2 = x_2 - x_2^*$, we can conclude that Conclusion (2) is true.

3. A Simulation Example

In this part, we consider the system:

$$\begin{aligned} \dot{x}_1 &= [x_2]^{\frac{6}{5}+\frac{1}{5}\sin t} + \tfrac{1}{2}x_1^3 + \tfrac{1}{2}x_1 \sin^2 x_1 \zeta(t), \\ \dot{x}_2 &= [u]^{\frac{7}{6}+\frac{1}{6}\sin t} + x_2^2 \sin x_1 + \tfrac{1}{2}x_2 \sin^2 x_1 \zeta(t), \\ y &= x_1. \end{aligned} \tag{56}$$

In the simulation, we choose $\zeta(t) = \sin(w(t))$, where $w(t)$ is white noise with limited bandwidth produced by MATLAB (noise power is 10 and sample time is 0.01). Obviously, $\zeta(t)$ is a second-order moment process with $E\zeta(t)^2 \leq 1$, which shows that Assumption 3 is satisfied.

Let $y_0 = \sin t$. Choosing $\bar{r} = 2$, $\underline{r} = 1$. Obviously, the assumptions are true.

By the calculation, we have

$$u = -(c_2 + \beta_{21} + \beta_{22} + \beta_{23})(\eta_2 + [\eta_2]^2), \tag{57}$$

where

$$
\begin{aligned}
\beta_{21} &= \frac{3}{2}((x_1+1)^2+1)^{\frac{15}{2}}(\alpha_1+1)^5+101\alpha_1^5+40,\\
\beta_{22} &= \frac{3}{4}\Big((1+x_2^4\sin^2 x_1)^{\frac{1}{2}}+\frac{\partial x_2^*}{\partial x_1}(x_1+1)+|\frac{\partial x_2^*}{\partial y_0}|\Big)^{\frac{5}{3}},\\
\beta_{23} &= \frac{3}{4}\eta_2^5\Big(\Big(1+\frac{1}{4}x_2^2\sin^4 x_1\Big)^{\frac{1}{2}}+\frac{\partial x_2^*}{\partial x_1}\Big(1+\frac{1}{4}x_1^2\sin^4 x_1\Big)^{\frac{1}{2}}\Big)^{\frac{10}{3}},\\
x_2^* &= -(\eta_1+[\eta_1]^2)\Big(1+\frac{3}{5}\Big(\Big(1+\frac{1}{4}x_1^6\Big)^{\frac{1}{2}}+1\Big)^{\frac{5}{3}}+3\eta_1^5\Big(1+\frac{1}{4}x_1^2\sin^4 x_1\Big)^{\frac{10}{3}}\Big),\\
\frac{\partial x_2^*}{\partial x_1} &= \frac{15}{16}((\frac{1}{4}x_1^6+1)^{1/2}+1)^{2/5}(\frac{1}{4}x_1^6+1)^{-1/2}x_1^5+\frac{3}{4}(\frac{1}{4}x_1^2\sin^4 x_1+1)^{10/3}\eta_1^4\\
&\quad+\frac{25}{3}(\frac{1}{4}x_1^2\sin^4 x_1+1)^3\eta_1^5(\frac{1}{2}x_1\sin^4 x_1+x_1^2\sin^3 x_1\cos x_1),\\
|\frac{\partial x_2^*}{\partial y_0}| &= 5\eta_1^4(\frac{1}{4}x_1^2\sin^4 x_1+1)^{10/3}(\eta_1+[\eta_1]^2)+\alpha_1(1+2\eta_1).
\end{aligned}
$$

By randomly choosing parameters $c_1 = 1.3$, $c_2 = 1$, and a set of initial values $(x_1(0), x_2(0))^T = (-0.5, 0.6)^T$. Through the actual simulation, the system responses of the tracking error, states, and controller are shown in Figures 1–3. It can be seen from Figure 1 that when $\forall t > T = 2s$, the error $|e| = |x_1 - y_0| < 0.1$. At the same time, the effectiveness of the design idea can be directly illustrated with Figure 2 and 3.

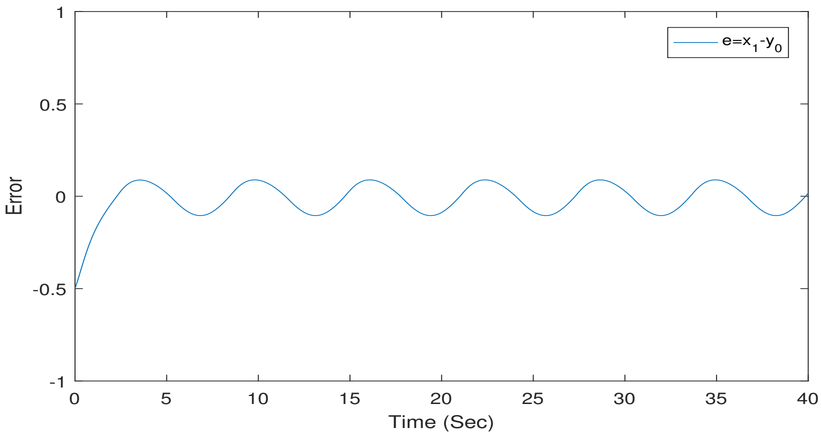

Figure 1. Response to the tracking error.

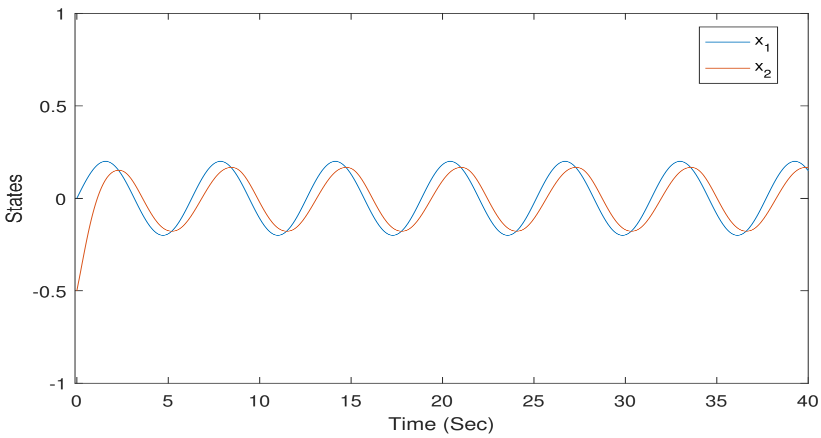

Figure 2. Response of the states.

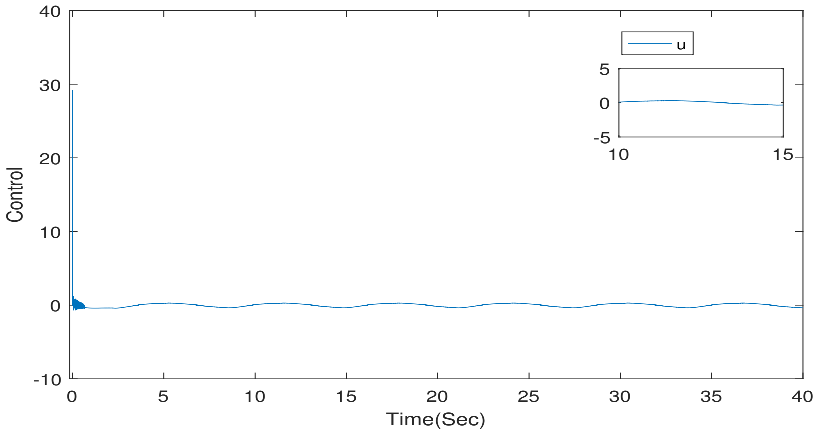

Figure 3. Response of the controller.

4. Conclusions

We studied the output tracking problem of RNSs with time-varying powers. The advantage of our control is that both time-varying power and SOMP are considered, which is more practical than the existing results, which only consider one factor. First, different from the deterministic systems considered in [15], the disturbance of the system studied in this paper is characterized by SOMP. The difference between the SOMP and Gaussian white noise is that white noise is an independent random process, while the SOMP is an interrelated random process. Secondly, the power of the system studied in this paper is a function of the time-varying order, which must be taken into account in the construction of the controller. The time-invariant controller was designed. It is concluded that the expectation of the fourth moment of the tracking deviation can be trimmed to be arbitrarily small, and all states are bounded in probability.

There are some future research topics, e.g., when there is uncertainty in the system, how to design the controller to ensure the tracking performance, or to generalize the results in this paper to more general systems in [28–31] or a more practical system [32].

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Abbreviations

The following abbreviations are used in this manuscript:

SNSs	stochastic nonlinear systems
RNSs	random nonlinear systems
SOMP	second-order moment process

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