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Editorial

Preface to the Special Issue on "Bayesian Predictive Inference and Related Asymptotics—Festschrift for Eugenio Regazzini's 75th Birthday"

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It is my pleasure to write this Preface to the Special Issue of Mathematics entitled "Bayesian Predictive Inference and Related Asymptotics—Festschrift for Eugenio Regazzini's 75th Birthday". As the title suggests, this Special Issue is dedicated to Professor Eugenio Regazzini to honor his more than quinquagenary career (which is still ongoing!). For many years, Eugenio served as both a lecturer and scholar of various Italian Universities (Torino, Bologna, Milano Statale, Bocconi, Pavia), visited a number of foreign academic institutions (including Stanford University in the US, where he lectured for a brief period) and organized various summer schools to promote advanced studies in probability and statistics around Italy. Indeed, more than one generation of Italian scholars has learned and consolidated the study of probability and mathematical statistics under his supervision. It is evident that, besides transmitting enthusiasm and expertise to his students, Eugenio created a solid bridge between the actual academic generation, working in probability and mathematical statistics, and the great Italian masters of the first half of the twentieth century, such as de Finetti, Cantelli and Gini. As a scholar, Eugenio's activity has received—and still receives—appreciation from both colleagues and academic institutions worldwide. Apropos of this, it would be remiss not to mention the prestigious IMS fellowship he received in July 2007.

To briefly outline his scientific contributions, MathSciNet includes 84 of his publications: of these, 41 are concerned with Mathematical Statistics, 9 with Pure Probability, 16 with Mathematical Physics and Economics, and 18 with historical issues. His most significant works can be thematically grouped as follows:

(a) *Bayesian Nonparametrics*: means of the Dirichlet process [1–4], means of normalized completely random measures [5], approximations of posterior distributions by mixtures of Dirichlet probability laws [6];
(b) *General Bayesian Inference*: Bayesian sufficiency [7–9], asymptotics for Bayesian predictive inference [10–12];
(c) *Classical Inference*: minimum distance estimation [13–16], classical point estimation, and testing theory [17,18];
(d) *Descriptive Statistics*: theory of concentration [19,20], theory of monotone dependence [21];
(e) *Abstract Probability Theory*: finitely additive probability [22–27], mixtures of distributions of Markov chains [28], CLT for exchangeable summands [29,30];
(f) *Mathematical Physics and Economics*: analysis of some kinetic Boltzmann-type equations [31–39].

Returning to the Special Issue, we present 11 papers, which are briefly summarized below.

In [40], the author reviews the historical position of Sir R.A. Fisher towards Bayesian inference, particularly regarding the classical Bayes–Laplace paradigm. The main focus of the paper is on Fisher's fiducial argument.

In [41], the author considers point estimation problems concerned with random quantities which depend on both observable and non-observable variables, starting from

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Citation: Dolera, E. Preface to the Special Issue on "Bayesian Predictive Inference and Related Asymptotics—Festschrift for Eugenio Regazzini's 75th Birthday". *Mathematics* **2022**, *10*, 2567. https://doi.org/10.3390/math10152567

Received: 21 July 2022
Accepted: 21 July 2022
Published: 23 July 2022

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decision-theoretical principles. A two-phase strategy is proposed, the former relying on estimation of the random parameter of the model, the latter concerning estimation of the original quantity sampled from the distinguished element of the statistical model after plug-in of the estimated parameter in the place of the random parameter. The asymptotic efficiency of the entire procedure is finally discussed.

In [42] the authors obtains explicit descriptions of properties of some Markov chains, called Mittag-Leffler Markov chains, conditioned with a mixed Poisson process when it equates to an integer n, which has interpretations in a species sampling context. This is equivalent to obtaining properties of the fragmentation operations when applied to mass partitions formed by the normalized jumps of a generalized gamma subordinator and its generalizations.

The author of [43] develops a parameter-free version of classical models for contingency tables, along the lines of de Finetti's notions of partial exchangeability.

In [44], the authors introduce a betting game where the gambler aims to guess the last success epoch in a series of inhomogeneous Bernoulli trials paced at random. At a given stage, the gambler may bet on either the event that no further successes occur, or the event that exactly one success is yet to occur, or may choose any proper range of future times (a trap). When a trap is chosen, the gambler wins if the final success epoch is the only one that falls in the trap. Then, the authors use this tool to analyse the best-choice problem, with random arrivals generated via a Pólya–Lundberg process.

In [45], the authors consider a sequence $\{X_n\}_{n\geq 1}$ of conditionally identically distributed random variables. They show that, under suitable conditions, the finite dimensional distributions of the empirical process stably converge to a Gaussian kernel with a known covariance structure.

In [46], the authors introduce mixtures of species sampling sequences and discuss how these sequences are related to various types of Bayesian models. They prove that mixtures of species sampling sequences are obtained by assigning the values of an exchangeable sequence to the classes of a latent exchangeable random partition. Using this representation, they give an explicit expression of the Exchangeable Partition Probability Function of the partition generated by a mixture of species sampling sequences. Finally, they discuss some special cases.

The authors of [47] pursue a project in which the authors introduce, study, and apply a variant of the Eggenberger–Pólya urn, called the "rescaled" Pólya urn. This variant exhibits a reinforcement mechanism based on the most recent observations, a random persistent fluctuation of the predictive mean, and the almost certain convergence of the empirical mean to a deterministic limit. Then, the authors show that the multidimensional Wright–Fisher diffusion with mutation can be obtained as a suitable limit of the predictive means associated with a family of rescaled Pólya urns.

In [48], the authors review "sufficientness" postulates for species-sampling models, and investigate analogous predictive characterizations for the more general feature-sampling models. In particular, they present a "sufficientness" postulate for a class of feature-sampling models referred to as Scaled Processes, and then discuss analogous characterizations in the general setup of feature-sampling models.

In [49], the authors study the asymptotic properties of the predictive distributions and the empirical frequencies of certain sequences $\{X_n\}_{n\geq 1}$ of random variables that are connected to the so-called measure-valued Pólya urn processes, under different assumptions on the weights. They also investigate a generalization of the above models via a randomization of the law of reinforcement.

Finally, in [50], the authors consider a generalization of the log-series compound Poisson sampling model, and they show that it leads to an extension of the compound Poisson perspective of the Ewens sampling model to the more general Ewens–Pitman sampling model. The interplay between the negative Binomial compound Poisson sampling model and the Ewens–Pitman sampling model is then applied to the study of the large

n asymptotic behavior of the number of blocks in the corresponding random partitions, leading to new proof of Pitman's α diversity.

Funding: This research received no external funding

Conflicts of Interest: The authors declare no conflict of interest.

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