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Stability of Switched Systems with Time-Varying Delays under State-Dependent Switching

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Abstract: This paper studies the stability of linear switched systems with time-varying delays and all unstable subsystems. According to the largest region function strategy, the state-dependent switching rule is designed. By bringing in integral inequality and multiple Lyapunov-Krasovskii functionals, the stability results of delayed switched systems with or without sliding motions under the designed state-dependent switching rule are derived for different assumptions on time delay. Several numerical examples are employed to show the effectiveness and superiority of the proposed results.

Keywords: stability; switched system; state-dependent switching; time delay

MSC: 93D20; 93C10

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1. Introduction

The dynamics of switched systems are affected by both subsystems and switching rules. For example, Decarlo R A has indicated that some appropriate switching rules can make switched systems unstable (or asymptotically stable) even if all subsystems are asymptotically stable (or unstable) [1]. Therefore, we must concentrate on both subsystems and switching rules to derive the stability results. In recent years, the stability issue of switched systems with unstable subsystems has been extensively investigated. For instance, in [2–7] the researchers have derived some stability results for switched systems with both stable and unstable subsystems. The main strategy of some literature is to ensure that the dwell time of stable subsystems is sufficiently large to compensate for the state divergence caused by unstable subsystems and switching behaviors. Obviously, if there is no stable subsystem to absorb the state divergence, these results proposed in [2–7] are disabled.

Because of the absence of stable subsystems, the stability analysis of switched systems with all unstable subsystems is more complicated. How to design appropriate switching rules to stabilize switched systems with all unstable subsystems has become an interesting and challenging problem. Ordinarily, switching rules can be designed by two strategies: time-dependent switching and state-dependent switching. The main idea of the first one is to use the stabilization of switching behaviors to stabilize switched systems and the designed switching rules usually have both upper and lower bounds. In [8–12], the time-dependent switching rules are designed to stabilize switched systems with or without time delay by using discretized Lyapunov function approach or bound maximum average dwell time. The time-dependent switching strategy requires that switching behaviors have a good characteristic of stabilization. Therefore, when all switching behaviors do not contain stabilization characteristics, the time-dependent switching strategy is invalid.

In many instances, time-dependent switching rules that can stabilize switched systems are hard to design or even non-existent, which signifies that the state-dependent switching strategy becomes the unique way to stabilize switched systems. Up to now, the state-dependent switching rules can be designed by two methods. The first one is based on the regional partition of state space. Its basic idea can be summarized as follows: (a) divide the state space into different switching regions; (b) determine the index of activated subsystems for each switching region; (c) derive the stability conditions for switched systems under the designed switching rule. Under the assumption that there exists a Hurwitz convex combination of system matrices, the state-dependent switching rules have been designed via the regional partition of state space and some significant stability results have been deduced by common Lyapunov function (functional) in [13–19]. Remarkably, this assumption is a severe prerequisite. In order to relax this assumption, by employing some free matrices, a more flexible Hurwitz convex combination is presented in [20]. In [21] the regional partition of state space is implemented directly by the negative definite of the time-derivative of common Lyapunov functional. To ensure the strict completeness of regional partition, one additional condition is introduced. Based on newly introduced symmetric matrices, Pettersson S has defined switching rules via the largest region function strategy and established the stability results by multiple Lyapunov functionals [22,23]. Some restrictions are also employed to guarantee the decrease of Lyapunov functional when switching events occur. However, the largest region function strategy has not been generalized to switched systems with time delay. The second one is that the switching rules are defined in terms of the set-valued function. One typical state-dependent switching rule is given by $\sigma(t) = \arg\min\{x^T(t)P_1x(t), \cdots, x^T(t)P_mx(t)\}$, where P_i is a symmetric positive determined matrix, m is number of subsystems. In [24–27], the authors have designed the switching rules by the set-valued function and given the stability conditions with the Lyapunov-Metzler inequalities. Although there are numerous results for state-dependent switching, it is noteworthy that this issue still needs to be further studied. Designing new state-dependent switching rules and getting lower conservative stability results is still our research motivation.

Up to now, the literatures on the stability of delayed switched systems with state-dependent switching rules include [15–21,27]. However, the assumption that there exists a Hurwitz convex combination of system matrices is serious, which affects the effectiveness of stability results presented in [15–20]. The additional condition on strict completeness of regional partition makes it difficult to get appropriate switching regions [21]. Additionally, the results presented in [27] are only available for switched systems with constant delay. Therefore, the stability of switched systems with time-varying delays under state-dependent switching rules still deserves further attention. The main objective of this paper is to derive some new stability results for this problem. Based on the largest region function strategy, we design a state-dependent switching rule. By using integral inequality and the Leibniz-Newton formula, novel asymptotic stability results under different assumptions on time delay are presented in the form of bilinear matrix inequalities (BMIs). The effectiveness of the proposed results is shown via several numerical examples.

***Notations*:** matrix $A > 0(<0)$ yields that A is symmetric positive(negative) matrix, R^n denotes the $n-$dimension Euclidean space, $arg\max S$ is defined as the index of maximum element of order set S.

2. Preliminaries

This paper considers the following switched systems with time-varying delay

$$\begin{cases} \dot{x}(t) = A_{\sigma(x(t))}x(t) + B_{\sigma(x(t))}x(t-d(t)), t > 0, \\ x(s) = \phi(s), s \in [-d, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $\sigma(x(t)) \in M = \{1, 2, \cdots, m\}$ is the switching rule, $A_p, B_p \in R^{n\times n}$, $p \in M$, are known matrices, $d(t)$ is the time-varying delay, $\phi(s)$ is a piecewise continuous function. If $\sigma(t) = p$, we say that the p-th subsystem $\dot{x}(t) = A_px(t) + B_px(t-d(t))$ is activated.

Remark 1. *$\sigma(x(t))$ is a state-dependent switching rule which is generated by switching device [13]. Similar to [13–23], in this paper we also assume that there is no delay produced in switching device.*

That is to say, the switching rule $\sigma(x(t))$ is one dependent on the current state but irrelevant to the delayed state.

We would like to design a state-dependent switching rule $\sigma(t)$ such that switched system (1) is globally asymptotically stable. We employ the state-dependent switching strategy introduced in [22,23], which is based on the appropriate choice of symmetric matrices Q_p, $p \in M$. Define the following regions

$$\Omega_p = \left\{ x \in R^n | x^T Q_p x \geq 0 \right\}, p \in M,$$
$$\Omega_{pq} = \left\{ x \in R^n | x^T Q_q x = x^T Q_p x \geq 0 \right\}, p, q \in M, p \neq q.$$

We hope that the p-th subsystem is activated if $x(t) \in \Omega_p$ and switching events occur at the region Ω_{pq}. The following properties should be satisfied to ensure that the switched system (1) is well-defined [22],

(a) Covering property: $\bigcup_{p \in M} \Omega_p = R^n$,
(b) Switching property: $\Omega_{pq} \subseteq \Omega_p \bigcap \Omega_q$.

The covering property points out that there is at least one activated subsystem on an arbitrary region of the state space. The switching property implies that the switch from subsystem p to q occurs only if regions Ω_p and Ω_q are adjacent. According to [22,23], the covering property is satisfied, if there exists $\theta_p > 0$, $p \in M$, such that for any $x \in R^n$,

$$\sum_{p \in M} \theta_p x^T Q_p x \geq 0. \tag{2}$$

The switching rule can be defined as the following largest region function strategy [22,23]

$$\sigma(x(t)) = arg\max \left\{ x^T(t) Q_1 x(t), \cdots, x^T(t) Q_m x(t) \right\}. \tag{3}$$

As can be seen from [22] we know that if (2) is true and the switching rule (3) is used, the switching property is also satisfied.

The main purpose of this work is to get the stability results under one of the following assumptions.

Assumption 1. *The time delay and its time-derivative are bounded. Namely, there exist nonnegative constants $d, \bar{d}$ and constant $\tilde{d}$ such that*

$$0 \leq d(t) \leq d, \tilde{d} \leq \dot{d}(t) \leq \bar{d}. \tag{4}$$

Assumption 2. *The time delay is bounded. Namely, there exists a nonnegative constant d such that*

$$0 \leq d(t) \leq d. \tag{5}$$

The following lemma is the core of this research.

Lemma 1 ([28]). *If matrix $M > 0$ and function $x : [a, b] \to R^n$ is differentiable, then the following inequality is satisfied*

$$(b - a) \int_a^b \dot{x}^T(s) M \dot{x}(s) ds \geq \beta^T diag(M, 3M, 5M) \beta,$$

where $\beta = \left(\beta_1^T, \beta_2^T, \beta_3^T \right)^T$, $\beta_1 = x(b) - x(a)$, $\beta_2 = x(b) + x(a) - \dfrac{2}{b-a} \int_a^b x(s) ds$, $\beta_3 = x(b) - x(a) + \dfrac{6}{b-a} \int_a^b x(s) ds - \dfrac{12}{(b-a)^2} \int_a^b \int_\theta^b x(s) ds d\theta.$

3. Main Results

This section presents the stability criteria for the switched system (1) under the state-dependent switching rule (3). Owing to the Leibniz-Newton formula, we have the following equation

$$x(t) - x(t-d(t)) = \int_{t-d(t)}^{t} \dot{x}(s)ds. \tag{6}$$

Some notations are given as follows

$$v_1 = \frac{2}{d-d(t)} \int_{t-d(t)}^{t} x(s)ds, v_2 = \frac{12}{(d-d(t))^2} \int_{t-d(t)}^{t} \int_{\theta}^{t} x(s)dsd\theta,$$
$$\eta(t) = \left(x^T(t), x^T(t-d(t)), x^T(t-d), \dot{x}^T(t-d(t)), \dot{x}^T(t-d), v_1^T, v_2^T\right)^T.$$

Theorem 1. *Under Assumption 1, assume that for any $p \in M$, there exist $n \times n$ matrices $P_p > 0$, $R_i > 0, S_i > 0, U > 0$, $(i = 1,2)$, $Q_p = Q_p^T$, positive constants μ_p, θ_p, constants $\eta_{p,q}$, $q \in M$, $q \neq p$, such that*

$$\begin{pmatrix} \Lambda_l^p + \mu_p e_1^T Q_p e_1 & \sqrt{d} e_1^T P_p B_p \\ \sqrt{d} B_p^T P_p e_1 & -U \end{pmatrix} < 0, l = 1,2, \tag{7}$$

$$P_p = P_q + \eta_{p,q}(Q_q - Q_p), q \in M, q \neq p, \tag{8}$$

$$\sum_{j \in M} \theta_j Q_j \geq 0, \tag{9}$$

where

$$\begin{aligned}
\Lambda_1^p =& \Phi_1^p + \Phi_2 + \Phi_3^p + \Phi_4^p + (1-\bar{d})(\Psi_2 + \Psi_3) - \frac{1}{d}\Xi_4, \Lambda_2^p = \Phi_1^p + \Phi_2 + \Phi_3^p + \Phi_4^p + \\
& (1-\tilde{d})(\Psi_2 + \Psi_3) - \frac{1}{d}\Xi_4, \Phi_1^p = e_1^T\left((A_p + B_p)^T P_p + P_p(A_p + B_p)\right)e_1, \\
\Phi_2 =& e_1^T R_1 e_1 - e_3^T R_2 e_3, \Phi_3^p = (A_p e_1 + B_p e_2)^T S_1 (A_p e_1 + B_p e_2) - e_5^T S_2 e_5, \\
\Phi_4^p =& d(A_p e_1 + B_p e_2)^T U (A_p e_1 + B_p e_2), \Psi_2 = e_2^T(R_2 - R_1)e_2, \Psi_3 = e_4^T(S_2 - S_1)e_4, \\
\Xi_4 =& (e_2 - e_3)^T U (e_2 - e_3) + 3(e_2 + e_3 - e_6)^T U (e_2 + e_3 - e_6) + 5(e_2 - e_3 + 3e_6 - e_7)^T U \times \\
& (e_2 - e_3 + 3e_6 - e_7), e_i = \left(0_{n\times(i-1)n}, I, 0_{n\times(7-i)n}\right), i = 1,2,\cdots,7.
\end{aligned}$$

Then, the switched system (1) is globally asymptotically stable under the state-dependent switching rule (3), if there is no sliding motion or there exist sliding motions on the switching surface Ω_{pq} with $\eta_{p,q} > 0$.

Proof. Condition (9) implies that (2) is true, which indicates that the covering property holds. Therefore, under the switching rule (3), the switched system (1) is well-defined.

Now we prove that the switched system (1) is globally asymptotically stable. Similar to [29,30], for each subsystem p, we choose the Lyapunov-Krasovskii functional as follows

$$V_p(t) = V_{p1}(t) + \sum_{i=2}^{4} V_i(t), \tag{10}$$

where

$$V_{p1}(t) = x^T(t) P_p x(t), V_2(t) = \int_{t-d(t)}^{t} x^T(s) R_1 x(s)ds + \int_{t-d}^{t-d(t)} x^T(s) R_2 x(s)ds,$$

$$V_3(t)=\int_{t-d(t)}^{t}\dot{x}(s)S_1\dot{x}(s)ds+\int_{t-d}^{t-d(t)}\dot{x}^T(s)S_2\dot{x}(s)ds, V_4(t)=\int_{-d}^{0}\int_{t+\theta}^{t}\dot{x}(s)U\dot{x}(s)dsd\theta.$$

In each region Ω_p, the time derivate of $V_{p1}(t)$, $V_i(t)$, $i=2,3,4$, along the trajectory of the subsystem p are given as follows

$$\begin{aligned}
&\dot{V}_{p1}(t)\\
=&x^T(t)\Big((A_p+B_p)^TP_p+P_p(A_p+B_p)\Big)x(t)-\int_{t-d(t)}^{t}\Big(\dot{x}^T(s)B_p^TP_px(t)+x^T(t)P_pB_p\dot{x}(s)\Big)ds\\
\le&x^T(t)\Big((A_p+B_p)^TP_p+P_p(A_p+B_p)+d(t)P_pB_pU^{-1}B_p^TP_p\Big)x(t)+\int_{t-d(t)}^{t}\dot{x}^T(s)U\dot{x}(s)ds\\
=&\eta^T(t)\Big(\Phi_1^p+d(t)\Theta_1^p\Big)\eta^T(t)+\int_{t-d(t)}^{t}\dot{x}^T(s)U\dot{x}(s)ds.
\end{aligned}\tag{11}$$

$$\begin{aligned}
\dot{V}_2(t)=&x^T(t)R_1x(t)+(1-\dot{d}(t))x^T(t-d(t))(R_2-R_1)x(t-d(t))-x^T(t-d)R_2x(t-d)\\
=&\eta^T(t)(\Phi_2+(1-\dot{d}(t))\Psi_2)\eta(t).
\end{aligned}\tag{12}$$

$$\begin{aligned}
&\dot{V}_3(t)\\
=&\dot{x}^T(t)S_1\dot{x}(t)-\dot{x}^T(t-d)S_2\dot{x}(t-d)+(1-\dot{d}(t))\dot{x}^T(t-d(t))(S_2-S_1)\dot{x}(t-d(t))\\
=&(A_px(t)+B_px(t-d(t)))^TS_1(A_px(t)+B_px(t-d(t)))-\dot{x}^T(t-d)S_2\dot{x}(t-d)\\
&+(1-\dot{d}(t))\dot{x}^T(t-d(t))(S_2-S_1)\dot{x}(t-d(t))\\
=&\eta^T(t)\Big(\Phi_3^p+(1-\dot{d}(t))\Psi_3\Big)\eta(t).
\end{aligned}\tag{13}$$

$$\begin{aligned}
&\dot{V}_4(t)\\
=&d\dot{x}^TU\dot{x}(t)-\int_{t-d(t)}^{t}\dot{x}^T(s)U\dot{x}(s)ds-\int_{t-d}^{t-d(t)}\dot{x}^T(s)U\dot{x}(s)ds\\
=&d(A_px(t)+B_px(t-d(t)))^TU(A_px(t)+B_px(t-d(t)))-\int_{t-d(t)}^{t}\dot{x}^T(s)U\dot{x}(s)ds\\
&-\int_{t-d}^{t-d(t)}\dot{x}^T(s)U\dot{x}(s)ds.
\end{aligned}\tag{14}$$

where $\Theta_1^p=e_1^TP_pB_pU^{-1}B_p^TP_pe_1$. Under Lemma 1, one can obtain

$$(d-d(t))\int_{t-d}^{t-d(t)}\dot{x}^T(s)U\dot{x}(s)ds\ge\xi_1^TU\xi_1+3\xi_2^TU\xi_2+5\xi_3^TU\xi_3=\eta^T(t)\Xi_4\eta(t),$$

where $\xi_1=x(t-d(t))-x(t-d)$, $\xi_2=x(t-d(t))+x(t-d)-v_1$, $\xi_3=x(t-d(t))-x(t-d)+3v_1-v_2$. Above inequality implies that (14) can be continued as

$$\dot{V}_4(t)\le\eta^T(t)\Big(\Phi_4^\sigma-\frac{1}{d-d(t)}\Xi_4\Big)\eta(t)-\int_{t-d}^{t-d(t)}\dot{x}^T(s)U\dot{x}(s)ds\tag{15}$$

Then, it follows from (10)–(13), (15) that

$$\begin{aligned}
&\dot{V}_p(t)\\
\le&\eta^T(t)\Big(\phi_1^p+\phi_2+\phi_3^p+\phi_4^p+d(t)\Theta_1^p+(1-\dot{d}(t))(\Psi_2+\Psi_3)-\frac{1}{d-d(t)}\Xi_4\Big)\eta(t)\\
=&\frac{1}{d-d(t)}\eta^T(t)\Big((d-d(t))\Big(\phi_1^p+\phi_2+\phi_3^p+\phi_4^p+d(t)\Theta_1^p+(1-\dot{d}(t))(\Psi_2+\Psi_3)\Big)-\Xi_4\Big).
\end{aligned}\tag{16}$$

Due to Schur complements [31], Condition (7) indicates that

$$\Lambda_l^p + d\Theta_1^p + \mu_p e_1^T Q_p e_1 < 0, l = 1, 2. \tag{17}$$

Namely,

$$\begin{cases} \phi_1^p + \phi_2 + \phi_3^p + \phi_4^p + (1-\bar{d})(\Psi_2 + \Psi_3) + d\Theta_1^p + \mu_p e_1^T Q_p e_1 - \dfrac{1}{d}\Xi_4 < 0, \\ \phi_1^p + \phi_2 + \phi_3^p + \phi_4^p + (1-\tilde{d})(\Psi_2 + \Psi_3) + d\Theta_1^p + \mu_p e_1^T Q_p e_1 - \dfrac{1}{d}\Xi_4 < 0. \end{cases} \tag{18}$$

The above inequalities declare that

$$\phi_1^p + \phi_2 + \phi_3^p + \phi_4^p + (1-\dot{d}(t))(\Psi_2 + \Psi_3) + d\Theta_1^p + \mu_p e_1^T Q_p e_1 - \frac{1}{d}\Xi_4 < 0. \tag{19}$$

Due to $0 \le d(t) \le d$ and $\Theta_1^p > 0$, it is clear from (19) that

$$d\Big(\phi_1^p + \phi_2 + \phi_3^p + \phi_4^p + (1-\dot{d}(t))(\Psi_2 + \Psi_3) + d(t)\Theta_1^p + \mu_p e_1^T Q_p e_1\Big) - \Xi_4 < 0. \tag{20}$$

Noting that $0 \le d - d(t) \le d$ and $\Xi > 0$, (20) shows that

$$(d-d(t))\Big(\phi_1^p + \phi_2 + \phi_3^p + \phi_4^p + (1-\dot{d}(t))(\Psi_2 + \Psi_3) + d(t)\Theta_1^p + \mu_p e_1^T Q_p e_1\Big) - \Xi_4 < 0. \tag{21}$$

Based on (16) and (21), one can derive that

$$\dot{V}_p(t) < -\mu_p \eta^T(t) Q_p \eta(t) \le 0, \tag{22}$$

where the fact $x^T(t)Q_\sigma x(t) \ge 0$ is used.

Note that for arbitrary $x \in \Omega_{pq}$, $x^T Q_p x = x^T Q_q x$. Then, due to Condition (8) we can derive that $V_p(t) = V_q(t)$ if $x(t) \in \Omega_{pq}$. Therefore, when the trajectory $x(t)$ traverses from Ω_p to Ω_q, the Lyapunov functional $V_\sigma(t)$ is not increasing. In particular, if the sliding motion does not occur, the Lyapunov functional $V_\sigma(t)$ will be approximate to zero and shows that the switched system (1) is globally asymptotically stable.

Now we consider the case of sliding motions. Assume that the sliding motions occur along the switching surface Ω_{pq} at the boundary of regions Ω_p and Ω_q. According to Filippov's definition [32], we get

$$\begin{aligned} \dot{x}(t) =& \alpha\big(A_p x(t) + B_p x(t-d(t))\big) + \tilde{\alpha}\big(A_q x(t) + B_q x(t-d(t))\big) \\ =& \alpha\Big((A_p + B_p)x(t) - B_p \int_{t-d(t)}^{t} \dot{x}(s)ds\Big) + \tilde{\alpha}\Big((A_q + B_q)x(t) - B_q \int_{t-d(t)}^{t} \dot{x}(s)ds\Big), \end{aligned} \tag{23}$$

where $\alpha \in (0,1)$,$\tilde{\alpha} = 1 - \alpha$. Under the analysis of sliding motions [33], the sliding motions on the surface Ω_{pq} state that

$$\begin{aligned} x^T\Big((A_p + B_p)^T Q_{pq} + Q_{pq}(A_p + B_p)\Big)x(t) - x^T(t) Q_{pq} B_p \int_{t-d(t)}^{t} \dot{x}(s)ds \\ - \int_{t-d(t)}^{t} \dot{x}^T(s)ds B_p^T Q_{pq} x(t) < 0, \end{aligned} \tag{24}$$

and

$$x^T\Big(\big(A_q+B_q\big)^T Q_{pq}+Q_{pq}\big(A_q+B_q\big)\Big)x(t)-x^T(t)Q_{pq}B_q\int_{t-d(t)}^{t}\dot{x}(s)ds \\ -\int_{t-d(t)}^{t}\dot{x}^T(s)dsB_q^T Q_{pq}x(t)>0 \tag{25}$$

hold, where $Q_{pq}=Q_p-Q_q$. Let $P_{qp}=P_q-P_p$. Owing to Condition (8) and $\eta_{p,q}>0$, we obtain

$$x^T\Big(\big(A_p+B_p\big)^T P_{qp}+\big(P_q-P_p\big)\big(A_p+B_p\big)\Big)x(t)-x^T(t)P_{qp}B_p\int_{t-d(t)}^{t}\dot{x}(s)ds \\ -\int_{t-d(t)}^{t}\dot{x}^T(s)dsB_p^T P_{qp}x(t)<0, \tag{26}$$

$$x^T\Big(\big(A_q+B_q\big)^T P_{qp}+\big(P_q-P_p\big)\big(A_q+B_q\big)\Big)x(t)-x^T(t)P_{qp}B_q\int_{t-d(t)}^{t}\dot{x}(s)ds \\ -\int_{t-d(t)}^{t}\dot{x}^T(s)dsB_q^T P_{qp}x(t)>0, \tag{27}$$

which are equivalent to

$$\begin{aligned} & x^T(t)\Big(\big(A_p+B_p\big)^T P_q+P_q\big(A_p+B_p\big)\Big)x(t)-x^T(t)P_qB_p\int_{t-d(t)}^{t}\dot{x}(s)ds \\ & -\int_{t-d(t)}^{t}\dot{x}^T(s)dsB_p^T P_q x(t) \\ <& x^T(t)\Big(\big(A_p+B_p\big)^T P_p+P_p\big(A_p+B_p\big)\Big)x(t)-x^T(t)P_pB_p\int_{t-d(t)}^{t}\dot{x}(s)ds \\ & -\int_{t-d(t)}^{t}\dot{x}^T(s)dsB_p^T P_p x(t), \end{aligned} \tag{28}$$

$$\begin{aligned} & x^T(t)\Big(\big(A_q+B_q\big)^T P_p+P_p\big(A_q+B_q\big)\Big)x(t)-x^T(t)P_pB_q\int_{t-d(t)}^{t}\dot{x}(s)ds \\ & -\int_{t-d(t)}^{t}\dot{x}^T(s)dsB_q^T P_p x(t) \\ <& x^T(t)\Big(\big(A_q+B_q\big)^T P_q+P_q\big(A_q+B_q\big)\Big)x(t)-x^T(t)P_qB_q\int_{t-d(t)}^{t}\dot{x}(s)ds \\ & -\int_{t-d(t)}^{t}\dot{x}^T(s)dsB_q^T P_q x(t). \end{aligned} \tag{29}$$

Note that the switching signal is not unique on sliding surface Ω_{pq}. If $\sigma(t)=p$, one can derive

$$\begin{aligned} & \dot{V}_{p1}(t) \\ =& \alpha x^T(t)\Big(\big(A_p+B_p\big)^T P_p+P_p\big(A_p+B_p\big)\Big)x(t)-\alpha x^T(t)P_pB_p\int_{t-d(t)}^{t}\dot{x}(s)ds \\ & -\alpha\int_{t-d(t)}^{t}\dot{x}^T(s)dsB_p^T P_p x(t)+\tilde{\alpha}x^T(t)\Big(\big(A_q+B_q\big)^T P_p+P_p\big(A_q+B_q\big)\Big)x(t) \\ & -\tilde{\alpha}\bigg(x^T(t)P_pB_q\int_{t-d(t)}^{t}\dot{x}(s)ds+\int_{t-d(t)}^{t}\dot{x}^T(s)dsB_q^T P_p x(t)\bigg) \\ \leq& \alpha x^T(t)\Big(\big(A_p+B_p\big)^T P_p+P_p\big(A_p+B_p\big)\Big)x(t)-\alpha x^T(t)P_pB_p\int_{t-d(t)}^{t}\dot{x}(s)ds \end{aligned} \tag{30}$$

$$
\begin{aligned}
&- \alpha \int_{t-d(t)}^{t} \dot{x}^T(s) ds B_p^T P_p x(t) \\
&+ \tilde{\alpha} x^T(t) \Big((A_q + B_q)^T P_q + P_q (A_q + B_q) \Big) x(t) - \tilde{\alpha} \Big(x^T(t) P_q B_q \int_{t-d(t)}^{t} \dot{x}(s) ds \\
&+ \int_{t-d(t)}^{t} \dot{x}^T(s) ds B_q^T P_q x(t) \Big) \\
\leq & \alpha \eta^T(t) e_1^T \Big(\Phi_1^p + d(t) \Theta_1^p \Big) e_1 \eta(t) + \tilde{\alpha} \eta^T(t) e_1^T \Big(\Phi_1^q + d(t) \Theta_1^q \Big) e_1 \eta(t) + \int_{t-d(t)}^{t} \dot{x}^T(s) U \dot{x}(s) ds.
\end{aligned}
$$

Under (7), (10)–(13), (21) and (30), it is easy to deduce that

$$
\dot{V}_p(t) < -\eta^T(t) \Big(\alpha e_1^T Q_p e_1 + \tilde{\alpha} e_1^T Q_q e_1 \Big) \eta(t) \leq 0.
$$

Similarly, when $\sigma(t) = q$, we can also obtain

$$
\begin{aligned}
\dot{V}_{q1}(t) \leq & \alpha \eta^T(t) e_1^T \Big(\Phi_1^p + d(t) \Theta_1^p \Big) e_1 \eta(t) + \tilde{\alpha} \eta^T(t) e_1^T \Big(\Phi_1^q + d(t) \Theta_1^q \Big) e_1 \eta(t) \\
&+ \int_{t-d(t)}^{t} \dot{x}^T(s) U \dot{x}(s) ds,
\end{aligned}
$$

which further yields $\dot{V}_q(t) < 0$. Therefore, the Lyapunov-Krasovskii functional $V_\sigma(t)$ is decreasing when the sliding motions occur on switching surface Ω_{pq}. According to (22) one can deduce that the switched system (1) under the switching rule (3) is also globally asymptotically stable if the sliding motions occur on switching surfaces Ω_{pq} with $\eta_{p,q} > 0$. □

Remark 2. *According to the Proof of Theorem 1, one can see that the chosen Lyapunov functional is function of $x(t)$ and $\dot{x}(t)$. Similar Lyapunov functionals have been employed to establish the stability results for delayed systems [29,30]. This is because such Lyapunov functionals can fully utilize the features of systems. Most noteworthy, the proposed Lyapunov functional can be viewed as a special case of that presented in [29,30].*

Remark 3. *Condition (7) ensures that the time derivate of Lyapunov functional along the trajectory of switched systems is less than zero for each region Ω_p. Condition (8) guarantees that the Lyapunov functional is not increasing when the switching event occurs in the absence of sliding motion. When sliding motions occur, Conditions (7) and (8) can warrant that the time derivate of Lyapunov functional along the trajectory is less than zero when the trajectory slides on the surfaces Ω_{pq}. Condition (9) ensures that the switched system is well-defined.*

Remark 4. *In [15–19], the researchers have also studied the stability of delayed switched systems under state-dependent switchings. However, these results assume that there exists a Hurwitz linear convex combination of $A_p + B_p$(or A_p). Generally speaking, this assumption is rigorous and may not be satisfied in some cases. Obviously, in Theorem 1 we have removed this restriction, which yields that our results are more flexible. Moreover, in the proof of Theorem 1 new inequality (Lemma 1) is employed, which states that Theorem 1 is less conservative.*

Remark 5. *When there exist infinite switching events in an arbitrary time interval, we call it Zeno-behaviors. The switching rule (3) cannot avoid Zeno-behaviors. However, Theorem 1 can also ensure stability when Zeno-behaviors occur. The reasons can be listed as follows: (a) If the switching event does not occur, it is obvious that the time derivate of Lyapunov functional along the trajectory is less than zero. (b) If the switching event occurs, there are two cases. The first one is that the sliding motion does not occur. Obviously, for this case, the Lyapunov functional is not increasing. The second one is that the sliding motions occur. For this case, we have that the time derivate of Lyapunov functional along the trajectory is still less than zero. Although Zeno-behaviors* *may lead to the accumulation of switches in finite time, the Lyapunov functional along the trajectory is always gradually decreasing.*

By restricting $R_1 = R_2 = R$ and $S_1 = S_2 = S$, one can obtain the stability results under Assumption 2.

Theorem 2. *Under Assumption 2, assume that for any $p \in M$, there exist $n \times n$ matrices $P_p > 0$, $R > 0, S > 0, U > 0$, $Q_p = Q_p^T$, positive constants μ_p, θ_p, constants $\eta_{p,q}$, $q \in M$, $q \neq p$, such that Conditions (8) and (9) and*

$$\begin{pmatrix} \Lambda^p + \mu_p e_1^T Q_p e_1 & \sqrt{d} e_1^T P_p B_p \\ \sqrt{d} B_p^T P_p e_1 & -U \end{pmatrix} < 0, \tag{31}$$

where $\Lambda^p = \Lambda_1^p$ with $R_1 = R_2 = R$ and $S_1 = S_2 = S$. Then, the switched system (1) is globally asymptotically stable under the state-dependent switching rule (3), if there is no sliding motion or there exist sliding motions on the switching surface Ω_{pq} with $\eta_{p,q} > 0$.

Due to the existence of the product of unknown scalars and matrices, the conditions in Theorems 1 and 2 are BMIs. Therefore, the standard semi-positive definite programming methods cannot work. One can adopt two strategies to get a feasible solution. The first one is to utilize directly BMI solvers (such as PENBMI) to obtain these undetermined scalars and matrices. The second one, which is similar to [22], is to grid up the unknown scalars μ_p, θ_p and $\eta_{p,q}$. While these parameters are fixed, the BMIs in Theorems 1 and 2 degenerate into ordinary linear matrix inequalities, which can be solved by standard solvers such as lmilab and mosek.

When the switched system (1) is composed of two subsystems, one can set $Q_1 = -Q_2 = Q = Q^T$, $\eta_{1,2} = \eta_{2,1} = \eta$, $P_2 = P$, $P_1 = P - 2\eta Q$, constants $\theta_1 = \theta_2 = 1$. Then, Conditions (8) and (9) are always satisfied. The following corollaries can be derived readily from Theorems 1 and 2.

Corollary 1. *When $M = \{1, 2\}$, under Assumption 1, assume that there exist $n \times n$ matrices $P > 0$, $R_i > 0, S_i > 0, U > 0$, $(i = 1, 2)$, $Q = Q^T$, positive constants μ_1, μ_2, constant η, such that*

$$\begin{pmatrix} \Lambda_l^{1*} + \mu_1 e_1^T Q e_1 & \sqrt{d} e_1^T (P - 2\eta Q) B_1 \\ \sqrt{d} B_1^T (P - 2\eta Q) e_1 & -U \end{pmatrix} < 0, \tag{32}$$

$$\begin{pmatrix} \Lambda_l^{2*} - \mu_2 e_1^T Q e_1 & \sqrt{d} e_1^T P B_2 \\ \sqrt{d} B_2^T P e_1 & -U \end{pmatrix} < 0, l = 1, 2, \tag{33}$$

where

$$\begin{aligned}
\Lambda_1^{1*} =& \Phi_1^{1*} + \Phi_2 + \Phi_3^1 + \Phi_4^1 + \left(1 - \bar{d}\right)(\Psi_2 + \Psi_3) - \frac{1}{d}\Xi_4, \\
\Lambda_1^{2*} =& \Phi_1^{2*} + \Phi_2 + \Phi_3^2 + \Phi_4^2 + \left(1 - \bar{d}\right)(\Psi_2 + \Psi_3) - \frac{1}{d}\Xi_4, \\
\Lambda_2^{1*} =& \Phi_1^{1*} + \Phi_2 + \Phi_3^1 + \Phi_4^1 + \left(1 - \tilde{d}\right)(\Psi_2 + \Psi_3) - \frac{1}{d}\Xi_4, \\
\Lambda_2^{2*} =& \Phi_1^{2*} + \Phi_2 + \Phi_3^2 + \Phi_4^2 + \left(1 - \tilde{d}\right)(\Psi_2 + \Psi_3) - \frac{1}{d}\Xi_4, \\
\Phi_1^{1*} =& e_1^T \left((A_1 + B_1)^T (P - 2\eta Q) + (P - 2\eta Q)\, (A_1 + B_1) \right) e_1, \\
\Phi_1^{2*} =& e_1^T \left((A_2 + B_2)^T P + P (A_2 + B_2) \right) e_1,
\end{aligned}$$

and the other notations are in agreement with the ones presented in Theorem 1. Then, the switched system (1) is globally asymptotically stable under the state-dependent switching rule (3) if there is no sliding motion or there exist sliding motions on switching surfaces with $\eta > 0$.

Corollary 2. *When $M = \{1, 2\}$, under Assumption 2, assume that there exist $n \times n$ matrices $P > 0$, $R > 0$, $S > 0$, $U > 0$, $Q = Q^T$, positive constants μ_1, μ_2, constant η, such that*

$$\begin{pmatrix} \bar{\Lambda}^{1*} + \mu_1 e_1^T Q e_1 & \sqrt{d} e_1^T (P - 2\eta Q) B_1 \\ \sqrt{d} B_1^T (P - 2\eta Q) e_1 & -U \end{pmatrix} < 0, \tag{34}$$

$$\begin{pmatrix} \bar{\Lambda}^{2*} - \mu_2 e_1^T Q e_1 & \sqrt{d} e_1^T P B_2 \\ \sqrt{d} B_2^T P e_1 & -U \end{pmatrix} < 0, \tag{35}$$

where $\bar{\Lambda}^{1*} = \Lambda_1^{1*}$, $\bar{\Lambda}^{2*} = \Lambda_1^{2*}$ with $R_1 = R_2 = R$ and $S_1 = S_2 = S$. Then, the switched system (1) is globally asymptotically stable under the state-dependent switching rule (3), if there is no sliding motion or there exist sliding motions on switching surfaces with $\eta > 0$.

4. Numerical Simulations

In this section, several numerical examples are employed to illustrate the validity of the proposed results.

Example 1. *Consider the switched system (1) with $M = \{1, 2\}$ and*

$$A_1 = \begin{pmatrix} 0.8 & -4 \\ 0 & 0.8 \end{pmatrix}, B_1 = \begin{pmatrix} 0.2 & -1 \\ 0 & 0.2 \end{pmatrix}, A_2 = \begin{pmatrix} 0.8 & 0 \\ 4 & 0.9 \end{pmatrix}, B_2 = \begin{pmatrix} 0.2 & 0 \\ 1 & 0.1 \end{pmatrix}.$$

By choosing $\mu_1 = \mu_2 = 1$, $\eta = -0.7$ and letting $\bar{d} = -\tilde{d} = \delta$, according to Corollaries 1 and 2, we can obtain the upper bound d for different δ, which is given in Table 1 (in order to avoid zero solution, the matrix inequalities $P, R_i, S_i, U > aI$ with $a = 10^{-7}$ are employed to replace $P, R_i, S_i, U > 0$, $i = 1, 2$). For numerical simulation, we choose $d(t) = 0.1 + 0.1\sin(10t)$, which shows $d = 0.2$ and $\bar{d} = -\tilde{d} = 1$. By solving the matrix inequalities in Corollary 1, we get

$$Q_1 = -Q_2 = Q = \begin{pmatrix} -0.2567 & 0.1996 \\ 0.1996 & 0.2565 \end{pmatrix}, P_1 = P - 2\eta Q = \begin{pmatrix} 0.0935 & 0.1402 \\ 0.1402 & 4516 \end{pmatrix},$$
$$P_2 = P = \begin{pmatrix} 0.4528 & -0.1393 \\ -0.1393 & 0.0925 \end{pmatrix}.$$

The stable dynamics and convergent time response curves with $\phi(s) = (-1, 2)^T$, $s = [-0.2, 0]$, are plotted in Figure 1. The corresponding switching rule (3) is also shown in the sub-figure of Figure 1. Numerical simulations indicate that there is no sliding motion.

Now we give some comparisons with the existing results for this example to validate the superiority of our results.

(a) Note that for any $\alpha \in [0, 1]$, the eigenvalues of $\alpha(A_1 + B_1) + (1 - \alpha)(A_2 + B_2)$ are $1 \pm 5i\sqrt{\alpha(1 - \alpha)}$, which yields that there is no Hurwitz linear convex combination of $A_1 + B_1$ and $A_2 + B_2$. Therefore, the stability results proposed in [15–17,19] are not available for this example. Additionally, the eigenvalues of $\alpha A_1 + (1 - \alpha)A_2$ are $0.85 + 0.05\alpha \pm 0.5\sqrt{64.01\alpha^2 - 63.98\alpha - 0.01}$, $\alpha \in [0, 1]$, which indicates that there is no Hurwitz linear convex combination of matrices A_1 and A_2. This shows that the stability results in [18] are also invalid for this example.

(b) The results derived in [20,21] are also applicable for the switched system (1). For comparison, by restricting $d = 0.01$ we solve the stability conditions in ([20] Corollary 2) and ([21] Theorem 3.1) for $\bar{d} = 0, 0.1, 0.2, 0.5, 0.8$ and 1, respectively. Unfortunately, there is no feasible solution, which demonstrates that the results in [20,21] are not flexible for this example.

(c) *For the case of constant time delay, by solving the matrix inequalities in ([27] Theorem 5), one can get the upper bound $d = 0.2455$, which is also less than $d = 0.2489$. Therefore, the restriction on the time delay of our results is weaker than that proposed in ([27] Theorem 5).*

Table 1. The upper bound d of time delay for different $\bar{d} = -\tilde{d} = \delta$.

0	0.1	0.2	0.5	0.8	1	δ is Unknown
0.2489	0.2445	0.2417	0.2411	0.2411	0.2411	0.2411

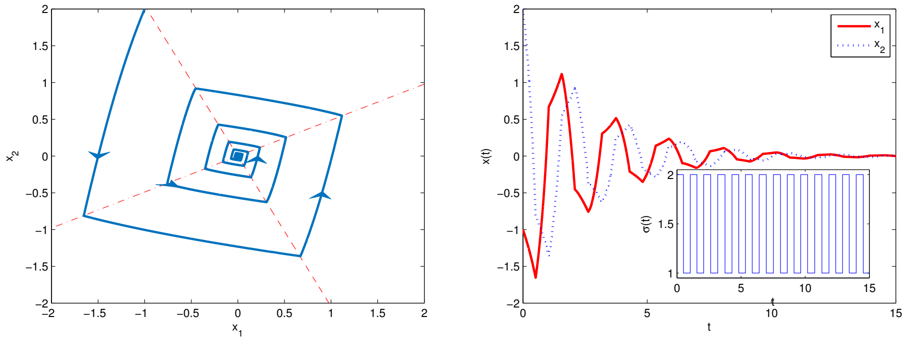

Figure 1. The stable dynamics (**Left**) and convergent response curves (**Right**) of the system in Example 1 with $d(t) = 0.1 + 0.1\sin(10t)$.

Example 2. *Consider the switched system (1) wiht $M = \{1,2\}$, $d(t) = 0.01 + 0.01\sin(50t)$, and*

$$A_1 = \begin{pmatrix} 0 & 1 \\ 2 & -8 \end{pmatrix}, B_1 = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 0.5 \\ -2 & 1 \end{pmatrix}, B_2 = \begin{pmatrix} 0 & 0.5 \\ 0 & 1 \end{pmatrix}.$$

It is easy to derive that $d = 0.02$, $\bar{d} = -\tilde{d} = 0.5$. By choosing $\eta = 0.01$, $\mu_1 = \mu_2 = 1$, according to Corollary 1, we get the following feasible solution

$$Q_1 = -Q_2 = Q = \begin{pmatrix} -0.02252 & 0.0251 \\ 0.0251 & 0.0287 \end{pmatrix}, P_1 = P - 2\eta Q = \begin{pmatrix} 0.0202 & 0.1940 \\ 0.0055 & 0.0028 \end{pmatrix},$$
$$P_2 = P = \begin{pmatrix} 0.0198 & 0.0060 \\ 0.0060 & 0.0034 \end{pmatrix}.$$

The sliding dynamics and stable response curves are shown in Figure 2. Numerical simulations indicate that there are sliding motions, which can make the trajectory approach the origin along the switching surfaces.

If we choose $\eta = -1$, $\mu_1 = \mu_2 = 1$, by solving the matrix inequalities in Corollary 1, we obtain

$$Q_1 = -Q_2 = Q = \begin{pmatrix} 0.0127 & -0.0964 \\ -0.0964 & 0.4049 \end{pmatrix}, P_1 = P - 2\eta Q = \begin{pmatrix} 0.0397 & -0.1802 \\ -0.1802 & 0.8545 \end{pmatrix},$$
$$P_2 = P = \begin{pmatrix} 0.0143 & 0.0125 \\ 0.0125 & 0.0446 \end{pmatrix}.$$

Numerical simulations show that there are unstable sliding motions for this case (see Figure 3), which is due to $\eta_{1,2} = \eta_{2,1} = \eta < 0$. This demonstrates that $\eta_{p,q} > 0$ is essential for the stability of the switched system (1) when sliding motions occur.

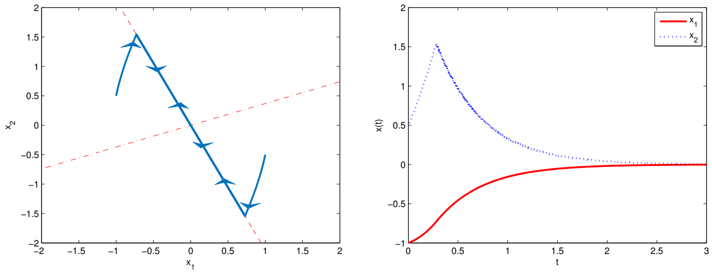

Figure 2. The stable dynamics (**Left**) and convergent response curves (**Right**) of the system in Example 2.

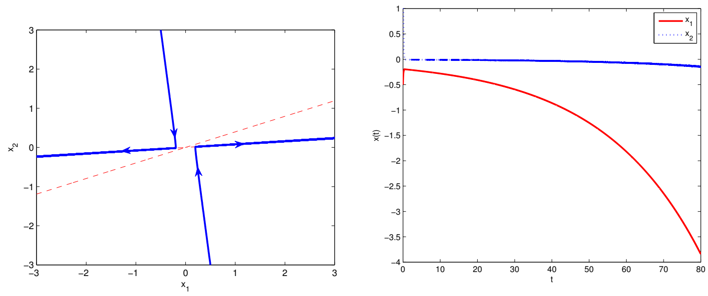

Figure 3. The unstable dynamics (**Left**) and unstable response curves (**Right**) of the system in Example 2.

5. Conclusions

This paper has investigated the stability of delayed switched systems with all unstable subsystems. Under the designed state-dependent switching rule, some stability results for different assumptions on time delay are derived via integral inequality and multiple Lyapunov-Krasovskii functionals. Numerical simulations demonstrate that the proposed results are more effective and less conservative than that presented in [15–21,27].

The main deficiency of this paper is that the condition that determines whether sliding motions occur is not employed. As a matter of fact, similar to [21,22], we have derived some conditions to verify the existence or non-existence of sliding motions. Unfortunately, if we introduce these conditions to the stability results, it is difficult to get a feasible solution. In desperation, we adopt the way which is used in [34,35]. Namely, the condition to determine whether sliding motions occur is not given and the existence or non-existence of sliding motions is revealed via numerical simulation. We hope some more feasible conditions on sliding motions can be deduced in the future.

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