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Highly Dispersive Optical Solitons in Fiber Bragg Gratings with Kerr Law of Nonlinear Refractive Index

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Abstract: This paper obtains highly dispersive optical solitons in fiber Bragg gratings with the Kerr law of a nonlinear refractive index. The generalized Kudryashov's approach as well as its newer version makes this retrieval possible. A full spectrum of solitons is thus recovered.

1. Introduction

One of the newly developed concepts in nonlinear optics, applicable to a variety of optoelectronic devices, is highly dispersive (HD) solitons. This emerges out of dire necessity when chromatic dispersion (CD) runs low. Thus, to replenish this low count, additional dispersion terms are taken into consideration. These are sixth-order dispersion (6OD); fifth-order dispersion (5OD); fourth-order dispersion (4OD); third-order dispersion (3OD); and inter–modal dispersion (IMD). The effect of soliton radiation, with such higher order dispersion terms to offset the low count of CD, is neglected to keep the model simple. Other means to compensate for the low count of CD is to introduce Bragg gratings in the fiber structure so that the dispersive reflectivity that it produces additionally replenishes this low count [1–32]. The current paper is the first of its kind to include both effects to offset this low CD. Such a model would also lead to soliton solutions.

The model would, therefore, be handled with the Kerr law of nonlinearity. The method of integrability would be two-fold and both due to Kudryashov. The first approach is the generalized Kudryashov's approach, followed by the lately developed enhanced

Kudryashov's scheme [9–16]. These two approaches can collectively yield a full spectrum of solitons, which are recovered and enumerated in the present paper. The parametric restrictions, also known as certain conditions, are extracted for the solitons to exist. The remaining details are presented in the rest of the paper via the unique integration tools that are discussed.

Governing Model

The perturbed HD nonlinear Schrödinger's equation is firstly introduced as below:

$$
\begin{aligned}
i\varphi_t + ia_1\varphi_x + a_2\varphi_{xx} + ia_3\varphi_{xxx} + a_4\varphi_{xxxx} + ia_5\varphi_{xxxxx} + a_6\varphi_{xxxxxx} + b|\varphi|^2\varphi \\
= i\big[\lambda\big(|\varphi|^2\varphi\big)_x + \mu\big(|\varphi|^2\big)_x\varphi + \theta|\varphi|^2\varphi_x\big],
\end{aligned}
\tag{1}
$$

such that θ, μ, λ, b and a_l, ($l = 1$–6) depict real-valued constant parameters, whereas $\varphi(x,t)$ purports a complex-valued function. Setting $\lambda = \mu = \theta = 0$ extracts the governing equation [16]. a_1 comes from the IMD, a_2 implies to the CD, a_3 is related to the 3OD, a_4 stems from the 4OD, a_5 purports the 5OD and a_6 stands for the 6OD. The first term arises from the temporal evolution, where $i = \sqrt{-1}$. μ and θ yield the nonlinear dispersions, b arises from Kerr law nonlinearity, λ comes from the self-steepening (SS) and $\varphi = \varphi(x,t)$ purports the soliton wave.

For the first time in fiber Bragg gratings, the strategic governing model derived from (1) reads as

$$
\begin{aligned}
iU_t + ia_{11}V_x + a_{12}V_{xx} + ia_{13}V_{xxx} + a_{14}V_{xxxx} + ia_{15}V_{xxxxx} + a_{16}V_{xxxxxx} \\
+ \big(b_{11}|U|^2 + b_{12}|V|^2\big)U + i\alpha_1 U_x + \beta_1 V + \sigma_1 U^* V^2 \\
= i\big[\lambda_1\big(|U|^2U\big)_x + \mu_1\big(|U|^2\big)_x U + \theta_1|U|^2 U_x\big],
\end{aligned}
\tag{2}
$$

and

$$
\begin{aligned}
iV_t + ia_{21}U_x + a_{22}U_{xx} + ia_{23}U_{xxx} + a_{24}U_{xxxx} + ia_{25}U_{xxxxx} + a_{26}U_{xxxxxx} \\
+ \big(b_{21}|V|^2 + b_{22}|U|^2\big)V + i\alpha_2 V_x + \beta_2 U + \sigma_2 V^* U^2 \\
= i\big[\lambda_2\big(|V|^2V\big)_x + \mu_2\big(|V|^2\big)_x V + \theta_2|V|^2 V_x\big],
\end{aligned}
\tag{3}
$$

such that σ_j, λ_j, μ_j, α_j, β_j, θ_j, b_{j1}, b_{j2} and a_{jl}, ($1 \le l \le 6$, $j = 1,2$) depict real-valued constant parameters, whereas $V(x,t)$ and $U(x,t)$ purport complex-valued functions. a_{j1} and α_j come from the IMD, a_{j2} imply the CD, a_{j3} are related to the 3OD, a_{j4} stem from the 4OD, a_{j5} purport the 5OD and a_{j6} stand for the 6OD. The first terms arise from the temporal evolution, where $i = \sqrt{-1}$. μ_j and θ_j yield the nonlinear dispersions, σ_j denote the four-wave mixing, b_{j1} arise from the self-phase modulation, β_j signify the detuning parameters, b_{j2} denote the cross-phase modulation, λ_j signify the SS, whilst $V(x,t)$ and $U(x,t)$ purport the soliton waves.

2. Mathematical Analysis

The governing model admits the analytical solutions

$$
U(x,t) = g_1(\xi)\exp[i\Omega(x,t)],
$$

$$
V(x,t) = g_2(\xi)\exp[i\Omega(x,t)],
\tag{4}
$$

such that

$$
\Omega(x,t) = -\kappa x + \omega t + \theta_0, \quad \xi = x - vt.
\tag{5}
$$

Here, $g_j(\xi)$ and $\Omega(x,t)$ signify real-valued functions, whereas v, k, ω and θ_0 purport real-valued constants. For the soliton wave, $\Omega(x,t)$ depicts the phase component, ξ depicts the wave variable, θ_0 arises from the phase constant, v stems from the velocity, ω denotes the wave number, $g_j(\xi)$ come from the amplitude components and κ depicts the frequency.

Placing (4) and (5) into (2) and (3) extracts the strategic equations

$$a_{16}g_2^{(6)} + \left(a_{14} - 5a_{15}\kappa - 15a_{16}\kappa^2\right)g_2^{(4)}$$
$$+ \left(a_{12} + 3a_{13}\kappa - 6a_{14}\kappa^2 - 10a_{15}\kappa^3 + 15a_{16}\kappa^4\right)g_2''$$
$$+ (\alpha_1 k - \omega)g_1 + \left(\beta_1 - a_{12}\kappa^2 + a_{11}\kappa + a_{14}\kappa^4 - a_{13}\kappa^3 - a_{16}\kappa^6 + a_{15}\kappa^5\right)g_2$$
$$+ [b_{11} - \kappa(\lambda_1 + \theta_1)]g_1^3 + (b_{12} + \sigma_1)g_1 g_2^2 = 0, \tag{6}$$

$$a_{26}g_1^{(6)} + \left(a_{24} - 5a_{25}\kappa - 15a_{26}\kappa^2\right)g_1^{(4)}$$
$$+ \left(a_{22} + 3a_{23}\kappa - 6a_{24}\kappa^2 - 10a_{25}\kappa^3 + 15a_{26}\kappa^4\right)g_1''$$
$$+ (\alpha_2 k - \omega)g_2 + \left(a_{21}\kappa + \beta_2 - a_{23}\kappa^3 - a_{22}\kappa^2 + a_{25}\kappa^5 + a_{24}\kappa^4 - a_{26}\kappa^6\right)g_1$$
$$+ [b_{21} - \kappa(\lambda_2 + \theta_2)]g_2^3 + (b_{22} + \sigma_2)g_1^2 g_2 = 0, \tag{7}$$

$$\left(a_{15} - 6a_{16}\kappa\right)g_2^{(5)} + \left(a_{13} - 4a_{14}\kappa - 10a_{15}\kappa^2 + 20a_{16}\kappa^3\right)g_2'''$$
$$- [3\lambda_1 + 2\mu_1 + \theta_1]g_1^2 g_1' + (\alpha_1 - v)g_1'$$
$$+ \left(a_{11} - 2a_{12}\kappa - 3a_{13}\kappa^2 + 4a_{14}\kappa^3 + 5a_{15}\kappa^4 - 6a_{16}\kappa^5\right)g_2' = 0, \tag{8}$$

$$\left(a_{25} - 6a_{26}\kappa\right)g_1^{(5)} + \left(a_{23} - 4a_{24}\kappa - 10a_{25}\kappa^2 + 20a_{26}\kappa^3\right)g_1'''$$
$$- [3\lambda_2 + 2\mu_2 + \theta_2]g_2^2 g_2' + (\alpha_2 - v)g_2'$$
$$+ \left(a_{21} - 2a_{22}\kappa - 3a_{23}\kappa^2 + 4a_{24}\kappa^3 + 5a_{25}\kappa^4 - 6a_{26}\kappa^5\right)g_1' = 0. \tag{9}$$

Set

$$g_2(\xi) = \Pi g_1(\xi), \quad \Pi \neq 0, \quad \Pi \neq 1, \tag{10}$$

where Π depicts real-valued constant parameters. Hence, Equations (6)–(9) appear as

$$a_{16}\Pi g_1^{(6)} + \left(a_{14} - 5a_{15}\kappa - 15a_{16}\kappa^2\right)\Pi g_1^{(4)}$$
$$+ \left(a_{12} + 3a_{13}\kappa - 6a_{14}\kappa^2 - 10a_{15}\kappa^3 + 15a_{16}\kappa^4\right)\Pi g_1''$$
$$+ \left[\alpha_1 k - \omega + \left(\beta_1 - a_{12}\kappa^2 + a_{11}\kappa + a_{14}\kappa^4 - a_{13}\kappa^3 - a_{16}\kappa^6 + a_{15}\kappa^5\right)\Pi\right]g_1$$
$$+ \left[b_{11} - \kappa(\lambda_1 + \theta_1) + (b_{12} + \sigma_1)\Pi^2\right]g_1^3 = 0, \tag{11}$$

$$a_{26}g_1^{(6)} + \left(a_{24} - 5a_{25}\kappa - 15a_{26}\kappa^2\right)g_1^{(4)}$$
$$+ \left(a_{22} + 3a_{23}\kappa - 6a_{24}\kappa^2 - 10a_{25}\kappa^3 + 15a_{26}\kappa^4\right)g_1''$$
$$+ \left[(\alpha_2 k - \omega)\Pi + a_{21}\kappa + \beta_2 - a_{23}\kappa^3 - a_{22}\kappa^2 + a_{25}\kappa^5 + a_{24}\kappa^4 - a_{26}\kappa^6\right]g_1$$
$$+ \left[b_{22} + \sigma_2 + b_{21}\Pi^2 - \kappa(\lambda_2 + \theta_2)\Pi^2\right]\Pi g_1^3 = 0, \tag{12}$$

$$\left(a_{15} - 6a_{16}\kappa\right)\Pi g_1^{(5)} + \left(a_{13} - 4a_{14}\kappa - 10a_{15}\kappa^2 + 20a_{16}\kappa^3\right)\Pi g_1'''$$
$$- [3\lambda_1 + 2\mu_1 + \theta_1]g_1^2 g_1'$$
$$+ \left[\alpha_1 - v + \left(a_{11} - 2a_{12}\kappa - 3a_{13}\kappa^2 + 4a_{14}\kappa^3 + 5a_{15}\kappa^4 - 6a_{16}\kappa^5\right)\Pi\right]g_1' = 0, \tag{13}$$

$$\left(a_{25} - 6a_{26}\kappa\right)g_1^{(5)} + \left(a_{23} - 4a_{24}\kappa - 10a_{25}\kappa^2 + 20a_{26}\kappa^3\right)g_1'''$$
$$- [3\lambda_2 + 2\mu_2 + \theta_2]\Pi^3 g_1^2 g_1'$$
$$+ \left[(\alpha_2 - v)\Pi + a_{21} - 2a_{22}\kappa - 3a_{23}\kappa^2 + 4a_{24}\kappa^3 + 5a_{25}\kappa^4 - 6a_{26}\kappa^5\right]g_1' = 0. \tag{14}$$

Equations (13) and (14) yield the certain restrictions

$$\kappa = \frac{a_{j5}}{6a_{j6}}, \tag{15}$$

$$a_{j3} - 4a_{j4}\kappa - 10a_{j5}\kappa^2 + 20a_{j6}\kappa^3 = 0, \tag{16}$$

$$3\lambda_j + 2\mu_j + \theta_j = 0, \tag{17}$$

$$v = -\alpha_1 + \left(a_{11} - 2a_{12}\kappa - 3a_{13}\kappa^2 + 4a_{14}\kappa^3 + 5a_{15}\kappa^4 - 6a_{16}\kappa^5\right)\Pi,$$

$$v = -\alpha_2 + \frac{1}{\Pi}\left(a_{21} - 2a_{22}\kappa - 3a_{23}\kappa^2 + 4a_{24}\kappa^3 + 5a_{25}\kappa^4 - 6a_{26}\kappa^5\right), \tag{18}$$

while Equation (18) extracts the constraint relation

$$\alpha_2 = \frac{\alpha_1\Pi + \left(a_{21} - 2a_{22}\kappa - 3a_{23}\kappa^2 + 4a_{24}\kappa^3 + 5a_{25}\kappa^4 - 6a_{26}\kappa^5\right)}{\Pi} \\ \frac{- \left(a_{11} - 2a_{12}\kappa - 3a_{13}\kappa^2 + 4a_{14}\kappa^3 + 5a_{15}\kappa^4 - 6a_{16}\kappa^5\right)\Pi^2}{\Pi}. \tag{19}$$

Moreover, Equations (11) and (12) admit the strategic constraints

$$\frac{a_{16}\Pi}{a_{26}} = \frac{\left(a_{14}-5a_{15}\kappa-15a_{16}\kappa^2\right)\Pi}{a_{24}-5a_{25}\kappa-15a_{26}\kappa^2}$$
$$= \frac{\left(a_{12}+3a_{13}\kappa-6a_{14}\kappa^2-10a_{15}\kappa^3+15a_{16}\kappa^4\right)\Pi}{a_{22}+3a_{23}\kappa-6a_{24}\kappa^2-10a_{25}\kappa^3+15a_{26}\kappa^4}$$
$$= \frac{\alpha_1 k-\omega+\left(\beta_1+a_{11}\kappa-a_{12}\kappa^2-a_{13}\kappa^3+a_{14}\kappa^4+a_{15}\kappa^5-a_{16}\kappa^6\right)\Pi}{(\alpha_2 k-\omega)\Pi+\beta_2+a_{21}\kappa-a_{22}\kappa^2-a_{23}\kappa^3+a_{24}\kappa^4+a_{25}\kappa^5-a_{26}\kappa^6}$$
$$= \frac{b_{11}-\kappa(\lambda_1+\theta_1)+(b_{12}+\sigma_1)\Pi^2}{\left[b_{22}+\sigma_2+b_{21}\Pi^2-\kappa(\lambda_2+\theta_2)\Pi^2\right]\Pi}, \tag{20}$$

and the certain parametric restrictions

$$\omega = \frac{\Pi\begin{bmatrix}17\kappa^2(a_{26}a_{12}-a_{16}a_{22})-11\kappa^3(a_{16}a_{23}-a_{26}a_{13})\\+20a_{16}(\Pi\alpha_2 k+\beta_2+a_{21}\kappa)-20a_{26}(\beta_1+\kappa a_{11})\end{bmatrix}-20a_{26}\alpha_1 k}{20\left(a_{16}\Pi^2-a_{26}\right)},$$
$$a_{24} = \frac{a_{16}a_{22}+3a_{16}a_{23}\kappa-a_{26}a_{12}-3a_{26}a_{13}\kappa+8a_{26}a_{14}\kappa^2}{8a_{16}\kappa^2},$$
$$a_{25} = \frac{40a_{26}a_{15}\kappa^3+a_{16}a_{22}+3a_{16}a_{23}\kappa-a_{26}a_{12}-3a_{26}a_{13}\kappa}{40a_{16}\kappa^3},$$
$$b_{22} = \frac{\begin{aligned}a_{26}b_{11}-a_{26}\kappa\lambda_1-a_{26}\kappa\theta_1+(a_{16}\kappa\lambda_2-a_{16}b_{21}+a_{16}\kappa\theta_2)\Pi^4\\+(a_{26}b_{12}+a_{26}\sigma_1-a_{16}\sigma_2)\Pi^2\end{aligned}}{a_{16}\Pi^2}. \tag{21}$$

Equation (11) is also extracted as

$$g_1^{(6)}+\Omega_4 g_1^{(4)}+\Omega_2 g_1''+\Omega_1 g_1+\Omega_3 g_1^3 = 0, \tag{22}$$

where

$$\Omega_4 = \frac{a_{14}-5a_{15}\kappa-15a_{16}\kappa^2}{a_{16}},$$
$$\Omega_2 = \frac{a_{12}+3a_{13}\kappa-6a_{14}\kappa^2-10a_{15}\kappa^3+15a_{16}\kappa^4}{a_{16}},$$
$$\Omega_1 = -\frac{\alpha_1 k-\omega+\left(\beta_1-a_{12}\kappa^2+a_{11}\kappa+a_{14}\kappa^4-a_{13}\kappa^3-a_{16}\kappa^6+a_{15}\kappa^5\right)\Pi}{a_{16}\Pi},$$
$$\Omega_3 = \frac{b_{11}-\kappa(\lambda_1+\theta_1)+(b_{12}+\sigma_1)\Pi^2}{a_{16}\Pi}. \tag{23}$$

From the standpoint of electromagnetic theory, Equations (1)–(3) are a far cry from the basic alphabets of electromagnetic theory, namely Maxwell's equation. It is well known that Maxwell's equation led to the derivation of the nonlinear Schrodinger's equation (NLSE) with the Kerr law of nonlinear refractive index by the aid of multiple scales. This is alternatively known as the cubic Schrodinger's equation. It is interesting to point out here that NLSE is a special case of the Schrodinger's equation that appears in Quantum Mechanics when the potential function is the intensity of light. This so happens since the refractive index of light is intensity dependent. Thus, there exists a close proximity between Schrodinger's equation in Quantum Mechanics and NLSE in Quantum Optics. The extended or perturbed version of NLSE is also derived from Maxwell's equation with the inclusion of higher order perturbation terms. These are typically some of the Hamiltonian type of perturbation terms that would include self-steepening effect, self-frequency shift, inter-modal dispersion, detuning effect, and others.

Later, it was realized that the CD alone turns out to be insufficient to maintain the much-needed delicate balance between CD and self-phase modulation (SPM) because of its depletion with trans-continental and trans-oceanic distance soliton transmission through optical fibers. This would lead to a catastrophic pulse collapse. Thus, to circumvent this situation, the concept of HD solitons was conceived a couple of years ago where the low count of CD would be supplemented with higher order dispersion terms. Another engineering marvel that was proposed a couple of decades ago is the introduction of the gratings structure by Bragg, which would lead to the arrest of the pulse collapse and introduce dispersive reflectivity which would maintain the necessary balance between CD and SPM. The current paper is a combination of both, namely introducing HD solitons as well as Bragg grating's structure to ensure the uninterrupted long-distance transmission of

solitons. Thus, Equations (2) and (3) can be derived from (1), just as the coupled equation for birefringent fibers are derived from the scalar version of the NLSE. Here, in (2) and (3), the variables U and V represent the forward and backward propagating waves in the cubic nonlinear core.

In this paper, the higher order dispersion terms as well as the nonlinear dispersion due to θ_j (j = 1, 2) are all taken to be strong dispersion. This would only slow down the soliton of the soliton and would introduce some constraints or connectivity between these dispersions and other Hamiltonian perturbation parameters. These are reflected in relations (15)–(17) and the velocity slowdown is reflected in (18) along the two core components. However, the integrability of model (2) and (3) would not be affected. Evidently, these dispersion terms would introduce a considerable amount of soliton radiation. This effect is discarded in the current paper since the study of soliton radiation falls in the continuous regime and can be handled as a separate project with the usage of the variational principle or the method of moments, or even by the theory of unfoldings. Finally, if the dispersive effect was taken to be weak, it would lead to the emergence of quasi-monochromatic solitons that can be recovered only with the usage of multiple scales [21]. However, again, this is outside the scope of the current work.

While the governing equation with Hamiltonian perturbation terms is integrable with the application of the inverse scattering transform which would have additionally revealed soliton radiation effects analytically, this paper focuses on the retrieval of bound state solitons only by the aid of the generalized Kudryashov's approach and the enhanced Kudryashov's method. The details of the retrieval of solitons using these two algorithms are presented in the subsequent sections.

3. Generalized Kudryashov's Method

The integration technique satisfies the analytical solution

$$g_1(\xi) = \frac{\sum_{k=0}^{N} A_k F^k(\xi)}{\sum_{h=0}^{M} B_h F^h(\xi)}, \quad A_N \neq 0, \ B_M \neq 0, \tag{24}$$

such that $F(\xi)$ admits the ancillary equation

$$F'(\xi) = F(\xi)[F(\xi) - 1] \ln H, \ 0 < H \neq 1, \tag{25}$$

and the explicit solutions

$$F(\xi) = \frac{1}{1 + \varepsilon \exp_H(\xi)}, \tag{26}$$

$$F(\xi) = \frac{1}{1 + \varepsilon[\cosh(\xi \ln H) + \sinh(\xi \ln H)]}. \tag{27}$$

Here, $\varepsilon = \pm 1$, $\exp_H(\xi) = H^{(\xi)}$, A_k ($k = 1 - N$) and B_h ($h = 1 - M$) denote constants, whereas N and M arise from the balance principle.

Setting $\varepsilon = 1$, Equation (27) evolves as the dark soliton

$$F(\xi) = \frac{1}{2}\left[1 - \tanh\left(\frac{1}{2}\xi \ln H\right)\right], \tag{28}$$

whilst setting $\varepsilon = -1$, Equation (27) yields the singular soliton

$$F(\xi) = \frac{1}{2}\left[1 - \coth\left(\frac{1}{2}\xi \ln H\right)\right]. \tag{29}$$

Balancing g_1^3 with $g_1^{(6)}$ extracts the restriction

$$N - M + 6 = 3(N - M) \implies N = 3 + M. \tag{30}$$

When $M = 1$, Equation (24) reads as

$$g_1(\xi) = \frac{A_4 F^4(\xi) + A_3 F^3(\xi) + A_2 F^2(\xi) + A_1 F(\xi) + A_0}{B_1 F(\xi) + B_0}, \quad A_4 \neq 0, \quad B_1 \neq 0. \tag{31}$$

Placing (31) with the usage of (25) into (22) leaves us the results

$$A_4 = 24B_1 \sqrt{-\frac{35}{\Omega_3}} \ln^3 H, \quad A_3 = 0, \quad A_2 = -54B_1 \sqrt{-\frac{35}{\Omega_3}} \ln^3 H,$$

$$A_1 = 6B_1 \sqrt{-\frac{35}{\Omega_3}} \ln^3 H, \quad A_0 = 9B_1 \sqrt{-\frac{35}{\Omega_3}} \ln^3 H, \quad B_1 = B_1, \quad B_0 = \tfrac{3}{2} B_1, \tag{32}$$

$$\Omega_4 = -83 \ln^2 H, \quad \Omega_2 = 946 \ln^4 H, \quad \Omega_1 = 1260 \ln^6 H, \quad \Omega_3 < 0. \tag{33}$$

Inserting (32) together with (27)–(29) into (31) acquires the explicit solutions:

(I) The combo bright-singular soliton solutions:

$$U(x,t) = \pm 9 \sqrt{-\frac{35}{\Omega_3}} \left(\ln^3 H\right) \left[1 + \frac{4 - 6 \left\{ \begin{array}{l} 1 + \varepsilon \sinh[(x-vt)\ln H] \\ +\varepsilon \cosh[(x-vt)\ln H] \end{array} \right\}}{\left\{ \begin{array}{l} 1 + \varepsilon \sinh[(x-vt)\ln H] \\ +\varepsilon \cosh[(x-vt)\ln H] \end{array} \right\}^3} \right]$$
$$\times \exp[i(-\kappa x + \omega t + \theta_0)], \tag{34}$$

$$V(x,t) = \pm 9\Pi \sqrt{-\frac{35}{\Omega_3}} \left(\ln^3 H\right) \left[1 + \frac{4 - 6 \left\{ \begin{array}{l} 1 + \varepsilon \sinh[(x-vt)\ln H] \\ +\varepsilon \cosh[(x-vt)\ln H] \end{array} \right\}}{\left\{ \begin{array}{l} 1 + \varepsilon \sinh[(x-vt)\ln H] \\ +\varepsilon \cosh[(x-vt)\ln H] \end{array} \right\}^3} \right]$$
$$\times \exp[i(-\kappa x + \omega t + \theta_0)]. \tag{35}$$

(II) The singular soliton solutions:

$$U(x,t) = \pm 3 \sqrt{-\frac{35}{\Omega_3}} \left(\ln^3 H\right) \left\{ \coth^2 \left[\tfrac{1}{2}(x-vt)\ln H \right] - 3 \right\} \coth \left[\tfrac{1}{2}(x-vt)\ln H \right]$$
$$\times \exp[i(-\kappa x + \omega t + \theta_0)], \tag{36}$$

$$V(x,t) = \pm 3\Pi \sqrt{-\frac{35}{\Omega_3}} \left(\ln^3 H\right) \left\{ \coth^2 \left[\tfrac{1}{2}(x-vt)\ln H \right] - 3 \right\} \coth \left[\tfrac{1}{2}(x-vt)\ln H \right]$$
$$\times \exp[i(-\kappa x + \omega t + \theta_0)]. \tag{37}$$

(III) The dark soliton solutions:

$$U(x,t) = \pm 3 \sqrt{-\frac{35}{\Omega_3}} \left(\ln^3 H\right) \left\{ \tanh^2 \left[\tfrac{1}{2}(x-vt)\ln H \right] - 3 \right\} \tanh \left[\tfrac{1}{2}(x-vt)\ln H \right]$$
$$\times \exp[i(-\kappa x + \omega t + \theta_0)], \tag{38}$$

$$V(x,t) = \pm 3\Pi \sqrt{-\frac{35}{\Omega_3}} \left(\ln^3 H\right) \left\{ \tanh^2 \left[\tfrac{1}{2}(x-vt)\ln H \right] - 3 \right\} \tanh \left[\tfrac{1}{2}(x-vt)\ln H \right]$$
$$\times \exp[i(-\kappa x + \omega t + \theta_0)]. \tag{39}$$

4. Enhanced Kudryashov's Method

The integration algorithm admits the explicit solution

$$g_1(\xi) = \sum_{j=0}^{N} K_j Z^j(\xi), \quad K_N \neq 0, \tag{40}$$

such that $Z(\xi)$ holds the ancillary equation

$$Z'^2(\xi) = Z^2(\xi) \left[1 - \pi Z^{2s}(\xi) \right] \ln^2 H, \quad 0 < H \neq 1, \tag{41}$$

and the analytical solution

$$Z(\xi) = \left[\frac{4\eta}{(4\eta^2 - \pi)\sinh(s\xi \ln H) + (4\eta^2 + \pi)\cosh(s\xi \ln H)} \right]^{\frac{1}{s}}. \tag{42}$$

Here π, K_j $(j = 0 - N)$, s and η depict real-valued constant parameters. Balancing g_1^3 and $g_1^{(6)}$ in (22) secures the certain restriction

$$3N = N + 6s \implies N = 3s. \tag{43}$$

Case 1 : When $s = 1$, Equation (40) evolves as

$$g_1(\xi) = K_3 Z^3(\xi) + K_2 Z^2(\xi) + K_1 Z(\xi) + K_0, \quad K_3 \neq 0. \tag{44}$$

Inserting (44) with the help of (41) into (22) leaves us the results:
Result 1:

$$K_3 = -24\pi\sqrt{\frac{35\pi}{\Omega_3}} \ln^3 H, \quad K_2 = 0, \quad K_1 = \frac{288}{17}\sqrt{\frac{35\pi}{\Omega_3}} \ln^3 H, \quad K_0 = 0, \tag{45}$$

$$\Omega_4 = \frac{581}{17} \ln^2 H, \quad \Omega_2 = \frac{92659}{289} \ln^4 H, \quad \Omega_1 = -\frac{102825}{289} \ln^6 H, \quad \pi\Omega_3 > 0. \tag{46}$$

Plugging (45) with the usage of (42) into (44) formulates the combo solitons

$$U(x,t) = \pm 24\sqrt{\frac{35\pi}{\Omega_3}}\left(\ln^3 H\right)\left(\frac{4\eta}{\begin{array}{c}(4\eta^2 - \pi)\sinh[(x - vt)\ln H]\\ +(4\eta^2 + \pi)\cosh[(x - vt)\ln H]\end{array}} \right)$$
$$\times \left\{ \frac{12}{17} - \pi\left(\frac{4\eta}{(4\eta^2 - \pi)\sinh[(x-vt)\ln H] + (4\eta^2 + \pi)\cosh[(x-vt)\ln H]} \right)^2 \right\} \times \exp[i(-\kappa x + \omega t + \theta_0)], \tag{47}$$

$$V(x,t) = \pm 24\Pi\sqrt{\frac{35\pi}{\Omega_3}}\left(\ln^3 H\right)\left(\frac{4\eta}{\begin{array}{c}(4\eta^2 - \pi)\sinh[(x - vt)\ln H]\\ +(4\eta^2 + \pi)\cosh[(x - vt)\ln H]\end{array}} \right)$$
$$\times \left\{ \frac{12}{17} - \pi\left(\frac{4\eta}{(4\eta^2 - \pi)\sinh[(x-vt)\ln H] + (4\eta^2 + \pi)\cosh[(x-vt)\ln H]} \right)^2 \right\} \times \exp[i(-\kappa x + \omega t + \theta_0)]. \tag{48}$$

When $\Omega_3 > 0$ and $\pi = 4\eta^2$, the bright solitons evolve as

$$U(x,t) = \pm\frac{24}{17}\sqrt{\frac{35}{\Omega_3}}\left(\ln^3 H\right)\left\{12 - 17\, sech^2[(x - vt)\ln H]\right\}$$
$$\times sech\,[(x - vt)\ln H] \exp[i(-\kappa x + \omega t + \theta_0)], \tag{49}$$

$$V(x,t) = \pm\frac{24}{17}\Pi\sqrt{\frac{35}{\Omega_3}}\left(\ln^3 H\right)\left\{12 - 17\, sech^2[(x - vt)\ln H]\right\}$$
$$\times sech\,[(x - vt)\ln H] \exp[i(-\kappa x + \omega t + \theta_0)], \tag{50}$$

where as setting $\Omega_3 < 0$ and $\pi = -4\eta^2$ secures the singular solitons

$$U(x,t) = \pm\frac{24}{17}\sqrt{-\frac{35}{\Omega_3}}\left(\ln^3 H\right)\left\{12 + 17\, csch^2[(x - vt)\ln H]\right\}$$
$$\times csch\,[(x - vt)\ln H] \exp[i(-\kappa x + \omega t + \theta_0)], \tag{51}$$

$$V(x,t) = \pm\frac{24}{17}\Pi\sqrt{-\frac{35}{\Omega_3}}\left(\ln^3 H\right)\left\{12 + 17\, csch^2[(x - vt)\ln H]\right\}$$
$$\times csch\,[(x - vt)\ln H] \exp[i(-\kappa x + \omega t + \theta_0)]. \tag{52}$$

Result 2:

$$K_3 = 24\pi\sqrt{\frac{35\pi}{\Omega_3}} \ln^3 H, \quad K_2 = 0, \quad K_1 = 0, \quad K_0 = 0, \tag{53}$$

$$\Omega_4 = -83 \ln^2 H, \quad \Omega_2 = 1891 \ln^4 H, \quad \Omega_1 = -11025 \ln^6 H, \quad \pi\Omega_3 > 0. \tag{54}$$

Placing (53) with the help of (42) into (44) formulates the combo solitons

$$U(x,t) = \pm 24\pi\sqrt{\frac{35\pi}{\Omega_3}} \left(\frac{4\eta \ln H}{(4\eta^2 - \pi)\sinh[(x-vt)\ln H] + (4\eta^2 + \pi)\cosh[(x-vt)\ln H]} \right)^3 \times \exp[i(-\kappa x + \omega t + \theta_0)], \tag{55}$$

$$V(x,t) = \pm 24\Pi\pi\sqrt{\frac{35\pi}{\Omega_3}} \left(\frac{4\eta \ln H}{(4\eta^2 - \pi)\sinh[(x-vt)\ln H] + (4\eta^2 + \pi)\cosh[(x-vt)\ln H]} \right)^3 \times \exp[i(-\kappa x + \omega t + \theta_0)]. \tag{56}$$

When $\Omega_3 > 0$ and $\pi = 4\eta^2$, the bright solitons read as

$$U(x,t) = \pm 24\sqrt{\frac{35}{\Omega_3}} \left(\ln^3 H \right) sech^3[(x-vt)\ln H] \exp[i(-\kappa x + \omega t + \theta_0)], \tag{57}$$

$$V(x,t) = \pm 24\Pi\sqrt{\frac{35}{\Omega_3}} \left(\ln^3 H \right) sech^3[(x-vt)\ln H] \exp[i(-\kappa x + \omega t + \theta_0)], \tag{58}$$

whereas $\Omega_3 < 0$ and $\pi = -4\eta^2$ retrieves the singular solitons

$$U(x,t) = \pm 24\sqrt{-\frac{35}{\Omega_3}} \left(\ln^3 H \right) csch^3[(x-vt)\ln H] \exp[i(-\kappa x + \omega t + \theta_0)], \tag{59}$$

$$V(x,t) = \pm 24\Pi\sqrt{-\frac{35}{\Omega_3}} \left(\ln^3 H \right) csch^3[(x-vt)\ln H] \exp[i(-\kappa x + \omega t + \theta_0)]. \tag{60}$$

Case 2 : When $s = 2$, Equation (40) reads as

$$g_1(\xi) = K_6 Z^6(\xi) + K_5 Z^5(\xi) + K_4 Z^4(\xi) + K_3 Z^3(\xi) + K_2 Z^2(\xi) + K_1 Z(\xi) + K_0, \quad K_6 \neq 0. \tag{61}$$

Plugging (61) with the help of (41) into (22) reveals the results:
Result 1:

$$K_6 = -\frac{192}{17}\pi\sqrt{\frac{10115\pi}{\Omega_3}} \ln^3 H, \quad K_5 = 0, \quad K_4 = 0, \quad K_3 = 0,$$
$$K_2 = \frac{2304}{289}\sqrt{\frac{10115\pi}{\Omega_3}} \ln^3 H, \quad K_1 = 0, \quad K_0 = 0, \tag{62}$$

$$\Omega_4 = \frac{2324}{17} \ln^2 H, \quad \Omega_2 = \frac{1482544}{289} \ln^4 H, \quad \Omega_1 = -\frac{6580800}{289} \ln^6 H, \quad \pi\Omega_3 > 0. \tag{63}$$

Inserting (62) with the usage of (42) into (61) extracts the combo solitons

$$U(x,t) = \pm 192\sqrt{\frac{35\pi}{\Omega_3}} \left(\ln^3 H \right) \left(\frac{4\eta}{(4\eta^2 - \pi)\sinh[2(x-vt)\ln H] + (4\eta^2 + \pi)\cosh[2(x-vt)\ln H]} \right)$$
$$\times \left\{ \frac{12}{17} - \pi\left(\frac{4\eta}{(4\eta^2-\pi)\sinh[2(x-vt)\ln H]+(4\eta^2+\pi)\cosh[2(x-vt)\ln H]} \right)^2 \right\} \times \exp[i(-\kappa x + \omega t + \theta_0)], \tag{64}$$

$$V(x,t) = \pm 192\Pi\sqrt{\tfrac{35\pi}{\Omega_3}}\left(\ln^3 H\right)\left(\frac{4\eta}{\begin{array}{c}(4\eta^2 - \pi)\sinh[2(x-vt)\ln H]\\ +(4\eta^2 + \pi)\cosh[2(x-vt)\ln H]\end{array}}\right)$$
$$\times\left\{\tfrac{12}{17} - \pi\left(\frac{4\eta}{(4\eta^2-\pi)\sinh[2(x-vt)\ln H]+(4\eta^2+\pi)\cosh[2(x-vt)\ln H]}\right)^2\right\}$$
$$\times\exp[i(-\kappa x + \omega t + \theta_0)]. \tag{65}$$

When $\Omega_3 > 0$ and $\pi = 4\eta^2$ the bright solitons come out as

$$U(x,t) = \pm\tfrac{192}{17}\sqrt{\tfrac{35}{\Omega_3}}\left(\ln^3 H\right)sech[2(x-vt)\ln H]\left\{12 - 17\,sech^{\,2}[2(x-vt)\ln H]\right\}$$
$$\times\exp[i(-\kappa x + \omega t + \theta_0)], \tag{66}$$

$$V(x,t) = \pm\tfrac{192}{17}\Pi\sqrt{\tfrac{35}{\Omega_3}}\left(\ln^3 H\right)sech[2(x-vt)\ln H]\left\{12 - 17\,sech^{\,2}[2(x-vt)\ln H]\right\}$$
$$\times\exp[i(-\kappa x + \omega t + \theta_0)], \tag{67}$$

whilst setting $\Omega_3 < 0$ and $\pi = -4\eta^2$ acquires the singular solitons

$$U(x,t) = \pm\tfrac{24}{17}\sqrt{-\tfrac{35}{\Omega_3}}\left(\ln^3 H\right)csch[2(x-vt)\ln H]\left\{12 + 17\,csch^{\,2}[2(x-vt)\ln H]\right\}$$
$$\times\exp[i(-\kappa x + \omega t + \theta_0)], \tag{68}$$

$$V(x,t) = \pm\tfrac{24}{17}\Pi\sqrt{-\tfrac{35}{\Omega_3}}\left(\ln^3 H\right)csch[2(x-vt)\ln H]\left\{12 + 17\,csch^{\,2}[2(x-vt)\ln H]\right\}$$
$$\times\exp[i(-\kappa x + \omega t + \theta_0)]. \tag{69}$$

Result 2:

$$K_6 = 192\pi\sqrt{\frac{35\pi}{\Omega_3}}\,\ln^3 H,\quad K_5 = 0,\ K_4 = 0,\quad K_3 = 0, K_2 = 0,\quad K_1 = 0,\quad K_0 = 0, \tag{70}$$

$$\Omega_4 = -332\ln^2 H,\quad \Omega_2 = 30256\ln^4 H,\quad \Omega_1 = -705600\ln^6 H,\quad \pi\Omega_3 > 0. \tag{71}$$

Putting (70) with the usage of (42) into (61) secures the combo solitons

$$U(x,t) = \pm 192\pi\sqrt{\tfrac{35\pi}{\Omega_3}}\left(\frac{4\eta\ln H}{\begin{array}{c}(4\eta^2 - \pi)\sinh[2(x-vt)\ln H]\\ +(4\eta^2 + \pi)\cosh[2(x-vt)\ln H]\end{array}}\right)^3$$
$$\times\exp[i(-\kappa x + \omega t + \theta_0)], \tag{72}$$

$$V(x,t) = \pm 192\Pi\pi\sqrt{\tfrac{35\pi}{\Omega_3}}\left(\frac{4\eta\ln H}{\begin{array}{c}(4\eta^2 - \pi)\sinh[2(x-vt)\ln H]\\ +(4\eta^2 + \pi)\cosh[2(x-vt)\ln H]\end{array}}\right)^3$$
$$\times\exp[i(-\kappa x + \omega t + \theta_0)]. \tag{73}$$

When $\Omega_3 > 0$ and $\pi = 4\eta^2$, the bright solitons shape up as

$$U(x,t) = \pm 192\sqrt{\frac{35}{\Omega_3}}\left(\ln^3 H\right)sech^{\,3}[2(x-vt)\ln H]\exp[i(-\kappa x + \omega t + \theta_0)], \tag{74}$$

$$V(x,t) = \pm 192\Pi\sqrt{\frac{35}{\Omega_3}}\left(\ln^3 H\right)sech^{\,3}[2(x-vt)\ln H]\exp[i(-\kappa x + \omega t + \theta_0)], \tag{75}$$

where as setting $\Omega_3 < 0$ and $\pi = -4\eta^2$ formulates the singular solitons

$$U(x,t) = \pm 192 \sqrt{-\frac{35}{\Omega_3} \left(\ln^3 H \right)} \, csch^3[2(x-vt)\ln H] \exp[i(-\kappa x + \omega t + \theta_0)], \qquad (76)$$

$$V(x,t) = \pm 192\Pi \sqrt{-\frac{35}{\Omega_3} \left(\ln^3 H \right)} \, csch^3[2(x-vt)\ln H] \exp[i(-\kappa x + \omega t + \theta_0)]. \qquad (77)$$

5. Conclusions

The current work is the first of its kind to combine the two compensatory means to offset the low count of CD that is being implemented in optoelectronics for the first time. HD solitons were implemented together with a Bragg gratings structure to produce dispersive reflectivity that would work together to create performance enhancement. The effect of soliton radiation and slowdown of solitons due to the presence of higher order dispersions are neglected. The retrieval of solitons for the model has been successfully achieved by the two Kudryashov approaches. The enhanced Kudryashov's approach turned out to be especially useful for bright solitons, while the generalized Kudryashov's scheme failed to recover the much-needed bright solitons.

This successful retrieval of solitons paves the way for further developments in this newly formulated model. An immediate thought would be to obtain the conservation laws to the governing model that would give a plethora of physical insight into the governing model, which would follow up with additional features such as the quasi-monochromatic soliton dynamics and others. Later, this model would also be taken up with additional forms of self–phase modulation. We are awaiting the results that align with the latest findings [17–20] and expect to receive them soon.

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