



# *Article* **Multi-Objective Instantaneous Center of Rotation Optimization Using Sensors Feedback for Navigation in Self-Reconfigurable Pavement Sweeping Robot**

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**Abstract:** Pavement in outdoor settings is an unstructured environment with sharp corners, varying widths, and pedestrian activity that poses navigation challenges while cleaning for autonomous systems. In this work, an approach towards navigating without collision in constrained pavement spaces using the optimal instantaneous center of rotation (ICR) is demonstrated using an in-house developed omnidirectional reconfigurable robot named Panthera. The Panthera reconfigurable design results in varying footprints, supported by passive linear joints along the robot width, with locomotion and steering action using four wheels independent steering drive (4WISD). The robot kinematics and perception sensors system are discussed. Further, the ICR selection is carried out using multi-objective optimization, considering functions for steering, varying width, distance, and clearance to avoid a collision. The framework is incorporated in a local navigation planner and demonstrated experimentally in real pavement settings. The results with optimal selection of ICR in two dimensional space within the robot footprint successfully perform smooth navigation in the constraint space. It is experimentally highlighted with four different scenarios, i.e., constraint conditions encountered by a robot during navigation. Moreover, the formulation of optimal selection of ICR while avoiding collision is generic and can be used for other mobile robot architectures.

**Keywords:** self-reconfigurable robot; multi-objective optimization; instantaneous center of rotation (ICR); local obstacle avoidance; sensors feedback control; autonomous pavement sweeping system

**MSC:** 65K10

## <span id="page-0-0"></span>**1. Introduction**

Pavement cleaning and maintenance is a repetitive task around the year and is essential for public health and safety. These requirements continue to drive research and development of the autonomous cleaning robot. The goal of environment navigation using mobile robots is to prevent collision with structures and obstacles  $[1-3]$  $[1-3]$ . Pavement sweeping using autonomous mobile robots in an unstructured environment presents hurdles in the form of sharp turns and obstacles that need to be avoided. Most commercial robots deployed for pavement cleaning and maintenance tasks are not omnidirectional and follow steering such as in cars. This architecture has larger turning radii and cannot move sideways, which is an essential feature for moving in a constrained space with varying widths



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and sharp turns like in pavement. The typical design of an omnidirectional robot with the ability to reconfigure named Panthera is presented in this work. The ability to maneuver in constrained spaces is aided by its enhanced instantaneous center of rotation (ICR) space supported with the reconfiguration for navigation in the local path planner presented.

The demand for robots to perform pavement cleaning and maintenance tasks increase to keep the environment clean and pedestrian-friendly. Pavement cleaning is a repetitive task and, at the same time, labor-intensive. To meet increasing pavement cleaning needs, multiple fixed shaped robots such as MN-E800W [\[4\]](#page-19-2), QS3008 [\[5\]](#page-19-3), and CN 201 [\[6\]](#page-19-4) have been developed. These robots are limited by their fixed morphologies and the minimum turning radius for the maneuver. The optimal trajectory identification with the car-like vehicles is studied in detail [\[7](#page-19-5)[,8\]](#page-19-6) with the autonomous parking demonstrated in [\[9\]](#page-19-7) which requires multiple maneuvers for parking. The ICR of these robots is constrained to the position of the back wheels. In constraint spaces, as shown in Figure [1c](#page-2-0), it might get stuck or perform multiple complex movements to navigate and avoid obstacles.

The scope for the application of reconfigurable robots in the domain of cleaning and maintenance in an unstructured environment is huge. The advantages of the reconfigurable robotic architecture include evolvability, multifunctionality, and survivability, along with its autonomy, which is detailed in [\[10\]](#page-20-0). The shape-changing robot for the floor cleaning task named hTetro is reported in [\[11\]](#page-20-1) along with the energy calculation during the reconfiguration in [\[12\]](#page-20-2), genetic algorithm-based complete path planning in [\[13\]](#page-20-3), and reinforcement learning-based and bio-inspired neural network completed path planning in [\[14](#page-20-4)[,15\]](#page-20-5), evolutionary based path planing [\[16\]](#page-20-6). The other tiling-based reconfigurable platforms were proposed such as hexagon-based hTrihex [\[17\]](#page-20-7), rhombus-based hTetran [\[18\]](#page-20-8), polyiamond based hTetrakis [\[19\]](#page-20-9) to overcome the coverage difficulties of the complex workspace shapes. The reconfigurable platform called Mantis [\[20\]](#page-20-10) with the ability to change between panels to clear the glass façade vertical structure of the building has also demonstrated the flexibility of reconfigurable mechanism. The constraints of 4 wheels independent steering with the flexibility of freedoms between robot shapes are addressed in [\[21\]](#page-20-11). The challenges during navigation of these reconfigurable robots under the disturbance by monitoring the instantaneous center of rotation (ICR) are presented in [\[22\]](#page-20-12). The experiments in [\[22\]](#page-20-12) showed the path tracking results for a single shape out of seven forms of hTetro under external disturbance. In the present work, the ICR coupled with the shape-changing ability of the Panthera robot across its width enhances the feasible space of ICR selection. This is exploited for selecting the optimal ICR while avoiding obstacles using sensory feedback during the navigation of the Panthera on the pavement settings.

The reconfigurable architecture for the pavement sweeping task was highlighted in [\[23](#page-20-13)[,24\]](#page-20-14), where the robot has differential wheels for the steering action. The design principles for the self-reconfigurable architecture for the modified design of the pavement sweeping robot using four wheel independent steering and drive (4WISD) [\[25–](#page-20-15)[28\]](#page-20-16) are detailed in [\[29](#page-20-17)[,30\]](#page-20-18). Here, the authors highlighted the transformation principles used to design the sub-systems of the Panthera robot presented in this work. For brevity, the design of the Panthera is also discussed in the present work. Panthera consists of the mechanism supporting the reconfiguration to change its width, allowing the robot to move through narrow pavements when required, and enhanced area coverage during the cleaning operations. Moreover, it has a 4WISD model where the ICR during the locomotion can be selected anywhere in a two-dimensional (2D) space except the singularity occurring near the axis of steering units. Appropriate selection of ICR is challenging with complex local and global path planning.

Most research has been performed on fixed shape robots with fixed kinematic models based on Ackerman steering [\[31–](#page-20-19)[33\]](#page-20-20) and differential drive kinematics model [\[34–](#page-20-21)[37\]](#page-21-0). Other examples of robots using ICR as part of their control and maneuvering using ICR are four independent steering drive and skid steering mobile robots [\[38](#page-21-1)[–40\]](#page-21-2). However, their kinematic models for local planners do not incorporate ICR information to perform rotation within the robot footprint by checking the obstacle in the local path planner. In autonomous mobile robots, local planners are built to look for locomotion based on the obstacles around the robot.

In pavement sweeping mobile robots, the objective of the robot is to clean a large area and tends to move in a zig-zag pattern Figure [1b](#page-2-0) in order to increase area coverage. In a zig-zag global planner [\[41](#page-21-3)[,42\]](#page-21-4), waypoints are given to the robot for path following. It also has to follow the orientation of the waypoint. The reconstruction of the outdoor pavement environment for navigation purposes is reported in [\[43\]](#page-21-5). A survey of the approaches for sensing systems of environmental perception technologies for the driver-less vehicle is reviewed in [\[44\]](#page-21-6). The path control of the four Omni wheels mobile platform are studied in [\[45](#page-21-7)[,46\]](#page-21-8). In the pavement environment, there are many obstacles, and some of these obstacles might inhibit the robot's ability to perform rotation to achieve the desired heading angle of the waypoint in the global path planner. There are studies [\[47–](#page-21-9)[49\]](#page-21-10) focused on local planners that enable robots to avoid obstacles moving from waypoint to waypoint. Therefore, there is scope to study the planners that account for robot rotation in the constraint spaces by avoiding collisions.

<span id="page-2-0"></span>

**Figure 1.** Scenarios for the navigation during pavement sweeping task using cleaning and maintenance vehicles with (**a**) turning radius, (**b**) Omnidirectional feature using independent steering action on pavement settings, (**c**) Constraints at sharp corners by the ICR selection, and (**d**) obstacle near the turning routine of the robot. S1, S2, S3, and S4 are the scenarios mentioned in Section [1.](#page-0-0)

The area coverage task using the global planner is prevalent in cleaning and maintenance robots. The modified A-star algorithm for the shape-shifting robot similar to hTetro was reported in [\[50\]](#page-21-11). Further, the incorporation of reinforcement learning-based techniques, including reconfiguration and locomotion planning, was studied in [\[14\]](#page-20-4). The optimization scheme for the efficient area coverage with the zig-zag motion is shown in [\[17\]](#page-20-7). However, the robot's footprint was not accounted for during the reconfiguration task as the shape of these robots was small. Several schemes for the wall-following behavior of the robots are discussed in literature [\[51](#page-21-12)[–53\]](#page-21-13). Wall following using sonar-based feedback control was demonstrated in [\[54\]](#page-21-14). These approaches focus on maintaining the constant distance from the side surface, whereas the fixed shape of the robot with the sharp turns and constraint spaces can result in the robot with situation S4 shown in Figure [1c](#page-2-0).

In this work, the varying footprint ranging from 0.9 to 1.3 m in width is accounted for the optimal selection of ICR in order to avoid the obstacle and select the appropriate steering angle to avoid the collision. It enables Panthera to avoid pedestrians while moving, but it fails to consider the waypoint heading angle required by the global planner. When Panthera reaches a waypoint from the global planner, it needs to perform static rotation in order to perform zig-zag motion for cleaning operations. However, as there are many obstacles in the real-world environment, whenever Panthera rotation is blocked, it gets

stuck and is unable to continue its cleaning operations. Panthera needs a rotation algorithm to enable it to navigate out of such scenarios.

There is little or no work reported on a local planner for anti-collision static rotation on a reconfigurable robot considering its varying footprint using the 4WISD model under the local planner. Without a proper local planner, a simple rotation of its axis can be potentially dangerous. If an obstacle in the form of construction around the pavement or pedestrian stands beside Panthera while performing static rotation about its axis, it will result in a collision. Taking into consideration the above description, the following objectives of the present paper:

- Kinematics modeling for the reconfiguring pavement cleaning and maintenance robot.
- Considering the region within the robot footprint for the selection of instantaneous center of rotation (ICR) in order to avoid collision in constrained spaces.
- Optimal selection of ICR for the static rotation to get the heading angle of the robot where there is no collision with the obstacles in the form of constrained spaces environment or standing pedestrians.
- Experimental demonstration of implemented ICR selection approach for reconfigurable robot performing rotation to navigate through tight spaces namely in four scenarios, (S1) Rotation while avoiding structural obstacles, (S2) Rotation while avoiding pedestrians, (S3) Rotation on a wide pavement, and (S4) No rotation possible, are shown in Figure [1.](#page-2-0)

The paper is structured into the following sections. Section [2](#page-3-0) gives a brief description of the reconfigurable design of the robot for the pavement sweeping task referred to as Panthera. Section [3](#page-5-0) describes the kinematics of Panthera, highlighting the modes of locomotion using ICR. Optimal selection of ICR from the feasible space within the robot footprint using multi-objective optimization approach, namely the Gradient Descent method in Section [4.](#page-6-0) The experimental results and discussion are detailed in Section [5.](#page-11-0) Section [6](#page-18-0) concludes the paper with the scope of future works.

#### <span id="page-3-0"></span>**2. Robot Architecture**

This section briefly highlights the layout of the self-reconfigurable robot Panthera in this work. The design of Panthera is categorized into the mechanical design, steering units, electrical system and sensors, reconfiguration design, and system design.

#### *2.1. Mechanical Design*

Panthera body consists of two parts, the left frame and the right frame, and joining the two side beams is a cylindrical joint, as seen in Figure [2a](#page-4-0), which allows the reconfiguration of the robot width. The cylindrical joint allows the robot body to slide for the expand or compress action and is actuated by the steering units. Based on the steering unit's direction of motion, it makes the cylindrical joint slide inwards for compression, outwards for expansion, or static, where the reconfiguration state of the robot remains the same. Two SIKO WV36M Wire Encoders are used to get the reconfiguration state of Panthera's width.

The cover of Panthera is in two parts, one attached to the left side frame while the other is attached to the right side frame as seen in Figure [2a](#page-4-0). Panthera also has a cleaning unit system that consists of a vacuum motor, a rubbish bin, and two sweeping brushes. The rubbish bin is connected to a vacuum inlet which is controlled by a linear actuator. The linear actuator moves the vacuum inlet up and down depending on the type of litter it needs to vacuum. In addition, there are two linear actuators for the sweeping brushes to move them in and out to change the range of the sweeping brushes. Overall, Panthera has a total of ten motors where four motors are used to perform independent steering of the steering units, four motors are used to perform translation of the wheels, and two motors are used for turning the sweeping brushes. Panthera has a four steering unit design, each containing an independent steering and independent translation motors.

<span id="page-4-0"></span>

**Figure 2.** Design or reconfigurable robot Panthera (**a**) Chassis, steering units (SU), and cylindrical joints connecting the left frame (LF) to the right frame (RF), (**b**) Skeleton of the robot on pavement, (**c**) Fully compressed state of the robot with width 0.9 m, and (**d**) Fully expanded state of the robot with width 1.3 m.

#### *2.2. Steering Unit Design*

Panthera has four independent steering units, *SU*1, *SU*2, *SU*<sup>3</sup> and *SU*4, which are controlled using an Oriental Motor BLV200. Each individual steering unit has a SIKO WV58MR absolute encoder for accurate steering angle feedback. The steering motor uses the angular feedback from the encoder to perform a Proportional Integral and Differential (PID) control. The omnidirectional movement is governed by each steering unit which can rotate in both clockwise and anti-clockwise directions while the translation motors help the robot move in the desired direction. Similarly, this enables Panthera to perform zero turning radius rotation and rotate about a given ICR. For Panthera to perform such complex movements, the steering units and translation motors have to work synchronously to prevent wheels from slipping or sliding, as accounted in the robot kinematics in Section [3.](#page-5-0)

## *2.3. Electrical System and Sensors*

Panthera is powered by two 24 V 206 Ah batteries. The two power supplies are decoupled with one battery powering locomotion and steering actuators while the other battery powers the suction and sweeping brush actuators. The locomotion system and the cleaning unit are designed to have two separate batteries as the cleaning unit is expected to use more power than the locomotion system. Therefore, when the battery that supports the cleaning unit system is low with charge, there is remaining power for Panthera to perform locomotion back to its charging station.

The battery that powers the locomotion system also powers all the essential sensors for Panthera's autonomy. These sensors include XSens MTI 680 IMU/GNSS, Velodyne, and Ouster LiDARs, UCC4000 Pepperl and Fuchs ultrasonic sensors, SIKO WV58MR absolute encoder, WV36M wire encoder, and ZED camera. All these sensors are used to perceive the surroundings, localization, vision detection, and control feedback.

#### <span id="page-4-1"></span>*2.4. System Design*

The Robot Operating System (ROS) is used for data transmission and processing of the information of the sensor. ROS Melodic is running on the onboard computer, which

has the following specifications of 8 CPU Cores, 16 GB RAM, and RTX1060 GPU. All the linear actuators and the two sweeping brushes are controlled via RoboClaw, connected to the onboard computer. All sensors are connected directly to the onboard computer via USB or Ethernet ports. All data from the sensors are published as ROS Topics while the data processing is computed in the ROS Nodes. Multiple ROS Nodes are running in parallel for different functions such as localization, cost map generation, and optimization of ICR. ROS allows Panthera to process data parallelly so that it can dynamically adapt to the requirements of the surroundings, such as in the case of an optimized steering angle for its independent steering units for safe maneuvering.

## <span id="page-5-0"></span>**3. Robot Kinematics**

This section discusses the kinematics of Panthera. The rotation kinematics can be used to position ICR in the two dimensional space within and outside the robot footprint space except for the steering axis location of the wheels.

#### *3.1. Kinematic Model*

The four steering units (SU) as *SU*1, *SU*2, *SU*<sup>3</sup> and *SU*<sup>4</sup> are shown in seen in Figure [3a](#page-5-1) where its location from the robot frame R are defined as  $(x_{SUI}^R, y_{SUI}^R) = (a, b)$ ;  $(x_{SUI2}^R, y_{SUI2}^R) =$  $(a, -b)$ ;  $(x_{\text{SU3}}^R, y_{\text{SU3}}^R) = (-a, -b)$ ;  $(x_{\text{SU4}}^R, y_{\text{SU4}}^R) = (-a, b)$ . Here the parameter of width, i.e., *b* is variable since the robot can reconfigure and takes the value from 0.35 to 0.65 m. The distance from the midpoint to the steering unit, *a*, is fixed at 0.45 m. The four wheel independent steering and locomotion are governed by the following Equation [\(1\)](#page-5-2):

<span id="page-5-2"></span>
$$
\dot{\mathbf{q}} = \begin{bmatrix} \frac{C(\delta_1 + \theta)}{4} & \frac{C(\delta_2 + \theta)}{4} & \frac{C(\delta_3 + \theta)}{4} & \frac{C(\delta_4 + \theta)}{4} & 0 & 0 & 0 & 0 \\ \frac{S(\delta_1 + \theta)}{4} & \frac{S(\delta_2 + \theta)}{4} & \frac{S(\delta_3 + \theta)}{4} & \frac{S(\delta_4 + \theta)}{4} & 0 & 0 & 0 & 0 \\ \frac{P_1}{P_1} & \frac{P_2}{P_2} & \frac{P_3}{P_3} & \frac{P_4}{P_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}
$$
(1)

where *C* and *S*, are sine and cosine functions, *q*˙ is defined as the velocity vector as  $[\dot{x}\dot{y}\dot{\theta}\dot{\delta}_1\dot{\delta}_2\dot{\delta}_3\dot{\delta}_4]^T$  in the world coordinate and  $P_i = (-y_{SUI}^R\cos(\delta_i) + x_{SUI}^R\sin(\delta_i))/z$  where  $i \in (1, 2, 3, 4)$  is the index of steering unit , and  $z = 4a^2 + 4b^2$ .

For interested readers, Panthera's dynamics modeling can be found in Appendix [A.](#page-18-1)

<span id="page-5-1"></span>

**Figure 3.** Panthera steering units and ICR.

#### *3.2. Instantaneous Center of Rotation*

The instantaneous center of rotation (ICR) is the point at which steering units rotate about. The ICR of each respective steering unit is  $ICR_i = (-r_i \sin \delta_i, r_i \cos \delta_i)$ , where  $r_i$  is the radius of rotation curve from the ICR to axis of the steering unit in the robot frame  ${R}$ . In terms of the robot coordinate frame, ICR for the respective steering unit is as follows Equation [\(2\)](#page-6-1):

<span id="page-6-1"></span>
$$
ICR_i^R = (x_{SUi}^R - r_i \sin \delta_i, y_{SUi}^R + r_i \cos \delta_i)
$$
 (2)

It can also be represented in the world coordinate frame [\[55\]](#page-21-15) as follows Equation [\(3\)](#page-6-2):

<span id="page-6-2"></span>
$$
ICR_i^W = [x_{ICRi}^W y_{ICRi}^W]^T = R^{-1}(\theta) [x_{ICRi}^R y_{ICRi}^R]^T
$$
\n(3)

where  $R^{-1}(\theta)$  is the rotation matrix about the *Z*-axis. In the robot coordinate frame, depending on the coordinate of the ICR of the respective steering units, it is able to achieve different modes of locomotion as discussed below:

- Mode 1: When all  $ICR_i^R = (\infty, 0), (-\infty, 0), (0, \infty)$  or  $(0, -\infty)$  robot performs a straight line movement. When *ICR*<sup>*R*</sup> = (∞, 0) or (-∞, 0), it moves along the robot *y*-axis. When *ICR*<sup>*R*</sup> = (0, ∞) or (0, -∞), it moves along the robot *x*-axis.
- Mode 2: When all  $ICR_i^R = (0,0)$ , robot performs a static rotation about the geometric center of the robot.
- Mode 3: When all  $ICR_i^R = (C_x, 0)$  or  $(0, C_y)$ , where  $C_x$  and  $C_y$  are real constants, robot performs a circular motion where the ICR is perpendicular or parallel to the robot orientation.
- Mode 4: When all  $ICR_i^R = (C_x, C_y)$ , where  $C_x$  and  $C_y$  are real constants, the robot performs a circular motion where the ICR can be anywhere in space except the position of its wheels.

Mode 1, 2, and 3 are subsets of Mode 4. In this work, constrained Mode 4 is interested where the ICR is within the robot's footprint for local planning. Other locomotion models, such as four wheel differential drive, skid drive, and Ackerman models, cannot perform Mode 4 locomotion. Mode 4 locomotion is advantageous as it enables Panthera to perform a static rotation where the ICR is not at the center of the robot, as seen in Figure [3b](#page-5-1). The circular lines are the circular path that each steering unit follows in Mode 4 locomotion.

In the next section, the optimization of the ICR using the constrained Mode 4 locomotion leveraging the reconfigurable abilities to enable Panthera to navigate through tight spaces is described.

## <span id="page-6-0"></span>**4. Multi-Objective Optimization for ICR Selection in Local Planner**

The proposed ICR optimization approach consists of deriving the optimal ICR based on the constraints of Panthera's footprint and environment and the objective function. The constraints of Panthera's footprint and environment are incorporated into the approach to quickly identify the feasible ICR space where Panthera can perform rotation using the constrained Mode 4 locomotion. Feasible ICR space is a space that includes possible ICR, which is deemed safe for the robot to perform the constrained Mode 4 locomotion, taking into account factors such as obstacles, rotation target angles, and safety buffer. Based on the set of feasible ICR space, a multi-objective function is used to evaluate the optimal ICR.

The flow chart in Figure [4](#page-7-0) consists of two main steps, including the identification of a feasible ICR space and the optimization of ICR based on a multi-objective function. The input to the proposed algorithm is  $\theta$ , which is the difference between the current heading angle of the robot and the global planner waypoint target heading angle. In the first step of our proposed algorithm, a feasible search space for ICR is identified by adjusting the discrete reconfigurable state (width of the robot footprint size inside the grid workspace). Then, after defining the multi-objective function for steering angle, clearance, and distance, the 2D coordinate of optimal ICR within the found candidate region is derived. The following two sub-sections describe the two steps of our proposed approach in detail.

<span id="page-7-0"></span>

**Figure 4.** Flowchart for proposed ICR optimization.

#### *4.1. Identification of Feasible ICR Space*

Besides reducing the searching area, identifying feasible ICR space is also to ensure that when the robot performs the locomotion about the ICR, it is safe and is anti-collision. The identification of feasible ICR is determined through the constraints of the robot and its environment perception from sensor feedback. In the Panthera, the objective of the robot is to perform cleaning while navigating through tight spaces that other fixed shape robots are incapable of implementing. To navigate out of tight spaces, it has to rotate in a fine manner by an input angle, *θ*. If the space is too tight for its reconfiguration state, it might need to reconfigure and reduce its width, *b*. Furthermore, it has to avoid obstacles based on the local cost map and the robot's footprint. Therefore, it is concluded that the feasible ICR space is determined by its input angle,  $\theta$ , and its reconfiguration state,  $b$ , and the local cost map.

## 4.1.1. Local Cost Map and Robot Footprint

The local cost map is built based on Panthera's perception sensors. Panthera uses three dimensional LIDAR to determine obstacles in its surroundings. Velodyne VLP 16 is used to build a grid-based cost map, as seen in Figure [5,](#page-8-0) where the grid size and resolution can be predefined. Smaller grid size and lower resolution is recommended to reduce computational complexity.

In Figure [5,](#page-8-0) the black region of the cost map is the obstacle, while the white part is the obstacle-free area. A and B are the regions of corridors The red-colored dotted points represent the point cloud from LIDAR, the green boundary box represents the robot footprint, and the red axis line arrow represents the robot's heading angle. Surrounding the local cost map is the global 2D cost map. In the local cost map, cost map size and resolution can be tuned. The robot is always in the middle of the local cost map. The robot ICR space  $C \subset \mathbb{R}^2$  is a mathematical space for all discrete values of ICR that Panthera has based on the grid base cost map. It consists of different kinds of regions: the feasible moving region  $C_F$ , where there are no obstacles, and non-feasible moving region,  $C_0^k$ ,  $k = 1, ..., n_0$ , which represents obstacles where

<span id="page-7-1"></span>
$$
C = C_F \cup \left( \bigcup_{k=1}^{n_0} C_0^k \right). \tag{4}
$$

<span id="page-8-0"></span>In Equation [\(4\)](#page-7-1), it is assumed that the ICR space can be discretized into a grid of squares. Based on the feasible region of locomotion, initialization of the ICR space by using the discrete points within the grid base cost map and the robot footprint. After which, grids with ICR points  $ICR_i^R = (a, b)$  or  $(-a, b)$  or  $(-a, -b)$  or  $(a, -b)$  are excluded as that is where singularity of the kinematics model occurs.



**Figure 5.** Local Cost Map (the black-colored points map with the obstacle boundary).

## 4.1.2. Panthera Reconfiguration State and Robot Footprint

Panthera has the ability to reconfigure in its width, and as a result, the footprint of the robot varies. Panthera has a wire encoder, SIKO WV36M SG21 Wire Encoder, to get the state of reconfiguration width, *b*. Based on the sensor feedback, the respective robot footprint is used as part of the robot constraint. The initialized ICR space is constrained by the footprint of the robot, as seen in the Equation [\(5\)](#page-8-1):

<span id="page-8-1"></span>
$$
x_{min} \leq C_x \leq x_{max} \text{ and } y_{min} \leq C_y \leq y_{max} \tag{5}
$$

where  $x_{min}$ ,  $x_{max}$ ,  $y_{min}$  and  $y_{max}$  represents the minimum and maximum dimensions of the robot footprint in the x and y robot coordinate based on the robot reconfiguration state, *b*. As seen in Figure [5,](#page-8-0) the green-colored rectangular box represents the robot footprint. Therefore, the obstacle region of the cost map should not intersect the robot footprint. Otherwise, it is considered a collision.

## 4.1.3. ICR Trajectory Collision Check

As shown in Panthera's rotation kinematics, Panthera is capable of performing rotation about an ICR with 4 modes of locomotion. To optimize Panthera's rotation in a tight environment, there is a need to further reduce the ICR space into a feasible space where the rotation of the robot footprint does not encounter any obstacles. To perform a rotation, the ICR have coordinates where  $ICR_i^R = (C_x, C_y)$ , where  $C_x$  and  $C_y$  are real values. Furthermore, based on the input angle, *θ*, it may or may not collide with surrounding obstacles. After initializing the ICR space and the robot reconfiguration state, *b*, the algorithm reduces the ICR space by performing checks when the robot footprint is rotated about the ICR by the change in robot heading angle, *θ*, as seen in Figure [6a](#page-9-0). The blue grids represent the discrete ICR space that is used to perform the trajectory collision check. For a clockwise rotation about the ICR, the robot has to check the cost map in the blue region as seen in Figure [6b](#page-9-0) by searching all the cells within the blue region for obstacles. If it encounters an obstacle, such as in Figure [6c](#page-9-0), the discrete ICR value is removed from the ICR Space. Rotation about ICR trajectory collision check removes all ICR where there is the possibility of collisions. This leaves an ICR Space where there is no possibility of collision.

If there is no possible ICR for the rotation to occur without collision, the ICR space is an empty set. Subsequently, it checks for its current reconfiguration state, *b*, and if it is not fully compressed, the algorithm iterates with a smaller reconfiguration state, *b*, where the robot footprint is smaller. If the algorithm still gives an empty set for the smallest reconfiguration state, the algorithm stops as there is no possible ICR for a safe static rotation. If the ICR space is not empty after the anti-collision check, it is passed to an optimizer described in the following sub-section.

<span id="page-9-0"></span>It is noted that Panthera is able to change its reconfiguration state, *b*, and has multiple modes of reconfiguration, which is not optimized in this paper. In our proposed algorithm, it is assumed that Panthera reconfigures to the center to the smallest reconfiguration state if it needs to perform reconfiguration during the identification of ICR space.



a) Clockwise rotation about ICR ( $\mathcal{C}_x$ ,  $\mathcal{C}_y$ ) b) Search for obstacles in blue region

c) Search for obstacles in blue region (obstacle)

**Figure 6.** Search for feasible ICR within the robot footprint and collision check.

#### *4.2. Minimising Objective Function Using Gradient Descent*

After determining the ICR space where there are no collisions at a discrete reconfiguration state, *b*, the optimal ICR is selected based on the minimization of a multi-objective function. The multi-objective function incorporates the steering angle of the steering units and the clearance from obstacles, and ICR's distance from the robot's geometric center, as seen in the Equation [\(6\)](#page-9-1):

<span id="page-9-1"></span>
$$
h(C_x, C_y, b) = k_s h_s(C_x, C_y, b) - k_c h_c(C_x, C_y, b) + k_d h_d(C_x, C_y, b)
$$
(6)

where the objective function,  $h(C_x, C_y, b)$ , is the combination of weighted steering angle function,  $h_s(C_x, C_y, b)$ , clearance function,  $h_c(C_x, C_y, b)$  and distance function,  $h_d(C_x, C_y, b)$ .  $k_s$ ,  $k_c$  and  $k_d$  are weights corresponding to the steering angle function,  $h_s(C_x, C_y, b)$ , clearance function,  $h_c(C_x, C_y, b)$  and distance function,  $h_d(C_x, C_y, b)$ , respectively. More details of the objective functions are described in the next few sub-sections. The robot feedbacks its current steering unit heading angle, position, cost map, and the multi-objective function trade-off between minimizing the change of steering unit heading angle and the clearance space that the robot has after rotation. Although it is possible to evaluate the multi-objective function using the ICR space to find the global minimum, it is not feasible for a high-resolution grid-based cost map where the number of discrete ICR points increases quadratically with the resolution of the map. Therefore, gradient descent is used to evaluate the optimal objective function.

4.2.1. Steering Angle Function

The steering angle function  $h_s(C_x, C_y, b)$  goal is to reduce the summation of the change of steering angle for each steering unit. As the robot is deployed, reducing unnecessary rotation reduces wear and tear of wheels and steering motors.

The SIKO WV58MR absolute encoder gives the current steering angle for the steering units. The steering angle function is defined in the Equation [\(9\)](#page-11-1).

$$
h_{\rm s}(C_x, C_y, b) = \sum_{i=1}^4 \Delta \delta_i(C_x, C_y, b) \tag{7}
$$

<span id="page-10-0"></span>where ∆*δ<sup>i</sup>* is the difference between the current steering angle and targeted steering angle based on the ICR as seen in Figure [7a](#page-10-0),b. This function minimizes unnecessary rotation of steering units and reduces the power required to make a safe rotation.



Figure 7. Visualisation of the objective functions.

4.2.2. Clearance Function

The clearance function,  $h_c(C_x, C_y, b)$  represents the maximum rotation about the ICR before the robot encounters the surrounding obstacles as seen in Figure [7d](#page-10-0). The Equation [\(8\)](#page-11-2) works similarly to the searching of ICR space for collision check to ensure the obstacle avoiding while rotating, as seen earlier in Figure [5.](#page-8-0)

<span id="page-11-2"></span>
$$
h_{c}(C_{x}, C_{y}, b) = \frac{\pi s}{180} \sum_{i=1}^{360/s} OBS_{i}(C_{x}, C_{y}, b)
$$
\n(8)

where:

$$
OBS_i(C_x, C_y, b) = \begin{cases} 1, & \text{if no collision detected} \\ 0, & \text{if collision detected} \end{cases}
$$

where *s* is the step size of the clearance checked in degrees and *OBS<sup>i</sup>* is the anti-collision check for an increment of *f* degrees of rotation about ICR. After which, it is converted to radians.

If there are no obstacles in the direction of rotation, the value of the function is set to 2*π*. This function acts as a safety for the robot and the surroundings. Since the constraints in rotation resolution of robot locomotion and to reduce the computational cost, Panthera performs the clearance function where it checks with anti-collision rotation of an increment step size in degree, *s*, of 5 degrees as the example in the experimental section.

#### 4.2.3. Distance Function

The distance function,  $h_d(C_x, C_y, b)$  represents distance between the ICR  $(C_x, C_y, b)$ and the geometric center of the robot  $(0,0)$  as seen in Figure [7c](#page-10-0). The function is the magnitude of the vector between the ICR and the geometric center of the robot, as seen in the Equation [\(9\)](#page-11-1):

<span id="page-11-1"></span>
$$
h_{\rm d}(C_x, C_y, b) = \sqrt{C_x^2 + C_y^2}
$$
\n(9)

This is to push the ICR closer to the center of the robot so that it does not deviate too far from its position during the rotation. This also ensures more safety as the robot's position after rotation is closer to its initial position and prevents the robot from swinging too far from its initial position.

The weights of the steering angle function, clearance function, and distance function can be tuned to give more importance to the function. For example, if the distance function is more important, set the value  $k_d > k_s$  and  $k_d > k_c$ . In application, a fixed set of weights are pre-tuned for operation.

In the next section, the experimental results of our proposed algorithm are discussed.

## <span id="page-11-0"></span>**5. Experimental Results and Discussion**

The performance and results of the proposed algorithm are evaluated through implementation on Panthera in real-world applications. Four experiments were conducted to demonstrate the performance of the optimal ICR with Panthera reconfigurable mechanism and result in scenarios as shown in Figure [1.](#page-2-0) In all the experiments, Panthera starts from a stationary position with the direction of the steering units parallel to the robot's body. It is assumed that all obstacles are static, including a pedestrian. For the robot to perform a rotation about the ICR, the steering units turn to the target angle based on the optimal ICR before the translation motor starts to perform the static rotation. To ensure that the robot performs the rotation about the ICR smoothly, a Proportional Integral and Derivative (PID) controller [\[56\]](#page-21-16) is used to ensure that there is little error between the input linear velocities and the actual linear velocity of Panthera as a huge error can cause a jerk to the robot, potentially damaging it. The experiment is performed by the onboard computer described in Section [2.4.](#page-4-1)

#### *5.1. Experiments*

Robotic operating system (ROS) master sets up the connection between multiple computational ROS nodes. A Velodyne VLP 16 lidar is used to generate the local cost map and publish/cost\_map topic. The local cost map generated has a grid size of 100 by 100 with a resolution of 0.05 m per grid. The inflation in the cost map is set low as Panthera needs to be close to walls to perform cleaning. It also has stable control with quality sensors to feedback on its position accurately. Based on the cost map and Equation [\(4\)](#page-7-1), the grid is defined as the ICR space *C*. The identification of feasible ICR space ROS node subscribes to the /cost\_map topic where it removes discrete ICR points outside of the robot footprint, singularity ICR points, and ICR points where collisions are detected. If an empty ICR space is detected, it repeats the identification of feasible ICR space with the smallest reconfiguration state to check if there are possible ICR in their smallest state. If a non-empty ICR space is derived, it performs a reconfiguration to the center of the robot. The identification of feasible ICR node publishes an array of ICR via the topic/ICR\_space topic. The optimization node subscribes from the/ICR\_space topic. It performs optimization based on predefined weights of the multi-objective function in Equation [\(6\)](#page-9-1) using the gradient descent method. The optimization node publishes the ICR to the locomotion node for Panthera to perform rotation. A high-level ROS architecture implemented in Panthera can be seen in Figure [8.](#page-12-0) In all experiments, to reduce computational cost in its implementation, the clearance function checks in a resolution of 5 degrees of rotation clearance per iteration.

<span id="page-12-0"></span>

**Figure 8.** ROS nodes and topics developed for self-reconfigurable robot Panthera.

In the following subsections, the results of the 4 experiments similar to scenarios found in Figure [1](#page-2-0) are shown and demonstrated. The supporting video for the experiment can be found in the footnote. [\(https://drive.google.com/file/d/18psIKtHy4O6UB-](https://drive.google.com/file/d/18psIKtHy4O6UB-6nPwIadKaYpZirZNp5/view?usp=sharing)[6nPwIadKaYpZirZNp5/view?usp=sharing,](https://drive.google.com/file/d/18psIKtHy4O6UB-6nPwIadKaYpZirZNp5/view?usp=sharing) accessed on 2 August 2022) or see Supplementary Materials.

5.1.1. Scenario 1: Rotation While Avoiding Structural Obstacles

This experiment aims to demonstrate Panthera's collision-free rotation performance when it is near two adjacent walls. Figure [9a](#page-13-0) shows the experiment setup and initial position of the experiment. Panthera is in the fully compressed state and is facing a wall with an adjacent wall to its right, as reflected in the local cost map seen in Figure [9b](#page-13-0). The experiments follow a similar representation as found in Figure [5.](#page-8-0) If Panthera performs an anti-clockwise rotation about its geometrical center, Panthera's back will hit the wall after a 90 degrees rotation.





<span id="page-13-0"></span>

a) Initial position b) Cost map at initial position



c) Final position d) Cost map at final position

**Figure 9.** Rotation near a wall depicting Scenario S2 (Figure [1\)](#page-2-0).

A weight of  $k_s = 2$ ,  $k_c = 5$  and  $k_d = 5$  is used in this experiment. In this experiment, Panthera is commanded to turn 90 degrees anti-clockwise using our proposed algorithm for collision check and ICR optimization. Panthera performs 90 degrees anti-clockwise rotation about the optimal ICR smoothly without hitting the adjacent wall to its right, as seen in Figure [9c](#page-13-0),d.

## 5.1.2. Scenario 2: Rotation while Avoiding Pedestrians

This experiment aims to demonstrate Panthera's collision-free rotation performance when it is near a pedestrian. Figure [10](#page-14-0) shows the experimental setup and initial position of the experiment. Panthera is in the fully compressed state and is along a walkway with a pedestrian on its left, as reflected in the local cost map seen in Figure [10b](#page-14-0). If Panthera performs a clockwise rotation about its geometrical center, it will hit the pedestrian after a 90 degrees rotation.







<span id="page-14-0"></span>

a) Initial position b) Cost map at initial position



c) Final position d) Cost map at final position

**Figure 10.** Rotation awhile avoiding pedestrians depicting Scenario S2 (Figure [1\)](#page-2-0).

A weight of  $k_s = 1$ ,  $k_c = 2$  and  $k_d = 5$  is used in this experiment. In this experiment, Panthera is commanded to turn 90 degrees clockwise using our proposed algorithm for collision check and ICR optimization. Panthera performs a 90 degrees clockwise rotation about the optimal ICR smoothly without hitting the pedestrian on the left as seen in Figure [10c](#page-14-0),d.

## 5.1.3. Scenario 3: Rotation on a Wide Pavement

This experiment aims to demonstrate Panthera's collision-free rotation performance when in a wide path. Figure [11](#page-15-0) shows the experimental setup and initial position of the experiment. Panthera is along a wide walkway as reflected in the local cost map seen in Figure [11b](#page-15-0).

A weight of  $k_s = 1$ ,  $k_c = 1$  and  $k_d = 10$  is used in this experiment. In this experiment, Panthera is commanded to turn 90 degrees anti-clockwise using our proposed algorithm for collision check and ICR optimization. Panthera performs a 90 degrees clockwise rotation about the optimal ICR smoothly without any collision as seen in Figure [11c](#page-15-0),d.







<span id="page-15-0"></span>

a) Initial position b) Cost map at initial position



c) Final position d) Cost map at final position

**Figure 11.** Rotation in a wide pavement depicting Scenario S3 (Figure [1\)](#page-2-0).

5.1.4. Scenario 4: No Rotation Possible

Unable to conduct the rotation is an issue for a fixed shape robot when it is completely surrounded by static and dynamic obstacles. However, using the 4WISD mechanism incorporated with reconfiguration ability while selecting the flexibility ICR within the robot footprint, Panthera is able to navigate out of the "impossible" scenario. Figure [12a](#page-16-0),b,d,e shows the experimental set up and its initial position of the experiment to demonstrate the infeasible rotation condition for other fixed shape robots even with the 4WISD kinematics model. Panthera is set in the fully expanded state and is close to two adjacent walls and a pedestrian, making any static rotation impossible at its current reconfiguration state. Figure [12i](#page-16-0) shows the zoom-in cost map at the start of the experiment. The cost map is zoomed in for better visualization of the robot's footprint and movements.

A weight of  $k_s = 1$ ,  $k_c = 1$  and  $k_d = 10$  is used in this experiment. Panthera is commanded to rotate 10 degrees clockwise using our proposed algorithm and then move straight to navigate out of the situation. In the first iteration of the identification of the ICR space algorithm, it finds no possible ICR. It checks its reconfiguration state, *b*. As Panthera is in the fully expanded state, it performs a static reconfiguration to its center to its fully compressed state, which reduces the robot's footprint as seen in Figure [12f](#page-16-0),j. It repeats the identification of feasible ICR space algorithm and optimizes the ICR based on the multi-objective function. Panthera performs static rotation based on the optimized ICR in Figure [12g](#page-16-0),k by 10 degrees. After completing the static rotation, it moves straight to



<span id="page-16-0"></span>navigate out of the tight situation, which is impossible for fixed shape robots as seen in Figure [12h](#page-16-0),l.



In this experiment, Panthera is made to reconfigure in width and subsequently a static rotation smoothly without any collision, which would not have been possible for a fixed shape pavement sweeper. It is noted that Panthera has the ability to perform different gaits of reconfiguration [\[56\]](#page-21-16) where it can compress to the left, compress to the right or any position between the two ends of Panthera. This ability gives Panthera flexibility in reducing its footprint, allowing rotation about an ICR to conduct smoothly. To simplify the optimization, the reconfiguration gait is selected for any position between the two ends of Panthera.

## *5.2. Discussion*

Table [1](#page-17-0) shows the values of the objective function with different weights for the experiments above. Experiments 1, 2, 3, and 4 represent the experiment for scenarios 1, 2, 3, and 4, respectively. The objective function is minimized such that the lower the value, the more optimal. Based on Equation [\(6\)](#page-9-1), the optimal objective functions  $h_s$ ,  $h_c$ ,  $h_d$ , and *h* are calculated using variations of weight factors  $w_s$ ,  $w_c$ , and  $w_d$  as shown in the third column. These weights are important as it affects how the robot behaves and tuning of weights directly affects how the robot behaves. For example, in Panthera's autonomy implementation, more weight is given to  $h_d$  to ensure that it remains close to the global planner target waypoint during rotation. For a holistic comparison, each optimal ICR objective function value is listed in Table [1.](#page-17-0) In a multi-objective function, increasing the weights of the desired objective function reduces the weightage of the remaining objective function. For example, for best clearance function optimization,  $w_s = w_d = 0$ . It is observed that in experiments 1 and 2, rotation about its geometric center results in a collision with an obstacle. Correspondingly, even with the best distance function where  $w_s = w_c = 0$ ,  $h_d$  is not 0. On the other hand, when Panthera is in a wide open path or when reconfiguration to compress occurs, the best distance function  $h_d = 0$ .



<span id="page-17-0"></span>**Table 1.** Multi-objective functions with suitable weights.

OF: Objective function, SA: Steering Angle, MO: Multi-objective.

In Table [2,](#page-17-1) the time cost for the identification of feasible ICR space (Step 1), gradient descent optimization, and a brute force search method to find optimal ICR based on the multi-objective function in a scenario similar to experiment 3 with input angle 10 degrees and is tabulated against the resolution of the local cost map which represents a 10 m by 10 m space in the real world. A weight of  $k_s = 1$ ,  $k_c = 1$  and  $k_d = 1$  is used. The time cost for identifying ICR space is 0.31 s and 109.4 s for the 0.05 m and 0.01 m resolution cost map, respectively. For the 0.05 m resolution local cost map, the optimization of the objective function was 1.3 s, and the search method was similar at 7.52 s. However, due to the much higher total number of ICR points found in the higher resolution cost map of 0.01 m, the gradient descent method takes about 1.9 s and outperforms the speed of the search method at 2417.1 s. For a higher resolution, the computational cost for the optimization goes up, and a gradient descent method offers much better computational speed as compared to a brute force minimization algorithm such as the search method. Although a higher resolution cost map potentially offers better ICR optimization, lower-cost map resolution is recommended as the optimized ICR is anti-collision in nature. Any solution, even a non-optimal solution, within the feasible ICR space is anti-collision in nature due to the design of the anti-collision check. It is noted that as the resolution is about 5 times different, the expected number of feasible ICR is also about 25 times bigger in the higher resolution cost map. However, this is not the case for the time cost for the identification of feasible ICR space, gradient descent method, and search method.

<span id="page-17-1"></span>**Table 2.** Time for computation during optimization using gradient-descent (GD) and brute force search (BFS) method.

Grid Size	Resolution	Step $1(s)$	<b>ICR Candidates</b>	GD (s)	BFS (s)
$200 \times 200$	0.05 m	0.32	1107	1.3	7.52
$1000 \times 1000$	$0.01 \; \mathrm{m}$	109.4	26.999	1.9	2417.1

During our ICR experiments, it is noted that our proposed algorithm require additional constraints based on Panthera's motor limitations where  $v_{min} < v_i < v_{max}$ . In order to compensate for the translation motor speed range, a higher distance function weight  $k_d$  is used so that all its translation motors experience a smaller range of speed. Furthermore, normalization of objective functions was considered but not implemented because tuning of weights still needs to be performed for different robots and environments. However, if required, one way to perform normalization is to normalize *h*<sup>s</sup> and *h*<sup>c</sup> against  $2\pi$  and  $h_d$  against the maximum distance ICR can have based on robot footprint. This

is due to the nature of the respective objective functions, which depend on rotation and distance, respectively.

It is also important to note that the implementation of the proposed algorithm requires robust robot control. In Panthera, a PID controller is used to ensure low error between control inputs and actual control measurements. If the control implemented is not stable, build of up errors can lead to dragging of wheels, rubbing of wheels, and even motor failure [\[29\]](#page-20-17).

#### <span id="page-18-0"></span>**6. Conclusions and Future Work**

This paper presented a 4WISD reconfigurable robot which performs navigation through tight and confined spaces using rotation about its instantaneous center of rotation (ICR). The ICR is selected optimally within the robot footprint. We implemented the proposed framework in the self-reconfigurable robot named Panthera as a case study. The kinematics for rotation about an ICR, identification of feasible ICR space, and the optimization of the ICR through gradient descent is described. Changing the weights on the gradient descent function helps to tailor the steering unit function, clearance function, and distance function. Moreover, the paper also demonstrated the advantages of how a reconfigurable robot is able to adjust attentively its footprint and perform static rotation in comparison with fixed form robots in unable navigation scenarios. Our future works focus on optimizing reconfiguration gait to maximize area coverage while performing static rotation about its ICR in different reconfigurable platforms. Other future works include analyzing the optimization methods using other types of modern optimization based on the multi-objective function so as to improve the computational cost required to achieve the optimal result. Optimal ICR for robots with more independent steering and driving is also potential future research.

**Supplementary Materials:** The following supporting information can be downloaded at: [https:](https://www.mdpi.com/article/10.3390/math10173169/s1) [//www.mdpi.com/article/10.3390/math10173169/s1.](https://www.mdpi.com/article/10.3390/math10173169/s1)

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## <span id="page-18-1"></span>**Appendix A. Panthera Dynamics Model**

Dynamics of Panthera [\[57\]](#page-21-17) is described by the following equations:

$$
M(q)\ddot{q} + C(q,\ddot{q})\dot{q} = L_{\tau} - A^{*T}\lambda^*
$$
\n(A1)

$$
M = \begin{bmatrix} m & 0 & mdsin\varphi & 0 & 0 \\ 0 & m & -mdsin\varphi & 0 & 0 \\ mdsin\varphi & -mdsin\varphi & \bar{I} & 0 & 0 \\ 0 & 0 & 0 & I_w & 0 \\ 0 & 0 & 0 & 0 & I_w \end{bmatrix}
$$
(A2)

$$
L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}
$$
 (A3)

$$
C(q,\dot{q})\dot{q} = \begin{bmatrix} md\dot{\varphi}^2 cos\varphi \\ md\dot{\varphi}^2 sin\varphi \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$
 (A4)

$$
\tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix} \tag{A5}
$$

where  $q \in \mathbb{R}^5 = x_c, y_c, \varphi, \theta_r, \theta_l, (\tau_r, \tau_l)$  represents the left and right wheel torques. *m* is the mass of Panthera,  $(x_c, y_c)$  is the center of mass of Panthera, and  $\overline{I}$  is the inertia of the platform. *I<sub>w</sub>* is the wheel inertia.  $A^*$  and  $\lambda^*$  are the matrix representing constraints [\[58\]](#page-21-18) and the vectors of Lagrange multipliers.



**Figure A1.** Schematic of the base of PANTHERA.

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