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Research on Manufacturers' Referral Strategy Considering Store Brand Retailers and Traditional Retailers

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Abstract: It has become a common commercial phenomenon for retailers to establish their own brands. The manufacturer referral strategy is studied through a model which includes a manufacturer, a traditional retailer and a store brand retailer. We conduct research on the three cooperation methods of the manufacturer: "no information referral", "exclusive referral" and "nonexclusive referral". The equilibrium wholesale price, the manufacturer's order quantity and the retailer's own product output are studied by constructing game models, and the best referral cooperation choice between the manufacturer and the retailer is analysed according to their profit. The results show that the manufacturer's referral level choice does not change the number of products, while the manufacturer's market loss rate leads to a change in product order quantity among different choices. Under the combined effect of the market loss rate and the intensity of market competition, the store brand retailer will change the output decision of its own products. When the market loss rate meets a certain range, the manufacturer's product sales can be maximized. For the manufacturer, any referral strategy is better than no referral strategy, and in most cases, the manufacturer prefers nonexclusive referrals. The traditional retailer is willing to accept the manufacturer's referral cooperation, and the traditional retailer's profit is better under the nonexclusive referrals; while most store brand retailers are willing to choose the nonexclusive referrals.

Keywords: supply chain; manufacturer referrals; store brand retailer; Stackelberg game

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1. Introduction

The development of the digital economy has created huge wealth for society [1]. The global digital economy reached $32.6 trillion in 2020, with a nominal year-on-year growth of 3.0% and accounting for 43.7% of GDP (Gross Domestic Product). In January 2021, the number of people using the Internet in the world reached 4.66 billion, and the global Internet penetration rate was 59.5%. According to a CNNIC (China Internet Network Information Center) report, in December 2021, the number of Internet users in China reached 1.032 billion, and the Internet penetration rate reached.

73.0%. (Accessed from: http://www.199it.com/archives/1405773.html accessed on 22 July 2022). In addition, the deep integration of the Internet and manufacturing industry has spawned new business forms [2]. With the emergence of new digital forms of business and technology, the scale of Internet users and policies provide convenient conditions for manufacturers to "access the network" to upgrade. At the same time, manufacturers also need to be able to distinguish themselves in the massive amount of information to attract consumers.

Many manufacturers set up their own official websites to meet consumers' needs for product information, but the biggest drawback of this approach is the conflict between the manufacturer and the retailer's sales channels [3]. To avoid competing with retailers, some manufacturers set up official websites but do not sell directly through them. The main function of the official website is to display the product information and the purchase link of the retailer channel to consumers [4]. Ennew et al. [5] refer to the way that manufacturers show consumers the URL links of authorized retailers on their official websites as "referrals". This model of manufacturers indirectly selling products through the network is called "manufacturer referral" [6]. Many well-known companies have taken the manufacturer's recommendation approach such as L'Oréal Paris, Ultimate Ears, Rene Furterer and Meifubao [7].

However, with the development of the retail market, an increasing number of retailers want to capture more profits from the vertical structure by developing their own brands. However, with the development of the retail market, there is increasing evidence that retailers are starting to establish their own brands in the hope of obtaining more profit.

They not only distribute the manufacturer's products but also build their own production bases or entrust related factories to produce products bearing their own trademarks after fully understanding consumer demand through sales data [8]. In this process, store brand retailers are fully responsible for a series of activities, such as product procurement, storage, advertising and promotion. In this way, store brand retailers and manufacturers are both cooperative and competitive. This model first appeared in large physical supermarket chains such as Metro, Walmart and Carrefour [9]. Later, many well-known companies gradually followed suit; for example, Amazon launched Amazon Basics, Amazon Essentials and other brands, and Costco developed the Kirkland brand [10,11]. With the needs of market development and the support of government policies, Chinese enterprises have also begun to attach importance to the establishment of private brands. Large platform retailers such as Taobao, Suning and JD.com have launched their own brands [12]. There is a great development space for store brands, but they are a serious threat to manufacturers. Therefore, manufacturers must re-examine the relationship with store brand owners in supply chain management.

The manufacturer's referral plays a positive role in supply chain management. On the one hand, manufacturers can prevent the development of retailers' own brands by promoting traditional retailers; on the other hand, they can also realize the coordination between the manufacturer's products and the retailer's own products. Interestingly, manufacturers are not only promoting traditional retailers that focus only on selling the manufacturer's products but are also increasingly involving store brand retailers [13]. For example, L'Oréal Paris, a well-known French brand, enables consumers to find the website link of Watsons from its "How to Buy" page after logging in to its official website. Watsons is the most representative private label retailer, with more than 1200 private brands covering multiple categories such as personal care, which compete with L'Oréal Paris [14]. However, there are also manufacturers that display only information about traditional retailers to consumers, such as Pampers. When consumers click "Buy now" on its official website, they enter the Suning.com website through the referral link to make purchases [15]. Of course, many manufacturers do not display any information about their retailers to consumers. Based on the above analysis, it can be found that there is a "cooperative–competitive" relationship between manufacturers and retailers. When there are store brand retailers, the manufacturer's choice of referral strategy and retailer's quantitative decision affect the operation and management of enterprises.

This paper aims to provide the manufacturer's promotion strategy considering store brand retailers and traditional retailers. Therefore, we constructed a supply chain composed of a manufacturer, a traditional retailer and a store brand retailer with a homogeneous private brand. Retailer 1 is the traditional retailer that sells only the manufacturer's products to consumers, while Retailer 2 is the store brand retailer that sells both the manufacturer's products and its own substitute products to consumers. Since the manufacturer is generally dominant in the supply chain, this paper establishes the Stackelberg model with the manufacturer as the leader and the two retailers as the followers in which the manufacturer decides the wholesale price. This paper focuses on the following three questions. (1) Will manufacturers' different referral strategies affect wholesale prices and retailers' optimal order quantity or output under an equilibrium strategy? (2) When a manufacturer promotes a store brand retailer, some consumers may give up buying the manufacturer's products; how will these consumers lost by the manufacturer affect the supply chain members' choice of a referral cooperation strategy? (3) Is there an optimal referral cooperation strategy for supply chain members?

This study contributes to the research on manufacturer referrals and the manufacturer's and retailer's most recommended cooperation choices. Compared with previous research conclusions, most of the retailers cooperating with the manufacturer are of equal strength, so we consider the Nash game between retailers, and they decide the order quantity or output simultaneously. The traditional retailer decides the order quantity of the manufacturer's products, while the store brand retailer must decide both the order quantity of the manufacturer's products and the output of its own products. This process is affected by the manufacturer's referral strategy and the manufacturer's loss rate in the consumer market. This paper compares and analyses the changes in the manufacturer's product order quantity and private label product output among different referral strategies. Through the analysis of profit performance, we discuss the influence of the manufacturer's referral level and the loss rate of the consumer market on the profit of the manufacturer and the retailers and provide a theoretical reference for the decision making of enterprises in the supply chain.

The rest of this article is structured as follows. Section 2 reviews the related literature. We construct the model based on the description in Section 3. We carry out this research from three cooperation modes: no information referral, exclusive referral and nonexclusive referrals. Section 4 analyses the changes in the order quantity of the manufacturer and the product quantity of the store brand retailer under different cooperation modes. Section 5 analyses the best promotion cooperation choice for the manufacturer, traditional retailer and store brand retailer one by one by comparing the profits under different levels of cooperation. Finally, we offer managerial implications and conclude this research in Section 6.

2. Literature Review

Our study is related to the following streams of literature: manufacturer referral and retailer private brands in the supply chain.

2.1. Manufacturer Referral in the Supply Chain

Most scholars' research on supply chain channel management found that customer preferences, information asymmetry, market environment and environmental concerns affect manufacturers' channel selection [16]. At present, there are few studies on manufacturer referral. Ghose et al. [17] mentioned the referral service of the Internet in an earlier study and explored four scenarios: no referral, retailer referrals from an infomediary, retailer referrals from both the infomediary and manufacturers and manufacturers clearing the infomediary. By studying the competition between the manufacturer referral and infomediary referral, it is found that manufacturers can deal with the infomediary through wholesale prices and capture the profits of the infomediary. However, in some cases, the manufacturer referral is not a complete substitute for the infomediary referral. Wu et al. [6] considered an e-commerce environment where a manufacturer sells its products through two heterogeneous retailers to study the manufacturer's referral strategy. They found that when the recommendation segment is large enough, the non-exclusive referral is the equilibrium choice of the manufacturer. Wei et al. [18] introduced the retailer prediction information sharing mechanism and discussed whether manufacturers should promote the official mall or promote both the official mall and the retailer. Li et al. [19] concentrated on the impact of online cost of dual channel manufacturing on its recommendation strategy. With a linear cost structure as the benchmark model, it is found that when the recommended market size is small, manufacturers are only willing to recommend official malls. When the market scale is large, the manufacturer adopts the strategy of both promotions. Li et al. [20] analyzed the manufacturer's best recommendation strategy under different risk aversion characteristics of retailers and manufacturers. However, none of these studies take into account the recommendation behavior of manufacturers when retailers have their own brands. In the actual business operation, many retailers establish their own brands, and this has a significant impact on manufacturers' referral behavior.

2.2. Retailer Private Brands in the Supply Chain

In recent years, scholars have focused on research on the introduction of private brands by retailers.

Karray et al. [21] found that co-advertising between manufacturers and store brand retailers can reduce the harm to manufacturers by store brands. Amrouche and Yan [22] suggested that opening an online store is an effective strategy for manufacturers to deal with retailers introducing their own brands. Huang et al. [23], studied the equilibrium solution of a supplier establishing a complementary direct selling channel and a retailer introducing private brand based on the four-stage dynamic game. It is found that direct selling is the dominant strategy for suppliers and brand introduction is the dominant strategy for retailers. Ru et al. [24] constructed a Stackelberg game model led by retailers, and the study finds that retailers can reduce the double marginalization effect of the supply chain by introducing private brands. Kim et al. [25] studied the relationship between manufacturers and retailers and found that the high power and knowledge specificity of retailers increased the dependence of manufacturers on private label retailers, thus increasing the growth of private label sales. Choi [26] focused on the price competition between national brand manufacturers and retailers, and found that if a retailer wants to promote its own brand, its retail price may eventually rise, thus motivating it to increase brand awareness of domestic manufacturers. Amrouche et al. [27] built a game model of an online store competition between a manufacturer's brand and a retailer's private brand. The results showed that the quality difference, potential and cross-price competition between the manufacturer's brand and the retailer's private brand are the important factors for the introduction of a private brand. Hara and Matsubayashi [28] found that when there were only store brand retailers, it was difficult for manufacturers and retailers to coordinate, but high-quality private label products could benefit both parties. Bauner et al. [29] analyzed manufacturer's coupons, retailer's national brand coupons and retailer's private label coupons to explore couponing strategies in the competition between manufacturer and retailer. They found that the products and characteristics of private brands play an important role in the couponing strategies of manufacturers and retailers. Huang and Feng [30] constructed a game model of a money-back guarantee on a manufacturer brand and a retail store brand and found that the money-back guarantee strategy can be used as a strategic tool to help retailers develop their own brands. Han et al. [4] used Stackelberg game modelling to explore the impact of different referral information levels on manufacturers' profits, professional agents' profits and private brands' decisions when retailers developed their own private brands. The study found that only when the information referral level satisfies certain conditions will the manufacturer promoting professional agents be good for both sides; otherwise, the profits may be damaged, and the level of information referral is affected by the professional agent market scale. However, if the information referral level reaches a certain threshold, the retailers' own product helps to obtain more profits. Luo et al. [31] developed a game model to study how manufacturers' online channel choices affect retailers' store brand strategies. The study shows that retailers will adopt defensive strategies when online channel threats are high, but when threats are low, retailers tend to cooperate. Li et al. [32] studied the encroachment strategy of private brands and constructed a one-to-one supply chain composed of national brand manufacturers and retailers. They found that high-quality private brands are more conducive to the development of retailers under certain conditions. Chen and Xu [33] studied private brands under asymmetric information. The results suggest that the establishment of private brands by retailers may expand the total product demand and improve the profits of retailers and national brand manufacturers.

The above studies mainly discussed the relationship between manufacturers and private brand retailers from the aspects of private brand pricing strategy, advertising strategy, return guarantee, etc. However, few studies explored manufacturer referrals as a cooperative strategy.

Most existing studies are based on vertical cooperation between manufacturers and exclusive retailers, and less attention is given to the influence of store brand retailers on manufacturer referral strategies. In fact, the relationship between store brand retailers and manufacturers is more complex because they are both vertical partnerships and horizontal competition. In addition, studies on manufacturers' strategies dealing with private brands are mostly focused on quality differences, price differences, channels and advertising strategies.

Although Han et al. [4] considered both manufacturer referral and nonreferral, they did not consider whether to exclusively recommend traditional retailers, nor did they deeply analyse the referral cooperation among supply chain members. Therefore, when private label retailers exist in the supply chain system, should manufacturers choose not to promote, exclusively promote traditional retailers or nonexclusively promote all retailers, how will the manufacturer's wholesale price and the retailer's order volume change because of different referral strategies? What are the best referral strategies for the quantity ordered by the manufacturer and the output of the retailer's own product? When a manufacturer promotes store brand retailers, some consumers give up buying the manufacturer's products. How will the loss of these customers by the manufacturer affect the referral cooperation choice of supply chain members? These problems will be the focus of this paper. Based on previous studies, this paper analyses the pricing or quantitative decision of supply chain members under the competition and cooperation relationship between manufacturers and retailers, studies the role of manufacturer referral and the choice of strategy, expands the results of this research field and provides some management inspiration.

3. Model Description

This section presents the model of a supply chain consisting of a manufacturer and two competing retailers. The manufacturer produces product X and sells it to consumers through retailers. Retailer 1 is a traditional retailer who sells only product X, while Retailer 2, as a store brand retailer, not only sells product X but also produces and sells its own-brand product Y. There are three options for the manufacturer's referral strategy: no referral (denoted by N), exclusive referral to the traditional retailer (denoted by R) and nonexclusive referrals to both retailers (denoted by B). Similarly, the traditional retailer is also faced with three choices of N, R and B, while the store brand retailer has only two choices of N and B. The choice can be implemented only if the two parties reach a cooperation agreement. Figure 1 shows the decision-making process for a referral cooperation between the manufacturer and retailers. Product X and product Y are substitutes for each other. The product X sold by Retailer 1 is called L1, and the product X sold by Retailer 2 is called L2. There is channel competition between L1 and L2. The product Y sold by Retailer 2 is called L3; then, L3, L1 and L2 are brand competitors. To simplify the model, the competition between chains is represented uniformly by θ.

The current multichannel and multibrand model provides consumers with diversified choices, which provides a market for the development of the retailers' private brands [34]. According to consumers' preferences for manufacturer brands versus the retailers' own brands, consumers can be divided into two categories: manufacturer loyalty and retailer loyalty [35,36].

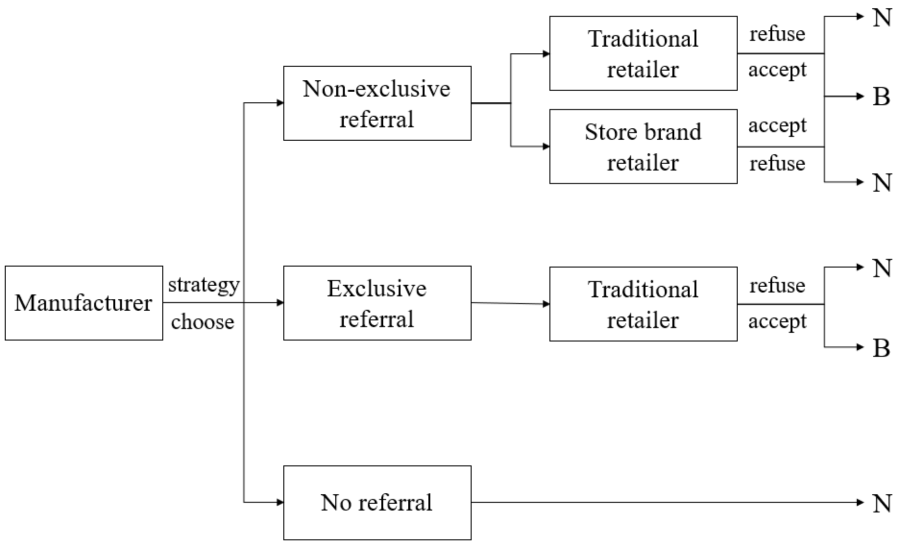

Figure 1. The process of the manufacturer's referral strategy.

(1) The consumer market loyal to the retailer is called the retailer's original market. Assuming its market size is 1 and the potential market size of the traditional retailer in this market is $s(0 < s < 1)$, the potential market size of the store brand retailer is $1 - s$. Since product X and product Y are substitutes for each other and consumers have no product preference in advance, it is assumed that the potential market sizes of product X and product Y sold by the store brand retailer are $\frac{1-s}{2}$ and $\frac{1-s}{2}$, respectively.

(2) Consumers who are loyal to the manufacturer browse product information on the manufacturer's official website and then are recommended by the manufacturer to the exclusive retailer for purchase, thus forming the manufacturer's referral market with a potential scale of $\alpha_r(\alpha_r > 0)$. In the manufacturer's referral market, consumers do not know any retailer and have no prior preferences.

(3) To avoid conflicts, retailers mostly adopt a unified pricing strategy. That is, in both markets, the retail price of L1 is p_1, the retail price of L2 is p_2, and the retail price of L3 is p_3.

For convenience, all the notations and their meanings in this paper are summarized in Table 1.

Table 1. Notations.

Parameter	Meaning
U	The consumer utility
s	The market size of the traditional retailer
α_r	The referral level of the manufacturer
θ	The intensity of market competition
w	The wholesale price
γ	The consumer market loss rate of the manufacturer
$p_i, i = 1,2,3$	The retail price in Li (i = 1, 2, 3)
$q_{io}^j, i = 1,2,3; j = N,R$	The product sales in the original market
$q_{ir}^R, i = 1,2,3$	The product sales in the referral market
$\pi_n^j, n = M,1,2; j = N,R$	The profit of the manufacturer/traditional retailer/store brand retailer

In this section, the Stackelberg game model is constructed for the supply chain system. When the manufacturer and the retailer choose the cooperation strategy, the manufacturer first decides the respective wholesale prices for the two retailers, and the two retailers have equal supply chain status and power. After knowing the wholesale price, the two retailers simultaneously decide their respective order quantity, and Retailer 2 decides the output of its own brand product Y at the same time. Finally, consumers make purchase choices according to their utility maximization, and the transaction is completed. Figure 1 shows the manufacturer's various referral strategies.

The following sections analyse each referral strategy in detail.

3.1. No Referral (N)

If the manufacturer does not recommend any retailer, the two retailers compete only in the original market. Referring to the previous relevant research [37,38] and combining the settings of this paper, the function of consumer surplus at this time is as follows.

$$U = sq_1 + \frac{1-s}{2}q_2 + \frac{1-s}{2}q_3 - \theta q_1 q_2 - \theta q_2 q_3 - \theta q_1 q_3 - \frac{q_1^2}{2} - \frac{q_2^2}{2} - \frac{q_3^2}{2} - p_1 q_1 - p_2 q_2 - p_3 q_3 \quad (1)$$

Note that U is the consumer utility in the original market; q_1 and q_2 represent the sales volume of product X sold by the traditional retailer and the private brand retailer, respectively. This utility function has been widely applied in marketing and operation management, such as in Cai et al. [39] and Chen et al. [40].

$$p_1 = s - q_1 - \theta q_2 - \theta q_3 \quad (2)$$

$$p_2 = \frac{1-s}{2} - q_2 - \theta q_1 - \theta q_3 \quad (3)$$

$$p_3 = \frac{1-s}{2} - \theta q_1 - \theta q_2 - q_3 \quad (4)$$

The profit functions of the manufacturer and retailer are as follows:

$$\pi_M^N = w^N\left(q_1^N + q_2^N\right) \quad (5)$$

$$\pi_1^N = \left(p_1^N - w^N\right)q_1^N \quad (6)$$

$$\pi_2^N = \left(p_2^N - w^N\right)q_2^N + p_3^N q_3^N \quad (7)$$

Note that $\pi_n^N, n = M, 1, 2$, respectively, represent the profits of the manufacturer, the traditional retailer and the store brand retailer without manufacturer information referral.

According to the profit functions Equations (5)–(7) of the manufacturer and the two retailers, the equilibrium result is obtained by inverse solution. First, the private brand retailer needs to simultaneously decide the order quantity of the manufacturer's products and the output of its own products; it is verified that its Hessian array $H = \begin{pmatrix} -2 & -2\theta \\ -2\theta & -2 \end{pmatrix}$. The first-order sequential principal minor is $-2 < 0$, and the second-order sequential principal minor is $\frac{4}{(1-\theta)^2(1+2\theta)} > 0$, which satisfies the negative definite requirement. That is, the profit function of the store brand retailer exhibits the maximum value. Similarly, since $\frac{4}{(1-\theta)^2(1+2\theta)} > 0$ and $\frac{\partial^2 \pi_M^N}{\partial (w^N)^2} = \frac{\theta^2+4\theta-8}{2(\theta^3-3\theta^2+2)} < 0$, the profit functions of both the traditional retailer and the manufacturer have the maximum value.

$\frac{\partial \pi_j}{\partial q_i} = 0, j = 1, i = 1; j = 2, i = 2, 3$ can be solved simultaneously to obtain the reaction function of product demand on wholesale price, which can be substituted back into the manufacturer's profit function to obtain the following equilibrium solution:

$$w^{*N} = \frac{(1-\theta)(1-\theta+s(1+2\theta))}{-\theta^2 - 4\theta + 8} \quad (8)$$

$$q_1^{*N} = \frac{4\theta^2 - 5\theta - 2 - s\left(\theta^3 + 11\theta^2 - 13\theta - 14\right)}{2(\theta^2 - 2\theta - 2)(\theta^2 + 4\theta - 8)} \quad (9)$$

$$q_2^{*N} = \frac{\theta^3 - 5\theta^2 - 2\theta + 12 + s\left(13\theta^2 - 14\theta - 20\right)}{4(\theta^4 + 2\theta^3 - 18\theta^2 + 8\theta + 16)} \quad (10)$$

$$q_3^{*N} = \frac{3\theta^3 - 11\theta^2 - 2\theta + 16 - s\left(4\theta^3 - 19\theta^2 + 2\theta + 16\right)}{4(\theta^2 - 2\theta - 2)(\theta^2 + 4\theta - 8)} \tag{11}$$

To ensure the nonnegativity of price, order quantity and output involved in this model, the market size needs to satisfy the condition $\underline{s} < s < \bar{s}$.

Proof. Analysis of the equilibrium solution shows that $w^{*N} > 0$ and $q_3^{*N} > 0$ are always valid. To ensure the nonnegativity of the retailer's selling price, order quantity and output, the critical values $\underline{s}$ and $\bar{s}$ of the market size can be obtained by solving $q_1^{*N} > 0$, $p_1^{*N} > 0$, $q_2^{*N} > 0$, $p_2^{*N} > 0$, $q_3^{*N} > 0$, and $p_3^{*N} > 0$. □

When $s > \underline{s}$, $q_1^{*N} > 0$, $p_1^{*N} > 0$; when $s < \bar{s}$, $q_2^{*N} > 0$, $p_2^{*N} > 0$; and in the interval $\underline{s} < s < \bar{s}$, there are always $p_3^{*N} > 0$ established. The expression of $\underline{s}$ and $\bar{s}$ is $\underline{s} = \frac{4\theta^2 - 5\theta - 2}{\theta^3 + 11\theta^2 - 13\theta - 14}$; $\bar{s} = \frac{\theta^3 - 5\theta^2 - 2\theta + 12}{-13\theta^2 + 14\theta + 20}$.

3.2. Exclusive Referral to the Traditional Retailer (R)

The superscript R indicates that the manufacturer exclusively promotes the traditional retailer to consumers. At this time, these consumers click on the manufacturer's referral link to enter the traditional retailer's store and make a purchase decision, so the manufacturer's recommendation increases the traditional retailer's potential consumer market [41]. By changing the potential consumer market of the traditional retailer, the consumer utility function is adjusted based on Equation (1), and Equation (12) is obtained as follows.

$$U = (s + \alpha_r)q_1 + \frac{1-s}{2}q_2 + \frac{1-s}{2}q_3 - \theta q_1 q_2 - \theta q_2 q_3 - \theta q_1 q_3 - \frac{q_1^2}{2} - \frac{q_2^2}{2} - \frac{q_3^2}{2} - p_1 q_1 - p_2 q_2 - p_3 q_3 \tag{12}$$

Maximizing the consumer surplus obtains the corresponding demand function.

$$p_1 = s + \alpha_r - q_1 - \theta q_2 - \theta q_3 \tag{13}$$

$$p_2 = \frac{1-s}{2} - \theta q_1 - q_2 - \theta q_3 \tag{14}$$

$$p_3 = \frac{1-s}{2} - \theta q_1 - \theta q_2 - q_3 \tag{15}$$

The profit functions of the manufacturer and the two retailers can be expressed as follows.

$$\pi_M^R = w^R(q_1^R + q_2^R) \tag{16}$$

$$\pi_1^R = (p_1^R - w^R)q_1^R \tag{17}$$

$$\pi_2^R = (p_2^R - w^R)q_2^R + p_3^R q_3^R \tag{18}$$

The equilibrium solution is obtained by reverse solving.

$$w^{*R} = \frac{(1-\theta)(1-\theta+s(1+2\theta)+\alpha_r(2+\theta))}{-\theta^2 - 4\theta + 8}, \tag{19}$$

$$q_1^{*R} = \frac{4\theta^2 - 5\theta - 2 - s\left(\theta^3 + 11\theta^2 - 13\theta - 14\right) - \alpha_r\left(\theta^3 + 7\theta^2 - 8\theta - 12\right)}{2(\theta^2 - 2\theta - 2)(\theta^2 + 4\theta - 8)}, \tag{20}$$

$$q_2^{*R} = \frac{\theta^3 - 5\theta^2 - 2\theta + 12 + s\left(13\theta^2 - 14\theta - 20\right) + \alpha_r\left(\theta^3 + 8\theta^2 - 16\theta - 8\right)}{4(\theta^2 - 2\theta - 2)(\theta^2 + 4\theta - 8)}, \tag{21}$$

$$q_3^{*R} = \frac{3\theta^3 - 11\theta^2 - 2\theta + 16 - s\left(4\theta^3 - 19\theta^2 + 2\theta + 16\right) - \alpha_r\left(\theta^2 - 8\theta + 4\right)\theta}{4(\theta^2 - 2\theta - 2)(\theta^2 + 4\theta - 8)} \tag{22}$$

The traditional retailer and the manufacturer enter into an exclusive referral cooperation on the premise that $w^{*R} > 0$ and $q_1^{*R} > 0$. When $\forall \alpha_r > 0$, there always exist $w^{*R} > 0$ and $q_1^{*R} > 0$. That is, the manufacturer and the traditional retailer can easily reach referral cooperation. The influence of the manufacturer's referral level on the private brand retailer needs further analysis. It is found that when $0 < \alpha_r < \alpha_{r,q2}^R$, there exist $q_2^{*R} > 0$ and $p_2^{*R} - w^{*R} > 0$; when $0 < \alpha_r < \alpha_{r,p3}^R$, there exist $q_3^{*R} > 0$ and $q_3^{*R} > 0$.

Note that $\alpha_{r,q2}^R = \frac{\theta^3 - 5\theta^2 - 2\theta + 12 + s\left(13\theta^2 - 14\theta - 20\right)}{-\theta^3 - 8\theta^2 + 16\theta + 8}$, $\alpha_{r,p3}^R = \frac{\theta^4 - 2\theta^3 - 13\theta^2 + 10\theta + 16 + s\left(9\theta^3 + 5\theta^2 - 22\theta - 16\right)}{-\theta(\theta^3 + 7\theta^2 - 8\theta - 12)}$.

It is found that $\alpha_{r,q2}^R < \alpha_{r,p3}^R$ by comparing $\alpha_{r,q2}^R$ and $\alpha_{r,p3}^R$, and Proposition 1 is obtained.

Proposition 1. *When the manufacturer chooses the exclusive referral, the change in the order quantity and output of the two retailers are as follows.*

When $0 < \alpha_r < \alpha_{r,q2}^R$, $q_i^{*R} > 0, i = 1,2,3$; when $\alpha_{r,q2}^R < \alpha_r < \alpha_{r,p3}^R$, $q_i^{*R} > 0, i = 1,3$ and $q_2^{*R} = 0$; when $\alpha_r > \alpha_{r,p3}^R$, $q_1^{*R} > 0$ and $q_2^{*R} = 0$, $q_3^{*R} = 0$.

Proposition 1 indicates that when the manufacturer and the traditional retailer reach exclusive referral cooperation, with the continuous improvement of the promotion level, the private brand retailer will change from operating products X and Y at the same time to gradually stripping the business of product X and eventually develop into an independent store brand retailer. However, when the manufacturer's referral level is higher than $\alpha_{r,p3}^R$, then the store brand retailer will completely withdraw from the market. This is because the manufacturer's referral intensifies the price war between the two products, and finally product Y will be discontinued due to price-cutting competition resulting in zero profit, while the traditional retailer will always sell product X.

3.3. Nonexclusive Referrals to Both Retailers (B)

Nonexclusive referrals mean that the manufacturer promotes both retailers. That is, the manufacturer shows the consumer the website links of both retailers. Since consumers do not know any retailer in advance and there is no prior preference, they will randomly click on a retailer link to make a purchase. Therefore, referring to the symmetrical settings of Wu et al. [6] and Yang and Gao [42], the market size of potential consumers added by the manufacturer's referral for both retailers is α_r. However, consumers' purchasing decisions are affected by many factors, such as the influence of the retailer's own products [43]. Therefore, considering the temptation of product Y, this paper makes the following assumptions. Among the consumers who enter the private brand store, some consumers who originally intended to buy product X will give up product X and choose product Y, which reduces the potential market size of product X operated by the store brand retailer. Here, γ represents the loss of product X and also represents the additional gain of product Y in this market. Therefore, the potential scale of product X changes to $(1-\gamma)\alpha_r$, and the potential scale of product Y changes to $\gamma\alpha_r (0 < \gamma < 1)$. To highlight the results of the collaboration between different referrals, this paper ignores the small impact of the referrals' locations because manufacturers often place a limited number of retailer links in a side-by-side manner. Therefore, the consumer surplus based on Equations (4)–(8) becomes:

$$\begin{aligned} U = &(s+\alpha_r)q_1 + \left(\tfrac{1-s}{2} + (1-\gamma)\alpha_r\right)q_2 + \left(\tfrac{1-s}{2} + \gamma\alpha_r\right)q_3 \\ &-\theta q_1 q_2 - \theta q_2 q_3 - \theta q_1 q_3 - \tfrac{q_1^2}{2} - \tfrac{q_2^2}{2} - \tfrac{q_3^2}{2} - p_1 q_1 - p_2 q_2 - p_3 q_3 \end{aligned} \tag{23}$$

Maximizing the utility function yields the inverse demand function as follows.

$$p_1 = s + \alpha_r - q_1 - \theta q_2 - \theta q_3 \tag{24}$$

$$p_2 = \frac{1-s}{2} - \theta(q_1 + q_3) - q_2 + (1-\gamma)\alpha_r \tag{25}$$

$$p_3 = \frac{1-s}{2} - \theta(q_1 + q_2) - q_3 + \gamma\alpha_r \tag{26}$$

Substituting the above results into the profit functions of the manufacturer and retailer obtains the final profit function as follows.

$$\pi_M^B = w^B\left(q_1^B + q_2^B\right) \tag{27}$$

$$\pi_1^B = \left(p_1^B - w^B\right)q_1^B \tag{28}$$

$$\pi_2^B = \left(p_2^B - w^B\right)q_2^B + p_3^B q_3^B \tag{29}$$

The reverse order solution obtains the equilibrium solutions:

$$w^{*B} = \frac{2(1-\theta)(1-\theta+s(1+2\theta)) - \left(\theta^2+4\theta-8-\gamma\left(2\theta^2-4\theta-4\right)\right)\alpha_r}{-2(\theta^2+4\theta-8)} \tag{30}$$

$$q_1^{*B} = \frac{\begin{array}{c} 2\left(4\theta^2-5\theta-2-s\left(\theta^3+11\theta^2-13\theta-14\right)\right) \\ -\alpha_r\left(\theta^2+4\theta-8+2\gamma\left(\theta^2-2\theta-2\right)\right)(2+\theta) \end{array}}{4(\theta^2-2\theta-2)(\theta^2+4\theta-8)} \tag{31}$$

$$q_2^{*B} = \frac{\left(\begin{array}{c} 2(1-\theta)\left(\theta^3-5\theta^2-2\theta+12+s\left(13\theta^2-14\theta-20\right)\right)- \\ \alpha_r\left(\theta^4+2\theta^3-12\theta^2+32\theta-32+6\gamma\left(\theta^4-10\theta^2+4\theta+8\right)\right) \end{array}\right)}{8(1-\theta)(\theta^2-2\theta-2)(\theta^2+4\theta-8)} \tag{32}$$

$$q_3^{*B} = \frac{\left(\begin{array}{c} 2(1-\theta)\left(-3\theta^3+11\theta^2+2\theta-16+s\left(4\theta^3-19\theta^2+2\theta+16\right)\right)+ \\ \alpha_r\left(3\theta\left(\theta^3+2\theta^2-16\theta+16\right)+2\gamma\left(\theta^4-16\theta^3+42\theta^2-4\theta-32\right)\right) \end{array}\right)}{-8(1-\theta)(\theta^2-2\theta-2)(\theta^2+4\theta-8)} \tag{33}$$

The prerequisite for the manufacturer to reach referral cooperation with both retailers is that the manufacturer is motivated to implement the nonexclusive referrals strategy and retailers will order product X, that is, to ensure that $w^{*B} > 0$,$q_1^{*B} > 0$,$q_2^{*B} > 0$.

1. The threshold of the referral level is obtained by solving $w^{*B} > 0$: $\alpha_{r,w}^B = \frac{-2(1-\theta)(1-\theta+s(1+2\theta))}{2\gamma(\theta^2-2\theta-2)-\theta^2-4\theta+8}$.

Since $-2(1-\theta)(1-\theta+s(1+2\theta)) < 0$, we need to analyse only the denominator. Then, let $2\gamma(\theta^2-2\theta-2)-\theta^2-4\theta+8=0$, and we can obtain the threshold $\gamma_1 = \frac{\theta^2+4\theta-8}{2(\theta^2-2\theta-2)}$.

When $0<\gamma<\gamma_1$, there exists $2\gamma(\theta^2-2\theta-2)-\theta^2-4\theta+8>0$, and the threshold of referral level at this time is $\alpha_{r,w}^B < 0$, so for $\forall\alpha_r > 0$, $\alpha_r > \alpha_{r,w}^B$ is satisfied.

When $\gamma_1 < \gamma < 1$, there exists $2\gamma(\theta^2-2\theta-2)-\theta^2-4\theta+8 < 0$, so $\alpha_{r,w}^B > 0$. Therefore, only when $\alpha_r > \alpha_{r,w}^B$ can $w^{*B} > 0$ be satisfied.

2. Next we will analyse q_1^{*B}. Since q_1^{*B} is complex, it is decomposed into $q_1^{*B} = \frac{2\left(4\theta^2-5\theta-2+s\left(-\theta^3-11\theta^2+13\theta+14\right)\right)}{4(\theta^2-2\theta-2)(\theta^2+4\theta-8)} + \frac{(2+\theta)\left(-\theta^2-4\theta+8+2\gamma\left(-\theta^2+2\theta+2\right)\right)\alpha_r}{4(\theta^2-2\theta-2)(\theta^2+4\theta-8)}$. It is easy to verify the nonnegativity of its denominator, so we need to check only the positive and negative of the molecule.

Through calculations, we can find that when $\underline{s} < s < \bar{s}$, there exists $4\theta^2-5\theta-2+s\left(-\theta^3-11\theta^2+13\theta+14\right) > 0$. When both $0<\theta<1$ and $0<\gamma<1$ are satisfied, $(2+\theta)\left(-\theta^2-4\theta+8+2\gamma\left(-\theta^2+2\theta+2\right)\right)\alpha_r > 0$ is always true. So for $\forall\alpha_r > 0$, $q_1^{*B} > 0$ is satisfied. Similarly, $p_1^{*B} - w^{*B} > 0$ can be obtained.

3. Through calculating $q_2^{*B} > 0$, we can find that the referral level needs to satisfy the condition $\alpha_r < \alpha_{r,q2}^B$, where $\alpha_{r,q2}^B = \frac{-2(1-\theta)\left(12-2\theta-5\theta^2+\theta^3+s\left(-20-14\theta+13\theta^2\right)\right)}{32-32\theta+12\theta^2-2\theta^3-\theta^4+6\gamma\left(-8-4\theta+10\theta^2-\theta^4\right)}$. In the interval, $\underline{s} < s < \bar{s}$, $12-2\theta-5\theta^2+\theta^3+s\left(-20-14\theta+13\theta^2\right) > 0$ is always true, so we can obtain the conclusion that $-2(1-\theta)\left(12-2\theta-5\theta^2+\theta^3+s\left(-20-14\theta+13\theta^2\right)\right) < 0$. In addition, when $32-32\theta+12\theta^2-2\theta^3-\theta^4+6\gamma\left(-8-4\theta+10\theta^2-\theta^4\right)=0$, we can solve this equation and obtain $\gamma_2 = \frac{(-1+s+s\theta)\left(8-4\theta-\theta^2\right)}{-24+8\theta+14\theta^2-4\theta^3+s(32+8\theta-34\theta^2+6\theta^3)}$.

When $0 < \gamma < \gamma_2$, we can obtain $32 - 32\theta + 12\theta^2 - 2\theta^3 - \theta^4 + 6\gamma(-8 - 4\theta + 10\theta^2 - \theta^4) > 0$, and by considering the numerator of the referral level threshold, it is known that $\alpha^B_{r,q2} < 0$ and $q_2^{*B} > 0$ are true for $\forall \alpha_r > 0$.

When $\gamma_2 < \gamma < 1$, we can obtain $32 - 32\theta + 12\theta^2 - 2\theta^3 - \theta^4 + 6\gamma(-8 - 4\theta + 10\theta^2 - \theta^4) < 0$ and $\alpha^B_{r,q2} > 0$, so only when the referral level can satisfy $\alpha_r < \alpha^B_{r,q2}$ can $q_2^{*B} > 0$ be guaranteed.

4. Here, $p_2^{*B} - w^{*B} > 0$ is analyzed in the same way as above, and we can obtain the threshold $\alpha^B_{r,(p2-w)}$ of the referral level and the critical value γ^B_{p2-w} of the manufacturer's market loss rate. When $0 < \gamma < \gamma^B_{p2-w}$, there exists $\alpha^B_{r,(p2-w)} < 0$. Thus, $p_2^{*B} - w^{*B} > 0$ is always true when $\forall \alpha_r > 0$. In addition, when $\gamma^B_{p2-w} < \gamma < 1$, there exists $\alpha^B_{r,(p2-w)} > 0$. Thus, $p_2^{*B} - w^{*B} > 0$ is always true when $\alpha_r < \alpha^B_{r,(p2-w)}$.

The conditions obtained from the above analysis are nonnegative for a single decision variable. To ensure the establishment of the nonexclusive referrals cooperation between the manufacturer and the retailer, comprehensive consideration should be given to ensure the nonnegativity of each variable.

First, to ensure $q_2^{*B} > 0$ and $p_2^{*B} - w^{*B} > 0$, we need to compare γ_2 and γ^B_{p2-w}. When $\gamma^B_{p2-w} < \gamma < 1$, there exists $\alpha^B_{r,q2} < \alpha^B_{r,(p2-w)}$, so $0 < \alpha_r < \alpha^B_{r,q2}$ is obtained, and both $q_2^{*B} > 0$ and $p_2^{*B} - w^{*B} > 0$ can be guaranteed. Then, we can obtain the following conclusion: when $0 < \gamma < \gamma_2$, there exist $q_2^{*B} > 0$ and $p_2^{*B} - w^{*B} > 0$ for $\forall \alpha_r > 0$; when $\gamma_2 < \gamma < 1$ and $0 < \alpha_r < \alpha_r^{q2,B}$, there exist $q_2^{*B} > 0$ and $p_2^{*B} - w^{*B} > 0$.

We can use the same method to analyse $q_2^{*B} > 0$, $p_2^{*B} - w^{*B} > 0$ and $w^{*B} > 0$, and we need to compare γ_1 and γ_2. Through calculations, we can obtain the expression of the market size s: $\frac{\theta^5+\theta^4-18\theta^3+7\theta^2+34\theta-40+s(-2\theta^5-4\theta^4+57\theta^3-59\theta^2-18\theta+56)}{(\theta^2-2\theta-2)(2\theta^3-7\theta^2-4\theta+12-s(3\theta^3-17\theta^2+4\theta+16))}$.

When $\underline{s} < s < \bar{s}$, the numerator and denominator of the above formula are both negative, so we can obtain $\gamma_2 < \gamma_1$. Figure 2 graphically shows the relationship between γ_1 and γ_2.

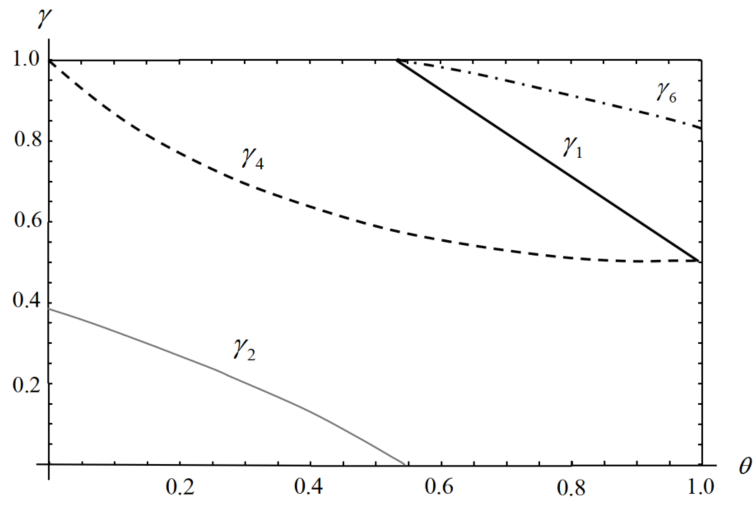

Figure 2. The relationship between $\gamma_i, i = 1, 2, 4, 6$.

In the interval $\gamma_1 < \gamma < 1$, we can obtain $\alpha^B_{r,q2} < \alpha^B_{r,w}$ by calculating $\alpha^B_{r,q2} - \alpha^B_{r,w}$. That is, when $0 < \gamma < \gamma_2$, there exist $w^{*B} > 0$, $q_2^{*B} > 0$ and $p_2^{*B} - w^{*B} > 0$ for $\forall \alpha_r > 0$; when $w^{*B} > 0$, $q_2^{*B} > 0$ and $p_2^{*B} - w^{*B} > 0$ can also exist as long as $0 < \alpha_r < \alpha^B_{r,q2}$.

Therefore, the following conclusion can be drawn:

When $0 < \gamma < \gamma_2$, there exist $w^{*B} > 0$, $q_1^{*B} > 0$, $p_1^{*B} - w^{*B} > 0$, $q_2^{*B} > 0$ and $p_2^{*B} - w^{*B} > 0$ for $\forall \alpha_r > 0$; when $\gamma_2 < \gamma < 1$, $w^{*B} > 0$, $q_1^{*B} > 0$, $p_1^{*B} - w^{*B} > 0$, $q_2^{*B} > 0$ and $p_2^{*B} - w^{*B} > 0$ can still exist as long as $0 < \alpha_r < \alpha^B_{r,q2}$.

This is also the prerequisite for the manufacturer's nonexclusive referrals of retailers. In this situation, whether the store brand retailer will develop its own product needs to verify the positive and negative of q_3^{*B} and p_3^{*B}, and we can obtain the threshold of the referral level $\alpha_{r,q3}^B = \frac{2(-1+\theta)\left(-16+2\theta+11\theta^2-3\theta^3+s\left(16+2\theta-19\theta^2+4\theta^3\right)\right)}{3\theta(16-16\theta+2\theta^2+\theta^3)+2\gamma\left(-32-4\theta+42\theta^2-16\theta^3+\theta^4\right)}$ and the critical value of market loss rate $\gamma_3 = \frac{-3\theta\left(\theta^3+2\theta^2-16\theta+16\right)}{2\left(\theta^4-16\theta^3+42\theta^2-4\theta-32\right)}$ by analysing $q_3^{*B} > 0$ and $p_3^{*B} > 0$. Therefore, the following conclusion can be drawn.

When $\gamma_3 < \gamma < 1$, there exist $q_3^{*B} > 0$ and $p_3^{*B} > 0$ for $\forall \alpha_r > 0$; when $0 < \gamma < \gamma_3$, $q_3^{*B} > 0$ and $p_3^{*B} > 0$ can be guaranteed if the referral level can satisfy condition $0 < \alpha_r < \alpha_{r,q3}^B$.

Proposition 2. *When the manufacturer provides nonexclusive referrals, the change in the retailers' order quantity or output is as follows.*

1. When $0 < \gamma < \gamma_3$, if $0 < \alpha_r < \alpha_{r,q3}^B$, then $q_i^{*B} > 0, i = 1,2,3$; if $\alpha_r > \alpha_{r,q3}^B$, then $q_i^{*B} > 0, i = 1,2$ and $q_3^{*B} = 0$.
2. When $\gamma_3 < \gamma < \gamma_2$, there exist $q_i^{*B} > 0, i = 1,2,3$ for $\forall \alpha_r > 0$.
3. When $\gamma_2 < \gamma < 1$, if $0 < \alpha_r < \alpha_{r,q2}^B$, then $q_i^{*B} > 0, i = 1,2,3$; if $\alpha_r > \alpha_{r,q2}^B$, then $q_i^{*B} > 0, i = 1,3$ and $q_2^{*B} = 0$.

It can be seen from the above analysis that, in this case, the market competition intensity and market size within the feasible region will not change the manufacturer's referral decision. With $\theta = 0.8$ and $s = 0.3$, the impacts of the consumer market loss rate and referral level on supply chain members are illustrated in Figure 3.

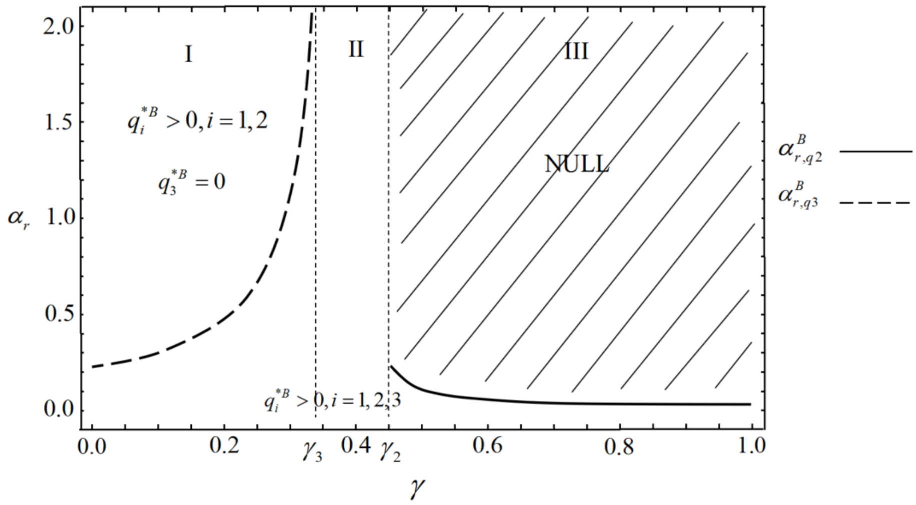

Figure 3. The impact of consumer market loss rate and referral level on supply chain members.

Define $0 < \gamma < \gamma_3$ as Region I. At this time, the consumer market loss rate is relatively low. If the manufacturer improves the referral level, it can form a simple cooperative relationship with the store brand retailer. That is, it will not produce its own product Y, i.e., $q_3^{*B} = 0$. When the market loss rate satisfies $\gamma_3 < \gamma < \gamma_2$ (defined as Region II), the manufacturer cannot control the production plan of the store brand retailer through the referral level, so any referral level at this time will encourage the store brand retailer to develop its own product Y. When the market loss rate exceeds γ_2 (defined as Region III), if the referral level is higher than $\alpha_{r,q2}^B$, it will lead the private brand owner to give up selling product X completely and concentrate on operating its own product Y. That is, when the market loss rate is below γ_3, the manufacturer can influence the output of product Y through the referral level, and when the market loss rate is higher than γ_2, the manufacturer needs to control the referral level within a reasonable range; otherwise, it will encourage the store brand retailer to develop its own product and reduce the sales of product X. This shows that the manufacturer can effectively deal with the invasion of the store brand retailer through information referral.

Based on the above analysis, the basic conditions for the existence of different referral cooperation can be obtained: the exclusive referral can hold for $\forall\alpha_r > 0$; when $0 < \gamma < \gamma_2$, the nonexclusive referrals can hold for $\forall\alpha_r > 0$; when $\gamma_2 < \gamma < 1$, the nonexclusive referrals can hold only when the referral level satisfies $0 < \alpha_r < \alpha^B_{r,q2}$.

Proposition 1 and Proposition 2 also show that different referral strategies have different effects on the retailers' order quantity and production quantity. Is it more beneficial for the manufacturer to exclusively introduce the traditional retailer to increase the order quantity of product X? Would the nonexclusive referrals of two retailers reduce the order quantity of product X and increase the output of product Y?

4. Comparative Analysis of Order Quantity

Next, we will analyse the change in the order quantity of the manufacturer's product X and the production of the store brand retailer's product Y. First, we compare the difference in the order quantity of product X under different cooperation forms. Here, $\Delta q^n_{1,2}, n = R,N;B,N;B,R$ represent the difference in output of product X between different cooperation pairs. By calculating $\Delta q^{R,N}_{1,2} = (q^R_1 + q^R_2) - (q^N_1 + q^N_2) > 0$, $\Delta q^{B,N}_{1,2} = (q^B_1 + q^B_2) - (q^N_1 + q^N_2) > 0$, we can obtain that the critical value of the loss rate of the consumer market is exactly γ_1. By calculating $\Delta q^{B,R}_{1,2} = (q^B_1 + q^B_2) - (q^R_1 + q^R_2) > 0$, we can obtain that the critical value of the market loss rate is $\gamma_4 = \frac{-\theta^2+2\theta-4}{2(\theta^2-2\theta-2)}$, and Proposition 3 is as follows.

Proposition 3. *Under the influence of the loss rate of the manufacturer's consumption market, the change in the order quantity of product X between different scenarios is as follows.*

1. When $0 < \gamma < \gamma_4$, there exists $q^B_1 + q^B_2 > q^R_1 + q^R_2 > q^N_1 + q^N_2$;
2. When $\gamma_4 < \gamma < \gamma_1$, there exists $q^R_1 + q^R_2 > q^B_1 + q^B_2 > q^N_1 + q^N_2$;
3. When $\gamma_1 < \gamma < 1$, there exists $q^R_1 + q^R_2 > q^N_1 + q^N_2 > q^B_1 + q^B_2$.

Proposition 3 indicates that the manufacturer's recommendation level does not affect the order quantity of product X, but for comparability between different cooperation forms, the following analysis meets $0 < \alpha_r < \alpha^B_{r,q2}$ to ensure the premise of cooperation. Specifically, when the consumer market loss rate is lower than γ_4, the referral of two retailers can effectively increase the order quantity of product X. However, as the consumer market loss rate increases, the advantage of exclusive referral gradually emerges. This indicates that the number of retailers recommended by manufacturers is not positively correlated with the sales volume of products, which may be because promotion increases the competition between channels or products. It is worth noting that regardless of the loss rate of the consumer market, the order quantity of product X with the manufacturer's referral is always higher than that without the referral. Only when the loss rate of the consumer market is very high and exceeds γ_1 will the order quantity of product X in the nonreferral situation be higher than that in the nonexclusive referrals situation. This is because a large number of consumers switch to purchase product Y, which makes the store brand retailer reduce the order quantity of product X and increase the production of product Y.

The above analysis shows that under different consumer market loss rates, the manufacturer's referral will always increase the order quantity of product X. Under the influence of different referrals of the manufacturer, will the output of product Y change similarly to the order quantity of product X? To solve this problem, this paper uses $\Delta q^n_3, n = R,N;B,N;B,R$ to represent the output difference of product Y under different cooperation forms. It is found that $\Delta q^{R,N}_3 > 0$ always holds for $\forall\alpha_r > 0$, $\Delta q^{B,N}_3$ is related to γ_3, and $\Delta q^{B,R}_3$ is related to γ_5 ($\gamma_5 = \frac{-\theta(5\theta^3-12\theta^2-24\theta+40)}{2(\theta^4-16\theta^3+42\theta^2-4\theta-32)}$) by solving $\Delta q_3^{R,N} = q_3^R - q_3^N = 0$, $\Delta q_3^{B,N} = q_3^B - q_3^N = 0$ and $\Delta q_3^{B,R} = q_3^B - q_3^R = 0$ simultaneously. Based on the above analysis, Proposition 4 can be obtained.

Proposition 4. *Under the influence of the manufacturer's consumption market loss rate, the output of product Y changes between different scenarios as follows.*

1. When $0 < \theta < 0.536$, if $0 < \gamma < \gamma_5$, then $q_3^{*N} > q_3^{*R} > q_3^{*B}$; if $\gamma_5 < \gamma < \gamma_3$, then $q_3^{*N} > q_3^{*B} > q_3^{*R}$; and if $\gamma_3 < \gamma < 1$, then $q_3^{*B} > q_3^{*N} > q_3^{*R}$.
2. When $0.536 < \theta < 1$, if $0 < \gamma < \gamma_3$, then $q_3^{*R} > q_3^{*N} > q_3^{*B}$; if $\gamma_3 < \gamma < \gamma_5$, then $q_3^{*R} > q_3^{*B} > q_3^{*N}$; and if $\gamma_5 < \gamma < 1$, then $q_3^{*B} > q_3^{*R} > q_3^{*N}$.

Compared with product X, the output of product Y is affected not only by the loss rate of the consumer market but also by the intensity of market competition. Surprisingly, when the market competition intensity is low ($0 < \theta < 0.536$), the output of product Y will be higher without referral if the consumer market loss rate is below γ_3. In other words, whether the manufacturer exclusively promotes the traditional retailer or promotes both retailers, it will prevent the store brand retailer from developing its own product. When market competition is fierce ($0.536 < \theta < 1$), the manufacturer's exclusive referral will increase the output of product Y if the market loss rate is below γ_3. This is because the manufacturer's referral increases the retail price of product X and loses its price advantage. Therefore, no matter what the intensity of market competition is, if the manufacturer's consumer market loss rate is at a low level, nonexclusive referrals are not conducive to the increase in the output of product Y, which is contrary to the intuitive guess. This is mainly because when the loss rate of the consumer market is low, consumers prefer the manufacturer's product X. Retailers are more willing to cooperate with the manufacturer to distribute product X instead of competing with the manufacturer. Otherwise, it will harm the interests of all parties because of vicious competition.

By analysing and comparing Proposition 3 and Proposition 4, Proposition 5 can be obtained.

Proposition 5. *When $\gamma_3 < \gamma < \gamma_4$, both the order quantity of product X and the output of product Y are maximized in the nonexclusive promotion scenario.*

Proposition 5 shows that although products X and Y are substitutes for each other, there is still a "win−win" area in terms of order quantity (output). That is, the order quantity of product X and the output of product Y can be maximized at the same time in region $\gamma_3 < \gamma < \gamma_4$. From Figure 4, Lemmas 1–4 can also be obtained as follows.

Lemma 1. *When $0 < \gamma < \gamma_3$, the order quantity of product X is always the lowest, and the output of product Y is the highest when there is no referral. However, as θ increases, exclusive referrals are more beneficial to product Y.*

In this case, referrals are beneficial to increase orders for product X, and referrals to two retailers are better than referrals to one retailer. Obviously, this is because consumers have high loyalty to product X and almost ignore the temptation of product Y. The manufacturer's referral of a private brand retailer is equivalent to adding a sales channel, which in turn is conducive to expanding the order quantity of its product X. However, for product Y, the result is the opposite. When the market competition intensity is $0 < \theta < 0.536$, the absence of referrals actually helps the store brand retailer increase the production of product Y. That is, any referral from the manufacturer will reduce the output of product Y. When $0.536 < \theta < 1$, the exclusive referral is more conducive to increasing the production of product Y than no referral. Because the substitutability between products X and Y increases, the difference decreases, and product Y has a price advantage and is more attractive to consumers. The manufacturer's referral will expand the scale of potential consumers of product Y, thus driving the output of product Y to increase.

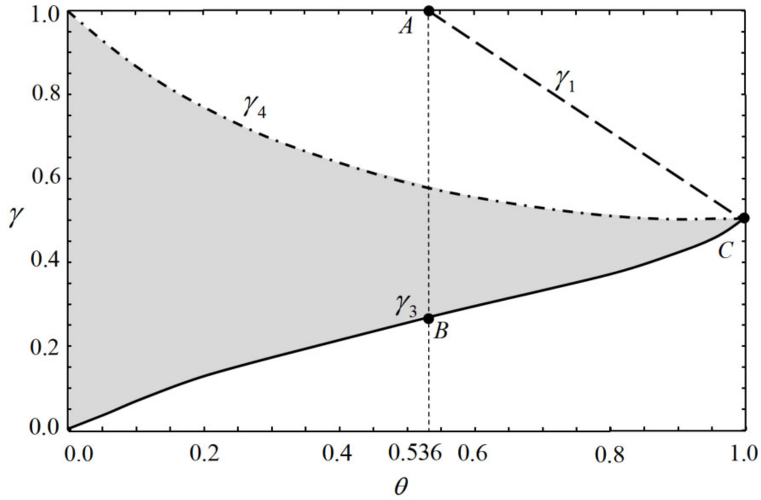

Figure 4. The relationship between consumption market loss rate and market competition intensity.

Lemma 2. *When $\gamma_4 < \gamma < 1$, the order quantity of product X is highest under exclusive referrals, while the volume of product Y is lowest.*

Compared with Lemma 1, the consumer market loss rate is higher under this condition, and the manufacturer can use the exclusive promotion strategy only to hit the production of product Y. Benefitting from the manufacturer's referral, the traditional retailer will increase its business credibility and widen the differentiation between itself and the store brand retailer, thereby attracting more consumers.

Lemma 3. *When $0.536 < \theta < 1$ and $\gamma_3 < \gamma < \gamma_1$, both product X and product Y have the lowest quantities in the no-referral situation.*

The conditions in Lemma 3 correspond to the triangular area ΔABC in Figure 4. Under these conditions, the competition between the two retailers is fierce, which indicates that both products are popular. Therefore, the manufacturer is motivated to expand sales channels. That is, the manufacturer is willing to introduce retailers to consumers. At this time, neither retailer will miss a business opportunity, and both will increase the order quantity.

Lemma 4. *When $\gamma_1 < \gamma < 1$, product Y has the largest production volume and product X has the lowest order quantity for nonexclusive referrals.*

This situation is realistic. At this time, product competition is fierce, and consumers are less loyal to product X. Product Y can easily attract more consumers through price advantages, which will lead to a decrease in the order quantity of product X and an increase in the output of product Y.

5. Comparative Analysis of Member Cooperation

By comparing the profits of members in different cooperation modes, we analyse the best promotion cooperation mode for the manufacturer, the traditional retailer and the private label retailer and explore whether there is a balanced cooperation option among supply chain members.

5.1. Analysis of the Manufacturer's Referral Strategy

The manufacturer chooses strategy $j, j = N, R, B$ because it is more profitable than other strategies. We use $\Delta\pi_M^{R,N}$, $\Delta\pi_M^{B,N}$ and $\Delta\pi_M^{B,R}$ to represent the profit difference of the manufacturer when pairwise cooperation is compared. It is found by calculation that $\Delta\pi_M^{R,N} = \pi_M^R - \pi_M^N > 0$ always holds for $\forall\alpha_r > 0$; $\Delta\pi_M^{B,N} = 0$ is calculated to obtain the following findings.

When $\gamma_1 < \gamma < 1$, there is a threshold $\alpha_{r,M}^{B,R}$ of the referral level and $\alpha_{r,q2}^B < \alpha_{r,w}^B < \alpha_{r,M}^{B,N}$, so the referral level needs to satisfy $0 < \alpha_r < \alpha_{r,q2}^B$ at this time; when $0 < \gamma < \gamma_1$, $\Delta\pi_M^{B,N} > 0$ always holds for $\forall\alpha_r > 0$; the threshold $\alpha_{r,M}^{B,R}$ can be obtained by calculating $\alpha_{r,q2}^B < \alpha_{r,w}^B < \alpha_{r,M}^{B,R}$ and $\Delta\pi_M^{B,R} = 0$. Thus, Proposition 6 can be drawn.

Proposition 6. *The changes in manufacturer profits among different scenarios are as follows.*

When $0 < \gamma < \gamma_2$ and $\forall\alpha_r > 0$, there exists $\pi_M^{*B} > \pi_M^{*R} > \pi_M^{*N}$;
When $\gamma_2 < \gamma < \gamma_4$ and $0 < \alpha_r < \alpha_{r,q2}^B$, there exists $\pi_M^{*B} > \pi_M^{*R} > \pi_M^{*N}$;
When $\gamma_4 < \gamma < \gamma_1$ and $0 < \alpha_r < \alpha_{r,q2}^B$, there exists $\pi_M^{*R} > \pi_M^{*B} > \pi_M^{*N}$;
When $\gamma_1 < \gamma < 1$ and $0 < \alpha_r < \alpha_{r,q2}^B$, there exists $\pi_M^{*R} > \pi_M^{*N} > \pi_M^{*B}$.

Take $s = 0.3$ in the feasible region and draw Figure 2 to illustrate $\gamma_i, i = 1,2,4,6$. Proposition 6 shows that the manufacturer's choice of referral cooperation is related to the loss rate of its consumer market. Specifically, with the increase in the loss rate of the consumer market, the disadvantage of nonexclusive referrals gradually emerges. When the loss rate exceeds γ_4, the manufacturer will not promote the store brand retailer to consumers. At this time, the most beneficial strategy for the manufacturer is $j = R$, but the manufacturer needs to control the referral level to not exceed $\alpha_{r,q2}^B$. Moreover, the best strategy for the manufacturer is $j = B$ when the loss rate is below γ_4, and the referral level needs to be controlled in a reasonable region when $\gamma_2 < \gamma < \gamma_4$. When the loss rate of the consumer market is low, the manufacturer's referral of two retailers is equivalent to adding a sales channel, while when the loss rate of the consumer market is high, the referral of the private brand retailer is more conducive to expanding the potential market scale of product Y. However, in any case, the manufacturer will always obtain more profit by promoting authorized retailers to consumers, and the manufacturer is willing to facilitate the referral cooperation with retailers.

5.2. The Influence of Different Referral Strategies on the Traditional Retailer

Intuitively, exclusive referrals are more beneficial to the traditional retailer. This section explores the traditional retailer's preference for referral cooperation by analysing the profit under different referral strategies.

First, the profits of the traditional retailer in the two kinds of cooperation of no referral and exclusive referral are compared and expressed by $\Delta\pi_1^{R,N} = \pi_1^R - \pi_1^N$. It is found that the exclusive referral is better than no referral because $\Delta\pi_1^{R,N} > 0$ for $\forall\alpha_r > 0$ through solving $\Delta\pi_1^{R,N}$.

Similarly, we analyse the change in profit for the traditional retailer between nonexclusive referrals and no referral through calculating $\Delta\pi_1^{B,N} = \pi_1^B - \pi_1^N$. It is found that $\Delta\pi_1^{B,N} > 0$ for $\forall\alpha_r > 0$, which means that the traditional retailer is more profitable under the nonexclusive referrals.

The profit difference of the traditional retailer between the manufacturer's nonexclusive referrals and exclusive referral is denoted by $\Delta\pi_1^{B,R} = \pi_1^B - \pi_1^R$. We can obtain a critical value $\gamma_6 = \frac{\theta^3+8\theta^2-16\theta-8}{2(\theta^3-6\theta-4)}$ of the consumer market loss rate and $\gamma_2 < \gamma_1 < \gamma_6$. Through collating the above analysis, the conclusions are summarized as Proposition 7.

Proposition 7. *Comparing the traditional retailer's profits under different scenarios, the following findings are obtained.*

When $0 < \gamma < \gamma_2$ and $\forall \alpha_r > 0$, or $\gamma_2 < \gamma < \gamma_6$ and $0 < \alpha_r < \alpha^B_{r,q2}$, there exists $\pi_1^{*R} > \pi_1^{*B} > \pi_1^{*N}$; when $\gamma_6 < \gamma < 1$ and $0 < \alpha_r < \alpha^B_{r,q2}$, there exists $\pi_1^{*B} > \pi_1^{*R} > \pi_1^{*N}$.

Figure 2 shows the relationship between $\gamma_i, i = 1,2,6$. Proposition 7 states that the traditional retailer is always least profitable when there is no referral, regardless of loss rates and referral levels. This is because no matter what kind of referral strategy the manufacturer adopts, it will always help to expand the scale of its potential consumers, so the traditional retailer is willing to accept the manufacturer's referral cooperation. Therefore, when the loss rate of the consumer market is lower than x, the traditional retailer is always willing to reach an exclusive promotion cooperation with the manufacturer. However, contrary to the previous conjecture, when the loss rate of the consumer market exceeds γ_6, the traditional retailer will be more inclined to engage in nonexclusive referral cooperation. To illustrate this problem, it must be emphasized that the loss of the consumer market occurs because after consumers enter the stores of private label retailers, a γ proportion of consumers will abandon product X and buy product Y. However, the size of the potential consumer market for traditional retailers is still α_r. When $\gamma > \gamma_6$, it means that the total potential consumers of product X have decreased. Lemma 4 proves that the order quantity of product X decreases at this time, so the manufacturer has to lower the wholesale price to achieve cooperation with retailers. It is easy to prove that $w^{*R} > w^{*N} > w^{*B}$ when $\gamma_6 < \gamma < 1$, so there is a counterintuitive result.

5.3. The Influence of Different Referral Strategies on the Store Brand Retailer

The choice faced by the store brand retailer is whether to accept the manufacturer's exclusive referral cooperation. To ensure comparability among different collaborations, all analyses are performed under the condition of $\alpha_r < \alpha^{*B}_{r,q2}$. Next, by calculating the profit difference of the private brand retailer in different cooperation modes, relevant conclusions are obtained to provide guidance for its business decisions.

The profit difference of the store brand retailer under the exclusive referral and the nonreferral is denoted by $\Delta\pi_2^{R,N} = \pi_2^R - \pi_2^N$. It can be obtained that when $0 < \alpha_r < \alpha^{R,N}_{r,R2}$, there exists $\Delta\pi_2^{R,N} > 0$ through solving $\Delta\pi_2^{R,N} > 0$, which means the exclusive referral is better than no referral for the store brand retailer when $0 < \alpha_r < \alpha^{R,N}_{r,R2}$. Otherwise, when $\alpha_r > \alpha^{R,N}_{r,R2}$, no referral is better.

The profit difference of the store brand retailer under the nonexclusive referrals and the nonreferral is denoted by $\Delta\pi_2^{B,N} = \pi_2^B - \pi_2^N$. A critical value γ_7 of the consumer market loss rate can be obtained by solving $\Delta\pi_2^{B,N} > 0$. When $0 < \gamma < \gamma_7$, there exists a critical value s_1 of the market size and a threshold $\alpha^{B,N}_{r,R2}$ of the referral level. The findings are as follows.

When $\underline{s} < s < s_1$, there exists $\Delta\pi_2^{B,N} > 0$ for $\forall \alpha_r > 0$; when $s_1 < s < \bar{s}$ and $0 < \alpha_r < \alpha^{B,N}_{r,R2}$, there exists $\Delta\pi_2^{B,N} < 0$ and when $\alpha_r > \alpha^{B,N}_{r,R2}$, then $\Delta\pi_2^{B,N} > 0$; and when $\gamma_7 < \gamma < 1$, there exists $\Delta\pi_2^{B,N} > 0$ for $\forall \alpha_r > 0$.

Similarly, the profit difference of the store brand retailer under the nonexclusive referrals and the exclusive referral is denoted by $\Delta\pi_2^{B,R} = \pi_2^B - \pi_2^R$. By solving $\Delta\pi_2^{B,R} = 0$, we can obtain the critical values γ_8 and γ_9 of the consumer market loss rate, and the results of the analysis are as follows.

In region $\Omega = \gamma_8 < \gamma < \gamma_9$, there exists a threshold $\alpha^{B,R}_{r,R2}$ of the referral level. When $0 < \alpha_r < \alpha^{B,R}_{r,R2}$, $\Delta\pi_2^{B,R} > 0$, and when $\alpha_r > \alpha^{B,R}_{r,R2}$, $\Delta\pi_2^{B,R} < 0$. In region $\overline{\Omega}$, there exists $\Delta\pi_2^{B,R} > 0$ for $\forall \alpha_r > 0$.

Take the manufacturer's consumer market loss rate $\gamma(0 < \gamma < 1)$ as the vertical axis and the market competition intensity $\theta(0 < \theta < 1)$ as the horizontal axis to make Figure 5. It should be noted that $0 < \gamma < \min\{\gamma_7, \gamma_9\}$ is defined as Region ①, the area enclosed by γ_2, γ_2 and γ_9 is defined as Region ②, and the rest area is defined as Region ③. The maximum profit of the retailer in each region is represented as $\pi_{R2}(\max)$, and Proposition 8 is obtained as follows.

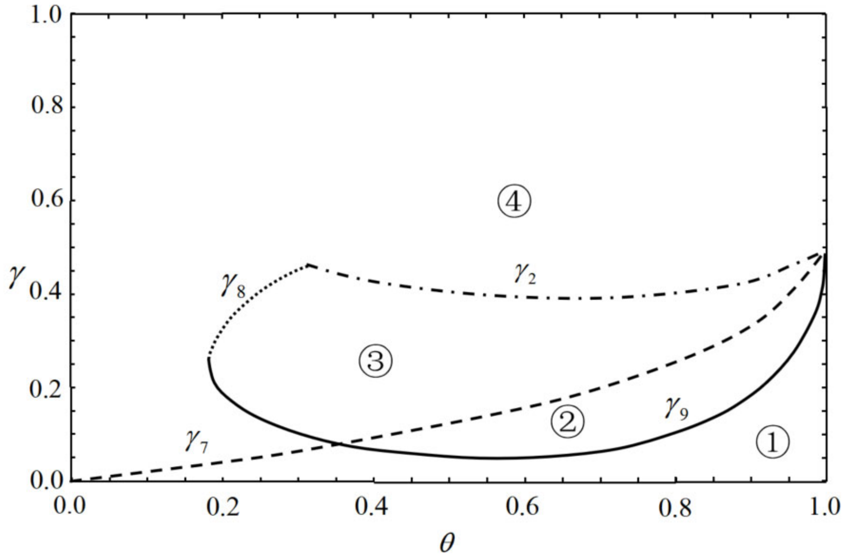

Figure 5. The $\gamma - \theta$ area graph.

Proposition 8. *Under different scenarios, as the level of referral increases, the profit of the store brand retailer changes as follows.*

1. In Region ①, when $\underline{s} < s < s_1$, the maximum profit is $\pi_{R2}(\max) = \pi_{R2}^{*B}$; when $s_1 < s < \bar{s}$, the maximum profit is $\pi_{R2}(\max) = \pi_{R2}^{*N} \to \pi_{R2}^{*B}$.
2. In Region ②, when $\underline{s} < s < s_1$, the maximum profit is $\pi_{R2}(\max) = \pi_{R2}^{*B} \to \pi_{R2}^{*R}$; when $s_1 < s < \bar{s}$, the maximum profit is $\pi_{R2}(\max) = \pi_{R2}^{*N} \to \pi_{R2}^{*B} \to \pi_{R2}^{*R}$.
3. In Region ③, the maximum profit is $\pi_{R2}(\max) = \pi_{R2}^{*B} \to \pi_{R2}^{*R}$.
4. In Region ④, the maximum profit is $\pi_{R2}(\max) = \pi_{R2}^{*B}$.

Note that $\pi_{R2}^{*N} \to \pi_{R2}^{*B}$ represents that as the referral level increases, the situation in which the store brand retailer is most profitable changes from nonreferral to nonexclusive referral, and $\pi_{R2}^{*N} \to \pi_{R2}^{*B} \to \pi_{R2}^{*R}$ means something similar. The specific proof process is as follows, and Figures 4–6 are made to analyse this proof.

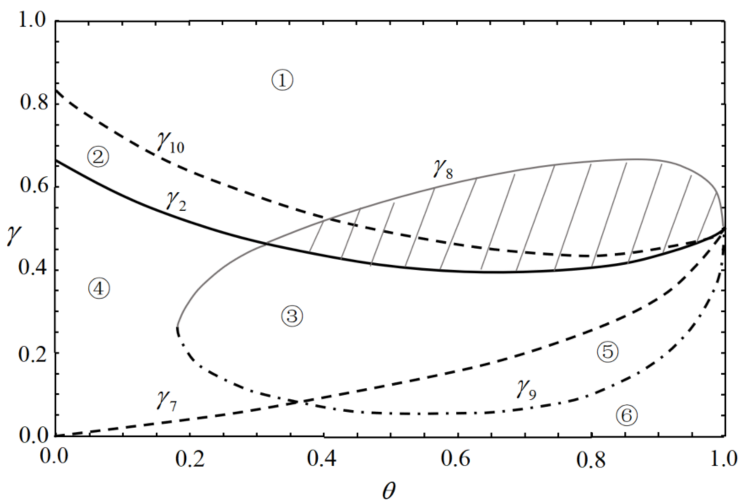

Figure 6. The $\gamma - \theta$ area graph.

1. There exist three thresholds $\alpha_{r,q2}^{B}$, $\alpha_{r,R2}^{R,N}$ and $\alpha_{r,R2}^{B,R}$ of the referral level in Region ① and Region ② at the same time, but $\alpha_{r,R2}^{B,R}$ exists only in the shaded area in the figure at this time.

By calculating $\alpha_{r,q2}^{B} - \alpha_{r,R2}^{B,R}$, we can obtain a function of s, γ, θ, denoted by $f_1(s, \gamma, \theta)$. The denominator of $f_1(s, \gamma, \theta)$ is greater than 0 in the shadow area. Then, we set the numerator part of this formula equal to 0 to solve for s, and we can obtain a critical value s_2. When $s > s_2$, the numerator is positive; otherwise, it is negative. To judge the relationship between the critical value and the feasible region of market size, $\bar{s} - s_2$ needs to be calculated, and we can obtain the function $g_1(\gamma, \theta)$. By analysing $g_1(\gamma, \theta)$, it is found that $\bar{s} < s_2$. Therefore, the numerator of $f_1(s, \gamma, \theta)$ is negative in the interval $(\underline{s}, \bar{s})$. That is, $f_1(s, \gamma, \theta) < 0$ and $\alpha_{r,q2}^{B} < \alpha_{r,R2}^{B,R}$.

2. Compare $\alpha_{r,q2}^{B}$ and $\alpha_{r,R2}^{R,N}$ in Regions ① and ②.

A similar method is used to analyse $\alpha_{r,q2}^{B} - \alpha_{r,R2}^{R,N}$, denoted by $f_2(s, \gamma, \theta)$. It is easy to find that the numerator of $f_2(s, \gamma, \theta)$ is negative. Let the numerator equal 0, and we can obtain the critical value s_3. By analysing $\underline{s} - s_3$ and $\bar{s} - s_3$, we can obtain the threshold γ_{10} of the consumer market loss rate, and the conclusions are as follows.

When $\gamma_{10} < \gamma < 1$, the numerator of $f_2(s, \gamma, \theta)$ is greater than 0, so $f_2(s, \gamma, \theta) < 0$, namely, $\alpha_{r,q2}^{B} < \alpha_{r,R2}^{R,N}$; when $0 < \gamma < \gamma_{10}$, if $\underline{s} < s < s_3$, then $f_2(s, \gamma, \theta) > 0$, namely, $\alpha_{r,q2}^{B} > \alpha_{r,R2}^{R,N}$; if $s_3 < s < \bar{s}$, then $f_2(s, \gamma, \theta) < 0$, namely, $f_2(s, \gamma, \theta) < 0$.

Therefore, in Region ①($\gamma_{10} < \gamma < 1$), there exists $\pi_{R2}^{*B} > \pi_{R2}^{*N} > \pi_{R2}^{*R}$ when $\alpha_{r,q2}^{B} < \alpha_{r,R2}^{R,N}$. In Region ②($\gamma_{10} < \gamma < 1$), if $\bar{s} < s < s_3$ and $0 < \alpha_r < \alpha_{r,R2}^{R,N}$, there exists $\pi_{R2}^{*B} > \pi_{R2}^{*N} > \pi_{R2}^{*R}$, and if $s_3 < s < \bar{s}$ and $\alpha_{r,R2}^{R,N} < \alpha_r < \alpha_{r,q2}^{B}$, there exists $\pi_{R2}^{*B} > \pi_{R2}^{*R} > \pi_{R2}^{*N}$.

3. There exist two thresholds $\alpha_{r,R2}^{R,N}$ and $\alpha_{r,R2}^{B,R}$ in Region ③. Using a similar method, it is found that $\alpha_{r,R2}^{B,R} > \alpha_{r,R2}^{R,N}$. Therefore, when $0 < \alpha_r < \alpha_{r,R2}^{R,N}; \pi_{R2}^{*B} > \pi_{R2}^{*N} > \pi_{R2}^{*R}$; when $\alpha_{r,R2}^{R,N} < \alpha_r < \alpha_{r,R2}^{B,R}; \pi_{R2}^{*B} > \pi_{R2}^{*R} > \pi_{R2}^{*N}$; and when $\alpha_r > \alpha_{r,R2}^{B,R}$, $\pi_{R2}^{*R} > \pi_{R2}^{*B} > \pi_{R2}^{*N}$.

4. There exists only a threshold $\alpha_{r,R2}^{R,N}$ in Region ④. Therefore, when $0 < \alpha_r < \alpha_{r,R2}^{R,N}$, there exists $\pi_{R2}^{*B} > \pi_{R2}^{*N} > \pi_{R2}^{*R}$; when $\alpha_r > \alpha_{r,R2}^{R,N}$, there exists $\pi_{R2}^{*B} > \pi_{R2}^{*R} > \pi_{R2}^{*N}$.

5. In Region ⑤, we can obtain $\alpha_{r,R2}^{B,R} > \alpha_{r,R2}^{R,N}$ by solving $\alpha_{r,R2}^{B,R} - \alpha_{r,R2}^{R,N}$ and can obtain $\alpha_{r,R2}^{R,N} > \alpha_{r,R2}^{B,N}$ by comparing $\alpha_{r,R2}^{R,N}$ and $\alpha_{r,R2}^{B,N}$. Therefore, when $\bar{s} < s < s_1$, if $0 < \alpha_r < \alpha_{r,R2}^{R,N}$, then $\pi_{R2}^{*B} > \pi_{R2}^{*N} > \pi_{R2}^{*R}$; if $\alpha_{r,R2}^{R,N} < \alpha_r < \alpha_{r,R2}^{B,R}$, then $\pi_{R2}^{*B} > \pi_{R2}^{*R} > \pi_{R2}^{*N}$; and if $\alpha_r > \alpha_{r,R2}^{B,R}$, then $\pi_{R2}^{*R} > \pi_{R2}^{*B} > \pi_{R2}^{*N}$. When $s_1 < s < \bar{s}$, if $0 < \alpha_r < \alpha_{r,R2}^{B,N}$, then $\pi_{R2}^{*N} > \pi_{R2}^{*B} > \pi_{R2}^{*R}$; if $\alpha_{r,R2}^{B,N} < \alpha_r < \alpha_{r,R2}^{R,N}$, then $\pi_{R2}^{*B} > \pi_{R2}^{*N} > \pi_{R2}^{*R}$; if $\alpha_{r,R2}^{R,N} < \alpha_r < \alpha_{r,R2}^{B,R}$, then $\pi_{R2}^{*B} > \pi_{R2}^{*R} > \pi_{R2}^{*N}$; and if $\alpha_r > \alpha_{r,R2}^{B,R}$, then $\pi_{R2}^{*R} > \pi_{R2}^{*B} > \pi_{R2}^{*N}$.

6. In Region ⑥, there exists $\alpha_{r,R2}^{R,N} > \alpha_{r,R2}^{B,N}$. Therefore, when $\bar{s} < s < s_1$, if $0 < \alpha_r < \alpha_{r,R2}^{R,N}$, then $\pi_{R2}^{*B} > \pi_{R2}^{*N} > \pi_{R2}^{*R}$; and if $\alpha_r > \alpha_{r,R2}^{R,N}$, then $\pi_{R2}^{*B} > \pi_{R2}^{*R} > \pi_{R2}^{*N}$. When $s_1 < s < \bar{s}$, if $0 < \alpha_r < \alpha_{r,R2}^{B,N}$, then $\pi_{R2}^{*N} > \pi_{R2}^{*B} > \pi_{R2}^{*R}$; if $\alpha_{r,R2}^{B,N} < \alpha_r < \alpha_{r,R2}^{R,N}$, then $\pi_{R2}^{*B} > \pi_{R2}^{*N} > \pi_{R2}^{*R}$; and if $\alpha_r > \alpha_{r,R2}^{R,N}$, then $\pi_{R2}^{*B} > \pi_{R2}^{*R} > \pi_{R2}^{*N}$. This is the end of the proof.

Proposition 7 shows that if the consumer market loss rate is high or the market competition is not fierce (Region IV), then the nonexclusive referrals are best for the store brand retailer. Under these conditions, the store brand retailer prefers the nonexclusive referral. By comparing Region III and Region IV, it can be found that as consumer market loss rates decrease and referral levels increase, the private label retailer also gains the most from exclusive referrals. Product Y increases production with its price advantage and thus increases profits, and the manufacturer can control the referral level to satisfy $0 < \alpha_r < \alpha_{r,R2}^{B,R}$, thus influencing the private brand retailer to choose a nonexclusive referral. By comparing Region II and Region III, it is found that as the loss rate in the consumer market decreases, the store brand retailer is likely to make more profit when there is no referral. This is mainly because when the loss rate is low, the referral level increased by the manufacturer intensifies the vicious competition between retailers. Similarly, in Region II, the manufacturer adjusts the referral level based on market size to influence the store brand retailer's referral choices. When $\underline{s} < s < s_1$ and $0 < \alpha_r < \alpha_{r,R2}^{B,R}$, or $s_1 < s < \bar{s}$ and $\alpha_{r,R2}^{B,N} < \alpha_r < \alpha_{r,R2}^{B,R}$, the store brand retailer prefers the nonexclusive referral. By comparing Regions I and II, it is found that in most cases, the store brand retailer prefers the nonexclusive referral. However, when $s_1 < s < \bar{s}$ and $0 < \alpha_r < \alpha_{r,R2}^{B,N}$ in Region I, the store brand retailer prefers the nonreferral. Because the referral would expand the store brand retailer's potential consumer market, the exclusive referral would increase the difference between the two retailers, and consumers are more loyal to product X. Thus, in this situation, the profit of the store brand retailer is low, even lower than in the no-referral scenario.

Summarizing Proposition 5, Proposition 6 and Proposition 7, it can be found that there is no balanced cooperation choice among the manufacturer, the traditional retailer and the store brand retailer. It needs to be studied by means of supply chain coordination, which is also an issue worthy of in-depth discussion in the future.

6. Discussion and Conclusions

This paper considers the cooperative model for the manufacturer to introduce retailers to consumers. The digital economy has become a huge driver of economic development, with complex cooperation and competition between manufacturers and retailers. Some manufacturers display links to authorized retailers on their official websites, such as Pechoin, Meifubao and L'Oréal Paris [10,11]. However, some retailers not only distribute manufacturers' products but also produce their own brand products after fully understanding consumer demand through sales data, such as Walmart and Carrefour [8,24]. Therefore, the following questions arise: how does the manufacturer make the choice of referral strategy, and how does the retailer make quantitative decisions? To explore these issues, this paper constructs a supply chain model consisting of a manufacturer, a traditional retailer, and a retailer with a homogeneous private brand.

Ghose et al. [17] proposed the referral service of the Internet in an earlier study and explored four scenarios: no referral, retailer referrals from an infomediary, retailer referrals from both the infomediary and manufacturers and manufacturers clearing the infomediary. This paper considers the three cooperation models: no referral, exclusive referral and nonexclusive referrals. Wu et al. [6] constructed a Stackelberg game model where a manufacturer sells its products through two heterogeneous retailers to study the manufacturer's referral strategy. They found that nonexclusive referrals are a balanced choice for the manufacturer if the referral market segment is large enough. Han et al. [4] constructed a Stackelberg model with a manufacturer, a professional retailer and a store brand retailer. The optimal price and profit of the manufacturer and both retailers when the manufacturer refers consumers to the professional retailer or not are solved and compared. It is found that the manufacturer's referral will be beneficial to both the manufacturer and the retailer if the referral level meets certain conditions; otherwise, it may reduce the profits of both parties, and the recommendation level is affected by the market size of the specialty retailer. In addition, most studies on the influence of retailers' private brands on manufacturers mainly focus on return guarantee, channel strategy, advertising strategy and other aspects [8,44]. By solving the models, the equilibrium wholesale price, the order quantity of the manufacturer's products and the output of the retailer's own products under different referral options are obtained. The following conclusions can be drawn from the analysis of the equilibrium results: the manufacturer's referral level will not change the product quantity. This paper studies the form of cooperation when the manufacturer promotes retailers to consumers. It is mainly divided into three types of cooperation: the manufacturer does not provide information referral, provides exclusive referral and provides nonexclusive referrals. By solving the models, the equilibrium wholesale price, the order quantity of the manufacturer's products and the output of the retailer's own products under different referral options are obtained. The following conclusions can be drawn from the analysis of the equilibrium results: the manufacturer's referral level will not change the product quantity among different referral options, while the manufacturer's market loss rate will change the order quantity of product X among different options. The combined effect of the market loss rate and competition intensity will prompt the store brand retailer to change the output decision of its own product Y. In addition, four important lemmas can be obtained according to the change in product quantity, providing theoretical guidance for supply chain members to choose referral cooperation.

1. When $0 < \gamma < \gamma_3$ and there is no referral, the order quantity of product X is always the lowest, and the output of product Y is the highest. However, with the increase of θ, exclusive referral is more beneficial to product Y. In this case, the referral is conducive to increasing the order quantity of product X, and the referral of two retailers is better than the referral of one retailer.

2. The loss rate of the market is high when $\gamma_4 < \gamma < 1$, and the order quantity of product X is the highest with the exclusive referral, while the output of product Y is the lowest.

3. When $0.536 < \theta < 1$, $\gamma_3 < \gamma < \gamma_1$, and there is no referral, the quantities of products X and Y are both the lowest. The manufacturer is willing to introduce retailers to consumers. At this time, neither retailer will miss a business opportunity, and both will increase the order volume.

4. When $\gamma_1 < \gamma < 1$, with the nonexclusive referrals, the production of product Y is the highest, and the order quantity of product X is the lowest. At this time, product competition is fierce, and consumers' loyalty to product X is low. Self-owned products can easily attract more consumers through price advantages.

Through profit analysis, the best referral cooperation option between the manufacturer and retailers is obtained under the conditions of different consumer market loss rates and referral levels. First, the manufacturer will always make more profits by promoting the authorized retailer to consumers, and the manufacturer is also willing to facilitate referral cooperation with the retailer. Second, the manufacturer's exclusive referral is better than no referral. The traditional retailer is willing to accept the manufacturer's referral cooperation, and the traditional retailer's profit is better with the nonexclusive referral. Third, in most cases, the store brand retailer is willing to choose nonexclusive referrals because the manufacturer's referral will expand the potential consumer market of the store brand retailer. However, the exclusive referral will increase the difference between the two retailers, and consumers have higher loyalty to product X, so the store brand retailer will make a lower profit in this situation, even lower than in the nonreferral situation.

In this paper, the study of the referral cooperation between the manufacturer and retailers is mainly based on their profit changes. However, the cooperation between manufacturers and retailers is more dependent on the incentives and constraints of the contract in reality. Therefore, how to design a contract to encourage manufacturers and retailers to reach a cooperative agreement has become a problem worthy of further study. When the choices of manufacturers and retailers diverge, more interesting conclusions may emerge by adjusting the choices of all parties through contract design.

We also found that there is no balanced cooperation option among the manufacturer, the traditional retailer and the store brand retailer, which needs to be studied by means of supply chain coordination. That is also worthy of further discussion in the future.

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