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h-Almost Ricci–Yamabe Solitons in Paracontact Geometry

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Abstract: In this article, we classify h-almost Ricci–Yamabe solitons in paracontact geometry. In particular, we characterize para-Kenmotsu manifolds satisfying h-almost Ricci–Yamabe solitons and 3-dimensional para-Kenmotsu manifolds obeying h-almost gradient Ricci–Yamabe solitons. Then, we classify para-Sasakian manifolds and para-cosymplectic manifolds admitting h-almost Ricci–Yamabe solitons and h-almost gradient Ricci–Yamabe solitons, respectively. Finally, we construct an example to illustrate our result.

Keywords: h-almost Ricci–Yamabe solitons; h-almost gradient Ricci–Yamabe solitons; paracontact geometry; para-Kenmotsu manifolds; para-Sasakian manifolds; para-cosymplectic manifolds

MSC: 53C25; 53D15; 53E20

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1. Introduction

In 1964, Eells and Sampson introduced the notion of a harmonic map heat flow on a Riemannian manifold. The first work of Hamilton on Ricci flow was inspired from the work of Eells and Sampson. In 1981, Hamilton [1] utilized the Ricci flow to gain insight into the geometrization conjecture of William Thurston. In 1988, Hamilton introduced the notion of Yamabe flow. The Ricci flow and Yamabe flow have many applications, especially in mathematics and physics. The notion of Ricci–Yamabe flow, a linear combination of Ricci and Yamabe flow, is defined in 2019 by Gular and Crasmareanu [2].

In a semi-Riemannian manifold, the Ricci–Yamabe soliton is defined by

$$\pounds_V g + (2\lambda - \beta r)g + 2\alpha S = 0, \tag{1}$$

where $\pounds$ denotes the Lie-derivative, S denotes the Ricci tensor, r denotes the scalar curvature and $\lambda, \alpha, \beta \in \mathbb{R}$. Ricci–Yamabe solitons are the special solutions of the Ricci–Yamabe flow

$$\frac{\partial g}{\partial t} = -2\alpha S + \beta r g, \tag{2}$$

which was introduced by Guler and Crasmareanu [2]. Equation (1) is called an almost Ricci–Yamabe soliton provided that λ is a smooth function. The Ricci–Yamabe soliton is said to be expanding, steady or shrinking according to $\lambda > 0$, $\lambda = 0$ or $\lambda < 0$, respectively.

In particular, for $\alpha = 1$ and $\beta = 0$, (1) implies

$$\pounds_V g + 2\alpha S + 2\lambda g = 0, \tag{3}$$

which represents the Ricci soliton equation for $\lambda \in \mathbb{R}$. Thus, almost Ricci–Yamabe solitons (respectively, Ricci–Yamabe solitons) are the natural generalizations of almost Ricci solitons (respectively, Ricci solitons). Several generalizations of Ricci solitons are almost Ricci solitons ([3–7]), η-Ricci solitons ([8–13]), $*$-Ricci solitons ([1,14–18]) and many others.

Recently, Gomes et al. [19] extended the concept of almost Ricci solitons to h-almost Ricci solitons on a complete Riemannian manifold by

$$\frac{h}{2}\pounds_V g + \lambda g + S = 0, \tag{4}$$

where $h : M \to \mathbb{R}$ is a smooth function. Specifically, a Ricci soliton is the 1-almost Ricci soliton endowed with constant λ.

Now, we introduce a new type of soliton named h-almost Ricci–Yamabe soliton (briefly, h-ARYS) which is an extended style of almost Ricci–Yamabe solitons, which are given by

$$\frac{h}{2}\pounds_V g + \alpha S + (\lambda - \frac{\beta}{2}r)g = 0, \tag{5}$$

where h is a smooth function on the manifold.

If V is a gradient of a function f on the manifold, then the foregoing concept is called h-almost gradient Ricci–Yamabe soliton (briefly, h-AGRYS) and (5) takes the form

$$h\nabla^2 f + (\lambda - \frac{\beta}{2}r)g + \alpha S = 0. \tag{6}$$

An h-AGRYS is named h-gradient Ricci–Yamabe soliton if λ is a constant.

An h-ARYS (or h-AGRYS) turns into:

(i) h-almost Ricci soliton (or h-almost gradient Ricci soliton), if $\beta = 0$ and $\alpha = 1$;
(ii) h-almost Yamabe soliton (or h-almost gradient Yamabe soliton), if $\beta = 1$ and $\alpha = 0$;
(iii) h-almost Einstein soliton (or h-almost gradient Einstein soliton), if $\beta = -1$ and $\alpha = 1$.

The h-ARYS (or h-AGRYS) is called proper if $\alpha \neq 0, 1$.

Recently, in ([20,21]), the first author and Sarkar studied Ricci–Yamabe solitons in Kenmotsu 3-manifolds and generalized Sasakian space forms, respectively. Furthermore, Sing and Khatri [22] studied Ricci–Yamabe solitons in perfect fluid spacetimes.

The above studies motivated us to study h-ARYS and h-AGRYS in paracontact geometry.

The paper is organized as follows:

After the introduction, the required preliminaries are mentioned in Section 2. In Section 3, we investigate h-ARYS and h-AGRYS in para-Kenmotsu manifolds. Then, we classify para-Sasakian manifolds admitting h-ARYS and h-AGRYS. In addition to these, we investigate h-ARYS and h-AGRYS in para-cosymplectic manifolds in Section 7. Finally, we construct an example to illustrate our result.

2. Preliminaries

An almost paracontact structure on a manifold M^{2n+1} consists of a (1,1)-tensor field ϕ, a vector field ζ and a one-form η obeying the subsequent conditions:

$$\phi^2 = I - \eta \otimes \zeta, \quad \eta(\zeta) = 1 \tag{7}$$

and the tensor field ϕ induces an almost paracomplex structure on each fiber of $\mathcal{D} = ker(\eta)$, that is, the ± 1-eigendistributions, $\mathcal{D}^{\pm} = \mathcal{D}_{\phi}(\pm 1)$ of ϕ have equal dimension n. Almost paracontact and almost parahodge structures on manifolds has been introduced by Kaneyuki and Williams [23].

A manifold M^{2n+1} with an almost paracontact structure is named an almost paracontact manifold. From the definition, it can be established that $\phi\zeta = 0$, $\eta \circ \phi = 0$ and rank of ϕ is $2n$. If the Nijenhuis tensor vanishes identically, then the manifold is said to be normal. M^{2n+1} is named an almost paracontact metric manifold if there exists a semi-Riemannian metric g such that

$$g(\phi Z_1, \phi Z_2) = -g(Z_1, Z_2) + \eta(Z_1)\eta(Z_2) \tag{8}$$

for all $Z_1, Z_2 \in \chi(\boldsymbol{M})$.

$(\boldsymbol{M}^{2n+1}, \phi, \zeta, \eta, g)$ is named a paracontact metric manifold if $d\eta(Z_1, Z_2) = g(Z_1, \phi Z_2) = \Phi(Z_1, Z_2)$, Φ being the fundamental 2-form of $\boldsymbol{M}^{2n+1}$.

An almost paracontact metric manifold $\boldsymbol{M}^{2n+1}$, with a structure (ϕ, ζ, η, g) is said to be an almost γ-paracosymplectic manifold, if

$$d\eta = 0, d\Phi = 2\gamma\eta \wedge \Phi, \tag{9}$$

where γ is a constant or function on $\boldsymbol{M}^{2n+1}$. If we put $\gamma = 1$ in (9), we obtain an almost para-Kenmotsu manifold. A para-Kenmotsu manifold satisfies [24]

$$\boldsymbol{R}(Z_1, Z_2)\zeta = \eta(Z_1)\, Z_2 - \eta(Z_2)\, Z_1, \tag{10}$$

$$\boldsymbol{R}(Z_1, \zeta)\, Z_2 = g(Z_1,\, Z_2)\zeta - \eta(Z_2)\, Z_1, \tag{11}$$

$$\boldsymbol{R}(\zeta, Z_1)Z_2 = -g(Z_1, Z_2)\zeta + \eta(Z_2)Z_1, \tag{12}$$

$$\eta(\boldsymbol{R}(Z_1, Z_2)Z_3) = -g(Z_2, Z_3)\eta(Z_1) + g(Z_1, Z_3)\eta(Z_2), \tag{13}$$

$$(\nabla_{Z_1}\phi)Z_2 = g(\phi Z_1, Z_2)\zeta - \eta(Z_2)\phi Z_1, \tag{14}$$

$$\nabla_{Z_1}\zeta = Z_1 - \eta(Z_1)\zeta, \tag{15}$$

$$\boldsymbol{S}(Z_1, \zeta) = -2n\eta(Z_1). \tag{16}$$

Lemma 1 ([24])**.** *In a three-dimensional para-Kenmotsu manifold* $\boldsymbol{M}^3$*,*

$$\zeta r = -2(r + 6). \tag{17}$$

In $\boldsymbol{M}^3$, we also have

$$\boldsymbol{Q}Z_1 = (\frac{r}{2} + 1)Z_1 - (\frac{r}{2} + 3)\eta(Z_1)\zeta, \tag{18}$$

which provides

$$\boldsymbol{S}(Z_1, Z_2) = -(\frac{r}{2} + 3)\eta(Z_1)\eta(Z_2) + (\frac{r}{2} + 1)g(Z_1, Z_2), \tag{19}$$

where $\boldsymbol{Q}$ denotes the Ricci operator defined by $\boldsymbol{S}(Z_1, Z_2) = g(\boldsymbol{Q}Z_1, Z_2)$.

3. *h*-ARYS on Para-Kenmotsu Manifolds

We assume that the manifold $\boldsymbol{M}^{2n+1}$ admits an h-ARYS $(g, \zeta, \boldsymbol{\lambda}, \boldsymbol{\alpha}, \boldsymbol{\beta})$. Then, from (5), we obtain

$$\frac{h}{2}(\pounds_\zeta g)(Z_1, Z_2) + \boldsymbol{\alpha}\boldsymbol{S}(Z_1, Z_2) + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)g(Z_1, Z_2) = 0, \tag{20}$$

which gives

$$\frac{h}{2}[g(\nabla_{Z_1}\zeta, Z_2) + g(Z_1, \nabla_{Z_2}\zeta)] + \boldsymbol{\alpha}\boldsymbol{S}(Z_1, Z_2) + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)g(Z_1, Z_2) = 0. \tag{21}$$

Using (15) in (21), we infer

$$\boldsymbol{\alpha}\boldsymbol{S}(Z_1, Z_2) = h\eta(Z_1)\eta(Z_2) - (h + \boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)g(Z_1, Z_2). \tag{22}$$

Putting $Z_1 = Z_2 = \zeta$ in the foregoing equation entails that

$$\frac{\boldsymbol{\beta}}{2}r = \boldsymbol{\lambda} - 2n\boldsymbol{\alpha}. \tag{23}$$

Equations (22) and (23) together give us

$$\boldsymbol{\alpha}\boldsymbol{S}(Z_1, Z_2) = -(h + 2n\boldsymbol{\alpha})g(Z_1, Z_2) + h\eta(Z_1)\eta(Z_2). \tag{24}$$

Thus, $\boldsymbol{M}^{2n+1}$ is an η-Einstein manifold. Hence, we have:

Theorem 1. *If a $\boldsymbol{M}^{2n+1}$ admits a proper h-ARYS, then the manifold becomes an η-Einstein manifold.*

Let $\boldsymbol{M}^3$ admit an h-AGRYS. Then, (6) implies

$$h\nabla_{Z_1}Df = -\boldsymbol{\alpha}\boldsymbol{Q}Z_1 - (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)Z_1. \tag{25}$$

Taking the covariant derivative of (25) with respect to Z_2, we obtain

$$\begin{aligned} h\nabla_{Z_2}\nabla_{Z_1}Df &= \frac{1}{h}(Z_2h)[\boldsymbol{\alpha}\boldsymbol{Q}Z_1 + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)Z_1] - \boldsymbol{\alpha}\nabla_{Z_2}\boldsymbol{Q}Z_1 \\ &\quad -(Z_2\boldsymbol{\lambda})Z_1 - (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)\nabla_{Z_2}Z_1 + \frac{\boldsymbol{\beta}}{2}(Z_2r)Z_1. \end{aligned} \tag{26}$$

Interchanging Z_1 and Z_2 in (26) entails that

$$\begin{aligned} h\nabla_{Z_1}\nabla_{Z_2}Df &= \frac{1}{h}(Z_1h)[\boldsymbol{\alpha}\boldsymbol{Q}Z_2 + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)Z_2] - \boldsymbol{\alpha}\nabla_{Z_1}\boldsymbol{Q}Z_2 \\ &\quad -(Z_1\boldsymbol{\lambda})Z_2 - (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)\nabla_{Z_1}Z_2 + \frac{\boldsymbol{\beta}}{2}(Z_1r)Z_2. \end{aligned} \tag{27}$$

Equation (25) provides

$$h\nabla_{[Z_1,Z_2]}Df = -\boldsymbol{\alpha}\boldsymbol{Q}([Z_1,Z_2]) - (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)([Z_1,Z_2]). \tag{28}$$

Equations (26)–(28) reveal that

$$\begin{aligned} h\boldsymbol{R}(Z_1,Z_2)Df &= \frac{1}{h}(Z_1h)[\boldsymbol{\alpha}\boldsymbol{Q}Z_2 + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)Z_2] \\ &\quad -\frac{1}{h}(Z_2h)[\boldsymbol{\alpha}\boldsymbol{Q}Z_1 + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)Z_1] \\ &\quad -\boldsymbol{\alpha}[(\nabla_{Z_1}\boldsymbol{Q})Z_2 - (\nabla_{Z_2}\boldsymbol{Q})Z_1] \\ &\quad +\frac{\boldsymbol{\beta}}{2}[(Z_1r)Z_2 - (Z_2r)Z_1] - [(Z_1\boldsymbol{\lambda})Z_2 - (Z_2\boldsymbol{\lambda})Z_1]. \end{aligned} \tag{29}$$

Moreover, Equation (18) implies

$$\begin{aligned} (\nabla_{Z_1}\boldsymbol{Q})Z_2 &= \frac{Z_1r}{2}[Z_2 - \eta(Z_2)\zeta] \\ &\quad -(3 + \frac{r}{2})[g(Z_1,Z_2)\zeta - 2\eta(Z_1)\eta(Z_2)\zeta + \eta(Z_2)Z_1]. \end{aligned} \tag{30}$$

Using (30) in (29), we obtain:

$$\begin{aligned} h\boldsymbol{R}(Z_1,Z_2)Df &= \frac{1}{h}(Z_1h)[\boldsymbol{\alpha}\boldsymbol{Q}Z_2 + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)Z_2] \\ &\quad -\frac{1}{h}(Z_2h)[\boldsymbol{\alpha}\boldsymbol{Q}Z_1 + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)Z_1] \\ &\quad -\boldsymbol{\alpha}\frac{(Z_1r)}{2}[Z_2 - \eta(Z_2)\zeta] + \boldsymbol{\alpha}\frac{(Z_2r)}{2}[Z_1 - \eta(Z_1)\zeta] \\ &\quad +\boldsymbol{\alpha}(3 + \frac{r}{2})[\eta(Z_2)Z_1 - \eta(Z_1)Z_2] - [(Z_1\boldsymbol{\lambda})Z_2 - (Z_2\boldsymbol{\lambda})Z_1] \\ &\quad +\frac{\boldsymbol{\beta}}{2}[(Z_1r)Z_2 - (Z_2r)Z_1]. \end{aligned} \tag{31}$$

If we take h = constant, then (31) turns into

$$\begin{aligned} h\boldsymbol{R}(Z_1, Z_2)Df &= -\boldsymbol{\alpha}\frac{(Z_1 r)}{2}[Z_2 - \eta(Z_2)\zeta] + \boldsymbol{\alpha}\frac{(Z_2 r)}{2}[Z_1 - \eta(Z_1)\zeta] \\ &+\boldsymbol{\alpha}(3 + \frac{r}{2})[\eta(Z_2)Z_1 - \eta(Z_1)Z_2] - [(Z_1\boldsymbol{\lambda})Z_2(Z_2\boldsymbol{\lambda})Z_1] \\ &+\frac{\boldsymbol{\beta}}{2}[(Z_1 r)Z_2 - (Z_2 r)Z_1]. \end{aligned} \tag{32}$$

Contracting (32), we infer

$$h\boldsymbol{S}(Z_2, Df) = (\frac{\boldsymbol{\alpha}}{2} - \boldsymbol{\beta})Z_2 r + 2(Z_2\boldsymbol{\lambda}). \tag{33}$$

Replacing Z_1 by Df in (19) and comparing with (33), we obtain

$$h[(1 + \frac{r}{2})Z_2 f - (3 + \frac{r}{2})(\zeta f)\eta(Z_2)] = (\frac{\boldsymbol{\alpha}}{2} - \boldsymbol{\beta})Z_2 r + 2(Z_2\boldsymbol{\lambda}). \tag{34}$$

Putting $Z_2 = \zeta$ in (34) entails that

$$h(\zeta f) = (\frac{\boldsymbol{\alpha}}{2} - \boldsymbol{\beta})(r + 6) - (\zeta\boldsymbol{\lambda}). \tag{35}$$

Taking the inner product of (32) with ζ, we have

$$\begin{aligned} h[\eta(Z_2)Z_1 f - \eta(Z_1)Z_2 f] &= -[(Z_1\boldsymbol{\lambda})\eta(Z_2) - (Z_2\boldsymbol{\lambda})\eta(Z_1)] \\ &+\frac{\boldsymbol{\beta}}{2}[(Z_1 r)\eta(Z_2) - (Z_2 r)\eta(Z_1)]. \end{aligned} \tag{36}$$

Setting $Z_2 = \zeta$ in (36) and using (35), we obtain

$$h(Z_1 f) = \frac{\boldsymbol{\beta}}{2}(Z_1 r) + \frac{\boldsymbol{\alpha}}{2}(r + 6)\eta(Z_1) - (Z_1\boldsymbol{\lambda}). \tag{37}$$

Let us assume that the scalar curvature r = constant. Then, from (17), we obtain $r = -6$. Therefore, the above equation implies

$$h(Z_1 f) = -(Z_1\boldsymbol{\lambda}), \tag{38}$$

which implies

$$h(Df) = -(D\boldsymbol{\lambda}). \tag{39}$$

Using (39) in (25) reveals that

$$-\nabla_{Z_1} D\boldsymbol{\lambda} = -\boldsymbol{\alpha}\boldsymbol{Q}Z_1 - (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)Z_1,$$

which shows that $\boldsymbol{M}^3$ is an almost gradient Ricci–Yamabe soliton whose soliton function is $-\boldsymbol{\lambda}$. Hence, we have:

Theorem 2. *If $\boldsymbol{M}^3$ with a constant scalar curvature admits an h-AGRYS, then the soliton becomes an almost gradient Ricci–Yamabe soliton whose soliton function is -$\boldsymbol{\lambda}$, provided the function h is a constant.*

4. Para-Sasakian Manifolds

A para-Sasakian manifold is a normal paracontact metric manifold. It is to be noted that a para-Sasakian manifold is a K-paracontact manifold and conversely (only in three dimensions) [25]. In a para-Sasakian manifold, the following relations hold [26]:

$$\boldsymbol{R}(Z_1, Z_2)\zeta = \eta(Z_1)Z_2 - \eta(Z_2)Z_1, \tag{40}$$

$$(\nabla_{Z_1}\phi)Z_2 = -g(Z_1, Z_2)\zeta + \eta(Z_2)Z_1, \tag{41}$$
$$\nabla_{Z_1}\zeta = -\phi Z_1, \tag{42}$$
$$\boldsymbol{R}(\zeta, Z_1)Z_2 = -g(Z_1, Z_2)\zeta + \eta(Z_2)Z_1, \tag{43}$$
$$\boldsymbol{S}(Z_1, \zeta) = -2n\eta(Z_1). \tag{44}$$

In a 3-dimensional semi-Riemannian manifold, the curvature tensor $\boldsymbol{R}$ is of the form

$$\begin{aligned}\boldsymbol{R}(Z_1, Z_2)Z_3 \;=\; & g(Z_2, Z_3)\boldsymbol{Q}Z_1 - g(Z_1, Z_3)\boldsymbol{Q}Z_2 + \boldsymbol{S}(Z_2, Z_3)Z_1 \\ & -\boldsymbol{S}(Z_1, Z_3)Z_2 - \frac{r}{2}[g(Z_2, Z_3)Z_1 - g(Z_1, Z_3)Z_2].\end{aligned} \tag{45}$$

Then, Equation (45) implies

$$\boldsymbol{Q}Z_1 = (\frac{r}{2} + 1)Z_1 - (\frac{r}{2} + 3)\eta(Z_1)\zeta, \tag{46}$$

which gives

$$\boldsymbol{S}(Z_1, Z_2) = (\frac{r}{2} + 1)g(Z_1, Z_2) - (\frac{r}{2} + 3)\eta(Z_1)\eta(Z_2). \tag{47}$$

Lemma 2 ([24])**.** *For a para-Sasakian manifold* $\boldsymbol{M}^3$*,*

$$\zeta r = 0. \tag{48}$$

5. *h*-ARYS on Para-Sasakian Manifolds

Let us assume that a para-Sasakian manifold $\boldsymbol{M}^{2n+1}$ admits an h-ARYS $(g, \zeta, \boldsymbol{\lambda}, \boldsymbol{\alpha}, \boldsymbol{\beta})$. Then, Equation (5) implies

$$\frac{h}{2}(\pounds_{\zeta}g)(Z_1, Z_2) + \boldsymbol{\alpha}\boldsymbol{S}(Z_1, Z_2) + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)g(Z_1, Z_2) = 0, \tag{49}$$

which gives

$$\frac{h}{2}[g(\nabla_{Z_1}\zeta, Z_2) + g(Z_1, \nabla_{Z_2}\zeta)] + \boldsymbol{\alpha}\boldsymbol{S}(Z_1, Z_2) + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)g(Z_1, Z_2) = 0. \tag{50}$$

Using (42) in (50) entails that

$$\boldsymbol{\alpha}\boldsymbol{S}(Z_1, Z_2) = (\frac{\boldsymbol{\beta}}{2}r - \boldsymbol{\lambda})g(Z_1, Z_2). \tag{51}$$

Putting $Z_1 = Z_2 = \zeta$ in (51), we obtain

$$\boldsymbol{\beta}r = 2\boldsymbol{\lambda} - 4n\boldsymbol{\alpha}. \tag{52}$$

Hence, from (51), we infer

$$\boldsymbol{S}(Z_1, Z_2) = -2ng(Z_1, Z_2),$$

since for proper h-ARYS, $\boldsymbol{\alpha} \neq 0$. Hence, it is an Einstein manifold. Therefore, we state:

Theorem 3. *If* $\boldsymbol{M}^{2n+1}$ *admits a proper h-ARYS, then the manifold becomes an Einstein manifold.*

If we take $\boldsymbol{\alpha} = 1$ and $\boldsymbol{\beta} = 0$, then (52) implies $\boldsymbol{\lambda} = 2n$. Hence, we obtain:

Corollary 1. *If* $\boldsymbol{M}^{2n+1}$ *admits a proper h-almost Ricci soliton, then the soliton is expanding.*

Suppose that an $\boldsymbol{M}^3$ admits an h-AGRYS. Then, Equation (6) implies

$$h\nabla_{Z_1}Df = -\boldsymbol{\alpha}\boldsymbol{Q}Z_1 - (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2}r)Z_1. \tag{53}$$

Using (46) in the above equation entails that

$$h\nabla_{Z_1}Df = -[\frac{(\alpha-\beta)}{2}r+\alpha+\lambda]Z_1 + \alpha(\frac{r}{2}+3)\eta(Z_1)\zeta. \tag{54}$$

Taking the covariant differentiation of (54), we obtain:

$$\begin{aligned} h\nabla_{Z_2}\nabla_{Z_1}Df &= \frac{1}{h}(Z_2h)[\{\frac{(\alpha-\beta)}{2}r+\alpha+\lambda\}Z_1-\alpha(\frac{r}{2}+3)\eta(Z_1)\zeta] \\ &-[\frac{(\alpha-\beta)}{2}Z_2r+Z_2\lambda]Z_1-[\frac{(\alpha-\beta)}{2}r+\alpha+\lambda]\nabla_{Z_2}Z_1 \\ &+\frac{\alpha}{2}(Z_2r)\eta(Z_1)\zeta+\alpha(\frac{r}{2}+3)[(\nabla_{Z_2}\eta(Z_1))\zeta-\eta(Z_1)\phi Z_2]. \end{aligned} \tag{55}$$

Swapping Z_1 and Z_2 in (55), we infer that:

$$\begin{aligned} h\nabla_{Z_1}\nabla_{Z_2}Df &= \frac{1}{h}(Z_1h)[\{\frac{(\alpha-\beta)}{2}r+\alpha+\lambda\}Z_2-\alpha(\frac{r}{2}+3)\eta(Z_2)\zeta] \\ &-[\frac{(\alpha-\beta)}{2}Z_1r+Z_1\lambda]Z_2-[\frac{(\alpha-\beta)}{2}r+\alpha+\lambda]\nabla_{Z_1}Z_2 \\ &+\frac{\alpha}{2}(Z_1r)\eta(Z_2)\zeta+\alpha(\frac{r}{2}+3)[(\nabla_{Z_1}\eta(Z_2))\zeta-\eta(Z_2)\phi Z_1]. \end{aligned} \tag{56}$$

Equation (54) implies

$$h\nabla_{[Z_1,Z_2]}Df = -[\frac{(\alpha-\beta)}{2}r+\alpha+\lambda]([Z_1,Z_2]) + \alpha(\frac{r}{2}+3)\eta([Z_1,Z_2])\zeta. \tag{57}$$

With the help of (55)–(57), we obtain

$$\begin{aligned} h\boldsymbol{R}(Z_1,Z_2)Df &= \frac{1}{h}(Z_1h)[\{\frac{(\alpha-\beta)}{2}r+\alpha+\lambda\}Z_2-\alpha(\frac{r}{2}+3)\eta(Z_2)\zeta] \\ &-\frac{1}{h}(Z_2h)[\{\frac{(\alpha-\beta)}{2}r+\alpha+\lambda\}Z_1-\alpha(\frac{r}{2}+3)\eta(Z_1)\zeta] \\ &-[\frac{(\alpha-\beta)}{2}Z_1r+Z_1\lambda]Z_2+[\frac{(\alpha-\beta)}{2}Z_2r+Z_2\lambda]Z_1 \\ &+\frac{\alpha}{2}[(Z_1r)\eta(Z_2)\zeta-(Z_2r)\eta(Z_1)\zeta] \\ &+\alpha(\frac{r}{2}+3)[2g(Z_1,\phi Z_2)\zeta-\eta(Z_2)\phi Z_1+\eta(Z_1)\phi Z_2]. \end{aligned} \tag{58}$$

Contracting the foregoing equation entails that

$$\begin{aligned} h\boldsymbol{S}(Z_2,Df) &= -\frac{1}{h}(Z_2h)[2\{\frac{(\alpha-\beta)}{2}r+\alpha+\lambda\}-\alpha(\frac{r}{2}+3)] \\ &-\frac{\alpha}{h}(\frac{r}{2}+3)(\zeta h)\eta(Z_2)+(\frac{\alpha}{2}-\beta)(Z_2r)+2(Z_2\lambda). \end{aligned} \tag{59}$$

Replacing Z_1 by Df in (47) and comparing with the above equation, we obtain

$$\begin{aligned} h[(\frac{r}{2}+1)Z_2f-(\frac{r}{2}+3)(\zeta f)\eta(Z_2)] &= -\frac{1}{h}(Z_2h)[2\{\frac{(\alpha-\beta)}{2}r+\alpha+\lambda\} \\ &-\alpha(\frac{r}{2}+3)]-\frac{\alpha}{h}(\frac{r}{2}+3)(\zeta h)\eta(Z_2) \\ &+(\frac{\alpha}{2}-\beta)(Z_2r)+2(Z_2\lambda). \end{aligned} \tag{60}$$

Setting $Z_2 = \zeta$ in (60) reveals that

$$h(\zeta f) \quad = \quad \frac{1}{h}[\frac{(\alpha-\beta)}{2}r + \alpha + \lambda](\zeta h) - (\zeta\lambda). \tag{61}$$

Taking inner product of (58) with ζ, we obtain

$$\begin{aligned} h[\eta(Z_2)Z_1 f - \eta(Z_1)Z_2 f] \quad = \quad & \frac{1}{h}(Z_1 h)[\{\frac{(\alpha-\beta)}{2}r + \alpha + \lambda\} - \alpha(\frac{r}{2}+3)]\eta(Z_2) \\ & -\frac{1}{h}(Z_2 h)[\{\frac{(\alpha-\beta)}{2}r + \alpha + \lambda\} - \alpha(\frac{r}{2}+3)]\eta(Z_1) \\ & -[\frac{(\alpha-\beta)}{2}Z_1 r + Z_1\lambda]\eta(Z_2) \\ & +[\frac{(\alpha-\beta)}{2}Z_2 r + Z_2\lambda]\eta(Z_1) \\ & +\frac{\alpha}{2}[(Z_1 r)\eta(Z_2) - (Z_2 r)\eta(Z_1)] + 2\alpha(\frac{r}{2}+3)g(Z_1, \phi Z_2). \end{aligned} \tag{62}$$

Substituting Z_1 by ϕZ_1 and Z_2 by ϕZ_2 in (62) gives

$$\alpha(r+6)g(\phi Z_1, Z_2) = 0. \tag{63}$$

Since for proper h-AGRYS, $\alpha \neq 0$, then the above equation implies $r = -6$. Therefore, from (47), we obtain

$$S(Z_1, Z_2) = -2g(Z_1, Z_2), \tag{64}$$

which gives us that M^3 is an Einstein manifold. In view of (45) and (64), we obtain

$$R(Z_1, Z_2)Z_3 = -[g(Z_2, Z_3)Z_1 - g(Z_1, Z_3)Z_2], \tag{65}$$

which represents the fact it is a space of constant sectional curvature -1. Hence, we have:

Theorem 4. *If M^3 admits a proper h-AGRYS, then the manifold is locally isometric to $\mathbb{H}^3(1)$.*

6. Para-Cosymplectic Manifolds

An almost paracontact metric manifold M^{2n+1} with a structure (ϕ, ζ, η, g) is named an almost γ-paracosymplectic manifold [27] if

$$d\eta = 0, d\Phi = 2\gamma \wedge \Phi. \tag{66}$$

Specifically, if $\gamma = 0$, we obtain an almost paracosymplectic manifold. A manifold is called paracosymplectic, if it is normal. We refer ([27,28]) for more details. Any paracosymplectic manifold satisfies

$$R(Z_1, Z_2)\zeta = 0, \tag{67}$$

$$(\nabla_{Z_1}\phi)Z_2 = 0, \tag{68}$$

$$\nabla_{Z_1}\zeta = 0, \tag{69}$$

$$S(Z_1, \zeta) = 0. \tag{70}$$

Lemma 3 ([24])**.** *For a 3-dimensional para-cosymplectic manifold M^3,*

$$QZ_1 = \frac{r}{2}[Z_1 - \eta(Z_1)\zeta], \tag{71}$$

$$S(Z_1, Z_2) = \frac{r}{2}[g(Z_1, Z_2) - \eta(Z_1)\eta(Z_2)]. \tag{72}$$

Lemma 4 ([24])**.** *In a para-cosymplectic manifold M^3*

$$\zeta r = 0. \tag{73}$$

7. *h*-ARYS on Para-Cosymplectic Manifolds

Assume that a para-cosymplectic manifold admits an h-ARYS $(g, \zeta, \boldsymbol{\lambda}, \boldsymbol{\alpha}, \boldsymbol{\beta})$. Then, (5) implies

$$\frac{h}{2}(\pounds_{\zeta} g)(Z_1, Z_2) + \boldsymbol{\alpha} \boldsymbol{S}(Z_1, Z_2) + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2} r) g(Z_1, Z_2) = 0, \tag{74}$$

which turns into

$$\frac{h}{2}[g(\nabla_{Z_1}\zeta, Z_2) + g(Z_1, \nabla_{Z_2}\zeta)] + \boldsymbol{\alpha} \boldsymbol{S}(Z_1, Z_2) + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2} r) g(Z_1, Z_2) = 0. \tag{75}$$

Using (69) in (75) gives

$$\boldsymbol{\alpha} \boldsymbol{S}(Z_1, Z_2) = -(\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2} r) g(Z_1, Z_2). \tag{76}$$

So the manifold is an Einstein manifold. Hence, we have:

Theorem 5. *If a para-cosymplectic manifold admits a proper h-ARYS, then the manifold becomes an Einstein manifold.*

Let $\boldsymbol{M}^3$ admits an h-AGRYS. Then from (10), we obtain:

$$h \nabla_{Z_1} Df = -\boldsymbol{\alpha} \boldsymbol{Q} Z_1 - (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2} r) Z_1. \tag{77}$$

Hence we have

$$\begin{aligned} h\boldsymbol{R}(Z_1, Z_2)Df &= \frac{1}{h}(Z_1 h)[\boldsymbol{\alpha} \boldsymbol{Q} Z_2 + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2} r) Z_2] \\ &\quad -\frac{1}{h}(Z_2 h)[\boldsymbol{\alpha} \boldsymbol{Q} Z_1 + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2} r) Z_1] \\ &\quad -\boldsymbol{\alpha}[(\nabla_{Z_1} \boldsymbol{Q}) Z_2 - (\nabla_{Z_2} \boldsymbol{Q}) Z_1] - (Z_1 \boldsymbol{\lambda}) Z_2 + (Z_2 \boldsymbol{\lambda}) Z_1 \\ &\quad +\frac{\boldsymbol{\beta}}{2}[(Z_1 r) Z_2 - (Z_2 r) Z_1]. \end{aligned} \tag{78}$$

Using (71) in (78) reveals that

$$\begin{aligned} h\boldsymbol{R}(Z_1, Z_2)Df &= \frac{1}{h}(Z_1 h)[\boldsymbol{\alpha} \boldsymbol{Q} Z_2 + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2} r) Z_2] \\ &\quad -\frac{1}{h}(Z_2 h)[\boldsymbol{\alpha} \boldsymbol{Q} Z_1 + (\boldsymbol{\lambda} - \frac{\boldsymbol{\beta}}{2} r) Z_1] \\ &\quad -\frac{\boldsymbol{\alpha}}{2}(Z_1 r)[Z_2 - \eta(Z_2)\zeta] + \frac{\boldsymbol{\alpha}}{2}(Z_2 r)[Z_1 - \eta(Z_1)\zeta] \\ &\quad -(Z_1 \boldsymbol{\lambda}) Z_2 + (Z_2 \boldsymbol{\lambda}) Z_1 + \frac{\boldsymbol{\beta}}{2}[(Z_1 r) Z_2 - (Z_2 r) Z_1]. \end{aligned} \tag{79}$$

If we take h = constant, then the above equation implies

$$\begin{aligned} h\boldsymbol{R}(Z_1, Z_2)Df &= -\frac{\boldsymbol{\alpha}}{2}(Z_1 r)[Z_2 - \eta(Z_2)\zeta] + \frac{\boldsymbol{\alpha}}{2}(Z_2 r)[Z_1 - \eta(Z_1)\zeta] \\ &\quad -(Z_1 \boldsymbol{\lambda}) Z_2 + (Z_2 \boldsymbol{\lambda}) Z_1 + \frac{\boldsymbol{\beta}}{2}[(Z_1 r) Z_2 - (Z_2 r) Z_1]. \end{aligned} \tag{80}$$

Contracting the foregoing equation entails that

$$h\boldsymbol{S}(Z_2, Df) = (\frac{\boldsymbol{\alpha}}{2} - \boldsymbol{\beta}) Z_2 r + 2(Z_2 \boldsymbol{\lambda}). \tag{81}$$

Substituting Z_1 by Df in (72) and equating with (81), we obtain

$$\frac{hr}{2}[Z_2 f - (\zeta f)\eta(Z_2)] = (\frac{\alpha}{2} - \beta)Z_2 r + 2(Z_2\lambda). \tag{82}$$

Putting $Z_2 = \zeta$ and using (73), we infer

$$\zeta\lambda = 0. \tag{83}$$

Taking the inner product of (80) with ζ and using (67) gives

$$-(Z_1\lambda)\eta(Z_2) + (Z_2\lambda)\eta(Z_1) + \frac{\beta}{2}[(Z_1 r)\eta(Z_2) - (Z_2 r)\eta(Z_1)] = 0. \tag{84}$$

Setting $Z_2 = \zeta$ in (84), we obtain

$$-(Z_1\lambda) + \frac{\beta}{2}(Z_1 r) = 0. \tag{85}$$

If we take r = constant, then (85) implies

$$Z_1\lambda = 0, \tag{86}$$

which implies that λ is a constant. Therefore, we have:

Theorem 6. *If M^3 with constant scalar curvature admits an h-AGRYS, then the soliton becomes an h-gradient Ricci–Yamabe soliton, provided that the function h is a constant.*

In particular, if we take $\alpha = 1$ and $\beta = 0$, then (85) implies $Z_1\lambda = 0$. Therefore λ is a constant. Hence, we have:

Corollary 2. *An h-almost gradient Ricci soliton in M^3 becomes an h-gradient Ricci soliton.*

8. Example

Let us consider $M^3 = \{(x, y, z) \in \mathbb{R}^3 : (x, y, z) \neq (0, 0, 0)\}$, where (x, y, z) are the standard coordinates of $\mathbb{R}^3$.

We consider three linearly independent vector fields

$$u_1 = e^{-z}\frac{\partial}{\partial x}, \quad u_2 = e^{-z}\frac{\partial}{\partial y}, \quad u_3 = \frac{\partial}{\partial z}.$$

Then

$$[u_1, u_2] = 0, \ [u_2, u_3] = u_2, \ [u_1, u_3] = u_1.$$

Let g be the semi-Riemannian metric defined by

$$g(u_1, u_1) = 1, \quad g(u_2, u_2) = -1, \quad g(u_3, u_3) = 1,$$

$$g(u_1, u_2) = 0, \quad g(u_1, u_3) = 0, \quad g(u_2, u_3) = 0.$$

Let η be the 1-form defined by $\eta(Z_1) = g(Z_1, u_3)$ for any $Z_1 \in \chi(M)$.
Let ϕ be the (1,1)-tensor field defined by

$$\phi u_1 = u_2, \quad \phi u_2 = u_1, \quad \phi u_3 = 0.$$

Using the above relations, we acquire

$$\phi^2 Z_1 = Z_1 - \eta(Z_1)u_3, \quad \eta(u_3) = 1,$$

$$g(\phi Z_1, \phi Z_2) = -g(Z_1, Z_2) + \eta(Z_1)\eta(Z_2)$$

for any $Z_1, Z_2 \in \chi(\boldsymbol{M})$. Hence, for $u_3 = \zeta$, the structure (ϕ, ζ, η, g) is an almost paracontact structure on $\boldsymbol{M}^{2n+1}$.

Using Koszul's formula, we have

$$\nabla_{u_1} u_1 = -u_3, \quad \nabla_{u_1} u_2 = 0, \quad \nabla_{u_1} u_3 = u_1,$$

$$\nabla_{u_2} u_1 = 0, \quad \nabla_{u_2} u_2 = u_3, \quad \nabla_{u_2} u_3 = u_2,$$

$$\nabla_{u_3} u_1 = 0, \quad \nabla_{u_3} u_2 = 0, \quad \nabla_{u_3} u_3 = 0.$$

Hence, the manifold is a para-Kenmotsu manifold.

With the help of the above results, we can easily obtain

$$\boldsymbol{R}(u_1, u_2)u_3 = 0, \quad \boldsymbol{R}(u_2, u_3)u_3 = -u_2, \quad \boldsymbol{R}(u_1, u_3)u_3 = -u_1,$$

$$\boldsymbol{R}(u_1, u_2)u_2 = u_1, \quad \boldsymbol{R}(u_2, u_3)u_2 = u_3, \quad \boldsymbol{R}(u_1, u_3)u_2 = 0,$$

$$\boldsymbol{R}(u_1, u_2)u_1 = u_2, \quad \boldsymbol{R}(u_2, u_3)u_1 = 0, \quad \boldsymbol{R}(u_1, u_3)u_1 = u_3$$

and

$$\boldsymbol{S}(u_1, u_1) = -2, \quad \boldsymbol{S}(u_2, u_2) = 2, \quad \boldsymbol{S}(u_3, u_3) = -2.$$

From the above results, we obtain $r = -6$.

Again, suppose that $-f = \boldsymbol{\lambda} = e^z$ and $2\boldsymbol{\alpha} - 3\boldsymbol{\beta} = 0$. Therefore $-Df = D\boldsymbol{\lambda} = e^z u_3$. Hence, we obtain that:

$$\nabla_{u_1} \mathcal{D}f = -e^z u_1,$$
$$\nabla_{u_2} \mathcal{D}f = -e^z u_2,$$
$$\nabla_{u_3} \mathcal{D}f = -e^z u_3.$$

Therefore, for $2\boldsymbol{\alpha} - 3\boldsymbol{\beta} = 0$ and $h = 1$, Equation (25) is satisfied. Thus, g is an h-AGRYS with the soliton vector field $V = \mathcal{D}f$, where $-f = \boldsymbol{\lambda} = e^z$ and $2\boldsymbol{\alpha} - 3\boldsymbol{\beta} = 0$. Since $-f = \boldsymbol{\lambda} = e^z$ and $-Df = D\boldsymbol{\lambda} = e^z u_3$, hence, Theorem 2 is verified.

9. Conclusions

In order to generalize Ricci and Yamabe solitons, Guler and Crasmareanu proposed the idea of Ricci–Yamabe solitons in 2019. The notion of almost Ricci solitons was recently expanded by Gomes et al. to include h-almost Ricci solitons on a complete Riemannian manifold. The h-almost Ricci–Yamabe soliton, which is a natural extension of the almost Ricci–Yamabe soliton, is a new one which we introduced in this study.

Here, we showed that if a para-Kenmotsu or a para-Sasakian manifold admits a proper h-almost Ricci–Yamabe soliton, then the manifold becomes an η-Einstein manifold whereas for a cosymplectic manifold, it is an Einstein manifold. Finally, we constructed an example of para-Kenmotsu manifolds which verifies our result.

In the near future, we or possibly other authors will investigate the properties of h-almost Ricci–Yamabe solitons in the general theory of relativity and cosmology, or in particular, in perfect fluid spacetimes.

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