mathematics

Article

The Mathematical Culture in Test Items of National College Entrance Examination in China from 1978 to 2021

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Abstract: As one of the most important examinations in China, the National College Entrance Examination (commonly known as the *Gaokao*, hereafter denoted *Gaokao*) has a long history and has attracted attention from the Chinese educational community, among others. This study focused on mathematics test items of the *Gaokao* from the perspective of mathematical culture and examined the national papers (drafted by the Ministry of Education of the People's Republic of China) of tests from 1978 to 2021 in order to investigate the content and time variation characteristics of mathematics test items reflecting mathematical culture since its restoration. A mathematical culture categorization conceptual framework was established based on previous studies and was applied to test item analysis. Mathematical culture in test items was classified using four categories: *Historical Topics*, *Interdisciplinary Connections*, *Social Roles*, and *Aesthetics & Recreation*. These were used for data coding and analysis. The results showed that mathematical culture in the mathematics test items of the *Gaokao* over a 44-year period, in terms of content categories, was diverse but uneven in distribution, with greater focus on demonstrating the social roles of mathematics and less on the historical development of mathematical knowledge. Moreover, the average number of items with mathematical culture during this period was small and fluctuated over time. The content of topics related to social context and technology also changed over time with the distinct characteristics of the times and changes in society. This study provides empirical evidence on how mathematical culture is integrated into test items and how they have changed over time.

Keywords: math exam items; National College Entrance Examination in China; *Gaokao*; mathematical culture; mathematics education

MSC: 97-03; 97A30; 97B10; 97U40

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1. Introduction

As early as the middle of the 20th century, Wilder proposed the idea of "Mathematics as a Cultural System" [1]. Mathematics, like other cultures, has its own system of practices and ideas, and mathematical culture has drawn the attention of mathematicians, mathematical historians, philosophers, and mathematics educators. In the field of international mathematics education, a cultural turn in research on mathematics education is underway as well, with many researchers examining the influence of culture on mathematics education (e.g., [2–4]). It should also be noted there are many benefits of incorporating mathematical culture into mathematics teaching. As M. Kline pointed out, "mathematics detached from its rich intellectual setting in the culture of our civilization and reduced to a series of techniques has been grossly distorted" [5] (p. viii). Therefore, it can be considered that there is a need to integrate mathematical culture in mathematics teaching activities. Furthermore, some researchers have summarized the roles of mathematical culture in mathematics teaching and learning, including improving students' knowledge, abilities, and attitudes (e.g., [6,7]). These studies have also shown that research pointing to the teaching of mathematical culture is valuable and can provide educators with certain insights about how to integrate mathematical culture into their teaching activities.

An important support for carrying out teaching activities comes from instructional resources [8], and the official instructional resources of a country or region play a very important role in guiding local teaching activities. It is also a reflection of the local culture [9]. One such important instructional resource is the national-level examination items. Especially in the Confucian cultural circle, entrance exams for higher education, which play a decisive role in admission, have an important place in education. In Chinese education, the National College Entrance Examination (commonly known as *Gaokao*, hereafter denoted *Gaokao*) is undoubtedly the most important entrance exam that attracts the attention of the entire educational community, among others [10–13]. As one of the long-standing compulsory subjects of the *Gaokao* [13], mathematics has always been important. Under the influence of the examination culture, the mathematics test items of the *Gaokao* provide an important orientation for mathematics education. Research on these test items is of great importance to improving mathematics teaching in high schools and universities [14].

As old textbooks and mathematics books can be a resource for understanding mathematics education and mathematical history [15], so can mathematics test items. At the same time, the test items themselves are also a kind of cultural artifacts, influenced by various factors such as politics, culture, and society in a specific time, with the test items in different temporal and spatial contexts presenting different characteristics. Currently, China is undergoing a new round of educational assessment reform, with the integration of mathematical culture being one of the goals in the reform of the mathematics test items of the *Gaokao* [16]. Many educators have also found that, in recent years, mathematical culture has become more obviously present in the mathematics items of the *Gaokao*, and they have begun to appreciate and analyze the mathematical culture in test items and propose teaching suggestions for these test items accordingly (e.g., [17,18]).

At the same time, in a globalized era, different countries, including Australia and Canada, have started to accept Chinese students' *Gaokao* scores as admission application materials [10], with other countries also referring to it. With the increasing cross-regional educational research exchanges, understanding the characteristics of *Gaokao* items in different cultural contexts can facilitate cross-cultural understanding and communication.

In the previous literature, the analysis of mathematical culture in the mathematics tests of the *Gaokao* is mainly limited to the review and evaluation of test items in a particular year (e.g., [17]), as well as simple statistics and summaries of the mathematical culture characteristics of test items over a period of time (e.g., [19–21]). These papers are essentially education papers summarizing teaching experience, and relatively few employ empirical methods to systematically code and analyze the mathematical culture in test items over a long time span.

In view of this, this study established a mathematical culture classification conceptual framework applicable to the analysis of test items based on previous studies and systematically analyzed the mathematical culture in mathematics test items of the *Gaokao* framed by the Ministry of Education of the People's Republic of China (hereafter denoted MOE of PRC) from 1978 to 2021, in order to explore the content and time variation characteristics of mathematical culture in these test items over a longer period.

The questions mainly of focus here were as follows:

(1) What are the characteristics of mathematical culture in terms of content distribution as reflected in mathematics items of the *Gaokao* from 1978 to 2021?
(2) What are the characteristics of the time variation in mathematical culture in the test items in the *Gaokao* from 1978 to 2021?

It is expected that this study will provide the international mathematics education community with knowledge of the characteristics of mathematical culture in items of the *Gaokao* and relevant educational researchers and educators with some empirical evidence on how mathematical culture is integrated into instructional resources, thus contributing to the role of mathematical culture in mathematics teaching activities.

2. Theoretical Background

2.1. Mathematical Culture

Although "culture" is an widely used word, it was not until 1871 that the British anthropologist Tylor defined it: "Culture or Civilization, taken in its wide ethnographic sense, is that complex whole which includes knowledge, belief, art, morals, law, custom, and any other capabilities and habits acquired by man as a member of society." [22] (p. 1), and in the following 80 years, anthropologists, sociologists, psychologists, and other scholars defined the concept of culture from various perspectives (e.g., [23–25]). The phenomenon of culture is so complex that it is difficult to find a definition that is applicable to all disciplines. In general efforts of different fields to do so, the work conducted by Kroeber and Kluckhohn is noteworthy, as they grouped 164 definitions of culture in academic communities into seven categories—descriptive, historical, normative, psychological, structural, genetic, and incomplete; in an attempt to further clarify what culture was, they argued that culture consists of explicit and implicit behavior patterns [26] (p. 181). In the 1960s, Wilder published the monograph *Evolution of Mathematical Concepts: An Elementary Study* (1968), followed by *Mathematics As a Cultural System* (1981), in which he argued that mathematics was a cultural system and proposed a cultural view of mathematics and its systematic theory, introducing a perspective of cultural anthropology to the study of mathematical culture [1] (pp. 14–20). M. Kline pointed out in *Mathematics in Western Culture* that mathematics had always been a cultural power in Western civilization. As an embodiment of the spirit of reason, mathematics has permeated areas previously dominated by authority, habits, and customs and replaced them as a guide to thought and action [5] (p. vii). Therefore, it can be argued that, on one hand, mathematical culture is an integral part of the overall human culture, further interacting with other cultures or whole civilizations. On the other hand, mathematics itself is a cultural system.

From 2012 to 2014, a series of three conferences with the theme of mathematical culture was held in London, and at that time there were two main academic orientations in the academic community towards the understanding of mathematical culture [27] (p. 2), one being the philosophy of mathematical practice, which means that mathematics is seen as a culture, and various different cultural systems will produce different mathematics (e.g., [28,29]). The other is a "cultural turn" in mathematics education research, which is to study the teaching and learning of mathematics as a cultural activity [30]. The former connects mathematicians, philosophers, and mathematical historians. A representative figure holding this view is Wilder, whose view of mathematics as a culture has also influenced later mathematicians [27] (p. 2), such as Hersh, who has studied the nature of mathematics from the perspective of professional mathematicians and has published relevant papers [31]. The latter view has guided the mathematics education research community to the study of the pedagogical activities of mathematics from a cultural perspective. Most scholars use cultural values to represent culture (e.g., [32]). Researchers have found that the influence of culture on mathematics education is manifested in various aspects such as curriculum, classrooms, and students (e.g., [2–4]). For example, Leung confirmed that Jewish and Chinese culture both exerted a strong influence on mathematics education [33], and Clivaz and Miyakawa compared the differences in mathematics lesson design between Switzerland and Japan and discussed the cultural factors that caused these differences [34]. Hu et al. demonstrated that cultural factors could explain the differences in students' mathematics scores in different countries [35]. Simamora argued that incorporating local culture into mathematics instruction could improve students' mathematical problem solving skills and self-efficacy [36]. Furthermore, some researchers summarized the relevance of integrating mathematical culture into mathematics classrooms, for example, the integration of mathematical history into mathematics classrooms can lead to the harmony of knowledge, the beauty of ideas or methods, the pleasure of inquiries, the improvement of capabilities, the charm of cultures, and the availability of moral education [6]. Demonstrating the connection between mathematics and the real world in the high school mathematics classroom can improve students' motivation, interest, and attitude toward learning and develop their mathematical reasoning, problem solving skills, and conceptual learning [7]. All these studies confirm that a cultural turn in mathematics education research has appeared, which is also reflected in the International Congress on Mathematical Education (ICME). Although ICME-14 did not have a Thematic Study Group (TSG) on the topic of mathematical culture, TSG27 (topic: the role of the history of mathematics in mathematics education) and TSG48 (topic: mathematics education in a multicultural environment) are both closely related to mathematical culture, representing the impact of this research area on the international mathematics education community.

It is worth noting that, perhaps because of the differences in research purpose and focus, most of the above studies did not develop detailed conceptual frameworks about mathematical culture. Among the few studies that included a conceptual framework of mathematical culture, Fan et al. developed a conceptual framework of cultural factors in mathematics textbooks based on beliefs, values, and ways of interacting with geography, artifacts, flora and fauna, organizations, ways of behaving, and customs, history, and identities [37]. Wang JianPan et al. summarized the cultural conceptual framework of mathematics applicable to Chinese and Western countries by analyzing four sets of textbooks from three countries, namely, China, France, and the United States, based on the experience of mathematicians, who proposed that the contents of mathematical culture could be divided into four areas: *mathematical history*, *mathematics and real life*, *mathematics and technology*, and *mathematics and arts* [38]. Based on both the philosophy of mathematical practice and the cultural view of mathematics teaching activities, Wang Xiaoqin concluded that the cultural value of mathematics and its history could be divided into five dimensions: *Historical Topics, Interdisciplinary Connections, Social Roles, Aesthetics & Recreation*, and *Multiculturalism* (see Table 1) from the perspective of the history and pedagogy of mathematics, with reference to a large amount of literature on mathematical history [39]. This conceptual framework has provided a more systematic and comprehensive analytical perspective for the cultural turn of mathematics education research, with many mathematics education researchers having generalized it for later use in the study of mathematical culture in mathematics teaching activities and textbooks (e.g., [40–42]).

Table 1. Cultural value conceptual framework of mathematics and its history.

Type	Connotation
Historical Topics	The development process of one knowledge point and the related people, events, and ideas.
Interdisciplinary Connections	The connections between mathematics and other disciplines.
Social Roles	The important role played by mathematics in cultural progress and social development.
Aesthetics & Recreation	Mathematical beauty and recreational mathematics.
Multiculturalism	The achievements and contributions made by different civilizations and regions in one mathematics subject.

The Chinese mathematics education community has attached great importance to the study of mathematical culture and developed fruitful research results. Since 2011, the National Mathematical Culture Forum Academic Conference, sponsored by the Chinese Mathematical Society, has been held once a year for researchers and front-line teachers to exchange research progress on mathematical culture at home and abroad. In the field of mathematics education, Chinese scholars have been accustomed to treating mathematical culture as a specific and independent concept. For example, Huang believed that "mathematical culture is a dynamic system with specific functions, with mathematical science as the core and the related cultural fields ranging from the ideas, spirit, and methods to the contents of mathematics as integral components" [43]. Some scholars have concluded that the study of mathematical culture in China can be divided into three main areas:

the connotations, the values, and the classroom integration of mathematical culture [44]. The value of mathematical culture has gained both recognition by Chinese mathematics education researchers and attention by the MOE of PRC. In the latest round of curriculum reform, the Ordinary High School Mathematics Curriculum Standards (2017 Edition) (hereafter denoted OHSMCS) states that the integration of mathematical culture should be emphasized in teaching activities and explains mathematical culture by suggesting that "mathematical culture refers to the ideas, spirit, language, methods, and perspectives of mathematics, as well as their formation and development, and it also includes the role of mathematics in human society and technology, the contribution and significance in the development of society, and the humanistic activities related to mathematics." [45] (p. 10). It is believed that, in this context, the Chinese mathematics education community will continue to focus on mathematical culture.

Overall, the connotations about mathematical culture are complex and have not yet been commonly agreed upon [46]. However, there is a clear sign of the cultural turn in mathematics education research, and mathematical culture has caught the interest of mathematics education researchers. Among them, the Chinese mathematics education community attaches great importance to the study of mathematical culture, and the MOE of PRC has promoted the integration of mathematical culture into mathematics classrooms as a key element of the new curriculum reform of elementary education.

2.2. The Gaokao

In China, there has been a long examination culture. Since the introduction of the imperial examination system in the Sui Dynasty [47], there has been a strong tendency to select talent in Chinese official institutions through examinations [48,49]. The selection of qualifications for admission to higher education is undoubtedly one of the most important stages of talent selection. In this context, the *Gaokao* was also created. This examination is aimed at testing students' mastery of the subjects studied in high school, whose results are scored using unified evaluation criteria. Each university will admit students in descending order based on the enrollment quota and the scores of the students who have applied for this school [50]. The exam began to undergo a preliminary exploratory phase in 1952, was suspended for a few years due to changes in talent selection methods, resumed officially in 1977, and today continues to be one of the most important exams in China [51]. As it concerns the fate of almost all Chinese students [52], the *Gaokao* is of interest to countless parents, educators, education researchers, and even the public [10–13,53]. In China, where an examination culture prevails, the evaluation orientation of the *Gaokao* influences the teaching direction of secondary schools, and it can even be said that it is the focus of teaching. The Evaluation System of Gaokao issued by National Education Examinations Authority (hereafter denoted NEEA) in 2019 also suggests that the *Gaokao* has the core function of "fostering virtue through education, serving talent selection, and guiding teaching" [54].

From the perspective of test designing, there are both individual tests drafted out by each province and national test items framed by the MOE of PRC, the former named after each province and the latter usually referred to as the "national paper". In most years, the papers drafted out by related departments at all levels are divided into two types: arts (for students applying for majors such as history, politics, and geography) and science (for students applying for majors such as physics, chemistry, biology, and medicine). Since the restoration of the *Gaokao*, according to the MOE of PRC, regardless of what type of paper it is, the scope of the test items and the assessment requirements should be made based on the secondary school syllabus (the teaching guidelines made by the MOE of PRC, sometimes called curriculum standards) [55]. Over the last many decades, China's high school education has undergone many reforms, with the secondary school syllabus having changed several times [56], and the examination objectives and corresponding item design of the *Gaokao* have also changed. Therefore, in some alternative years of education reform, there were examinations drafted out based on two standards, with the names of such papers distinguished by the "syllabus edition" and the "curriculum standard edition".

2.3. Previous Research

The Chinese mathematics education community has begun to focus on mathematical culture in *Gaokao* items, especially when the NEEA has intentionally emphasized the value of mathematical culture in test items in its reviews about *Gaokao* items in recent years [57]. The previous literature can be divided into two main categories. The first category is an analysis of the integration of mathematical culture in *Gaokao* items, including the review of test items and the statistics and analysis of multi-year data. The former usually provides a review of typical items appearing in the *Gaokao*, such as Chen's review of the mathematics tests of the 2015 edition, considered to highlight the fine traditional Chinese mathematical culture [17]; the latter conducts statistics and analyses of mathematical culture in *Gaokao* items over a period of time, usually studying data within 5 or 10 years. (e.g., [20,21]) For example, some researchers have analyzed national, provincial, and municipal *Gaokao* items between 2011 and 2020 using famous mathematical problems, famous mathematics classics, mathematicians, and mathematical beauty as the four carrier forms of mathematical culture, and found the highest prevalence of items with mathematical beauty as a carrier. They also found that the material selection of mathematical culture in China and the West has different backgrounds, with most selected materials in China being mathematical masterpieces (e.g., *Nine Chapters on Mathematical Art*) [20]. The second category is the teaching suggestions for the *Gaokao* items incorporating mathematical culture. For example, by evaluating six *Gaokao* items with mathematical culture in 2018, some researchers have put forward four types of teaching suggestions [18]. Such studies have a strong teaching practice orientation and are usually published in journals of mathematics teaching that share teaching experiences.

It is noteworthy that some researchers investigated 1007 Chinese high school students by asking the question, "Do you like items with a mathematical cultural background? Why?" The results showed that more than 60% of the students said that they did not like the items, mainly because "the items are too long and it is difficult to understand the meaning of the items due to the limited test time" [58]. This indicates that the mathematical culture in the *Gaokao* items seems not to exert an expected value, and its presentation and content need to be improved.

From an international perspective, although few studies have focused on mathematical culture in test items, the existing studies can be reviewed from a broader research perspective. For example, in terms of the context in which a mathematics item arises, such as the mathematical literacy test of PISA, the contexts proposed in this test are personal, occupational, societal, and scientific [59]. Since they are real situations that students may face [60], they actually reflect the role of mathematics in human civilization and scientific development. Similarly, Dogbey and Dogbey analyzed the context characteristics in the West African Examination Council's Core Mathematics assessment from 1993 to 2013, dividing the contexts of test items into three types: pure mathematics, semi-reality, and real-life reference [61], where the semi-reality and real-life contexts are also related to mathematical culture. Expanding the study object to mathematics textbooks and having a review could indicate that examples and exercises as important parts of textbooks have also attracted the attention of scholars. In addition to the contexts of items, previous studies have also begun to focus on more general cultural factors.(e.g., [37,62]) For example, Fan et al. compared the Chinese edition and the English edition of the textbook *One Lesson One Exercise*, published in China and the UK, respectively, and found that it was the cultural factors that caused differences in the context of the exercises in the two sets of books, with most differences related to "ways of behaving and customs" and "artifacts, flora, and fauna," followed by "identities" and "geography," and with "organizations" and "history" being scarce [37]. As for the study of mathematical culture in textbook exercises and examples, Li and Wang Xiaoqin employed Wang Xiaoqin's conceptual framework in the literature [39] to analyze mathematical culture in the textbook exercises of junior high school in France in a more systematic way, and found that the textbook had a high application level of mathematical culture, manifested in the rich materials of mathematical history and an emphasis on the connections between mathematics and subjects, such as physics, personal life, and social life [42], which provided a reference for studies on mathematical culture in test items.

In general, the present research on test items that integrate mathematical culture is still in its infancy, as manifested by the inconsistent classification of mathematical culture, the short time span, and the scarcity of focused and systematic empirical studies. Given the importance of the *Gaokao* in the Chinese education system and its influence on mathematics education in China and even on the global spread and exchange of mathematical culture, it is necessary to study the content characteristics and the time variation in mathematical culture in *Gaokao* test items over several decades.

3. Materials and Methods

3.1. Data Sources

The test items of the *Gaokao* not only include the national papers drafted out by the MOE of PRC but also the papers drafted out by each province. Except for the first year since the restoration of the *Gaokao* (1977), when the MOE of PRC did not have time to draft out a national paper due to limited time, and only the provinces had their own papers drafted out that year, all other years saw national papers. Based on the purpose of this study, the national papers of mathematics tests of the *Gaokao* from 1978 to 2021 were selected as the data sources for this study. For the alternative years of curriculum reform, the study did not use some national papers in the statistics and analysis because they were only piloted on a small scale in individual provinces as certain pilot papers. Therefore, the final data sources for this study were the national papers of mathematics tests of the *Gaokao* from 1978 to 2021, totaling 147 sets of papers, and the specific information of the selected papers is shown in Appendix A.

3.2. Methods

3.2.1. Research Methods

This study used document analysis methods, a research method used to review and evaluate documents, mainly involving skimming (superficial examination), reading (thorough examination), and interpretation [63]. The content analysis was used as an analytical technique [64], which is a widely used qualitative analysis technique in mathematics education research [65] (p. 365). In conducting the content analysis, this study first coded each specific item as the smallest analysis unit and then interpreted and descriptively analyzed the coded results.

3.2.2. Analysis Conceptual Framework

First, the researcher divided all the test items into pure mathematical items and the items with mathematical culture. The former refers to the items without any cultural elements or features, i.e., the items did not contain any historical or realistic context other than mathematics [66] (p. 30); [67] (p. 40), nor did they exhibit any specific element of mathematical beauty, containing only mathematical symbols, concepts, formulas, etc.

For example: (*pure mathematics items*) It has been known that the set

$$U = \{-2, -1, 0, 1, 2, 3\},$$

$$A = \{-1, 0, 1\}, B = \{1, 2\},$$

Then

$$\complement_U(A \cup B) = (\ldots.)$$

A. $\{-2, 3\}$ B. $\{-2, 2, 3\}$ C. $\{-2, -1, 0, 3\}$ D. $\{-2, -1, 0\}$,

Based on the purpose of the study, only the test items with mathematical culture were coded and analyzed in this study. Therefore, after excluding pure mathematical items, the study established an analysis conceptual framework, according to which all test items with mathematical culture were classified and labeled as such, with each item belonging to only one classification.

Definition of Mathematical Culture

Different scholars have different definitions of mathematical culture. The *Gaokao* items selected in this study were drafted out according to the Chinese curriculum standards, so the OHSMCS was used to define mathematical culture: "mathematical culture refers to the mathematical ideas, spirit, language, methods, and perspectives, as well as their formation and development; it also includes the contributions and significance which mathematics has made and produced in human life, science and technology, and social development, and the humanistic activities related to mathematics." [45] (p. 10).

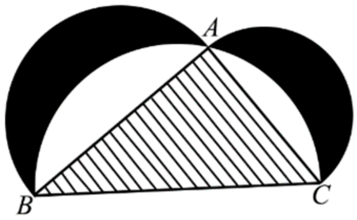
A Conceptual Framework for Analyzing Mathematical Culture Based on *Gaokao* Items

Due to the complexity of the concept of mathematical culture, this study did not intend to, and could not, establish a universal conceptual framework to discuss all components of mathematical culture, and the study only hoped to establish a mathematical culture conceptual framework that can be used to effectively analyze test items. Considering that Wang Xiaoqin's conceptual framework incorporates the requirements of the Chinese curriculum, which fits the cultural context in which *Gaokao* items should abide by curriculum standards, and that some studies have shown that such conceptual framework has operability [40–42,68], this study considered it suitable for the analysis of *Gaokao* items. An exploratory analysis was conducted in this study under this conceptual framework in an earlier stage. Since Wang Xiaoqin's operational definition, researchers have conducted a pre-analysis of mathematical culture in *Gaokao* items and found that the category of *Multiculturalism* overlaps with other categories, such as a question with a background in mathematical history in ancient Greece (see Example 1 in Table 2), which could be classified as *Historical Topics* and *Multiculturalism*. This suggests that the conceptual framework needed to be revised in this study, but this could be understood because the conceptual framework was initially the generalization of the components of mathematical culture, while the test items were what this study coded for analysis. After discussion, the study decided to delete the category of *Multiculturalism* as a modification to the conceptual framework, as this category was the analysis of mathematical culture from a different perspective and could be replaced by other categories in the coding. Further exploratory research revealed that the contents about mathematical culture present in the test items could be clearly categorized without overlap under the modified Wang Xiaoqin's conceptual framework, and that all mathematical culture present in the test items could be covered. Therefore, this conceptual framework was applied in the present study.

Thus, based on Wang Xiaoqin's theory, mathematical culture in the test items was classified into four categories: *Historical Topics*, *Interdisciplinary Connections*, *Social Roles*, and *Aesthetics & Recreation*. The classification of contexts of mathematics items from PISA [59] and the classification of contexts of mathematics items by Wang Jianpan [38] and some descriptions in the OHSMCS were also drawn upon to establish a conceptual framework for classifying mathematical culture for an analysis of *Gaokao* items. Some subcategories under this conceptual framework were also discussed and were approved by experts in the field of mathematics education research (including experts in the history and pedagogy of mathematics and experts in mathematics problem research), and the finalized conceptual framework is shown in Figure 1.

Table 2. Examples of *Gaokao* items with mathematical culture.

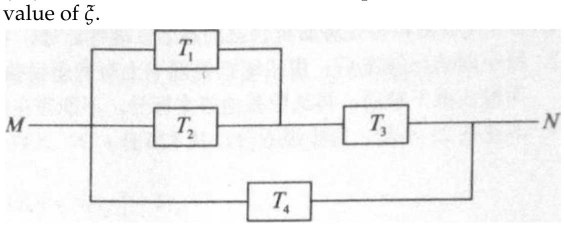
<table>
<tr><th>Category</th><th>Examples</th></tr>
<tr><td>Historical Topics</td><td>
Example 1 (People and Events) : The figure below is from the shape studied by the ancient Greek mathematician Hippocrates. The shape consists of three semicircles, the diameters of which are the hypotenuse BC and the sides AB and AC of the right triangle ABC. The region enclosed by the three sides of $\triangle ABC$ is denoted I, the black part is denoted II, and the rest is denoted III. One point is randomly picked from the whole figure. The probabilities of taking a random point from I, II, and III are denoted as p_1, p_2, and p_3, respectively.



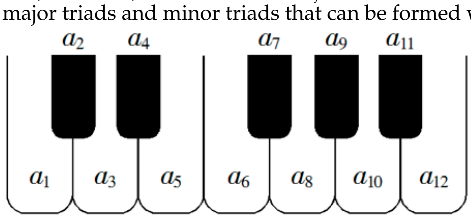
A. $p_1 = p_2$ B. $p_1 = p_3$ C. $p_2 = p_3$ D. $p_1 = p_2 + p_3$

Example 2 (Problems and Solutions): The Nine Chapters on the Mathematical Art is a mathematics classic with rich content related to ancient China, and the book has the following problem: “In a house, one man should stack rice at the corner of the wall (as shown in the figure, the rice pile is a quarter of a cone). The arc length of the bottom of the rice pile is 8 chi (Chi is a unit of length in ancient China, 3 chi = 1 m), the height of the rice pile is 5 chi. The question is what is the volume of the rice pile and how large is the pile of rice?” Knowing that the volume of 1 hu (Hu is a dry measure used in former times, originally equal to 10 dou, later 5 dou, 1 dou = 10 L) of rice is about 1.62 cubic chi and that Pi is about 3, estimate the size of the pile of rice.

A. 14 hu B. 22 hu C. 36 hu D. 66 hu
</td></tr>
<tr><td>Interdisciplinary Connections</td><td>
Example 3 (Mathematics and Technology): As shown in the figure, there are four components in the circuit from M to N, labeled T_1, T_2, T_3, and T_4, respectively. The probabilities of a current passing through T_1, T_2, and T_3 are the same, p, and the probability of passing through T_4 is 0.9. The current pass through each component is independent. It is given that the probability that the current can pass through at least one of T_1, T_2, and T_3 is 0.999.
(I) Find p.
(II) Find the probability that a current can pass between M and N.
(III) Given that ξ is the number of components in T_1, T_2, T_3, and T_4 through which the current can pass, find the expected value of ξ.



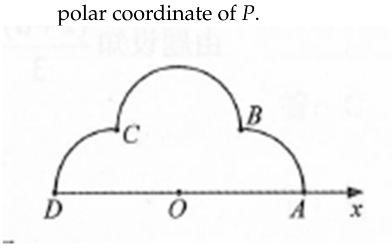
Example 4 (Mathematics and Arts) : As shown in the figure, the 12 keys on a piano are denoted $a_1, a_2, \ldots, a_{12}$.
Let $1 \le i < j < k \le 12$. If $k - j = 3$ and $j - i = 4$, then a_i, a_j, and a_k are called major triads on the root position; if $k - j = 4$ and $j - i = 3$, then a_i, a_j, and a_k are called minor triads on the root position. What is the sum of the number of major triads and minor triads that can be formed with these 12 keys?



A. 5 B. 8 C. 10 D. 15
</td></tr>
</table>

Table 2. *Cont.*

<table>
<tr><th>Category</th><th>Examples</th></tr>
<tr><td>Social Roles</td><td>Example 5 (Occupational): During the prevention and control of the COVID-19 epidemic, a supermarket opened an online sales business and was able to fulfill 1200 orders per day. A significant increase in orders has caused a backlog. To solve this difficulty, many volunteers enthusiastically signed up for the distribution work. It is known that the supermarket has a backlog of 500 orders on a certain day, and the probability that the number of new orders exceeds 1600 on the next day is 0.05. Each volunteer can complete 50 orders per day. To make sure that the probability of completing the backlog and the new orders the next day is not less than 0.95, how many volunteers are needed?
A. Ten B. Eighteen C. Twenty-four D. Thirty-two

Example 6 (Societal): A place has 10,000 hectares of arable land, and it is planned that, in 10 years, the grain yield per unit area will increase by 22% and the per capita occupancy of grain will increase by 10%. If the annual population growth rate is 1%, then how many hectares of arable land will be reduced on average per year at most (accurate to 1 ha)?
Grain Yield Per Unit Area $= \frac{\text{Total Grain Yeild}}{\text{Arable Land Area}}$,
Per Capita Occupancy of Grain $= \frac{\text{Total Grain Filed}}{\text{Total Number of Population}}$</td></tr>
<tr><td>Aesthetics & Recreation</td><td>Example 7 (Mathematical Beauty): As shown in the figure, $A(2,0), B(\sqrt{2}, \frac{\pi}{4}), C(\sqrt{2}, \frac{3\pi}{4})$, and$D(2,\pi)$ are four points in polar coordinate system Ox. The centers of the circles where the arcs AB, BC, and CD are located are $(1,0)$, $(1, \frac{\pi}{2})$, and $(1,\pi)$. Assume that the curve M_1 is the arc AB, the curve M_2 is the arc BC, and the curve M_3 is the arc CD.
(1) Find the polar equations of M_1, M_2, and M_3, respectively.
(2) The curve M consists of M_1, M_2, and M_3. Suppose that P is on the curve M and that $|OP| = \sqrt{3}$, find the polar coordinate of P.


Example 8 (Recreational Mathematics): How many different ways can the numbers 1, 2, and 3 be filled in a 3×3 square, without duplicate numbers in either each row or each column. The figure shown below is one example.
<table><tr><td>1</td><td>2</td><td>3</td></tr><tr><td>3</td><td>1</td><td>2</td></tr><tr><td>2</td><td>3</td><td>1</td></tr></table>A. Six B. Twelve C. Twenty-four D. Forty-eight</td></tr>
</table>

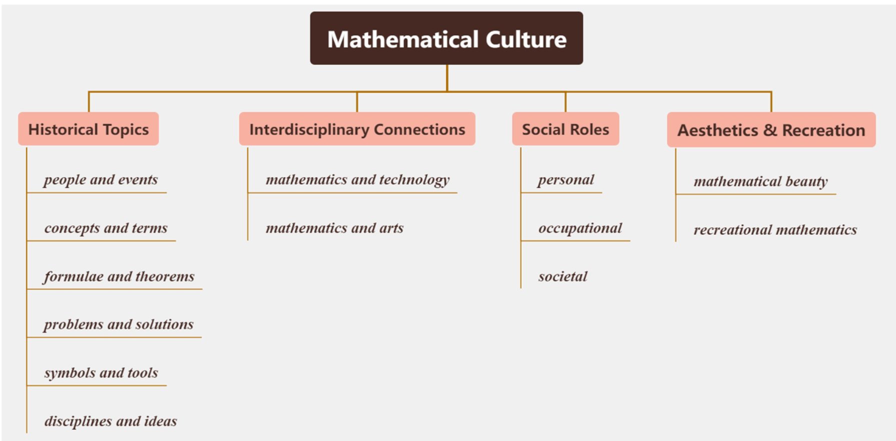


Figure 1. A conceptual framework for the analysis of mathematical culture based on *Gaokao* items.

The details of the conceptual framework are shown as follows (see Table 1 for some examples):

(1) *Historical Topics*

This category refers to mathematical history, focusing on the historical development of a certain knowledge point and its related people, events, and ideas [39]. It is usually used as a background introduction in the *Gaokao* items, and it also applies when the items require students to prove a formula or theorem. The definition of this category in the OHSMCS is "the mathematical ideas, spirit, language, methods, and perspectives, and their formation and development" [45] (p. 10). This category can be further divided into six specific subcategories: *people and events*, *concepts and terms*, *formulae and theorems*, *problems and solutions*, *symbols and tools*, and *disciplines and ideas* [69].

People and events. This subcategory includes mathematicians and their related profiles, mathematical work, historical events, and mathematics classics. In the mathematics items of the *Gaokao*, it mainly refers to the introduction of mathematicians and their related events that appear in the background of the items, such as the question stem in Example 1, which introduces the geometric figures studied by the Greek mathematician Hippocrates.

Concepts and terms. This subcategory includes the history of mathematical concepts and the origin of mathematical terms. In the mathematics items of the *Gaokao*, it usually appears in the background introduction of the question.

Formulae and theorems. This subcategory includes the history of formulae, theorems, axioms, laws, and their deduction and proof of historical methods. In the *Gaokao*, it is both included in the historical introduction to the source of formulae and theorems in the question stem and in the task where the question asks students to derive and prove the source of the formulae or theorems.

Problems and solutions. This subcategory includes historical mathematical problems as well as solutions to mathematical problems in history. In the mathematics items of the *Gaokao*, this category primarily refers to historical problems or solutions that are explicitly introduced in the question stem, such as the problem of stacking rice in the *Nine Chapters on the Mathematical Art* shown in Example 2.

Symbols and tools. This subcategory includes the origin of mathematical symbols, the use of mathematics as a drawing tool, a measuring tool, and a computational tool throughout history. It mainly refers to the historical background introduction of symbols and tools given by the items.

Disciplines and ideas. This subcategory includes a historical overview of the subdisciplines of mathematics and the sources of mathematical thought and methods.

Broadly speaking, all mathematical knowledge exists in history and can be considered as a part of mathematical history. However, this study only analyzed items with distinct historical features because they are intentionally presented by the test makers, and these historical elements can be used to identify and analyze the cultural orientation of the test makers.

(2) *Interdisciplinary Connections*

Interdisciplinary connections are the connections between mathematics and other subjects [39]. M. Kline once pointed out that mathematics should be taught in a way that links mathematics with the knowledge of other subjects [70]. The OHSMCS has mentioned that "mathematics is an important foundation of the natural science and plays a growing role in social science. [...] Emphasis should be put on the integration of mathematical culture and the connections between mathematics and life as well as other subjects." [45] (pp. 1–2) Specifically, this study divides the connections between mathematics and other disciplines into the connections between mathematics and technology and between mathematics and the arts [38].

Mathematics and technology. This connection is divided according to the study type of technology, and the test items include connections between mathematics and biological science, earth sciences, materials science, high technology, and architecture science, such as the circuit problem in Example 3.

Mathematics and arts. According to the differences in art forms, the test items include connections between mathematics and the humanities, fine arts, music (Example 4), and architectural arts.

(3) *Social Roles*

Social roles are the roles played by mathematics in human life, civilization progress, and social development [39]. The OHSMCS states that "mathematics is closely linked to human life and social development. [...] The application of mathematics has integrated into all aspects of modern society and people's daily life. [...] Mathematics directly creates value for society and promotes the development of social productivity." [45] (p. 1) According to PISA's [59] classification of the context in which a mathematics item arises, this category is further divided into personal, occupational, and societal contexts, which all reflect the role of mathematics in real life and social development.

Personal. This refers to the test contexts which can reflect the application of mathematics in daily life, mainly involving the activities of oneself, one's family, or one's peer group, which are accessible and familiar to every student. In *Gaokao* items, the contexts include applications of mathematics in personal, family, and school life. In personal life, students may encounter problems that are most closely related to individuals; for example, individual users buy software and disks, and individuals visit different cities. In family life, students may encounter problems that easily occur in households, such as issues with the water supply. In school life, students will face common situations on campus such as quizzes and student representative elections.

Occupational. This is about the role that mathematics may play in students' future work situations. In *Gaokao* items, it mainly refers to problems in the workplace that may be encountered in various occupations set by the test items, including problems of product production, sales, quality, and profit maximization. An order arrangement problem in a supermarket is shown in Example 5. They are distant from students' present lives, but potentially encountered in students' future working lives.

Societal. This is about the application of mathematics in the students' communities or in the larger social environment, which is related to the public affairs of society. In *Gaokao* items, it includes applications of mathematics in social, entertainment, and economic life. In social life, students may encounter macro social issues such as population growth and number of cars. In entertainment life, students will encounter problems related to culture and entertainment, including sports competitions as well as leisure and cultural activities, such as shooting competitions, tourism, and knowledge competitions. In economic life, it consists of problem situations related to national economic activities such as freshwater fish farming subsidies, the output of farmland used for food production (Example 6), and vegetable sowing areas.

(4) *Aesthetics & Recreation*

This mainly refers to mathematical beauty and recreational mathematics [39]. It is mentioned in the OHSMCS that "students should be guided to observe the world through mathematical visions [. . .] to recognize the aesthetic value of mathematics [. . .] to portray common features of aesthetics from a mathematical perspective, including simplicity, symmetry, periodicity, harmony, etc.," and "suitable teaching situations should be created [. . .] to stimulate interest in learning mathematics." [45] (pp. 2–3, 8, 69). In *Gaokao* items, it can be specifically divided into two categories, *mathematical beauty* and *recreational mathematics*.

Mathematical beauty. This refers to the beauty in mathematics. In a larger sense, *mathematical beauty* exists in all aspects. Hardy claimed that "there is no permanent place in the world for ugly mathematics" [71]. Each person also perceives and evaluates beauty differently [72], and it seems difficult to find a universal standard to define *mathematical beauty*. According to some classical descriptions of *mathematical beauty* [73–75], specifically in secondary school mathematics test questions, *mathematical beauty* includes the beautiful and pretty feelings brought to the senses by the mathematical objects in the questions due to their symmetry, harmony, and simplicity, as well as the wonderful feelings brought by the conclusions of the questions. Specifically in this study, the study only coded the obvious and typical *mathematical beauty* features presented in the test items for the characteristics of the mathematics test items of the *Gaokao*, combined with the descriptions of the OHSMCS. For example, some special geometric figures have an intuitive beauty of symmetry (e.g., Example 7), some algebraic formulae also have a formal beauty of symmetry, the fixed values in some problems of moving points represent a kind of stable beauty, and some conclusions involving concurrent, collinear, and special trajectories have the beauty of coincidence, all of which are very common in mathematics tests of the *Gaokao*.

Recreational mathematics. This refers to problems that reflect the fun of mathematics. Some interesting mathematical games and mathematical riddles are included in *Gaokao* items, such as the number game shown in Example 8.

3.2.3. Coding Process

Two rounds of coding were conducted in this study employing content analysis [64]. In Step 1, after the research conceptual framework was established, Coder A (a PhD student in history and pedagogy of mathematics) consulted a professor and two PhD students in mathematics education, and Coder A then independently conducted the coding. In Step 2, by employing mathematical software, Coder A randomly selected approximately 20% of the data (8 years) between 1978 and 2021, from which Coder B (a PhD student in mathematics problem research) independently conducted the coding. In Step 3, a consistency check between the two coders was performed. At this stage, a total of 669 *Gaokao* items underwent a coding check, among which 7 items were found to be inconsistently coded by two coders, 4 were inconsistent in the categorization of mathematical culture, and 3 were inconsistent in whether they were mathematical culture questions, with the final calculated coding consistency up to 98.80%. In Step 4, due to the high degree of coding consistency, after the two coders consulted each other and met consistent criteria, the second round of coding was independently conducted again by Coder A, and the data for this study were finally obtained. Throughout the coding process, the coders used a code book for recording.

4. Findings

4.1. General Features

The items with mathematical culture in the mathematics tests of the *Gaokao* from 1978 to 2021 were counted, and a total of 403 items were found to have mathematical culture reflected in them. The data are described below in terms of the content distribution and time variation in the test items.

4.1.1. Content Distribution of Mathematical Culture in the Test Items

According to the conceptual framework of mathematical culture, the test items with mathematical culture according to four categories were counted, and a pie chart was obtained (see Figure 2). Combined with the statistical data, there was an extremely uneven content distribution of mathematics culture in the mathematics tests of the *Gaokao*, and more than half of the content of mathematics culture focused on the category of *Social Roles*, with 221 items in aggregate, accounting for 54.8% of the total items, followed by *Interdisciplinary Connections* and *Aesthetics & Recreation*, both accounting for about 20%. The category of *Historical Topics* was less frequent, only accounting for 5.0%. From the overall data, it seems that the mathematical culture in the *Gaokao* items focused more on the application of mathematics in real life and less on the process of mathematical knowledge generation and development.

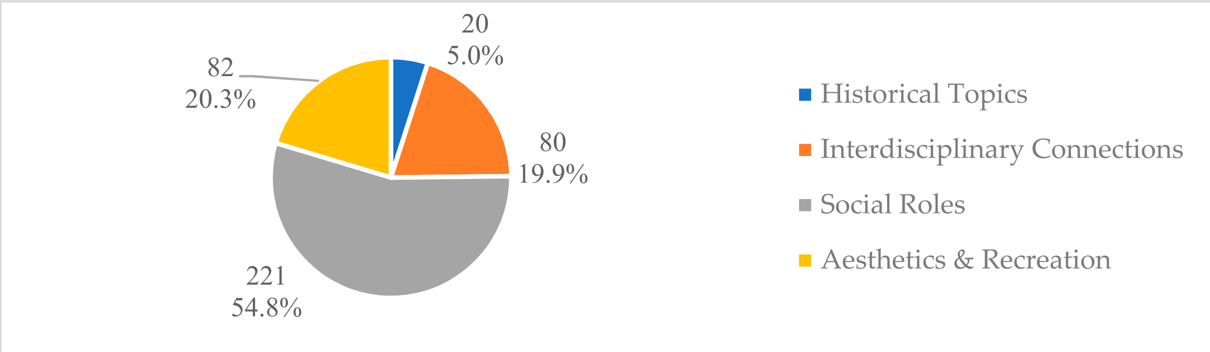


Figure 2. Pie chart of content distribution of mathematical culture.

4.1.2. Time Variation in Mathematical Culture in the Test Items

Considering that the number of national papers varied from year to year, the average quantity of test items with mathematical culture for each year was calculated (the number of test items divided by the number of the test papers) (see Appendix B, Figure 3). First, it can be seen that the number of items with mathematical culture was generally low. Specifically, the annual mean of the number of items with mathematical culture was 2.51 in the national papers of mathematics tests of the *Gaokao* in the past 44 years, with a median of 2.50. The mean of the number of items was three or less in more than half of the years (65.9%), with only two years (1999 and 2019) having five and the year of 1992 having zero. Second, the number of items with mathematical culture was volatile, with a significant increase in recent years. Figure 3 shows that the number of test items with mathematical culture has been more volatile over the past 44 years, having peaked in 1979, 1999, and 2019 for a short period.

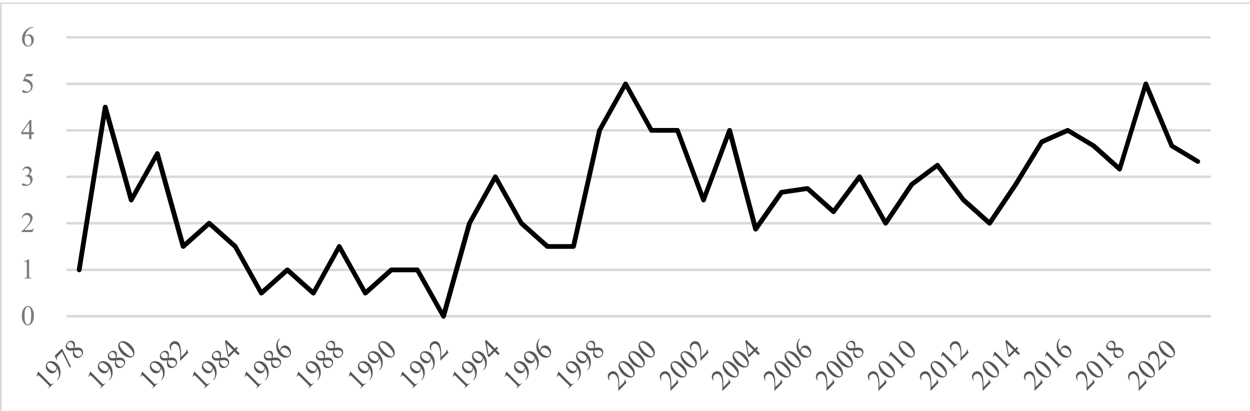


Figure 3. Line graph of the changes in the number of test items with mathematical culture.

Combined with a line graph of the changes in the number of test items with mathematical culture in different categories (Figure 4), it can be inferred from the vertical axis that the changes in the number of test items in the categories of *Social Roles*, *Interdisciplinary Connections*, *Aesthetics & Recreation*, and *Historical Topics* showed fluctuation, although the test items of each category had their own characteristics in history. It can also be inferred from the horizontal axis that the content distribution of mathematical culture in test items of different years was also similar to the overall content distribution of mathematical culture in test items. In almost all years, the number of items in the category of *Social Roles* was the highest, followed by the categories of *Interdisciplinary Connections* and *Aesthetics & Recreation*.

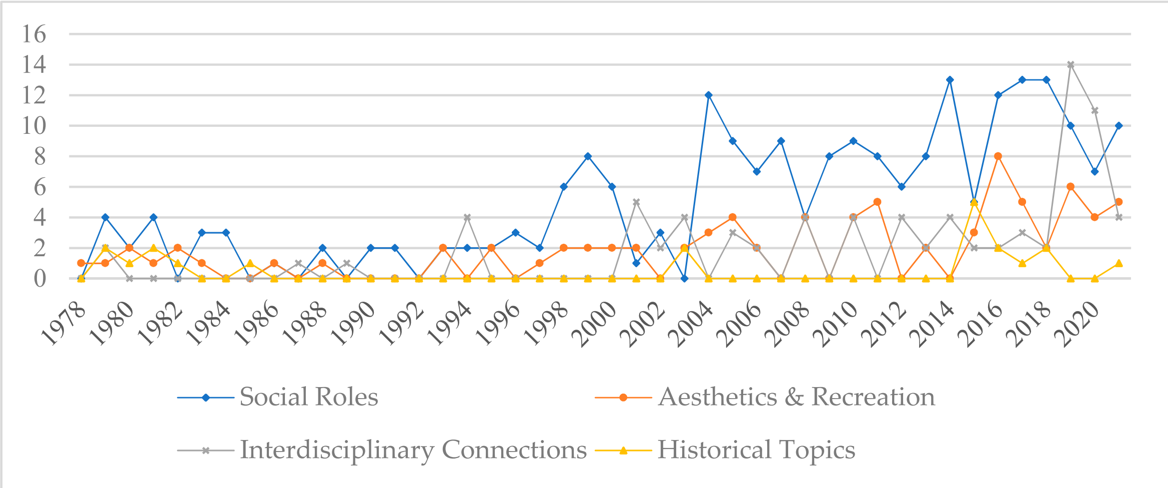


Figure 4. Line graph of the changes in the number of test items with mathematical culture in different categories.

4.2. The Specific Features of Different Categories

4.2.1. Historical Topics

The number of items with mathematical culture in each sub-category of the *Historical Topics* category in the national papers of mathematics tests of the *Gaokao* in the past 44 years were counted. Table 3 shows that the category of *Historical Topics* had the following characteristics in terms of content distribution: First, items in this category were relatively small in number but more diverse in their content settings. Second, the content distribution was uneven. More than 80% of the items in this category were related to historical problems, formulas, and theorems in history, such as mutual subtraction and the Qin Jiushao algorithm (the Horner algorithm) in the *Nine Chapters on the Mathematical Art*. Third, each sub-category was very similar in the selected materials, with materials mainly from the proofs of several famous formulas and theorems (including the Gougu Theorem (the Pythagorean Theorem), the Law of Sines, the Law of Cosines, and the Formula for the Change of Base of Logarithms) and the problems in the famous Chinese Mathematics Classics, namely *Nine Chapters on the Mathematical Art*, *Suanfa Tongzong (A Comprehension Book on Calculation*) and *Haidao Suanjin* (*Mathematical Manual of Sea Islands*).

Table 3. The content distribution of the category of *Historical Topics*.

Sub-Category	Number	Percentage
People and Events	2	10.0
Concepts and Terms	1	5.0
Formulae and Theorems	11	55.0
Problems and Solutions	5	25.0
Symbols and Tools	1	5.0
Disciplines and Ideas	0	0
Subtotal	20	100.0

The category of *Historical Topics* was also characterized by time variation in its history of 44 years (see Figure 5). Firstly, after a down period of 28 years, mathematical history regained attention. The number of items in this category was high from 1978 to 1986, was almost zero from 1987 to 2014, and then increased rapidly after 2015. Secondly, the materials of mathematical history gradually became enriched and began to focus on promoting the achievements of ancient Chinese mathematics. Before 2015, items concerning mathematical history were mostly about formulas and theorem proofs. However, starting in 2015, the materials of mathematical history became enriched, and the national papers of mathematics would incorporate traditional Chinese mathematics into the test items almost every year, introducing the mathematical achievements made by Chinese mathematicians, the contributions made by ancient Chinese mathematics, and the applications of ancient Chinese mathematics.

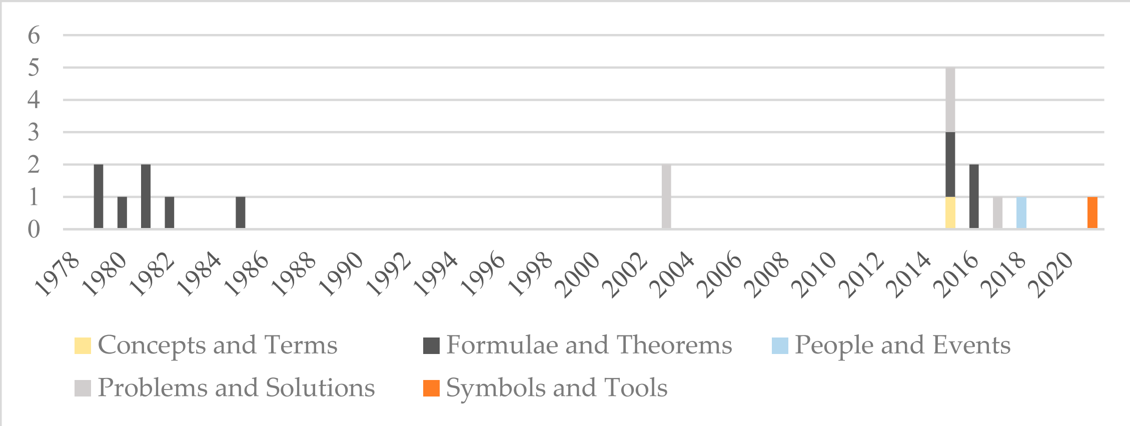


Figure 5. Histogram of the changes in the number of items in different sub-categories of the category of *Historical Topics*.

4.2.2. Interdisciplinary Connections

Table 4 summarizes the content distribution of *Interdisciplinary Connections* in the test items. As can be seen, the content distribution of mathematical culture in this category had the following characteristics: First, it covered a wide range of topics with rich and detailed content. Second, the content setting was emphasized, highlighting the extensive connection to *mathematics and technology*. The statistical analysis found that the most common items were related to *mathematics and technology*, accounting for 82.5% of the items in this category, which covered traditional science such as materials science, architecture science, and hi-tech technology, such as information technology and space technology. Third, the materials were chosen to consciously display elements with Chinese characteristics. Although items related to *mathematics and arts* were relatively small in quantity, they were quite rich in background, including the traditional paper cut and images of the hexagrams in *Zhouyi* (*Book of Changes*). At the same time, many items concerning science and technology are integrated with a Chinese background, such as items on the mortise–tenon connection in ancient China, China's high-speed railway, and Chang'e 4 (lunar probe), introducing China's scientific and technological achievements.

Table 4. The content distribution of the category of *Interdisciplinary Connections*.

Sub-Category	Specific Category	Number	Percentage
Mathematics and Technology	Bioscience	18	22.5
	Geoscience	10	12.5
	Material science	22	27.5
	Hi-tech	12	15.0
	Architecture science	4	5.0
	Subtotal	66	82.5
Mathematics and Arts	Arts	6	7.5
	Fine arts	4	5.0
	Music	1	1.3
	Architecture art	3	3.8
	Subtotal	14	17.5
	Total	80	100.0

Based on the changes in the number of items in the categories of *mathematics and technology* and *mathematics and arts* over the past 44 years (see Figure 6), the items in the category of *Interdisciplinary Connections* had the following characteristics over time. First, the number of items is gradually on the rise, the topics of which are becoming richer as well. The number of items in this category was relatively small in quantity between 1978 and 2000, with items in only four years involving a connection to *mathematics and technology*, and they were relatively monotonous in terms of topic. The year of 2001 saw a gradual increase in the number of items, with the number reaching a new high in 2019. At the same time, the themes have become more diverse, gradually shifting from the more concentrated subject themes of biology, physics, and chemistry in earlier years to more varied themes such as arts, computers, and temperature. Second, items fit the background of the times. The science and technology embodied in the test items have been renewed with the progress of society. Third, the artistic value of mathematics has begun to gain importance in recent years. Since 2017, the number of items in the category of *mathematics and arts* has continued to rise, with items related to traditional Chinese culture having appeared almost every year since then.

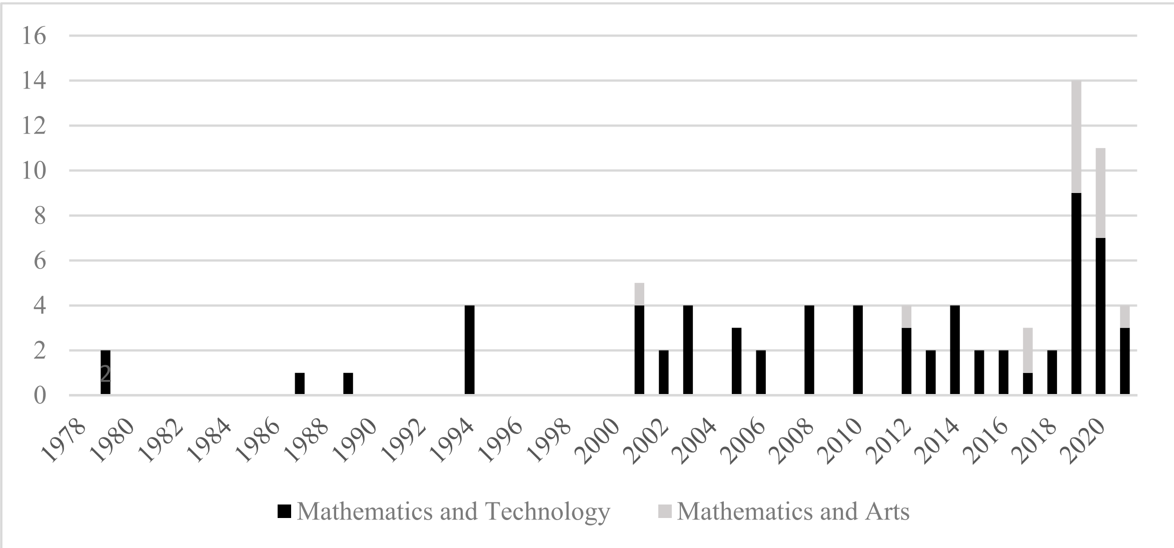


Figure 6. Histogram of the changes in the number of items in different sub-categories of the category of *Interdisciplinary Connections*.

4.2.3. Social Roles

Table 5 shows a summary of the specific categories in the category of *Social Roles*, from which the following can be seen: items in this category were high in quantity, with rich social and cultural contexts. First, the most involved sub-category was *societal*, the items of which accounted for 40.7%, covering social life, entertainment, and economics. Second, the occupational role of mathematics was emphasized, with the category of *occupational* life as the second most popular theme in terms of quantity (36.7%). It integrated items under job contexts that people may encounter in different industries, including factory output, the sampling of inferior products in factories, teacher assignments, and machine monitoring. Third, items in *personal* contexts appeared less frequently, focusing mainly on students' school life, including the common contexts that occur on campus, such as electing student representatives, taking physical fitness tests, and course selection.

Combined with the histogram of the changes in the number of items in different sub-categories (Figure 7), items in the category of *Social Roles* showed the following characteristics with the development of the era: First, items of the *Gaokao* in 84.1% of 44 years reflected the *Social Roles* of mathematics. Second, the themes reflecting real life have changed from simple to rich. Between 1978 and 2003, the topics were relatively simple, mostly related to *social* situations. Since 2003, real-life topics have shown a diversified distribution, covering at least real-life contexts of two categories almost every year, and items in the category of *personal* situations have begun to increase. Third, the test materials kept up with current events, with distinctive characteristics of the times. For example, the context in which a mathematics item arose was about the rising prices in the United States in 1979, the data of China's population growth in 1981, and the COVID-19 epidemic in 2020. At the same time, elements such as the Road and Belt Initiative and the Beijing Winter Olympics were also reflected in the context of the test items, integrating China's state affairs.

Table 5. The content distribution of the category of *Social Roles*.

Sub-Category	Specific Category	Number	Percentage
Personal	Personal life	16	7.2
	Family life	1	0.5
	School life	33	14.9
	Subtotal	50	22.6
Occupational	Occupational life	81	36.7
Societal	Social life	29	13.1
	Entertainment life	29	13.1
	Economic life	32	14.5
	Subtotal	90	40.7
	Total	221	100.0

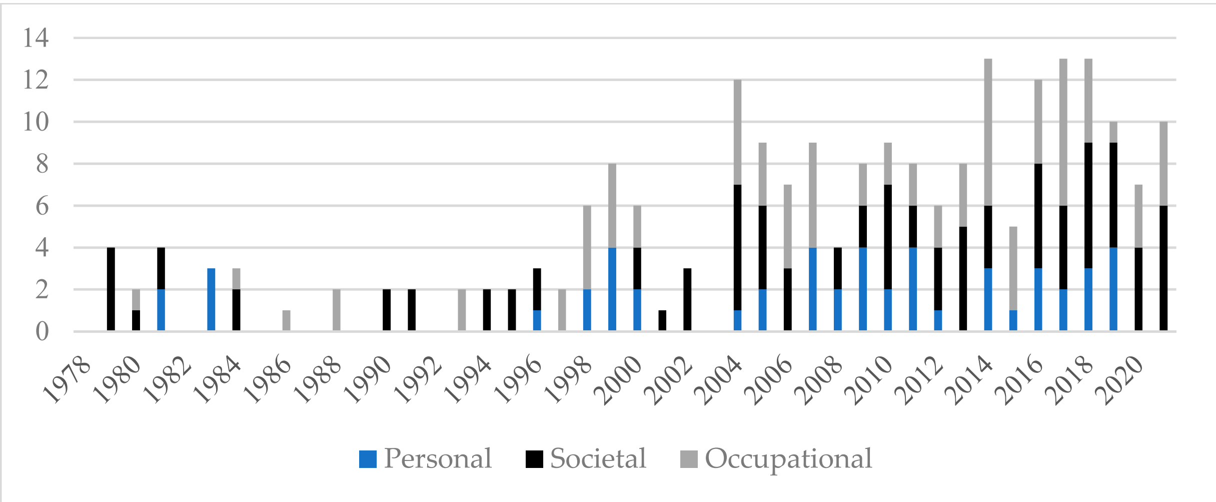


Figure 7. Histogram of the changes in the number of items in different sub-categories of the category of *Social Roles*.

4.2.4. Aesthetics & Recreation

The content distribution of the category of *Aesthetics & Recreation* is shown in Table 6. The items in this category accounted for the second largest proportion of the overall mathematical culture items, and its content distribution had the following characteristics: First, it highlights the aesthetic value of mathematics. Many items in the *Gaokao* emphasize *mathematical beauty*, including the intuitive symmetry and harmony of geometric figures involved in the items, the coincidental beauty of question conclusions, and the stable beauty of geometric transformations. Second, the items adapted from recreational mathematical problems in the *Gaokao* were lower in quantity and simpler in topics, which included simulated ball touch games, card games, and number games.

Table 6. The content distribution of the category of *Aesthetics & Recreation*.

Sub-Category	Number	Percentage
Mathematical beauty	73	89.0
Recreational mathematics	9	11.0
Subtotal	82	100.0

Combined with the histogram of changes in the number of items in the category of *Aesthetics & Recreation* (Figure 8), the following characteristics of this category have emerged over time: First, it appears that items in 66.0% of 44 years reflected *mathematical beauty*. Second, mathematics items reflecting *mathematical beauty* were similar to each other in design ideas, with no obvious time variation characteristics. For example, among the items reflecting the beauty of symmetry, items in 1988 and 2020 had similar topics, i.e., function image symmetry. Third, items have gradually created recreational mathematical situations, reflecting the fun of mathematics. There were few recreational mathematics items before 1995, but they have appeared occasionally since 2004.

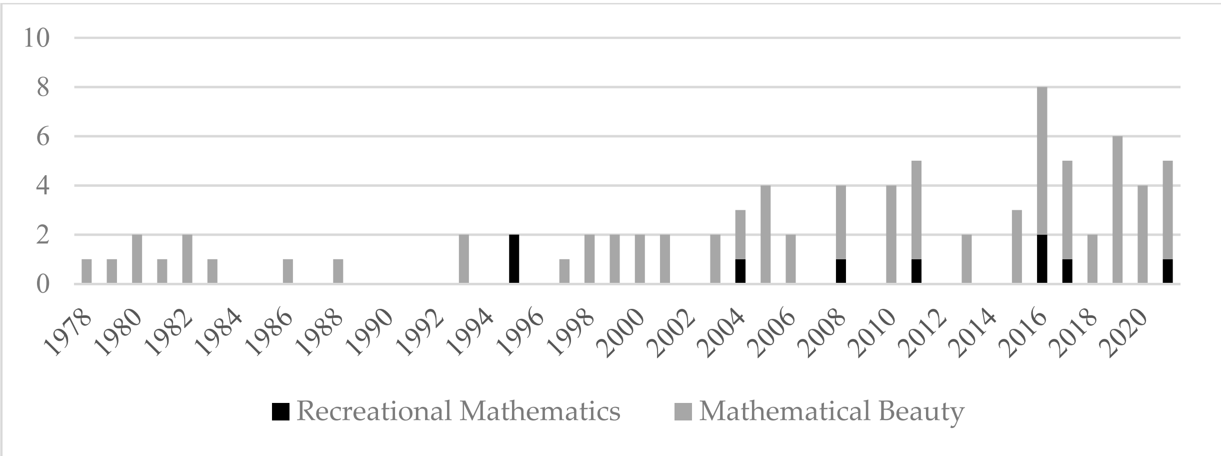


Figure 8. Histogram of the changes in the number of items in different sub-categories of the category of *Aesthetics & Recreation*.

5. Discussion

This study took the philosophy of mathematical practices and the cultural turn in mathematics education into account, and the conceptual framework of mathematical culture proposed in [39] was adapted and refined for the mathematics test items of the *Gaokao* in response to the characteristics of *Gaokao* test items. This conceptual framework considered some richer perspectives, such as the historical development process of mathematical practices, the original culture that mathematical practices have, the role that mathematics plays in human civilization, and the connections between mathematics and other disciplines. The conceptual framework of these categories was richer than that of those categories studying mathematical culture in *Gaokao* test items in the past (e.g., [19–21]). More importantly, this study analyzed the national papers of *Gaokao* mathematics test items with a larger sample size and a longer time span and based on the research process employing content analysis techniques of coding one by one and conducting descriptive statistics, the changing characteristics of test item design over times was systematically and clearly studied. From these perspectives, this study yielded some major findings, which are discussed below.

This study found that mathematical culture in *Gaokao* items has been very uneven in terms of the distribution of different categories for more than 40 years, with items in the category of mathematical culture concentrating mostly on the *Social Roles* of mathematics and less on the historical development of mathematical knowledge. A Chinese scholar obtained similar conclusions when analyzing and counting mathematics test items with mathematical culture in the 2008–2016 *Gaokao* in various Chinese provinces [21]. The preferred category of *Social Roles* is consistent with the curriculum standards on which the *Gaokao* items are based. Since China's 1978 syllabus, mathematics education has placed great emphasis on the role of mathematics in modern production life [45,56], while the emphasis on mathematical history in the curriculum standards have only been apparent since 2003 [76].

In the category of *Historical Topics*, the *Gaokao* tended to show the history of mathematical problems, formulas, and theorems, which was consistent with its characteristic as an examination question. In terms of the material selection about mathematical history, the materials for the *Gaokao* items largely originated from ancient Chinese mathematical classics or mathematical achievements made by famous people, and such findings have also been confirmed in other analyses with regard to *Gaokao* items, especially as many scholars have found a high frequency of the ancient Chinese mathematical work *Nine Chapters on the Mathematical Art* (e.g., [20,21]), which may be related to the Chinese government's emphasis on showcasing the country's long history and culture in education and fostering students' national confidence [77]. Researchers from NEEA have also proposed in their papers to increase the content of the traditional Chinese culture of ancient times in *Gaokao* test items [19].

In terms of the category of *Interdisciplinary Connections*, the *Gaokao* items showed more connections between mathematics and technology, especially with physics and biological science, and fewer connections between mathematics and arts. This is also related to the orientation in the Chinese curriculum standards. Since the 1978 syllabus, the MOE of PRC has required students to master the mathematical knowledge necessary for learning modern technology [56], while the connections between mathematics and arts have only been reflected in the curriculum standards probably in the 21st century [76]. This feature was also rather similar to other well-known international tests, such as the famous PISA [59], which has a specific science context that reflects the connections between mathematics and science, but has no context reflecting the connections between mathematics and arts. However, some researchers have found that textbooks in other countries, such as the United States and France, contain rich artistic content, reflecting the emphasis on the role of mathematics in arts subjects (e.g., painting) [38], the exercise section of which may provide some insights for Chinese test designers.

In terms of the category of *Social Roles* that reflect the connections between mathematics and real life, mathematics test items of the *Gaokao* were more about *societal* and *occupational* contexts and less about *personal* contexts, which was also consistent with the findings of studies about Chinese high school textbooks [38]. However, when analyzing test items or exercises in textbooks from different countries with reference to the context of mathematics items from PISA, other researchers have found that the most common realistic context in the Mathematics Admissions Test in the British Mathematical Talent Recruitment Examination in the United Kingdom is about students' personal lives [78], and exercises in French textbooks also emphasize cultural contexts related to students' daily lives [42]. This difference may be due to the fact that traditional Chinese education has always emphasized the cultivation of students' patriotism and sense of responsibility [48], so its mathematics education also expects students to employ mathematical knowledge to solve problems in social contexts. Furthermore, such a virtue-orientation model of learning behavior is different from the mind-orientated model in western countries [79] (p. 112), It may also explain some characteristics of mathematics education in East Asia. For example, some researchers have suggested that Chinese students perform well academically but have low levels of self-confidence, which may also be related to their weak "self-consciousness", who tend to view learning as a responsibility and obligation to their families and country [80]. In addition, other scholars who are studying the TIMSS-R have found that students in East Asian countries have a low self-concept in mathematics compared to those in Western countries, which may be related to their cultural traditions that emphasizes the virtue of humility, i.e., students should not overrate themselves [81] (p. 40).

As for the cultural value of mathematics in *Aesthetics & Recreation*, *Gaokao* items more often reflected the beauty of mathematical practice itself, which was similar to the conclusions obtained from some previous studies (e.g., [20]) and in line with mathematicians' understanding that mathematical structures have an inherent sense of beauty (e.g., [71]). However, recreational mathematical items were low in frequency in comparison with those in other countries, such as Russia, a traditional mathematics power, where many interesting items are couched in the setting of a game rich in recreational mathematics [82] (p. 249), which is quite different from Chinese test items. Some studies have shown that recreational mathematics can promote students' mathematical skills, learning achievement, and learning motivation [83,84]. In the future, test designers in China could try to design more test items with interesting contexts.

In terms of time variation, the study found that test items reflecting mathematical culture were low in overall quantity, with an annual mean of only 2.51 items. This was similar to the findings of other Chinese scholars who analyzed the mathematics tests of Gaokao in ten years of Chinese provincial and national papers, and the results showed that the annual mean number of items with mathematical culture was two to three items [20,21]. In comparison with test items from other countries, some researchers have also found that *Gaokao* items have fewer items with cultural contexts (including personal, societal, and scientific contexts) [85], which attests to the findings of this study.

This study also found that the number of items with mathematical culture fluctuated, having peaked in 1979, 1999, and 2019 for a short period. The exploration of the educational context behind these changes may all be related to the educational reform of that year. These three peaking periods (1978, 1996, and 2017) all came when curriculum reform was underway, where the corresponding syllabus (curriculum standards) requiring mathematics education needed to integrate mathematical culture or apply mathematics in real life [45,56]. Mathematical culture, therefore, was also more frequently found in the test items in these years.

The study also found that there were distinctive characteristics of the times in the materials reflecting social context and technology. This also reflected the fact that mathematics test items themselves, as artifacts, can be influenced by changes in society.

6. Conclusions

The conclusions are shown as follows.

First, the mathematical culture in the national mathematics tests of the *Gaokao* over 44 years was diverse in content categories but uneven in distribution. They focused more on demonstrating the *Social Roles* of mathematics and less on the historical development of mathematical knowledge.

Second, the quantity of items with mathematical culture was low on average over the 44-year period and fluctuated over time, and specific contents about social contexts and technology having also changed over time, with distinct characteristics of the times.

Finally, the conceptual framework for classifying mathematical culture in this study can be used as a reference for other studies on mathematical culture in test items and may provide some food for thought or insight for test designers. As mentioned, mathematical test items from different eras and cultures are themselves cultural artifacts, which can not only be influenced by culture but can also become a part of and even influence it. Therefore, it is necessary to continue this research. However, it is worth noting that the classification framework of mathematical culture in this study mainly took into account the specific cultural contexts such as Chinese curriculum standards and test items, thus it is not the most complete one. Mathematical culture could also be categorized and analyzed from different perspectives, which is subject to further refinement in this study and by researchers in the future. Future researchers can carry out analyses of mathematical culture in national test items from different cultural contexts and draw more diverse research conclusions.

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Appendix A

Table A1 lists the national papers of mathematics tests of the *Gaokao* from 1978 to 2021 and the specific information of the selected papers.

Table A1. Specific information on data sources.

Year	Type of Paper	Quantity of Papers
1978	National Paper (does not distinguish arts and science)	1
1979–2003	National Paper (arts) National Paper (science)	2 × 25 = 50
2004	Arts: National Paper I, National Paper II, National Paper III, National Paper IV Science: National Paper I, National Paper II, National Paper III, National Paper IV	8
2005	Arts: National Paper I, National Paper II, National Paper III Science: National Paper I, National Paper II, National Paper III	6
2006–2009	Arts: National Paper I, National Paper II Science: National Paper I, National Paper II	4 × 4 = 16
2010	Arts: National Paper I, National Paper II, National Paper Under New Curriculum Science: National Paper I, National Paper II, National Paper Under New Curriculum	6
2011–2012	Arts: National Paper (Syllabus Edition), National Paper Under New Curriculum Science: National Paper (Syllabus Edition), National Paper Under New Curriculum	2 × 4 = 8
2013–2014	Arts: National Paper (Syllabus Edition), National Paper I Under New Curriculum, National Paper II Under New Curriculum Science: National Paper (Syllabus Edition), National Paper I Under New Curriculum, National Paper II Under New Curriculum	6 × 2 = 12
2015	Arts: National Paper I, National Paper II Science: National Paper I, National Paper II	4
2016–2020	Arts: National Paper I, National Paper II, National Paper III Science: National Paper I, National Paper II, National Paper III	6 × 5 = 30
2021	Arts: National Paper I, National Paper II Science: National Paper I, National Paper II National Paper I of the New *Gaokao*, National Paper II of the New *Gaokao* (does not distinguish arts and science)	6

Appendix B

Table A2 lists the number of test items with mathematical culture in each year.

Table A2. Number of test items with mathematical culture.

Year	Number of Test Papers	Number of Test Items with Mathematical Culture	Average Quantity	Year	Number of Test Papers	Number of Test Items with Mathematical Culture	Average Quantity
1978	1	1	1.00	2000	2	8	4.00
1979	2	9	4.50	2001	2	8	4.00
1980	2	5	2.50	2002	2	5	2.50
1981	2	7	3.50	2003	2	8	4.00
1982	2	3	1.50	2004	8	15	1.88
1983	2	4	2.00	2005	6	16	2.67
1984	2	3	1.50	2006	4	11	2.75
1985	2	1	0.50	2007	4	9	2.25
1986	2	2	1.00	2008	4	12	3.00
1987	2	1	0.50	2009	4	8	2.00
1988	2	3	1.50	2010	6	17	2.83
1989	2	1	0.50	2011	4	13	3.25
1990	2	2	1.00	2012	4	10	2.50
1991	2	2	1.00	2013	6	12	2.00
1992	2	0	0.00	2014	6	17	2.83
1993	2	4	2.00	2015	4	15	3.75
1994	2	6	3.00	2016	6	24	4.00
1995	2	4	2.00	2017	6	22	3.67
1996	2	3	1.50	2018	6	19	3.17
1997	2	3	1.50	2019	6	30	5.00
1998	2	8	4.00	2020	6	22	3.67
1999	2	10	5.00	2021	6	20	3.33

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