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Economic Performance and Stock Market Integration in BRICS and G7 Countries: An Application with Quantile Panel Data and Random Coefficients Modeling

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Abstract: The interest in studies aimed at understanding the integration of the stock market with the economic performance of countries has been growing in recent years, perhaps driven by the recent economic crises faced by the world. Although several studies on the topic have been carried out, the results are still far from a meaningful conclusion. In this sense, this paper considered the dual objective of investigating whether there is significant variance in the economic performance of developed and emerging markets' countries and whether the global risk factors are statistically significant in explaining the variations in their future economic performance over time. From a sample of (i) gross domestic products from BRICS and G7 countries (total of twelve countries), and (ii) returns of the risk factors of developed and emerging stock markets for the period 1993 to 2019, we applied longitudinal regression modeling for five distinct percentiles, and random coefficients modeling (RCM) with repeated measures. We found that risk factors explain the future economic performance, there is significant variation in economic performance over time among countries, and the temporal variation in the random effects of intercepts can be explained by RCM. The results of this study confirm that stock markets follow an integration process and that moderately integrated markets may have the same risk factors. Furthermore, considering that risk factors are related to future GDP growth, they act as proxies for unidentified state variables.

Keywords: GDP growth; BRICS and G7; five-factor asset pricing model; panel data; quantile models; random coefficient models

MSC: 60-11

1. Introduction

Emerging markets have long been a challenge to finance [1], and there has been extensive debate about the relationship between the real economy and stock market performance, especially in the context of emerging markets [2]. As a result of recent global economic crises such as the COVID-19 pandemic, research interest in the field of market integration has increased considerably in the last decade [3]. According to [4], financial integration intensifies during sovereign debt crises, being mainly driven by macroeconomic variables, market capitalization, political uncertainty and technological developments. In view of the process of integrating stock markets at the regional and global levels, an increasing number of studies on asset pricing have been carried out in developed markets [5–8] and emerging markets [9–12]. Many studies use multifactor models such as those of [13,14], in which diversified portfolios of stocks are formed based on characteristics such as size (SMB, small minus big), book-to-market index (HML, high minus low), operating profit (RMW, robust minus weak) and investment (CMA, conservative minus aggressive). These portfolios produce risk and return different, from market beta risk (MKT), and reflect unidentified state variables consistent with Merton's intertemporal capital asset pricing model (ICAPM) [15], such as [13,14,16]. In general, empirical evidence indicates that multi-factor models present positive risk premiums and better explain the variation of expected returns than the single-factor model, capital asset pricing model (CAPM) and there is segmentation between developed and emerging stock markets.

Fama [17] and Aylward and Glen [18] verified a positive and statistically significant relationship between returns on stock market portfolios and the future economic growth of the United States and of twenty-three countries with developed and emerging stock markets, respectively. Liew and Vassalou [19], motivated by [17,18], using data from ten developed stock market countries (Australia, Canada, France, Germany, Italy, Japan, Netherlands, Switzerland, United Kingdom and United States), demonstrated returns on the SMB and HML risk factors considered in [13] contain information about future GDP growth. SMB and HML are related to changes in the set of investment opportunities, and act as substitutes for two sources of the real economy's risk, consistent with the ICAPM state variables.

Although empirical evidence suggests that variations in expected returns reflect business cycle exposures [19–22], and that a considerable part of the risk and return of domestic stock markets can be attributed to the co-movement and interdependence of regional and global stock markets [6,7,23,24], given the integration process of stock markets and the real economy, few studies have analyzed the relationship between future domestic economic growth and risk factors of a global nature from a temporal perspective that allows us to investigate whether there is variability in economic performance between different countries and whether risk factors of a global nature help to explain the variations in economic performance between different countries.

The size risk factor (SMB) of [14] represents the average of three elementary risk factors formed from diversified portfolios of stocks grouped in (i) size and book-to-market (B/M) ($SMB_{B/M}$, difference between returns of diversified portfolios of stocks of small and large companies with high and low B/M ratio), (ii) size and operating income (SMB_{OP}, difference between returns of diversified portfolios of stocks of small and large companies with high and low operating income) and (iii) size and investment (SMB_{INV}, difference between the returns of diversified portfolios of stocks of small and large companies with low and high investment). The decomposition of the SMB risk factor into three elementary risk factors makes it possible to explore the three dimensions of systematic risk of size effect, through the magnitude and sign of the respective risk premiums and offers a new understanding of the behavior of each return parcel in the average SMB premium, and, in our study, its relationship to future economic growth.

Although many studies have been carried out in different regions and countries that analyzed the influence of bank financing or the stock market on economic growth, the findings are still far from meaningful conclusions [25]. Our study seeks to fill this gap in the literature by proposing a new model in terms of variables, aggregating the elementary risk factors of SMB ($SMB_{B/M}$, SMB_{OP} and SMB_{INV}) in explaining the future economic growth in addition to methodological differences to consider (i) the asymmetric distribution of the GDP growth rate and (ii) the heterogeneity of GDP growth rates between developed and emerging countries.

The present study has the dual objective of investigating whether the global risk factors considered in [14] are statistically significant to explain the variations in countries' future economic performance over time, measured by the growth rate of the gross domestic product (GDP), and whether there is significant variance in the economic performance of developed and emerging markets' countries. To achieve these objectives, we applied panel data and random-coefficient models to a sample of developed (G7: Germany, Canada, United States, France, Italy, Japan, United Kingdom) and emerging (BRICS: South Africa, Brazil, China, India, and Russia) countries, considering their GDP data and the return of global risk factors in the period from 1993 to 2019.

The first objective is to analyze whether the global risk factors of developed and emerging stock markets considered in [14], including the decomposed SMB factor in its elementary risk factors ($SMB_{B/M}$, SMB_{OP} and SMB_{INV}), capture information that helps to explain the variation in future economic performance, represented by GDP of 12 analyzed countries (G7 and BRICS). We estimate longitudinal regression models for panel data, using the quantile regression technique for the percentiles 0.05; 0.25; 0.50; 0.75; and 0.95 to accomplish this objective.

The second objective is to analyze whether there is significant variance in the economic performance of BRICS and G7 countries over time, and across countries over time, and whether the global, developed and emerging stock market risk factors from [14] help to explain variation in future economic performance over time. We estimate a two-level model with repeated measures to accomplish this objective.

The present study is structured in six sections. Section 2 presents the literature review; Section 3 presents the methodologies and the hypothesis; Section 4 presents the description of the data, followed by the analysis and discussion of the results in Section 5, and finally, the conclusion is drawn in Section 6.

2. Literature Review

Research on financial development and economic growth has been growing comprehensively for a long time in the theoretical and empirical literature [25]. Fama and French [5–7] observed that moderately integrated stock markets may have the same risk factors that reflect important dimensions of systematic risk in returns not priced by market beta risk, which condition future investment opportunities.

Positive and statistically significant relationships between the return of the stocks of the market portfolio and the future economic growth of the United States, and of twenty-three countries of developed and emerging stock markets were verified by [17,18], respectively. Motivated by them, Liew and Vassalou [19] were pioneers in demonstrating that the returns of the SMB and HML risk factors considered in [13] are related to changes in the set of investment opportunities and act as substitutes for two sources of the real economy's risk, consistent with the ICAPM state variables. The authors estimated simple and multiple regression models, with data on the returns of domestic risk factors and the GDP growth rate from 1978 to 1996 for ten developed countries (Germany, Australia, Canada, United States, France, Netherlands, Italy, Japan, United Kingdom and Switzerland). They found that the risk factors SMB and HML independently present positive and statistically significant relationships with the future GDP growth rate. Additionally, each risk factor, SMB and HML, in the presence of the MKT risk factor, maintains the positive relationship and magnitude of the regression coefficient.

Several studies followed [19], such as:

(1) Neves and Leal [26] verified a positive relationship between the SMB and HML risk factors and the future economic growth of Brazil for the period from 1986 to 2001.
(2) Font-Belaire and Grau-Grau [27] provided evidence on the positive and statistically significant relationship between future GDP growth and the SMB risk factor of the Spanish market during the period from 1995 to 2000.
(3) Hanhardt and Ansotegui [28] used data from 1990 to 2008 and found that the SMB risk factor has an explanatory capacity for the future economic growth of the Euro Zone.
(4) Fajardo and Fialho [29], using Brazilian market data from 1995 to 2008, observed that the risk factor SMB and HML are positively related to economic growth and negatively related to inflation.

(5) Liu and Di Iorio [30] provided evidence of the explanatory power of SMB and HML risk factors in predicting future Australian economic growth for the period 1993 to 2010.
(6) Boamah [31] confirmed the ability of [13] in predicting the economic growth of South Africa for the period 1996 to 2016.
(7) Ali, He and Jiang [32] reported that the MKT and SMB risk factors help to predict the future economic growth of Pakistan in the period 2002 to 2016.

Although the empirical evidence is supports [13] in relation to the CAPM, in capturing the expected return, Fama and French [14] extended the model from three to five risk factors that outperform the model of three factors in describing average returns. For this purpose, the authors added the (i) operating profit factor (RMW) that results from the difference between the returns on diversified portfolios of stocks of companies with high and low operating profits and (ii) the investment factor (CMA) that results from the difference between the returns of diversified portfolios of stocks of companies with low and high investment. In this context, Lalwani and Chakraborty [33], using data from the period 1992 to 2017, analyzed the ability of [14] to explain the future economic growth of five developed countries (Australia, United States, Canada, Japan and the United Kingdom) and four emerging countries (China, South Korea, India and Taiwan). The authors observed that in the presence of MKT, the additional risk factors (SMB, HML, RMW and CMA) remained positive and statistically significant for Canada, the United Kingdom, South Korea, and India, respectively.

In view of the process of integrating the stock markets, Ferreira and Gama [34], using data from the period from 1991 to 2018, confirmed the evidence that the risk factors of a regional nature considered in [14] help to predict the future economic growth of six developed markets namely, Germany, Canada, the United States, France, Hong Kong and Singapore. Ferreira et al. [35] reported that global risk factors capture information that helps explain the future economic performance of each emerging BRICS country (South Africa, Brazil, China, India and Russia).

Economic Performance and Stock Market's Integration

Regarding the integration between stock market and economic performance, the academic literature has several studies. Bekaert and Harvey [1] explored the financial effects of market integration as well as the impact on the real economy and presented results on political risk and liberalization, the volatility of capital flows and the performance of investments in emerging markets.

Tripathi and Seth [2] examined causal relationships between stock market performance and macroeconomic variables in India. The authors used various statistical approaches to data analysis and found that there is a significant correlation between stock market variables and macroeconomic factors, with the exception of the exchange rate.

Sehgal et al. [36] studied the dynamic nature of stock market integration in some Asian countries. The authors used the Copula GARCH models to study the intertemporal process of stock market integration and found that fiscal position, stock market performance, external position, governance and trade linkages appear to be the fundamental drivers of the integration of the stock market in that region.

Saji [3] analyzed the dynamics of price integration among Asian financial markets during the post-2008 financial crisis period. The authors analyzed monthly stock index data from five Asian economies from April 2009 to March 2020. The results did not yield any conclusive evidence of long-term relationships between stock markets. According to the authors, the asymmetric pattern of price behavior of Asian markets has important implications for the price efficiency of domestic markets and offers arbitrage potential for global investors to optimize returns through market diversification in a long-term perspective.

Olubiyi [37] assessed the relationship between economic integration and stock market performance in Nigeria alongside its main trading partners. The author found a negative relationship between US stock price and trade integration with Nigeria. The study made it possible to verify sectors that positively drive the Nigerian stock market, which could be prioritized by the country's trade policies.

Chukwuma et al. [38] carried out a study to demonstrate how forensic accounting can be used to predict future financial performance. The authors used OLS data analysis, unit root test and cointegration analysis. The results obtained revealed that forensic accounting indicators are statistically significant and have a significant positive impact on the growth of financial performance.

Jamil et al. [39] examined the impact of corporate social responsibility, leverage on assets and company age on the performance of organizations. The study considered the OLS model to estimate the impact and the use of the robustness factor so that the result was reliable. The results showed that sustainable corporate social responsibility is the main factor that enhances the company's performance.

Abdelkafi et al. [40] investigated the dynamic relationship between pandemics and government actions, such as government response rates and economic support packages. The authors used a panel dataset to analyze the effect of government actions on stock market returns. The empirical results showed the harmful effect of the COVID-19 pandemic on stock prices, hence the risk-adverse behavior of investors.

According to [25], several studies have been carried out in different regions and countries, analyzing the influence of banking or stock market finance on economic growth. However, the results are still far from a meaningful conclusion. Therefore, our study proposes a deeper and more detailed analysis of the topic. The methodology proposed in this article presents a new model in terms of variables, adding the elementary risk factors of SMB ($SMB_{B/M}$, SMB_{OP} and SMB_{INV}) in explaining the future economic growth; besides methodological differences to consider (i) the asymmetric distribution of the GDP growth rate and (ii) the heterogeneity of GDP growth rates between developed and emerging countries.

3. Methodology

Given the process of integration of stock markets, this study analyzes, based on a longitudinal quantile regression model and a two-level model with repeated measures, whether the risk factors MKT, SMB, HML, RMW and CMA considered in [14], as well as the three elementary risk factors ($SMBB_{/M}$, SMBOP and SMB_{INV}) of the SMB risk, capture information that helps to explain the differences in GDP growth rates for a total of twelve countries composed of G7 developed countries (Germany, Canada, United States, France, Italy, Japan and United Kingdom) and BRICS emerging countries (South Africa, Brazil, China, India and Russia) and whether these differences occur over time.

Thus, the longitudinal regression models for long panel data are estimated using the quantile regression technique for the percentiles 0.05; 0.25; 0.50; 0.75; and 0.95. The percentiles 0.05 and 0.25 represent the lowest growth rates, the percentile 0.50 denotes median growth rate; and the percentiles 0.75 and 0.95 represent the highest growth rates. For the purpose of comparing the magnitudes and signs of the coefficients, regression models are estimated by Pooled Ordinary Least Squares (POLS). Two-level model with repeated measures is estimated to verify whether there is variance in the economic performance over time, and between countries over time, explained by the risk factors of [14] model.

We chose a sample that includes countries from different continents and sub-regions that, in the set of all developed and emerging stock market countries, according to the Morgan Stanley Capital International (MSCI) classification, represent the countries with high economic development (G7) and high potential for economic development (BRICS). We chose methodologies that considers: (i) that the statistical distribution of the dependent variable—GDP growth rate presents an asymmetric distribution, in addition, the error terms of the regression models do not show adherence to normality, which allows exploring the different behaviors for the different percentiles of the conditional distribution, not observable in the regression models to the mean estimated by the ordinary least squares (OLS) method, and (ii) the technological development focused on computer science and analysis software of data offers new approaches to panel data that allows estimating not only parameters by fixed effects, but also investigating the interaction between individual explanatory variables and the random effects of intercept and slope [41] whose models estimate parameters that present the best fit between actual and predicted values.

Hypothesis

Following the proposed objectives, using panel data, this study:

(1) First explores the relationship between the global risk factors MKT, SMB, HML, RMW and CMA, as well as the three elementary risk factors ($SMB_{B/M}$, SMB_{OP} and SMB_{INV}) of the SMB, considered in the five-factor model by Fama and French (2015) and the future economic performance of the BRICS and G7 countries.

(2) In the second moment, this study analyses (i) if there are significant differences, over the years, in the economic performance of the G7 and BRICS countries, as well as (ii) if these differences can be explained by the global risk factors of the model developed by [14].

The studies that analyzed the relationship between future economic growth and the risk factors of the models [13,14], such as [28,34,35], attested that such risk factors, of regional and global nature, individually or in association with each other, help to predict future domestic economic growth. The risk factors of a global nature of the developed and emerging stock markets will be used to test the following investigation hypotheses:

Hypothesis 1. *The global risk factors of [14] asset pricing model, individually or in association with each other, explain the variability in future economic growth in BRICS and G7 countries.*

Hypothesis 2. *There is significant variability in the economic growth rates of BRICS and G7 countries over time.*

Hypothesis 3. *There is significant variability in the economic growth rates of BRICS and G7 countries over time across countries.*

Hypothesis 4. *The economic growth rates of BRICS and G7 countries follow a linear trend over time, and there are differences in this trend between countries.*

Hypothesis 5. *The global risk factors of [14] asset pricing model help to explain the variability in the future economic growth rate over time.*

Hypothesis 6. *Elementary size-effect risk factors associated with market beta risk help to explain the variability in the rate of future economic growth over time.*

In order to answer the first objective, simple and multiple quantile regression models are estimated [35]. The analysis was carried out in three stages. The first stage consists of the estimation of eight simple regression models, having each of the risk factors as an explanatory variable, to assess whether the return in period $t - 1$ of each of the risk factors individually explains the variability between the percentiles of the annual economic performance, measured by the logarithmic growth rate of GDP for the twelve countries under analysis. Equation (1) represents this first stage.

$$GDP_{i,t} = \alpha_i + \beta_{ij} Factor_{ij,t-1} + \varepsilon_{i,t}, \quad j = 1, 2, \ldots, 8. \qquad (1)$$

where, $GDP_{i,t}$ denotes the growth rate of the Gross Domestic Product of each observation in the sample for period t, calculated logarithmically; Factor represents the returns of each of the five (MKT, SMB, HML, RMW and CMA) and three elementary risk factors ($SMB_{B/M}$, SMB_{OP} and SMB_{INV}) in the previous period ($t - 1$) of each observation; and $\varepsilon_{i,t}$ represents the error terms.

The second stage consists of estimating three multiple regression models, with two explanatory variables represented by the Equation (2), which includes the MKT risk factor and each elementary risk factor of the size effect (SMB).

$$\mathrm{GDP}_{\mathrm{i,t}} = \alpha_{\mathrm{i}} + \beta_1\mathrm{MKT}_{\mathrm{i,t-1}} + \beta_2\mathrm{Factor}_{\mathrm{i,t-1}} + \varepsilon_{\mathrm{i,t}} \quad (2)$$

$\mathrm{Factor}_{i,t-1} \in \{\mathrm{SMB}_{\mathrm{B/M},i,t-1}, \mathrm{SMB}_{\mathrm{OP},i,t-1}, \mathrm{SMB}_{\mathrm{INV},i,t-1}\}$.

where, $\mathrm{MKT}_{\mathrm{it-1}}$ represents the market returns in the previous period (t − 1) of each observation; and Factor represents the global market returns of each of the three elementary risk factors, $\mathrm{SMBB}_{/\mathrm{M}}$, $\mathrm{SMB}_{\mathrm{OP}}$ and $\mathrm{SMB}_{\mathrm{INV}}$ in the previous period (t − 1) of each observation.

The third stage consists of estimating a multiple regression model represented by the Equation (3), which includes the five risk factors of [14] model.

$$\mathrm{GDP}_{\mathrm{i,t}} = \alpha_{\mathrm{i}} + \beta_1\mathrm{MKT}_{\mathrm{i,t-1}} + \beta_2\mathrm{SMB}_{\mathrm{i,t-1}} + \beta_3\mathrm{HML}_{\mathrm{i,t-1}} + \beta_4\mathrm{RMW}_{\mathrm{i,t-1}} + \beta_5\mathrm{CMA}_{\mathrm{i,t-1}} + \varepsilon_{\mathrm{i,t}} \quad (3)$$

The estimates of the regression models by OLS are made using the statistical software Gretl version 2021d. The estimates of the simple and multiple quantile regression models offer results that allow us to reject or not the investigation hypothesis (H1), thus concluding the first objective.

The second objective is to verify whether there is variability in the economic performance of the countries under analysis over time, and between countries over time, explained by risk factors. Thus, random coefficients models are estimated for two-level data with repeated measures, in which the nesting of the data will be characterized by the presence of repeated measures, that is, the existence of temporal evolution in the behavior of GDP growth rates, following the procedures by [41–43].

Random coefficients models represent a generalization of regression methods, which allow estimating the parameters of the fixed effects component (intercept and slopes) and, simultaneously, estimating parameters of random effects of intercepts and slopes of different subgroups of the sample, given certain individual and group characteristics [41–43].

In this study, a two-level model with repeated measures is applied, where the same observation is evaluated in more than one period. The two levels of analysis are formulated in two sub-models that represent, respectively, individual variability in the economic performance of countries over time (level 1) and variability in economic performance (represented by the GDP growth rate) between countries (level 2).

Based on [41–43], the models to be estimated follow the step-up strategy procedure, which consists, at first, of analyzing the variance decomposition from the definition of a null model with repeated measures (to access the existence of temporal evolution of the distribution of the dependent variable) which is characterized by the absence of explanatory variables and presents estimates of the parameters of fixed and random effects, of which the variance component between the two levels (variance in time and between countries) provides an intraclass correlation index that measures the proportion of total variance that is due to levels 1 and 2, and serves as a comparison for the estimates of conditional models (models with explanatory variables).

In this sequence, models with random intercepts and a model with intercepts and random slopes are estimated. The comparison of the performance of the estimations is based on the restricted likelihood ratio test—Log restricted-likelihood, obtained by the difference of the logarithms of the two restricted likelihood functions. Finally, from the identification of the random character of the error terms (intercept or intercept and slope) the complete model is formulated with the inclusion of explanatory variables of level 2. The final model must be estimated according to the statistical significance of the explanatory variables that result of the complete model. For this study, given the absence of level 2 data, the analysis of the complete model focuses only on the interaction between level 1 risk factors and level 2 random effects, to capture any contextual heterogeneities. Thus, in order to obtain the best estimator, random coefficients models will be estimated without the explanatory variables in the fixed effects component, however, with random slopes precisely in the temporal evolution.

Thus, the null model to be estimated is expressed in the Equation (6).

Level 1 (Repeated Measure) of the null model is expressed in Equation (4):

$$GDP_{t,i} = \beta_{0i} + r_{t,i}, \qquad r_{t,i} \sim N(0,\ \sigma_i^2) \tag{4}$$

where, t = 1, 2, ... , T_i (years) and i = 1, 2, ... , n (countries); β_{0i} denotes the expected (average) GDP growth rate of country i in year 1; and σ^2 is the variance "within" the country.

Level 2 (Country) of the null model is expressed in Equation (5):

$$\beta_{0i} = \gamma_{00} + u_{0i} \qquad u_{0i} \sim N(0,\ \tau_{0i}) \tag{5}$$

where, γ_{00} is the general average of GDP growth rates; τ_{0i} is the variance between expected GDP growth rates of each country.

Thus, the null model (combining Equations (4) and (5)) is expressed in Equation (6):

$$GDP_{t,i} = \gamma_{00} + u_{0i} + r_{t,i} \tag{6}$$

Given the existence of two proportions of variance (σ^2 and τ_{00}), the level 2 intraclass correlation index (ρ), which measures the relationship between the idiosyncratic and group error terms, is calculated according to Equation (7).

$$\rho = \frac{\tau_{00}}{\tau_{00} + \sigma^2} \tag{7}$$

The intraclass correlation coefficient (ρ) varies between 0 and 1. A null value means that there was no variance of individuals between the level 2 groups (country), so estimates from random coefficients models are not appropriate; and a positive value indicates the presence of at least one statistically significant error term of level 2, therefore the estimations of regression parameters by OLS are not adequate [41,43], and a random coefficients model should be adopted. To this end, the likelihood ratio test (LR test) is analyzed in order to verify whether the error terms of the variance components of the random effects of intercepts (τ_{00}) are statistically different from zero.

The null model allows that hypotheses 2 and 3 to be tested. If the investigation hypotheses (H2) and (H3) are statistically supported, for the verification of the hypothesis (H4), two random coefficients models are estimated that include a trend component, variation over time at level 1.

The first model, represented by Equation (10) considers only random intercept effects.

Level 1 (Repeated Measure) is expressed in Equation (8):

$$GDP_{t,i} = \beta_{0i} + \beta_{1i}YEAR_{t,i} + r_{t,i} \qquad r_{t,i} \sim N(0,\ \sigma_i^2) \tag{8}$$

where, β_{1i} is the country i GDP growth rate; e $YEAR_{ti}$ is the explanatory variable of level 1, which represents the repeated measure of the temporal variable. A repeated measure is defined by the temporal evolution within the multilevel panel.

Level 2 (Country) is expressed in Equation (9):

$$\beta_{0i} = \gamma_{00} + u_{0i} \qquad \beta_{1i} = \gamma_{10} \qquad u_{0i} \sim N(0,\ \tau_{0i}) \tag{9}$$

where, γ_{10} is the overall average of expected GDP growth rates.

Thus, the random intercept model (combining Equations (8) and (9)) is expressed in Equation (10):

$$GDP_{t,i} = \gamma_{00} + \gamma_{10i}YEAR_{t,i} + u_{0i} + r_{t,i} \tag{10}$$

The second model, represented by the Equation (13), includes the random effects of the slopes, therefore, considering the random effects of intercepts and slopes.

Level 1 (Repeated Measure) is expressed in Equation (11):

$$GDP_{t,i} = \beta_{0i} + \beta_{1i}YEAR_{t,i} + r_{t,i}, \qquad r_{t,i} \sim N\left(0,\ \sigma_i^2\right) \tag{11}$$

Level 2 (Country) is expressed in Equation (12):

$$\beta_{0i} = \gamma_{00} + u_{0i} \qquad \beta_{1i} = \gamma_{10} + u_{1i} \qquad u_{0i} \sim N(0,\ \tau_{00}) \qquad u_{1i} \sim N(0,\ \tau_{11}) \tag{12}$$

where, τ_{11}, variance between expected growth rates across countries.

Thus, the random intercept and slope model (combining Equations (11) and (12)) is expressed in Equation (13a):

$$GDP_{t,i} = \gamma_{00} + \gamma_{10i}YEAR_{t,i} + u_{0i} + u_{1i}YEAR_{t,i} + r_{t,i} \tag{13a}$$

Given the existence of three proportions of variance (σ^2, τ_{00} and τ_{11}), the level 2 intraclass correlation index (ρ) is calculated according to Equation (13b).

$$\rho = \frac{\tau_{00} + \tau_{11}}{\tau_{00} + \tau_{11} + \sigma^2} \tag{13b}$$

The best fit between the estimates of the models with random intercepts and with random intercepts and slopes is given by the result of the restricted-likelihood ratio test (Log restricted-likelihood), obtained by the difference of the logarithms of the two restricted likelihood functions.

Once the randomness of the error terms has been identified, that is, a model with only random intercepts, or a model with random intercepts and slopes is selected, which supports the research hypothesis (H4), a complete model is proposed that includes the interaction between the risk factors and the random effects of intercepts and slopes at level 2, for the verification of the investigation hypothesis (H5), as represented in the Equations (14) and (15).

Level 1 (Repeated Measure) is expressed in Equation (14):

$$GDP_{t,i} = \beta_{0i} + \beta_{1i}YEAR_{t,i} + \beta_{2i}MKT_{t,i-1} + \beta_{3i}SMB_{t,i-1} + \beta_{4i}HML_{t,i-1} + \beta_{5i}RMW_{t,i-1} + \beta_{6i}CMA_{t,i-1} + r_{t,i} \tag{14}$$

Level 2 (Country) is expressed in Equation (15):

$$\begin{gathered}\beta_{0i} = \gamma_{00} + u_{0i} \quad \beta_{1i} = \gamma_{10} + u_{1i} \quad \beta_{2i} = \gamma_{20} + u_{2i} \quad \beta_{3i} = \gamma_{30} + u_{3i} \quad \beta_{4i} = \gamma_{40} + u_{4i} \\ \beta_{5i} = \gamma_{50} + u_{5i} \quad \beta_{6i} = \gamma_{60} + u_{6i}\end{gathered} \tag{15}$$

To answer the research hypothesis (H6), a complete model will be estimated, represented by the (i) Equations (16) and (19), (ii) Equations (17) and (19) and (iii) Equations (18) and (19), respectively, (i) considering the elementary risk factor B/M of the size risk factor: size B/M ($SMB_{B/M}$), (ii) considering the elementary risk factor operating profit of the size risk factor: size operating profit (SMB_{OP}) and (iii) considering the elementary risk factor investment of the size risk factor: size investment (SMB_{INV}). This formulation is discussed by [24,38]. Level 1 (Repeated Measure) are expressed in Equations (16)–(18):

Considering size B/M (Equation (16)):

$$\mathbf{GDP_{t,i} = \beta_{0i} + \beta_{1i}YEAR_{t,i} + \beta_{2i}MKT_{t,i-1} + \beta_{3i}SMB_{B/M,t,i-1} + r_{t,i}} \tag{16}$$

Considering size operating profit (Equation (17)):

$$GDP_{t,i} = \beta_{0i} + \beta_{1i}YEAR_{t,i} + \beta_{2i}MKT_{t,i-1} + \beta_{3i}SMB_{OP,t,i-1} + r_{t,i} \tag{17}$$

Considering size investment (Equation (18)):

$$GDP_{t,i} = \beta_{0i} + \beta_{1i}YEAR_{t,i} + \beta_{2i}MKT_{t,i-1} + \beta_{3i}SMB_{INV,t,i-1} + r_{t,i} \tag{18}$$

Level 2 (Country) is expressed in Equation (19):

$$\beta_{0i} = \gamma_{00} + u_{0i}\ \beta_{1i} = \gamma_{10} + u_{1i}\ \beta_{2i} = \gamma_{20} + u_{2i}\ \beta_{3i} = \gamma_{30} + u_{3i} \tag{19}$$

The composition of the final complete model will be done through the stepwise procedure, which consists of the step-by-step inclusion of each explanatory variable, in which a statistical significance of 10% is assumed [41]. This formulation is discussed by [24,38].

The fixed effects parameters and the error terms variances of the random effects component of the random coefficients model are estimated by the maximum likelihood method that produces the z test, to measure statistical significance of the fixed effect component and, Wald's z test, to measure the variance component of random effects. Model estimations are obtained using SPSS 22 and Stata 14 statistical software.

Table 1 presents the research hypotheses as well as the methods to be used for their validation.

Table 1. Research hypotheses and methodologies for their validation.

Research Hypotheses	Methodology
H1: The global risk factors of [14] asset pricing model, individually or in association with each other, explain the variability in future economic growth in BRICS and G7 countries.	Quantile regression modeling for longitudinal repeated measures data Longitudinal models of simple and multiple regression, with five explanatory variables, the risk factors of [14] model
H2: There is significant variability in the economic growth rates of BRICS and G7 countries over time.	Random coefficients modeling Null Model
H3: There is significant variability in the economic growth rates of BRICS and G7 countries over time across countries.	
H4: The economic growth rates of BRICS and G7 countries follow a linear trend over time, and there are differences in this trend between countries.	Random coefficients modeling Linear trend model with random intercept effects Linear trend model with random intercept and slope effects
H5: The global risk factors of [14] asset pricing model help to explain the variability in the future economic growth rate over time.	Random coefficients modeling Full model—Linear trend model with random effects and interaction of explanatory variables at level 1, risk factors, from [14] model and the random effects of slope at level 2 in order to capture differences in rates of economic growth of each country
H6: Elementary size-effect risk factors associated with market beta risk help to explain the variability in the rate of future economic growth over time.	

Table 2 summarizes the variables definitions and the expected relationships to the output variable according to the literature review.

The confirmation of a positive relationship between future economic growth and risk factors supports the arguments of [13,14,16] that risk factors obtained from company characteristics reflect proxies of variables of unidentified states that produce non-diversifiable risks in returns not estimated by the CAPM and represent innovations that affect the set of future investment opportunities, in the context of the ICAPM, and have three implications: (1) in the face of the risk-based explanation, there is a dual function of the asset pricing models, that is, they act as (1.i) instruments for analyzing the company's cost of capital and the investment portfolio management, and (1.ii) auxiliary indicator for forecasting economic growth, which, according to [19], in periods of expected economic growth the shares of small companies, with high B/M ratios and operating profit and with low investment index are better able to prosper than the stocks of large companies, with low B/M ratios and operating income and with high investment ratio, so when the market signals that the business cycle is unfavorable, investors seek to hold stock portfolios with good growth opportunities and a low debt ratio, (2) they act as substitutes for sources of risk in the real economy and (3) in view of the integrating process, moderately integrated markets [5,6,14], selectively offers complementary information to investors for decision making on the selection and formation of the investment portfolio.

Table 2. Variables definition and the expected relationship according to the literature review.

Variables	Variable Definition	Expected Signal	Reference
	Dependent Variable		
Economic performance (GDP)	GDP growth rate		[19,26–35]
	Independent Variables—Stock market risk factor		
Market beta risk factor (MKT)	Difference between the market portfolio rate of return and the risk-free rate	Positive	[19,26–35]
Size B/M ($SMB_{B/M}$) [13]	Difference between the returns of diversified portfolios of stocks of small and large companies with high and low B/M ratio	Positive	[19,26–34]
Size operating profit (SMB_{OP}) [14]	Difference between returns of diversified portfolios of stocks of small and large companies with high and low operating income	Positive	N.A.
Size investment (SMB_{INV}) [14]	Difference between the returns of diversified portfolios of stocks of small and large companies with low and high investment	Positive	N.A.
Size (SMB) [14]	Difference between the returns of diversified portfolios of stocks of small and large companies	Positive	[33–35]
B/M ratio (HML)	Difference between the returns of diversified portfolios of high and low B/M ratio stocks	Positive	[19,26–35]
Operating profitability (RMW)	Difference between returns on diversified portfolios of stocks of companies with high and low operating income	Positive	[33–35]
Investment (CMA)	Difference between returns on diversified portfolios of stocks of low and high investment companies	Positive	[33–35]

4. Data

4.1. Sample

For the present study, historical series of annual data valued in US dollars were collected for the period between January 1993 and December 2019 referring to the Gross Domestic Product (GDP), at constant prices and base year 2010, from a total of twelve countries among developed (Germany, Canada, United States, France, Italy, Japan and United Kingdom) and emerging (South Africa, Brazil, China, India and Russia), according to Morgan Stanley Capital International (MSCI) classification, extracted from the World Bank database; and global risk factor returns for developed and emerging stock markets, obtained from the Kenneth French database.

4.2. Univariate Analysis

Table 3 presents the descriptive statistics of the variance decomposition of the variables, dependent (GDP growth rate) and explanatory (risk factors of developed and emerging countries), for a data structure in a balanced longitudinal panel with 26 periods, year (from 1994 to 2019, GDP; 1993 to 2018, risk factors), and for each of the 12 countries under analysis, totaling 312 observations.

Given the panel data structure of the sample under analysis, overall (general), within (variation over time for a given individual) and between (variation between individuals) variances are reported.

Table 3. Descriptive statistics—Decomposition of variance.

Variable		Mean	Std Deviation	Minimum	Maximum	Observations	
GDP	Overall	0.02827	0.03243	−0.13433	0.13305	N.T =	312
	Between		0.02350	0.00734	0.08780	N =	12
	Within		0.02332	−0.12599	0.10365	T =	26
MKT	Overall	0.08472	0.26108	−0.55360	0.86370	N.T =	312
	Between		0.01842	0.06982	0.10560	N =	12
	Within		0.26048	−0.57447	0.84283	T =	26
SMB	Overall	0.02072	0.09402	−0.17240	0.44850	N.T =	312
	Between		0.00573	0.01608	0.02721	N =	12
	Within		0.09386	−0.16777	0.44201	T =	26
$SMB_{B/M}$	Overall	0.00741	0.09019	−0.17730	0.34440	N.T =	312
	Between		0.00077	0.00679	0.00828	N =	12
	Within		0.09019	−0.17668	0.34353	T =	26
SMB_{OP}	Overall	0.03244	0.09265	−0.16873	0.43013	N.T =	312
	Between		0.00734	0.02651	0.04075	N =	12
	Within		0.09238	−0.16280	0.42182	T =	26
SMB_{INV}	Overall	0.02230	0.10512	−0.18020	0.57103	N.T =	312
	Between		0.00907	0.01497	0.03258	N =	12
	Within		0.10476	−0.17286	0.56076	T =	26
HML	Overall	0.06319	0.14011	−0.30320	0.50870	N.T =	312
	Between		0.03347	0.03611	0.10111	N =	12
	Within		0.13638	−0.27612	0.47078	T =	26
RMW	Overall	0.02801	0.09050	−0.51730	0.12860	N.T =	312
	Between		0.01633	0.00951	0.04122	N =	12
	Within		0.08914	−0.49881	0.13839	T =	26
CMA	Overall	0.03167	0.09953	−0.26800	0.30940	N.T =	312
	Between		0.01144	0.02242	0.04463	N =	12
	Within		0.09892	−0.25874	0.29644	T =	26

Obs.: N.T: total observations; N: number of countries; T: number of periods.

For the dependent variable, GDP growth rate, the variation between countries (between effect) is slightly higher than the variation over time for a given country (within effect), which indicates the existence of variation in economic performance between the countries. With respect to the explanatory variables, the risk factors showed greater variation over time (within effect) than between individuals (between effect). The minimum and maximum values, respectively, indicate that the economic performance (GDP) of the between effect was in the range from 0.734% to 8.78%, and in relation to the performance of the within effect, the it was in the range from −12.599% to 10.365%.

5. Multivariate Analysis

5.1. Quantile Regression Analysis

This section analyzes the relationship between risk factors and GDP in twelve developed and emerging countries that are part of G7 (Germany, Canada, the United States, France, Italy, Japan and the United Kingdom) and BRICS (South Africa, Brazil, China, India and Russia), through the estimation of linear longitudinal regression models for long panel data, using the quantile regression technique for the percentiles 0.05; 0.25; 0.50; 0.75; and 0.95. For the purpose of comparing the magnitudes and signs of the parameters, the POLS regression model is also used. Thereafter, beginning at Table 4, the existence of variation in economic performance is analyzed, through the decomposition of the variance, based on random coefficients modeling.

Table 4. Simple and multiple regression estimates.

Model			Pooled OLS	Quantile Regression				
				0.05	**0.25**	**0.50**	**0.75**	**0.95**
Panel A: $GDP_{i,t} = \alpha + \beta Fator_{i,t-1} + \varepsilon_{i,t}$								
1	MKT	Coef	0.036 ***	0.079 ***	0.045 ***	0.038 ***	0.037 ***	0.019 **
		SE	0.006	0.011	0.003	0.004	0.011	0.008
2	SMB	Coef	0.020	0.124 *	0.042 **	0.036 **	0.015	0.089 **
		SE	0.019	0.067	0.016	0.016	0.024	0.039
3	HML	Coef	0.032	0.079 **	0.020 *	0.036 ***	0.045 ***	0.088 ***
		SE	0.013	0.035	0.010	0.010	0.016	0.018
4	RMW	Coef	−0.037	0.190 **	−0.035 **	−0.052 ***	−0.098 ***	−0.071
		SE	0.020	0.082	0.015	0.014	0.031	0.045
5	CMA	Coef	−0.019	−0.168 ***	−0.029 **	−0.016	0.004	0.035
		SE	0.018	0.028	0.014	0.014	0.021	0.049
6	$SMB_{B/M}$	Coef	0.013	0.122	0.033 *	0.030 **	−0.010	0.048 *
		SE	0.020	0.084	0.019	0.015	0.021	0.026
7	SMB_{OP}	Coef	0.027	0.133 **	0.044 ***	0.034 **	0.026	0.099 ***
		SE	0.019	0.064	0.016	0.017	0.023	0.021
8	SMB_{INV}	Coef	0.018	−0.095	0.034 **	0.033 **	0.018	0.074 ***
		SE	0.017	0.062	0.012	0.015	0.021	0.021
Panel B: $GDP_{i,t} = \alpha_i + \beta_1 MKT_{i,t-1} + \beta_2 Fator_{i,t-1} + \varepsilon_{i,t}$								
9	MKT	Coef	0.039 ***	0.081 ***	0.045 ***	0.040 ***	0.043 ***	0.017 *
		SE	0.007	0.011	0.004	0.005	0.011	0.009
	$SMB_{B/M}$	Coef	−0.025	−0.045	−0.010	−0.009	−0.029	0.014
		SE	0.020	0.033	0.010	0.015	0.032	0.025
10	MKT	Coef	0.037 ***	0.08 1 ***	0.045 ***	0.037 ***	0.036 ***	0.005
		SE	0.007	0.014	0.004	0.005	0.012	0.008
	SMB_{OP}	Coef	−0.010	−0.039	0.000	0.016	−0.012	0.089 ***
		SE	0.020	0.039	0.011	0.014	0.033	0.023
Model			Pooled OLS	Quantile regression				
				0.05	0.25	0.50	0.75	0.95
11	MKT	Coef	0.039 ***	0.083 ***	0.044 ***	0.038 ***	0.039 ***	0.009
		SE	0.007	0.011	0.004	0.006	0.011	0.018
	SMB_{INV}	Coef	−0.019	−0.033	−0.005	0.008	−0.013	0.056
		SE	0.018	0.027	0.009	0.014	0.026	0.044
Panel C: $GDP_{i,t} = \alpha_i + \beta_1 MKT_{i,t-1} + \beta_2 SMB_{i,t-1} + \beta_3 HML_{i,t-1} + \beta_4 RMW_{i,t-1} + \beta_5 CMA_{i,t-1} + \varepsilon_{i,t}$								
12	MKT	Coef	0.039 ***	0.089 ***	0.049 ***	0.046 ***	0.025 ***	0.007 ***
		SE	0.008	0.015	0.004	0.008	0.010	0.002
	SMB	Coef	−0.017	0.035	−0.016	−0.001	−0.025	0.107 ***
		SE	0.023	0.041	0.012	0.023	0.026	0.005
	HML	Coef	0.038 ***	0.019	0.026 ***	0.039 **	0.055 ***	0.050 ***
		SE	0.017	0.029	0.009	0.016	0.019	0.003
	RMW	Coef	0.037	0.235 ***	0.016	0.028	−0.044	0.020 ***
		SE	0.028	0.048	0.014	0.027	0.031	0.006
	CMA	Coef	−0.017	0.039	0.004	−0.025	−0.028	0.067 ***
		SE	0.023	0.040	0.012	0.022	0.025	0.005

Obs.: Coef: coefficient; SE, standard error; ***, **, *, $p < 1\%$, 5% and 10%.

Through Panel A of Table 4, it is observed that the estimates obtained by the simple quantile regression, represented by the Equation 1, indicate that all models presented a positive and statistically significant relationship, at least in one of the five percentiles under analysis, except model 5, estimated with the explanatory variable CMA. The magnitude of the coefficients varied between 1.9% (MKT—0.95 percentile) and 7.9% (MKT—0.05 percentile); 3.6% (SMB—0.50 percentile) and 12.4% (SMB—0.05 percentile); 2% (HML—0.25 percentile) and 8.8% (HML—0.95 percentile); −9.8% (RMW—0.75 percentile) and 19% (RMW—0.05 percentile);

3% (SMBB/M—0.50 percentile) and 4.8% ($SMB_{B/M}$—0.95 percentile); 3.4% (SMB_{OP}—0.50 percentile) and 13.3% (SMB_{OP}—0.05 percentile); and between 3.3% (SMB_{INV}—0.50th percentile) and 7.4% (SMB_{INV}—0.95 percentile), compared to the mean value of 3.6% (MKT), the only statistical significant coefficient (at the 5% level) of the POLS estimation.

Regarding the results of multiple quantile regression, with two explanatory variables composed of (i) each elemental risk factor of size effect associated with the (ii) beta market risk, represented by the Equation 2 (Panel B of Table 4), it can be seen that differently from the negative mean values of POLS estimation, in the presence of the MKT risk factor, the coefficients estimated by the risk factors $SMB_{B/M}$, SMB_{OP} and S SMB_{INV} present themselves positive at least in one percentile of the entire conditional distribution of economic performance. Within the five percentiles under analysis, SMB_{OP} presented a positive and statistically significant coefficient with a magnitude of 8.9% (SMB_{OP}—0.95 percentile). The risk factor SMB_{INV} showed positive and statistically significant coefficients between the 0.86 (6.83%) and 0.99 (9.85%) percentiles, as illustrated in Figure 1.

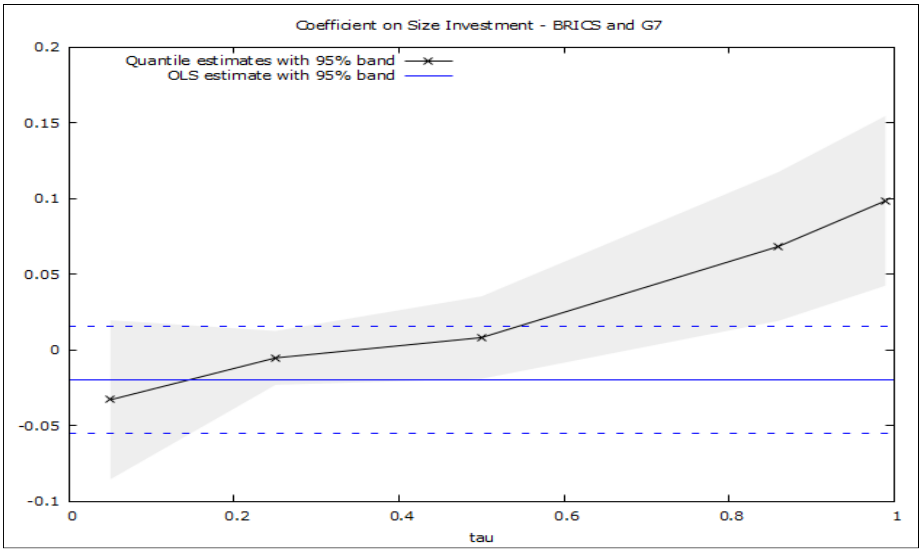

Figure 1. Performance of the SMB_{INV} risk factor on the percentiles and conditional average of the GDP growth rate of the BRICS and G7 countries.

Figure 1 illustrates the individual performance of the risk factor SMB_{INV}, on the conditional quantile distribution, as well as the estimation by OLS, of the GDP growth rate of BRICS and G7 countries. The vertical and horizontal lines show, respectively, the risk factor coefficients and the percentiles (tau) from 0 to 1. The solid blue line represents the estimated mean coefficient obtained through OLS, and the dotted blue lines the respective confidence intervals at 95%. The shaded area represents the confidence intervals of the parameters obtained through the estimation of the quantile regression models, with the black line being the average estimation of the parameters for each of the percentiles under analysis. Thus, through Figure 1, it can be seen that for the conditional performance to the risk factors MKT and SMB_{INV}, ceteris paribus, the positive coefficients were observed between the percentiles 0.43 and 0.58, 0.63 and 0.66, and between 0.84 and 0.99, however, statistically significant values were estimated between the 0.86 and 0.99 percentiles, as illustrated in Figure 1.

Regarding the multiple quantile regression model, with five explanatory variables being the global risk factors of developed and emerging countries considered in the asset pricing model of [14], represented by the Equation (3), it can be noted in Panel C of Table 4, with statistical significance at the 5% level, that risk factors help to explain the variation in the future economic performance of BRICS and G7 countries, at least in one percentile of the conditional distribution of GDP growth rates. Ceteris paribus, the MKT risk factor remained the central element of the explanation for the variability of economic growth in the five percentiles under analysis, with the magnitude of the coefficients varying between 2.5% (MKT—0.75 percentile) and 8.9% (MKT—0.05 percentile), compared to the mean value of 3.9% (MKT) of statistical significance, at the 5% level, obtained through the POLS estimation.

The SMB and CMA factors, respectively, showed positive and statistically significant coefficients, in one percentile, with magnitudes of 10.7% (SMB—0.95 percentile) and 6.7% (CMA—0.95 percentile), and the risk factors, respectively, RMW and HML, showed positive coefficients of statistical significance at two and four percentiles, respectively, ranging between 2% (RMW—0.95 percentile) and 23.7% (RMW—0.05 percentile), and between 2.6% (HML—0.25 percentile) and 5.5% (HML—0.50 percentile), compared to the mean value of 3.8% for HML, of statistical significance at the 5% level. As a consequence, the results presented here support the observations of [19] on the ability of risk factors to predict future economic growth, and in line with the study by [35] sheds light on the performance of global risk factors of developed and emerging equity markets in forecasting domestic economic performance, given the process of integration of stock markets. Based on the above, the research hypothesis (H1) is supported.

As noted by [41,44,45], the main utility of longitudinal data modeling is the fact that it allows the analysis of possible differences in the performance behaviors of individuals over time. However, without the effect of the panel structure on the data, the parameter estimators can be analyzed through OLS estimation, considering individual time series regression models. Thus, for the purpose of comparing the magnitudes and signs of the coefficients of the explanatory variables, Table A1 in Appendix A presents, for each country under analysis, the parameter estimates of six linear models of simple and multiple regression, with two explanatory variables being (i) each elemental risk factor of size effect associated with (ii) market beta risk, for time series data, using the quantile regression technique for the percentiles 0.05; 0.25; 0.50; 0.75; and 0.95.

For the purpose of comparing the magnitudes and signs of the parameters, the OLS regression model is also used. For the models estimated by OLS that showed autocorrelation and heteroscedasticity of residuals, the robust estimators of [46,47] were applied, which, although they do not correct the standard error, adjust the significance bands for the estimation, eventually, of more parsimonious models. Through Panel A of Table A1 in Appendix A, it is observed that for the estimates obtained by the simple quantile regression, the three models presented a positive and statistically significant relationship, at least in one of the five percentiles under analysis, of all countries, except for South Africa (SMB_{OP}), Brazil ($SMB_{B/M}$ and SMB_{OP}) and Russia ($SMB_{B/M}$, SMB_{OP} and SMB_{INV}).

For the three models, the asymmetry of the GDP growth rate vis-à-vis the explanatory variable varied between 2.9% (India, SMB_{INV}—0.75 percentile) and 29.6% (Canada, $SMB_{B/M}$—0.05 percentile), compared to the mean value of 7.8% (Japan, $SMB_{B/M}$) of statistical significance, at the 5% level of estimation by OLS. However, beyond the five percentiles under analysis, the risk factors $SMB_{B/M}$ and SM_{BOP}, showed a positive and statistically significant relationship to explain future economic performance, at least in one quantile for South Africa, 10.1% (SMB_{OP}—percentile 0.01) and for Brazil, 10.4% ($SMB_{B/M}$—percentile 0.30) and 16.8% (SMB_{OP}—percentile 0.80), as illustrated in Figures A1–A3 in Appendix A. In general, within the five percentiles under analysis, for the three risk factors, the positive coefficients of statistical significance and with high magnitude were in the percentiles below the median, with a variation between 3.3% (India, $SMB_{B/M}$,—0.75th percentile) and 29.6% (Canada, $SMB_{B/M}$—0.05 percentile), 3% (India, SMB_{OP}—0.75th percentile) and 29.4% (Canada, SMB_{OP}—0.05 percentile), and 2.9% (India, SMB_{INV}—0.05 percentile) and 27.5% (Canada, SMB_{INV}—0.05 percentile), for $SMB_{B/M}$, SMB_{OP} and SMB_{INV}, respectively.

Regarding the results of three multiple quantile regression models (Panel B of Table A1 in Appendix A), with two explanatory variables, as expected, it can be seen that within the five percentiles under analysis, in the presence of the MKT risk factor, the elementary risk factors $SMB_{B/M}$, SMB_{OP} and SMB_{INV} remained positive at least in percentile of the conditional distribution of economic performance of all countries under analysis, except for South Africa, Brazil, United Kingdom and Russia ($SMB_{B/M}$ and SMB_{INV}). Thus, ceteris paribus, the magnitude of the positive and statistically significant coefficients of the risk factors, respectively, $SMB_{B/M}$, SMB_{OP} and SMB_{INV}, varied between 2.4% (India, $SMB_{B/M}$—0.95 percentile) and 9.4% (China, $SMB_{B/M}$—0.25 percentile), 3.1% (India, SMB_{OP}—0.95 percentile) and 9% (Italy, SMB_{OP}—0.05 percentile), and 3% (India, SMB_{INV}—0.95 percentile) and 8.2% (Italy, SMB_{INV}—0.05 percentile), compared to the mean values of 4.8% (SMB_{OP}, Japan) and 4.5% (SMB_{INV}, China) of statistical significance at the 10% level, obtained through the estimation by OLS.

However, beyond the five percentiles under analysis, the risk factors $SMB_{B/M}$, SMB_{OP} and SMB_{INV}, respectively, showed a positive and statistically significant relationship to explain future economic performance, in a South African percentile, 2.1% ($SMB_{B/M}$—0.01 percentile) and 2% (SMB_{OP}—0.01 percentile), Brazil, 3.9% (SMB_{OP}—0.12 percentile) and 3.1% (SMB_{INV}—0.13 percentile), and United Kingdom, 2% (SMB_{OP}—0.01 percentile).

Thereafter, Table A2 in Appendix B, presents the results of each of the seven linear longitudinal regression models for long panel data, through the estimation of fixed effects, random effects, POLS, fixed effects with AR(1) error terms, random effects with AR(1) error terms, POLS with AR(1) error terms, and model with GLS (General Least Squares) estimation method with AR(1) error terms. It can be noted that the estimated parameters vary between models.

In general, it is observed that the fixed effects, random effects and POLS models present slightly higher standard errors compared to those obtained by the respective AR(1) error term models. The estimations with the GLS method are the most adequate, the parameters have slightly lower standard errors compared to those obtained by the other models. All risk factors showed positive coefficients in at least four models, except for CMA. The MKT risk factor was statistically significant in all models. With the exception of SMB and CMA risk factors, all risk factors showed statistical significance (in positive coefficients) in at least three models. The Hausman test applied to fixed and random effects models with AR(1) error terms support the null hypothesis that regression models with random effects provide consistent estimators of the parameters.

All these complementary analyses based on tables and figures in Appendices A and B, also support hypothesis (H1).

5.2. Random Coefficients Modeling

The existence of heterogeneity of within and between effects on economic performance between BRICS and G7 countries, according to the results presented in Table 3 on the variance decomposition, offers an opportunity through random coefficients modeling to investigate whether in fact there is significant variability, over time, in the economic growth rates of BRICS and G7 countries and whether this variability occurs between countries as a function of risk factors, considering the random variability of intercepts and slopes. As a consequence, the research hypotheses (H2, H3, H4, H5 and H6) stated will be verified. Table 5 presents the estimates of the two-level random coefficients models (Level 1: time or repeated measure and Level 2: country) with repeated measures, for a balanced panel data structure with 26 annual periods (from 1994 to 2019) for each of the countries under analysis, totaling 312 observations.

Table 5 presents the estimated results for the null model, without any explanatory variable, represented by the Equation 6. This estimate aims to analyze the existence or not of variability of error terms and the decomposition of variance between levels. If the intraclass correlation is different from zero, OLS estimates do not offer the best estimator of statistical significance other than zero, which justifies the application of random coefficients modeling [41–43].

Table 5. Variance Decomposition—Null Model.

Fixed Effect	**Coefficient**	**Std Error**	**z**
Global Mean—GDP	0.028 ***	0.007	4.17
Random Effect	Variance Components (%)	Std Error (%)	z
Level 1 (time)			
Temporal Variation (r_{ti})	0.056 ***	0.005	12.25
Level 2 (country)			
Country Variation—Intercept (u_{0i})	0.053 **	0.024	2.25
Variance Decomposition	% per Level		
Level 1 (time)	51.512		
Level 2 (country)	48.488		
LR test vs. OLS	158.32 ***		
Log restricted-likelihood	701.36		

Obs.: ***; ** $p < 1\%$ and 5%.

Through the analysis of the results presented in Table 5, there are significant differences in economic performance (GDP growth rates) between BRICS and G7 countries and these differences also occur over time, that is, in the period of 1994 to 2019. The parameter of the fixed effects component (global mean of expected economic performances, γ_{00}) and the estimates of the variance component of the error terms (rti and u0i) different from zero, at a significance level of 5%, which is why random coefficients modeling is justified. Regarding the coefficients of random effects, the variance decomposition indicates that 51.512% ($z = 12.25$; $p < 0.01$) of the variability in economic performance was due to the temporal evolution in each country, however, 48.488% ($z = 2.25$; $p < 0.05$) of the total variance of economic performance is due to differences between countries.

The result of the likelihood ratio test (LR test; LR test = 158.32; p $X^2 = 0.00 < 0.01$), which compares the robustness of the estimate (in terms of values expected) of random coefficients model in relation to linear regression by OLS (LR test vs. OLS), indicates that at the significance level of 5%, the random intercepts are not equal to zero, thus proving that for the repeated measures data for the analyzed period, estimation of a linear regression model by OLS, which produces only fixed effects coefficients, is not the most indicated. The null model results support the research hypotheses (H2) and (H3).

With the verification of the existence of significant variances in economic performance (i) over time, and (ii) over time, between countries, a temporal explanatory variable, YEAR at level 1, is included according to the proposed model, represented by the Equation (10). This model seeks to analyze whether the variable corresponding to time (linear trend) is statistically significant to explain the temporal variability in performance.

Table 6 presents the results of the linear trend model with random intercept effects.

Through the analysis of Table 6, it can be seen that with the inclusion of the explanatory variable of linear trend, YEAR at level 1, the parameters of intercept fixed effects, global mean of GDP ($z = 4.88$, $p < 0.01$) and the global means of the rates of change of GDP growth (parameter of the linear trend variable, YEAR; $z = -2.85$; $p < 0.05$) are statistically different from zero, at the significance level of 5%. The random intercept coefficients ($\sigma2 = 0.055\%$; $z = 12.23$; $p < 0.01$; $\tau00 = 0.053\%$; $z = 2.26$, $p < 0.05$) are statistical significant, at 5% significance level. Indeed, the intraclass correlation (ρ) indicates that 50.9% of the variance is due to the time evolution in each country and 49.1% of the total variance in economic performance is due to differences between countries.

There is a slight increase in the proportion of the variance component of the level 2 intercept in relation to the null model ($\rho = 48.488\%$).The result of the likelihood ratio test (LR test = 161.46; p $X^2 = 0.00 < 0.01$) at a significance level of 5%, indicates the rejection of the null hypothesis that the intercepts of random effects are equal to zero, so the random coefficients model with repeated measures offers better estimates than the linear fit model by OLS.

Table 6. Variance Decomposition—Linear Trend Model with Random Intercepts Effects.

Fixed Effect	**Coefficient**	**Std Error**	**z**
Global Mean—GDP	0.035 ***	0.007	4.88
YEAR	−0.001 ***	1.77×10^{-4}	−2.85
Random Effect	Variance Component (%)	Std Error (%)	z
Level 1 (time)			
Temporal Variation (r_{ti})	0.055 ***	0.005	12.23
Level 2 (country)			
Country Variation—Intercept (u_{0i})	0.053 **	0.024	2.26
Variance Decomposition	% per Level		
Level 1 (time)	50.900		
Level 2 (country)	49.100		
LR test vs. OLS	161.46 ***		
Log restricted-likelihood	697.67		

Obs.: ***, ** $p < 1\%$ and 5%.

Table 7 presents the expected values of the random effects temporal intercept terms for the economic performance (GDP) of the twelve countries under analysis. These results are illustrated in Figure 2.

Table 7. Expected Values of Intercepts of Random Effects by Country Estimated by the Linear Trend Model of Explanatory Variable Level 1—YEAR.

Country	**Random Intercept**	**Country**	**Random Intercept**
Brazil	−0.00412	Italy	−0.02012
Canada	−0.00366	Japan	−0.01827
China	0.05725	Russia	−0.00802
France	−0.01126	South Africa	−0.00216
Germany	−0.01361	UK	−0.00654
India	0.03371	United States	−0.00319

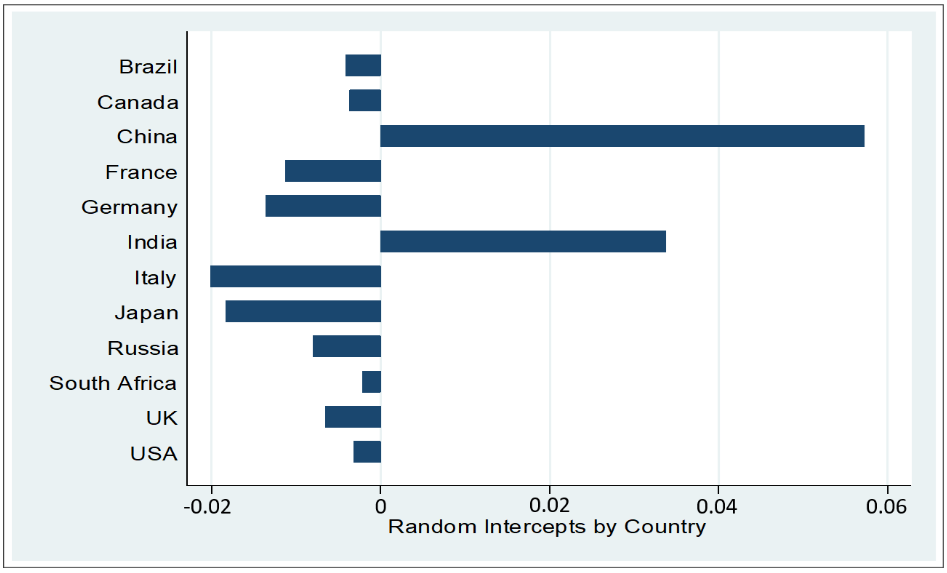

Figure 2. Expected values of intercepts of random effects by country estimated by the linear trend model of explanatory variable level 1—YEAR.

The expected error terms of random intercepts for the GDP of the same country do not vary over time, however, they vary between the GDP of each country, so, according to [41–43], it establishes the existence of an intercept for each country. Through the analysis of Table 7 and Figure 2, it can be seen that, on average, the initial economic performance dependent on the explanatory variable YEAR was of in the range from −2.012% (Italy) to 5.725% (China). Two countries, China and India, presented positive random temporal intercept terms, with a minimum magnitude of 3.371% (India).

Table 8 presents the results of the linear trend model with random effects of intercepts and slopes, represented by the Equation (13a).

Table 8. Variance Decomposition—Linear Trend Model with Random Intercepts and Slopes Effects.

Fixed Effect	Coefficient	Std Error	z
Global Mean—GDP	0.035 ***	0.007	4.86
YEAR	−0.001 ***	1.941×10^{-4}	−2.60
Random Effect	Variance Component (%)	Std Error (%)	z
Level 1 (time)			
Temporal variance (r_{ti})	0.055 ***	0.005	11.96
Level 2 (country)			
Country Variance—Intercept (u_{0i})	0.054 **	0.024	2.20
Country Variance—Slope (u_{1i})	7.9×10^{-8}	1.78×10^{-7}	0.44
Variance Decomposition	% per Level		
Level 1 (time)	50.411		
Level 2 (country)	49.589		
LR test vs. OLS	161.71 ***		
Log restricted-likelihood	697.79	X^2	p
LR test—Random Intercept Model vs. Random Intercept and Slope Model		0.26	0.61

Obs.: ***, ** $p < 1\%$ and 5%.

The statistical results support that for the analyzed period, there were no significant variances of slopes in economic performance over time between different countries. It is observed that the estimates of the fixed effects parameters (global means of the intercept, $\gamma_{00} = 0.035$), the global mean of the GDP growth rate ($\gamma_{10} = -0.001$) and of the residual variance ($\sigma^2 = 0.055$) in the model with intercept and random slopes do not differ from those obtained in the model with only random intercepts (see Table 6), because the variance component of the random slope terms (u1i) has statistical significance ($z = 0.44$; $p > 0.05$) equal to zero. In fact, the result of the likelihood ratio test, applied to compare the estimates of linear trend models with random intercepts (Log restricted-likelihood = 697.67) and with random intercepts and slopes (Log restricted-likelihood = 697.79), indicates that the values obtained by the difference of the logarithms of the two restricted likelihood functions (LR test = −0.26; $p\ X^2 = 0.61 > 0.05$) of the models are statistically equal, so that a linear trend model with only random intercepts is the most suitable.

From the analysis made through Tables 6 and 8, it is concluded that the investigation hypothesis (H4) is supported.

With the identification of the random character of the error terms (linear trend of random intercept), a final complete model of linear trend will be built, with the inclusion of explanatory variables at level 1, the risk factors considered in [14], where the interaction between level 1 and the random effects of slopes, at level 2, allows to capture the differences in GDP growth rates of each country, and offers the best fit model.

Of the five risk factors in [14], only the MKT and RMW risk factors showed statistical significance, at the 10% level, to explain the variation in GDP growth rates in BRICS and G7 countries. Thus, Table 9 presents the results of the final complete linear trend model with the inclusion of two risk factors MKT and RMW in the fixed effects component that capture the intercept random effects at level 2, represented by the Equation (20).

$$GDP_{t,i} = \gamma_{00} + \gamma_{10i} YEAR_{t,i} + \gamma_{11i} MKT_{t,i-1} + \gamma_{12i} RMW_{t,i-1} + u_{0i} + r_{t,i} \quad (20)$$

Table 9. Decomposition of variance—Linear Trend Model with Random Intercepts and MKT and RMW Explanatory Variables, Which Captures the Level 2 Random Effects—Final Complete Model.

Fixed Effect	**Coefficient**	**Std Error**	**z**
Global Mean—GDP	0.030 ***	0.007	4.24
YEAR	−0.001 ***	1.627×10^{-4}	−3.40
MKT	0.043 ***	0.005	7.98
RMW	0.059 ***	0.016	3.69
Random Effect	Variance Component (%)	Std Error (%)	z
Level 1 (time)			
Temporal variance (r_{ti})	0.046 ***	0.004	12.19
Level 2 (country)			
Country variance—Intercept (u_{0i})	0.054 **	0.024	2.27
Variance Decomposition	% per Level		
Level 1 (time)	45.807		
Level 2 (country)	54.193		
LR test vs. OLS	188.51 ***		
Log restricted-likelihood	719.15		

Obs.: ***, ** $p < 1\%$ and 5%.

However, a random coefficients model without the MKT and RMW risk factors in the fixed effects component, yet, with level 2 random slopes in the temporal evolution, presents the best estimators. The estimates of this model, represented by the Equation (21), are presented in Table 10.

$$GDP_{t,i} = \gamma_{00} + \gamma_{10i}YEAR_{t,i} + u_{0i} + u_{1i}MKT_{t,i-1} + u_{2i}RMW_{t,i-1} + r_{t,i} \quad (21)$$

Table 10. Decomposition of Variance—Linear Trend Model with Random Intercepts Without the Risk Factors MKT and RMW in the Fixed Effects Component, yet, with Random Slopes in the Temporal Evolution.

Fixed Effect	**Coefficient**	**Std Error**	**z**
Global Mean—GDP	0.032 ***	0.008	4.21
Global mean GDP growth rate (γ_{10})	−0.001 ***	1.465×10^{-4}	−3.79
Random Effect	Variance Component (%)	Std Error (%)	z
Level 1			
Temporal variance (r_{ti})	0.037 ***	0.003	11.71
Level 2			
Country Variance—Intercept (u_{0i})	0.062 **	0.027	2.28
Country Variance—Slope MKT (u_{1i})	0.243 **	0.115	2.11
Country Variance—Slope RMW (u_{1i})	1.018 **	0.487	2.09
Variance Decomposition	% per Level		
Level 1 (time)	2.699		
Level 2 (country)	97.301		
LR test vs. OLS	234.60 ***		
Log restricted-likelihood	734.24		

Obs.: ***, ** $p < 1\%$ and 5%.

From the analysis of Tables 9 and 10, it can be seen that the parameters of fixed effects and of the random coefficients of intercepts, present statistical significance different from zero, at a significance level of 5%. The global mean of economic performance (GDP) was adjusted to 3%. The explanatory variables of level 1, MKT (z = 7.98, $p < 0.01$) and RMW (z = 3.69, $p < 0.01$), showed positive coefficients and predicted an increase in performance economic growth between countries of 4.3% and 5.9%.

The variance decomposition between levels indicates that 45.807% (z = 12.19, $p < 0.01$), against 2.695% (z = 11.71, $p < 0.01$), of the model without the risk factors MKT and RMW in the fixed effects component of the variability of economic performance is due to the temporal evolution in each country, however, a significant portion of variance in the order of 54.193% (z = 2.27, $p < 0.05$), against 97.301% (z = 2.09, $p < 0.05$) of the model without the MKT and RMW risk factors in the fixed effects component, is due to differences in economic performance between countries.

The result of the likelihood ratio test (LR test = 188.51, $p\ X^2 = 0.00 < 0.01$ against LR test = 234.60, $p\ X^2 = 0.00 < 0.01$) indicates statistical significance, at the level of 5%, suggesting that the intercepts of random effects are in fact different from zero, so that the estimates of a linear regression model by OLS are discarded, however, a model without the explanatory variables MKT and RMW in the fixed effects, but in the random effects of slopes produces estimates with less distortions, as illustrated in Figure 3, which complements the result of the LR test, and illustrates the superiority of the random coefficients model with repeated measures in relation to the regression model estimated by OLS.

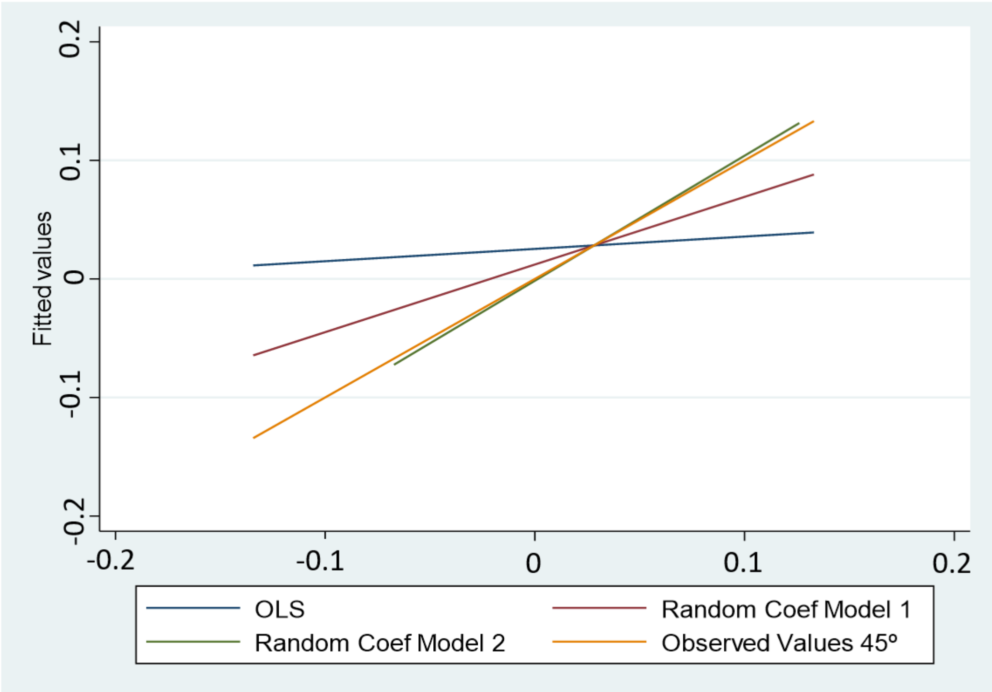

Figure 3. Values predicted by OLS and random coefficients vs. observed values of economic performance.

Indeed, it compares the predicted values of future economic performance estimated by the random coefficients modeling with the predicted values estimated by OLS, using the same explanatory variables in the fixed effects components (YEAR, MKT and RMW) and with observed real values of economic performance, represented by the sample GDP growth rates.

As shown in Figure 3, the yellow line at 45° indicates the observed values of the economic performance of each of the countries in the sample, in each of the analyzed periods. The red line indicates the values estimated by the random coefficients model, considering the explanatory variables YEAR, MKT and RMW in the fixed effects component, the green line indicates the values estimated by the random coefficients model without the explanatory variables MKT and RMW in the component of fixed effects, however, with random slopes of MKT and RMW precisely in the temporal evolution. Finally, the blue line denotes the fixed effects estimates of the multiple regression model by OLS.

It is found that, in relation to the OLS regression model, the random coefficients model, with a linear trend with explanatory variables YEAR, MKT and RMW and with random intercepts at level 2, presents a better adjustment in capturing random contexts of intercepts, however, it does not outperforms the random coefficients model with random slopes that presents less distortions in the adjustments of the expected values.

Table 11 presents the expected values of the random effects intercept terms, considering the linear trend models with explanatory variables YEAR, MKT and RMW in the fixed effects component, and without the explanatory variables, MKT and RMW in the fixed effects component, however, on the random effects of level 2 slopes on temporal evolution. The magnitudes and signs of the expected values of intercepts and random slopes, respectively, are illustrated in Figures 4 and 5.

Table 11. Expected Values of Intercepts of Random Effects by Country Estimated by Linear Trend Models of Explanatory Variables YEAR, MKT and RMW in the Fixed Effects Component and without MKT and RMW in the Fixed Effects Component, but rather in the Random Effects of the Slopes.

Country	Random Intercept		Random Slope	
	YEAR MKT RMW as Fixed Effect Components	YEAR as Fixed Effect Components	MKT	RMW
Brazil	−0.00397	−0.00557	0.04922	0.01287
Canada	−0.00382	−0.00105	0.04290	−0.04448
China	0.05784	0.06062	0.01705	−0.03760
France	−0.01147	−0.01059	0.03852	0.00791
Germany	−0.01384	−0.01564	0.06489	0.02938
India	0.03413	0.03878	−0.00447	−0.02460
Italy	−0.02040	−0.01977	0.04492	0.00136
Japan	−0.01853	−0.01873	0.04975	0.01434
Russia	−0.00790	−0.01492	0.07647	0.28924
South Africa	−0.00200	−0.00177	0.02975	0.03371
UK	−0.00671	−0.00730	0.04081	0.04156
USA	−0.00334	−0.00405	0.04767	0.03408

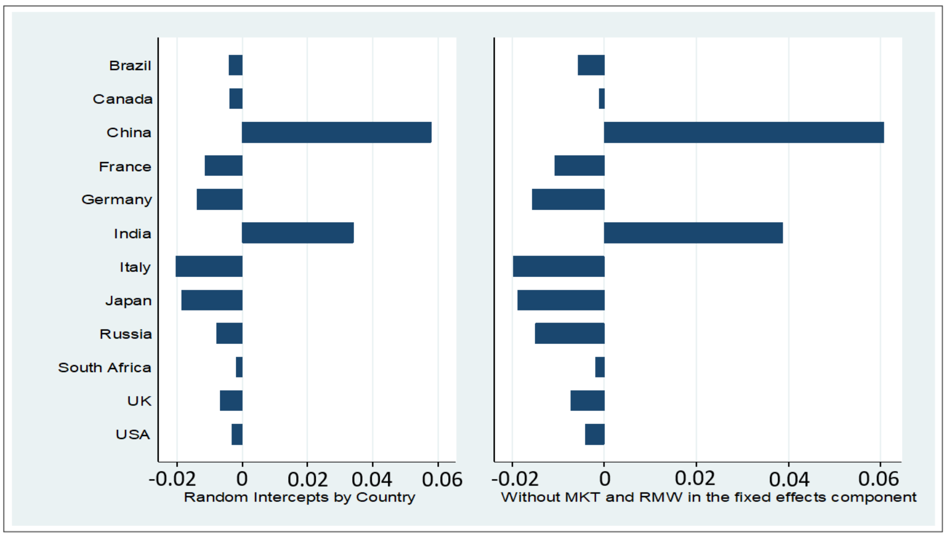

Figure 4. Expected values of random effects intercepts by country estimated by the linear trend model with and without the explanatory variables MKT and RMW in the fixed effects component.

Through the analysis of Table 11 and of Figure 4, it is found that considering the variables YEAR, MKT and RMW in the fixed effects component, the expected average of economic performance between countries varied between −2.040% and −1.977 (Italy), and 5.784% and 6.062% (China). Two countries, China and India preserved the terms of positive temporal intercepts, however, the minimum magnitude was adjusted to 3.413% (India), as illustrated in Figure 4.

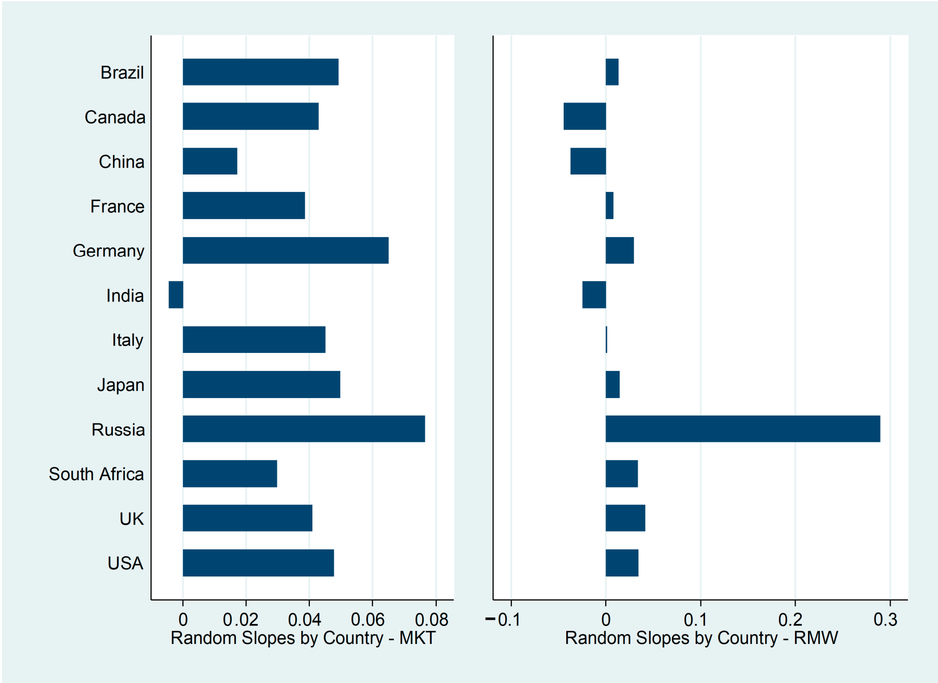

Figure 5. Expected values of random effects slopes by country estimated by the risk factors MKT and RMW for the linear trend model only with the explanatory variable YEAR in the fixed effects component.

Regarding the random slopes, through the analysis of Table 11 and of Figure 5, it is found that the expected average of economic performance between countries, considering the explanatory variables, respectively, MKT and RMW in the random effects component was of the order of −0.447% and −2.46% (India) to 7.647% and 28.924% (Russia). For the MKT risk factor, eleven countries, South Africa, Germany, Brazil, Canada, China, the United States, France, Italy, Japan, the United Kingdom and Russia presented positive slope terms, with a minimum magnitude of 1.705% (China). Regarding the RMW risk factor, nine countries, South Africa, Germany, Brazil, the United States, France, Italy, Japan, the United Kingdom and Russia, presented positive slope terms, with a minimum magnitude adjusted to 0.136% (Italy), as illustrated in Figure 5.

From the analysis made through Tables 10 and 11, it is concluded that the investigation hypothesis (H5) is supported.

Table 12 presents the decomposition of the variance between levels of three random coefficients models with three explanatory variables, YEAR, MKT and one of the three elementary risk factor ($SMB_{B/M}$, SMB_{OP} and SMB_{INV}). Panel A, B and C, respectively, correspond to the final complete model YEAR, MKT and $SMB_{B/M}$, YEAR, MKT and SMB_{OP} and YEAR, MKT and $SMB_{B/M}$, represented by the Equations (22)–(24).

$$GDP_{t,i} = \gamma_{00} + \gamma_{10i}ANO_{t,i} + \gamma_{11i}MKT_{t,i-1} + \gamma_{12i}SMB_{B/M,\ t,i-1} + u_{0i} + r_{t,i} \quad (22)$$

$$GDP_{t,i} = \gamma_{00} + \gamma_{10i}ANO_{t,i} + \gamma_{11i}MKT_{t,i-1} + \gamma_{12i}SMB_{OP,t,i-1} + u_{0i} + r_{t,i} \quad (23)$$

$$GDP_{t,i} = \gamma_{00} + \gamma_{10i}ANO_{t,i} + \gamma_{11i}MKT_{t,i-1} + \gamma_{12i}SMB_{INV,t,i-1} + u_{0i} + r_{t,i} \quad (24)$$

From the analysis of Table 12, which presents the results of the linear trend model with random intercept effects, it can be seen that the fixed effect parameters and the random intercept coefficients have a statistical significance different from zero. Although the parameters of the variables $SMB_{B/M}$, SMB_{OP} and SMB_{INV} showed a negative sign due to the presence of the other explanatory variables (YEAR and MKT), the correlation between economic performance and each elemental risk factor is positive, as seen in Table 4. The global mean (intercept) of economic performance was adjusted to 3.2% (Panel A), 3.3% (Panel B) and 3.3% (Panel C). The market beta risk factor of the three models, respectively, MKT (z = 7.22; $p < 0.01$, panel A), MKT (z = 7.16; $p < 0.05$, panel B) and MKT (z = 7.54; $p < 0.01$, panel C), showed positive coefficients and predicted an increase in economic performance of 3.6%, 3.6% and 3.8%, ceteris paribus.

Table 12. Decomposition of variance—Linear Trend Model with Random Intercepts and Explanatory Variables that Capture Level 2 Random Effects—Final Complete Model.

Panel A				Panel B				Panel C			
Fixed Effect	**Coef**	**SE**	**z**	**Fixed Effect**	**Coef**	**SE**	**z**	**Fixed Effect**	**Coef**	**SE**	**z**
Global Mean (Gm)—GDP	0.032 ***	0.007	4.57	Gm GDP	0.033 ***	0.007	4.65	Gm GDP	0.033 ***	0.007	4.68
YEAR	−0.001 ***	1.64×10^{-4}	−3.07	YEAR	−0.001 ***	1.66×10^{-4}	−3.13	YEAR	−0.001 ***	1.65×10^{-4}	−3.33
MKT	0.036 ***	0.005	7.22	MKT	0.036 ***	0.005	7.16	MKT	0.038 ***	0.005	7.54
$SMB_{B/M}$	−0.028 *	0.015	1.92	SMB_{OP}	−0.026 *	0.014	−1.79	SMB_{INV}	−0.035 **	0.013	2.73
Random Effect	VC (%)	SE (%)	z		VC (%)	SE (%)	z		VC (%)	SE (%)	z
Level 1 (time) Temporal Variance (r_{ti})	0.047 ***	0.004	12.19		0.047 ***	0.004	12.20		0.047 ***	0.004	12.19
Level 2 (country) Country Variance—Intercept (u_{0i})	0.052 ***	0.025	2.27		0.052 ***	0.023	2.27		0.052 ***	0.023	2.27
Variance Decomposition	% per Level				% per Level				per Level		
Level 1 (time)	47.738				47.551				47.066		
Level 2 (country)	52.262				52.449				52.934		
LR test vs. OLS	178.57 ***				179.51 ***				182.29 ***		
Log restricted-likelihood	714.24				713.99				715.97		

Obs.: Gm: global mean; Coef: coefficient; VC: variance component; SE: standard error; ***, **, * p < 1%, 5% and 10%.

The decomposition of the variance between levels indicates that 47.738% (z = 12.19; $p < 0.01$), 47.551% (z = 12.2; $p < 0.01$) and 47.066% (z = 12.19; $p < 0.01$) of the variability of economic performance is due to the temporal evolution in each country, however, a significant portion of variance of 52.262% (z = 2.27; $p < 0.01$), 52.449% (z = 2.27; $p < 0.01$) and 52.934% (z = 2.78; $p < 0.01$) is due to the difference in economic performance between countries. It is observed that the coefficients of the three models present equal performance to explain the differences in random intercepts in economic growth. The result of the likelihood ratio test (LR test = 178.57; $p < 0.01$), for Panel A, (LR test = 179.51; $p < 0.01$), for Panel B, and for Panel C (LR test = 182.29; $p < 0.01$) indicates statistical significance, at the 5% level, and supports the evidence that the intercepts of random effects are in fact different from zero, therefore discarding the estimates of a linear regression model by OLS.

Table 13 presents the expected values of the intercept terms of random effects of the economic performance of each of the twelve countries, estimated by the final complete model. Panels A, B and C, respectively, present the expected values of the intercept terms estimated with the explanatory variables YEAR, MKT and $SMB_{B/M}$, YEAR, MKT and SMB_{OP}, and YEAR, MKT and SMB_{INV}.

Through the analysis of Table 13, it can be seen that the variation of the expected average of economic performance between countries was of the order of −1.971% (Italy) to 5.681% (China), Panel A, −1.985% (Italy) to 5.701% (China), Panel B and −1.993% (Italy) to 5.714% (China), Panel C. Two countries, India and China kept positive temporal intercepts, however, the minimum magnitude was adjusted to 3.315% (India), Panel A, 3.335% (India), Panel B and 3.347% (India), Panel C.

As expected, the results presented in Tables 12 and 13 attest that, for the analyzed period, the economic performance of BRICS and G7 countries follows a linear trend over time and there are significant differences in random intercepts between countries. Size effect elementary risk factors $SMB_{B/M}$, SMB_{OP} and SMB_{INV} help to explain the variability in the future economic growth rate over time, so the research hypothesis (H6) is supported.

Table 13. Expected Values of Intercepts of Random Effects by Country Estimated by the Linear Trend Model with Explanatory Variables Regional Risk Factors Associated with the Final Complete Model.

Country	Panel A Independent Variable YEAR MKT $SMB_{B/M}$	Panel B Independent Variable YEAR MKT SMB_{OP}	Panel C Independent Variable YEAR MKT SMB_{INV}
Brazil	−0.00484	−0.00466	−0.00457
Canada	−0.00318	−0.00331	−0.00338
China	0.05681	0.05701	0.05714
France	−0.01081	−0.01094	−0.01102
Germany	−0.01317	−0.01330	−0.01338
India	0.03315	0.03335	0.03347
Italy	−0.01971	−0.01985	−0.01993
Japan	−0.01785	−0.01798	−0.01807
Russia	−0.00876	−0.00858	−0.00849
South Africa	−0.00288	−0.00270	−0.00260
UK	−0.00606	−0.00620	−0.00627
USA	−0.00270	−0.00283	−0.00290

6. Conclusions

In this study, in view of the integrating process of stock markets at the regional and global level, we explore the existence of variability in GDP growth rates, and the explanatory power of global risk factors of developed and emerging stock markets of [14] model as an indicator of the change in economic growth of a total of twelve countries, developed G7 countries and emerging BRICS countries, using GDP data and risk factor returns for a 26-year period, from 1993 to 2019.

The results show that global risk factors, from developed and emerging markets, considered in [14] help to explain the differences in GDP growth rates of the analyzed countries (developed from G7 and emerging from the BRICS). They also show that the temporal variation of the random effects of the intercepts can be explained by random coefficients models formed by a set of two risk factors: (i) MKT and RMW, (ii) MKT and $SMB_{B/M}$; (iii) MKT and SMB_{OP}, and (iv) MKT and SMB_{INV}.

The univariate analysis of the descriptive statistics of the decomposition of the variance of the return of the risk factors allowed to verify that all the risk factors presented positive average returns. The three elementary risk factors of the size effect ($SMB_{B/M}$, SMB_{OP} and SMB_{INV}) contributed positively to the average size risk premium; $SMB_{B/M}$ and SMB_{OP}, respectively, presented the lowest and highest average value of premiums. Risk factors showed greater variation over time than between countries. The variation in the GDP growth rate between countries was slightly greater than the variation over time. This result was confirmed through multivariate analysis, considering longitudinal quantile regression modeling for panel data that evaluated the existence of differences in future economic performance explained by global risk factors, as well as random coefficients modeling with repeated measures, which sought to investigate the existence of differences in economic performance across countries and over time, and the reasons for such differences.

The results obtained support that for the analyzed period, there are significant differences in the behavior of the conditional asymmetric distribution of the GDP growth rate, explained by the returns of the global risk factors of [14]. Like [19], we report that the market risk factors, $SMB_{B/M}$, SMB_{OP} e SMB_{INV}, SMB, HML, RMW and CMA individually, contain information about the future GDP growth variability, and jointly, the predictive capacity of these risk factors is independent of the information contained in the market risk factor beta.

By decomposing the variance through the estimation of a null model, it was possible to attest that economic performance follows a linear trend over analyzed period, there is significant variability in economic performance, over time, and between countries, 48% of the total variability of the GDP growth rate is due to the existence of differences between countries. For the analyzed period, the temporal variation of random effects of intercepts can be explained by four random coefficients models formed by a set of two explanatory variables, the fundamental risk factors, respectively, (i) MKT e RMW, (ii) MKT e $SMB_{B/M}$; (iii) MKT e SMB_{OP}, and (iv) MKT and SMB_{INV}, which explain approximately 54%, 52%, 52% and 53% of the total variability in the future GDP growth rate of BRICS and G7 countries.

The results of this study confirm that stock markets follow an integration process, and support the arguments of [5–7] that moderately integrated markets may have the same risk factors, and just like [13,14,16,19], since risk factors are related to future GDP growth, they act as proxies for unidentified state variables, consistent with the ICAPM pricing model of [15].

Given that the total country-effect GDP growth rate variability was considerable and variations in expected returns reflect business cycle exposures [20–22], a study that includes macroeconomic factors of each country provides a new understanding of the performance of economic growth in the face of elementary risk factors of the size effect, $SMB_{B/M}$, SMB_{OP} and SMB_{INV}.

Future studies may address the influence of other types of market besides stocks on the economic performance of countries. In addition, these studies can estimate such relationships by other statistical techniques to enable the comparison of results between different models.

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Appendix A

Table A1 presents the estimates of three simple and multiple quantile regression models for the percentiles 0.05; 0.25; 0.50; 0.75; and 0.95, having as explanatory variables the market beta risk factors, MKT and the elementary size effects $SMB_{B/M}$, SMB_{OP} and SMB_{INV}, represented by the Equations (1) (Panel A) and (2) (Panel B).

Table A1. Simple and Multiple Regression Estimates with Two Explanatory Variables Risk Factors MKT, $SMB_{B/M}$, SMB_{OP} and SMB_{INV}.

Model			**OLS**	**Quantile Regression**				
				0.05	**0.25**	**0.50**	**0.75**	**0.95**
Panel A: $GDP_{it} = \alpha + \beta Factor_{it-1} + \varepsilon_{i,t}$								
Brazil								
1	$SMB_{B/M}$	Coef	0.045	−0.215 **	−0.044 ***	0.084	0.040	0.061
		SE	0.048	0.075	0.015	0.058	0.062	0.046
2	SMB_{OP}	Coef	0.071 **	−0.269 ***	0.071	0.082	0.033	0.073
		SE	0.034	0.018	0.066	0.050	0.058	0.055
3	SMB_{INV}	Coef	0.057 **	−0.274 ***	0.076	0.047	0.025	0.069 **
		SE	0.021	0.020	0.050	0.050	0.049	0.032

Table A1. *Cont.*

Model			OLS	Quantile Regression				
				0.05	**0.25**	**0.50**	**0.75**	**0.95**
				Canada				
1	$SMB_{B/M}$	Coef	0.075	0.296 **	0.099 ***	0.065	0.056	−0.111 ***
		SE	0.049	0.104	0.023	0.043	0.067	0.010
2	SMB_{OP}	Coef	0.062	0.294 **	0.096 ***	0.066 **	0.037	−0.114 **
		SE	0.049	0.142	0.027	0.027	0.030	0.049
3	SMB_{INV}	Coef	0.064	0.275 *	0.089 **	0.110 ***	0.040	−0.081 *
		SE	0.049	0.163	0.031	0.027	0.050	0.044
				China				
1	$SMB_{B/M}$	Coef	0.060 *	0.054 ***	0.044	0.107 **	0.072	0.009
		SE	0.032	0.013	0.033	0.038	0.045	0.113
2	SMB_{OP}	Coef	0.075 ***	0.060 ***	0.113 **	0.098 ***	0.058	0.007
		SE	0.019	0.014	0.050	0.031	0.042	0.108
3	SMB_{INV}	Coef	0.066 ***	0.059 ***	0.086 ***	0.066 **	0.050	0.005
		SE	0.011	0.019	0.027	0.028	0.036	0.069
				France				
1	$SMB_{B/M}$	Coef	0.016	0.122 ***	0.089 ***	0.041 *	0.002	0.011
		SE	0.035	0.015	0.006	0.023	0.014	0.043
2	SMB_{OP}	Coef	0.048	0.213 ***	0.064 **	0.030	−0.016	−0.021
		SE	0.036	0.051	0.026	0.022	0.022	0.052
3	SMB_{INV}	Coef	0.005	0.034 ***	0.073 ***	0.041 **	−0.015	−0.019
		SE	0.035	0.004	0.014	0.015	0.018	0.054
				Germany				
1	$SMB_{B/M}$	Coef	0.043	0.116 ***	0.023	0.060 *	0.034	0.076 ***
		SE	0.044	0.039	0.032	0.033	0.029	0.004
2	SMB_{OP}	Coef	0.026	0.202 ***	−0.005	−0.018	0.031	0.077 ***
		SE	0.043	0.057	0.042	0.040	0.041	0.007
3	SMB_{INV}	Coef	0.039	0.057	0.029	0.023	0.025	0.078 ***
		SE	0.029	0.036	0.021	0.017	0.024	0.019
				India				
1	$SMB_{B/M}$	Coef	0.018	0.014	0.039	0.031	0.033 ***	0.039 **
		SE	0.034	0.061	0.063	0.025	0.005	0.016
2	SMB_{OP}	Coef	0.010	0.030	0.038	−0.017	0.030 ***	0.076 ***
		SE	0.033	0.060	0.085	0.019	0.008	0.013
3	SMB_{INV}	Coef	0.016	0.023	0.026	−0.013	0.029 ***	0.077 ***
		SE	0.026	0.048	0.056	0.014	0.005	0.005
				Italy				
1	$SMB_{B/M}$	Coef	0.045	0.167 ***	0.073 ***	0.007	−0.001	0.064 **
		SE	0.034	0.054	0.019	0.035	0.018	0.025
2	SMB_{OP}	Coef	0.034	0.175 ***	0.071 **	0.007	−0.001	−0.066 ***
		SE	0.035	0.031	0.026	0.038	0.013	0.013
3	SMB_{INV}	Coef	0.033	0.150 ***	0.064 *	0.006	−0.001	0.064 **
		SE	0.033	0.045	0.036	0.032	0.018	0.028

Table A1. *Cont.*

Model			OLS	Quantile Regression				
				0.05	0.25	0.50	0.75	0.95
Japan								
1	$SMB_{B/M}$	Coef	0.078 *	0.049 *	0.079 **	0.062 **	0.070 **	0.088 **
		SE	0.039	0.025	0.034	0.024	0.027	0.031
2	SMB_{OP}	Coef	0.066	0.224 ***	0.085 ***	0.059 *	0.022	0.074 ***
		SE	0.039	0.047	0.015	0.034	0.027	0.026
3	SMB_{INV}	Coef	0.064	0.053	0.081 ***	0.054	0.057	0.088 ***
		SE	0.039	0.035	0.016	0.033	0.059	0.025
Russia								
1	$SMB_{B/M}$	Coef	−0.234 **	−0.185	−0.360 **	−0.316 *	−0.094	−0.284 ***
		SE	0.104	0.193	0.134	0.160	0.109	0.012
2	SMB_{OP}	Coef	−0.179	−0.169	−0.320 ***	−0.145	−0.102	−0.225 ***
		SE	0.105	0.163	0.132	0.137	0.102	0.026
3	SMB_{INV}	Coef	−0.191 **	−0.133	−0.240 **	−0.137	−0.106	−0.152 ***
		SE	0.078	0.124	0.084	0.095	0.066	0.026
South Africa								
1	$SMB_{B/M}$	Coef	0.010	0.105 *	0.053	0.012	−0.026	0.019
		SE	0.026	0.063	0.046	0.032	0.035	0.023
2	SMB_{OP}	Coef	0.026	0.080	0.047	0.011	−0.022	−0.050
		SE	0.023	0.071	0.062	0.033	0.040	0.051
3	SMB_{INV}	Coef	0.016	0.056 **	0.036	0.008	−0.015	0.082 ***
		SE	0.015	0.021	0.053	0.026	0.030	0.010
United Kingdom								
1	$SMB_{B/M}$	Coef	0.012	0.090 ***	0.030	−0.002	−0.009	−0.001
		SE	0.032	0.027	0.040	0.026	0.046	0.013
2	SMB_{OP}	Coef	0.022	0.089 ***	0.031**	−0.002	−0.008	0.008
		SE	0.035	0.018	0.011	0.019	0.038	0.010
3	SMB_{INV}	Coef	0.015	0.082 **	0.029	−0.002	−0.020	0.007
		SE	0.033	0.034	0.023	0.027	0.028	0.024
United States								
1	$SMB_{B/M}$	Coef	0.012	0.084 ***	0.056 **	0.051 ***	−0.022	−0.021 ***
		SE	0.043	0.015	0.024	0.016	0.018	0.005
2	SMB_{OP}	Coef	0.001	0.214 ***	−0.009	0.031	−0.035	−0.024 **
		SE	0.045	0.023	0.022	0.033	0.045	0.011
3	SMB_{INV}	Coef	−0.001	0.089 **	−0.011	0.041	−0.035	−0.022 ***
		SE	0.044	0.035	0.040	0.030	0.043	0.006
Panel B: $GDP_{i,t} = \alpha + \beta_1 MKT_{i,t-1} + \beta_2 Fator_{it-1} + \varepsilon_{i,t}$								
Brazil								
4	MKT	Coef	0.052 ***	0.121 ***	0.050 ***	0.048 ***	0.051 ***	0.033 ***
		SE	0.010	0.018	0.010	0.010	0.005	0.004
	$SMB_{B/M}$	Coef	−0.021	−0.053	−0.011	−0.038	0.010	−0.023
		SE	0.034	0.064	0.037	0.036	0.017	0.027
5	MKT	Coef	0.053 ***	0.124 ***	0.050 ***	0.048 ***	0.050 ***	0.036 **
		SE	0.011	0.022	0.009	0.011	0.005	0.013
	SMB_{OP}	Coef	−0.021	−0.050	−0.010	−0.031	0.013	−0.029
		SE	0.032	0.076	0.032	0.031	0.018	0.046

Table A1. *Cont.*

Model			OLS	Quantile Regression				
				0.05	**0.25**	**0.50**	**0.75**	**0.95**
6	MKT	Coef	0.052 ***	0.095 ***	0.049 ***	0.047 ***	0.050 ***	0.035 **
		SE	0.010	0.013	0.009	0.009	0.008	0.012
	SMB_{INV}	Coef	−0.011	−0.015	−0.006	−0.023	0.013	−0.030
		SE	0.020	0.036	0.025	0.026	0.021	0.034
				Canada				
4	MKT	Coef	0.051 **	0.072 ***	0.063 ***	0.020	0.021 **	0.049 ***
		SE	0.024	0.010	0.011	0.021	0.009	0.003
	$SMB_{B/M}$	Coef	0.037	0.050 **	0.031	0.052	0.056 **	−0.133 ***
		SE	0.049	0.021	0.023	0.044	0.019	0.005
5	MKT	Coef	0.054 **	0.073 ***	0.061 ***	0.018	0.031	0.033 **
		SE	0.023	0.009	0.007	0.015	0.024	0.011
	SMB_{OP}	Coef	0.045	0.046 **	0.034 **	0.054 *	0.058	−0.115 ***
		SE	0.046	0.018	0.014	0.031	0.049	0.023
6	MKT	Coef	0.054 **	0.075 ***	0.057 ***	0.017	0.029	0.040 ***
		SE	0.046	0.008	0.019	0.018	0.027	0.009
	SMB_{INV}	Coef	0.043	0.039 **	0.033	0.058	0.059	−0.124 ***
		SE	0.046	0.017	0.039	0.037	0.056	0.019
				China				
4	MKT	Coef	0.021 **	0.032 ***	0.021 ***	0.026	0.010	0.052 ***
		SE	0.009	0.003	0.007	0.018	0.019	0.013
	$SMB_{B/M}$	Coef	0.034	0.085 ***	0.094 ***	0.026	0.048	−0.111 **
		SE	0.031	0.010	0.024	0.063	0.067	0.046
5	MKT	Coef	0.018 *	0.027 ***	0.021 **	0.005	0.012	0.057 ***
		SE	0.010	0.001	0.009	0.010	0.024	0.013
	SMB_{OP}	Coef	0.044	0.087 ***	0.086 ***	0.089 **	0.033	−0.104 **
		SE	0.032	0.003	0.030	0.034	0.083	0.045
6	MKT	Coef	0.016 *	0.023 ***	0.026 ***	0.012	0.006	0.057 ***
		SE	0.009	0.003	0.005	0.008	0.017	0.006
	SMB_{INV}	Coef	0.045 **	0.068 ***	0.054 ***	0.055 **	0.039	−0.079 ***
		SE	0.018	0.007	0.013	0.022	0.047	0.017
				France				
4	MKT	Coef	0.049 **	0.051 ***	0.015	0.035 ***	0.041 ***	0.050 ***
		SE	0.018	0.014	0.010	0.011	0.008	0.011
	$SMB_{B/M}$	Coef	−0.020	0.074 **	0.066 ***	−0.012	−0.038 **	−0.016
		SE	0.026	0.028	0.020	0.023	0.017	0.023
5	MKT	Coef	0.046 ***	0.051 ***	0.023 *	0.035 ***	0.036 ***	0.042 ***
		SE	0.016	0.003	0.012	0.011	0.005	0.004
	SMB_{OP}	Coef	−0.010	0.072 ***	0.049 *	−0.013	−0.034 ***	−0.021 **
		SE	0.027	0.006	0.024	0.022	0.011	0.009
6	MKT	Coef	0.047 ***	0.053 ***	0.024 *	0.035 ***	0.038 ***	0.043 ***
		SE	0.017	0.011	0.013	0.012	0.007	0.004
	SMB_{INV}	Coef	−0.014	0.064 ***	0.048 *	−0.012	−0.033 **	−0.021 **
		SE	0.026	0.021	0.026	0.024	0.014	0.009

Table A1. *Cont.*

Model			OLS	Quantile Regression				
				0.05	0.25	0.50	0.75	0.95
Germany								
4	MKT	Coef	0.076 ***	0.106 ***	0.047 ***	0.068 ***	0.056 **	0.044 ***
		SE	0.026	0.008	0.014	0.006	0.024	0.013
	$SMB_{B/M}$	Coef	−0.013	−0.057 ***	−0.044	−0.012	0.030	0.074 **
		SE	0.029	0.017	0.029	0.013	0.049	0.026
5	MKT	Coef	0.070 ***	0.107 ***	0.041 ***	0.066 ***	0.046 ***	0.069 ***
		SE	0.024	0.010	0.008	0.012	0.007	0.008
	SMB_{OP}	Coef	0.033	−0.058 ***	−0.040 **	−0.009	0.052 ***	0.077 ***
		SE	0.023	0.019	0.017	0.023	0.015	0.015
6	MKT	Coef	0.075 ***	0.106 ***	0.043 ***	0.067 ***	0.050 ***	0.066 ***
		SE	0.025	0.012	0.011	0.016	0.012	0.003
	SMB_{INV}	Coef	−0.008	−0.053 **	−0.035	−0.010	0.039	0.077 ***
		SE	0.029	0.024	0.022	0.033	0.024	0.007
India								
4	MKT	Coef	−0.003	−0.016 ***	−0.002	−0.011	0.001	−0.004
		SE	0.010	0.004	0.031	0.008	0.001	0.004
	$SMB_{B/M}$	Coef	0.022	0.024	0.049	0.021	0.031 ***	0.024 *
		SE	0.037	0.015	0.113	0.031	0.009	0.013
5	MKT	Coef	−0.002	−0.018 ***	0.013	−0.012	0.001	−0.006 *
		SE	0.011	0.005	0.016	0.012	0.005	0.003
	SMB_{OP}	Coef	0.014	−0.020	−0.046	0.011	0.028	0.031 **
		SE	0.039	0.016	0.057	0.043	0.018	0.011
6	MKT	Coef	−0.004	−0.014	−0.016	−0.012	−0.001	−0.007 *
		SE	0.011	0.011	0.026	0.012	0.002	0.003
	SMB_{INV}	Coef	0.022	0.037	0.046	0.009	0.031 ***	0.030 ***
		SE	0.029	0.031	0.072	0.033	0.006	0.009
Italy								
4	MKT	Coef	0.053 **	0.062 ***	0.030 **	0.031 **	0.045 ***	0.002
		SE	0.027	0.019	0.011	0.014	0.014	0.018
	$SMB_{B/M}$	Coef	0.006	0.087 **	0.005	−0.019	−0.005	0.063 *
		SE	0.027	0.038	0.023	0.029	0.029	0.037
5	MKT	Coef	0.053 *	0.061 ***	0.019	0.031 **	0.035 **	0.040 *
		SE	0.025	0.018	0.012	0.012	0.013	0.021
	SMB_{OP}	Coef	0.017	0.090 **	0.056 **	−0.014	−0.001	−0.013
		SE	0.027	0.036	0.024	0.024	0.026	0.042
6	MKT	Coef	0.053 **	0.061 ***	0.029 **	0.031 **	0.035 ***	0.041 *
		SE	0.025	0.020	0.010	0.012	0.012	0.020
	SMB_{INV}	Coef	0.011	0.082 *	0.004	−0.015	−0.001	−0.027
		SE	0.026	0.040	0.021	0.024	0.023	0.041
Japan								
4	MKT	Coef	0.051 *	0.081 ***	0.013	0.021	0.029 *	0.050 ***
		SE	0.028	0.018	0.022	0.015	0.014	0.008
	$SMB_{B/M}$	Coef	0.040	0.060	0.074	0.033	0.074 **	0.010
		SE	0.031	0.038	0.045	0.031	0.029	0.016

Table A1. *Cont.*

	Model		OLS	Quantile Regression				
				0.05	**0.25**	**0.50**	**0.75**	**0.95**
5	MKT	Coef	0.054 **	0.081 ***	0.023 *	0.036 ***	0.040	0.052 ***
		SE	0.025	0.004	0.013	0.016	0.025	0.014
	SMB_{OP}	Coef	0.048 *	0.059 ***	0.075 **	0.001	0.059	0.007
		SE	0.026	0.008	0.026	0.032	0.051	0.028
6	MKT	Coef	0.054 **	0.082 ***	0.017 ***	0.035 ***	0.034 ***	0.041 ***
		SE	0.026	0.007	0.003	0.006	0.005	0.011
	SMB_{INV}	Coef	0.042	0.052 ***	0.072 ***	0.001	0.067 ***	0.024
		SE	0.028	0.014	0.006	0.012	0.011	0.022
Russia								
4	MKT	Coef	0.057 *	0.129 ***	0.110 ***	0.055 **	0.035	0.037 ***
		SE	0.031	0.038	0.026	0.026	0.025	0.003
	$SMB_{B/M}$	Coef	−0.306 **	−0.504 ***	−0.204 **	−0.148	−0.250 **	−0.017
		SE	0.141	0.139	0.092	0.092	0.089	0.012
5	MKT	Coef	0.070 **	0.125 ***	0.137 **	0.079 *	0.023	0.046 **
		SE	0.031	0.003	0.054	0.045	0.017	0.017
	SMB_{OP}	Coef	−0.302 *	−0.449 ***	−0.277	−0.129	−0.171 ***	0.100 *
		SE	0.152	0.009	0.189	0.159	0.058	0.059
6	MKT	Coef	0.078 **	0.127 ***	0.125 ***	0.072 ***	0.039 **	0.027
		SE	0.029	0.013	0.042	0.025	0.016	0.021
	SMB_{INV}	Coef	−0.294 ***	−0.338 ***	−0.404 ***	−0.209 ***	−0.231 ***	−0.120 *
		SE	0.079	0.037	0.115	0.068	0.045	0.059
South Africa								
4	MKT	Coef	0.028 ***	0.027 ***	0.035 **	0.018 *	0.024 *	0.040 ***
		SE	0.009	0.004	0.016	0.009	0.015	0.003
	$SMB_{B/M}$	Coef	−0.025	0.021	−0.025	−0.038	−0.075	−0.106 ***
		SE	0.029	0.016	0.059	0.032	0.052	0.012
5	MKT	Coef	0.029 ***	0.027 ***	0.030 ***	0.019 *	0.038 ***	0.042 ***
		SE	0.010	0.005	0.007	0.011	0.005	0.004
	SMB_{OP}	Coef	−0.024	0.020	−0.011	−0.035	−0.098 ***	−0.065 ***
		SE	0.034	0.019	0.024	0.038	0.018	0.013
6	MKT	Coef	0.029 ***	0.027 ***	0.044 ***	0.019 *	0.035 ***	0.049 ***
		SE	0.009	0.006	0.010	0.010	0.012	0.002
	SMB_{INV}	Coef	−0.023	0.015	−0.033	−0.023	−0.065 *	−0.082 ***
		SE	0.021	0.016	0.029	0.029	0.032	0.006
United Kingdom								
4	MKT	Coef	0.047	0.085 ***	0.040 **	0.011	0.010	0.016 *
		SE	0.031	0.001	0.014	0.014	0.020	0.009
	$MB_{B/M}$	Coef	−0.022	−0.002	−0.043	−0.016	−0.009	−0.017
		SE	0.027	0.001	0.029	0.029	0.042	0.018
5	MKT	Coef	0.042	0.085 ***	0.029 ***	0.007	0.007	0.014 **
		SE	0.029	0.000	0.005	0.016	0.013	0.006
	SMB_{OP}	Coef	0.008	−0.001	−0.014	0.008	−0.008	0.002
		SE	0.031	0.001	0.009	0.033	0.027	0.012
6	MKT	Coef	0.043	0.085 ***	0.030 ***	0.002	0.008	0.014
		SE	0.030	0.002	0.005	0.014	0.013	0.009
	SMB_{INV}	Coef	−0.003	−0.002	−0.014	−0.003	−0.008	0.002
		SE	0.029	0.005	0.009	0.028	0.027	0.018

Table A1. *Cont.*

Model			OLS	Quantile Regression				
				0.05	0.25	0.50	0.75	0.95
				United States				
4	MKT	Coef	0.057 ***	0.058 ***	0.031	0.052 ***	0.055 ***	0.040 ***
		SE	0.018	0.017	0.025	0.021	0.014	0.002
	$MB_{B/M}$	Coef	−0.030	0.058 *	0.008	−0.033	−0.047	−0.069 ***
		SE	0.035	0.035	0.052	0.044	0.029	0.005
5	MKT	Coef	0.053 ***	0.059 ***	0.035 ***	0.053 ***	0.044 ***	0.029 ***
		SE	0.017	0.010	0.006	0.011	0.014	0.006
	SMB_{OP}	Coef	−0.017	0.058 ***	0.041 ***	−0.034	−0.047	−0.029 **
		SE	0.036	0.019	0.011	0.022	0.028	0.013
6	MKT	Coef	0.054 ***	0.061 ***	0.033	0.052 ***	0.048 ***	0.031 ***
		SE	0.017	0.015	0.020	0.018	0.006	0.007
	SMB_{INV}	Coef	−0.023	0.061 *	0.044	−0.028	−0.045 ***	−0.029 **
		SE	0.034	0.031	0.041	0.036	0.012	0.013

Obs.: Coef, coefficient; SE, standard error; ***, **, *, $p < 1\%$, 5% and 10%.

Figures A1–A3 respectively, illustrate the individual performance of the risk factors SMBOP, SMBB/M and SMBOP on the conditional quantile distribution, as well as the estimation by OLS, of the GDP growth rate of South Africa, Brazil and Brazil.

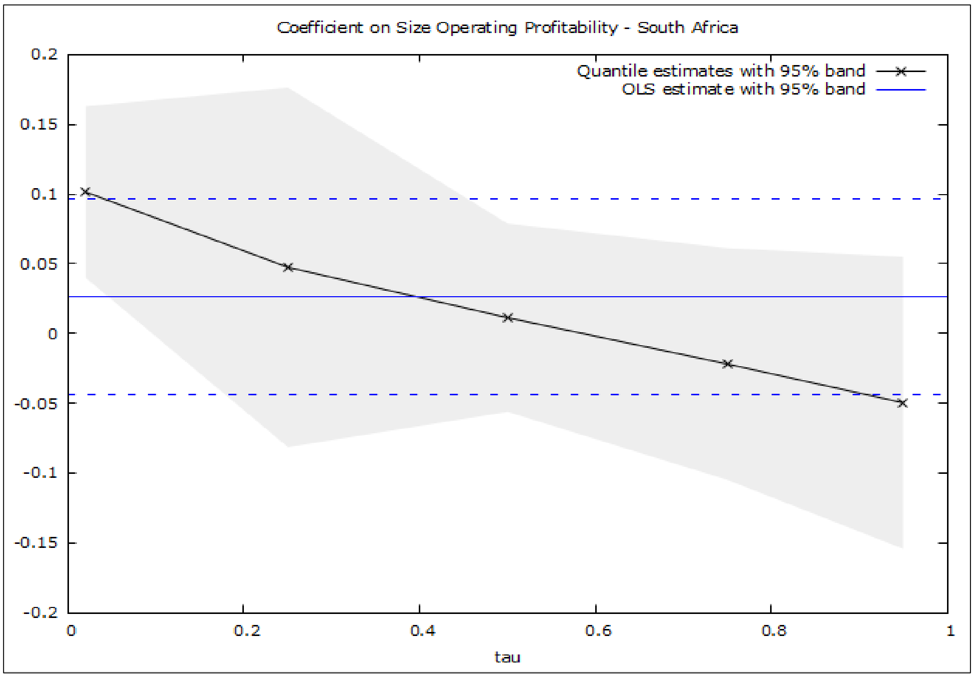

Figure A1. Performance of the SMB_{OP} risk factor on the percentiles and conditional average of South Africa's GDP growth rate.

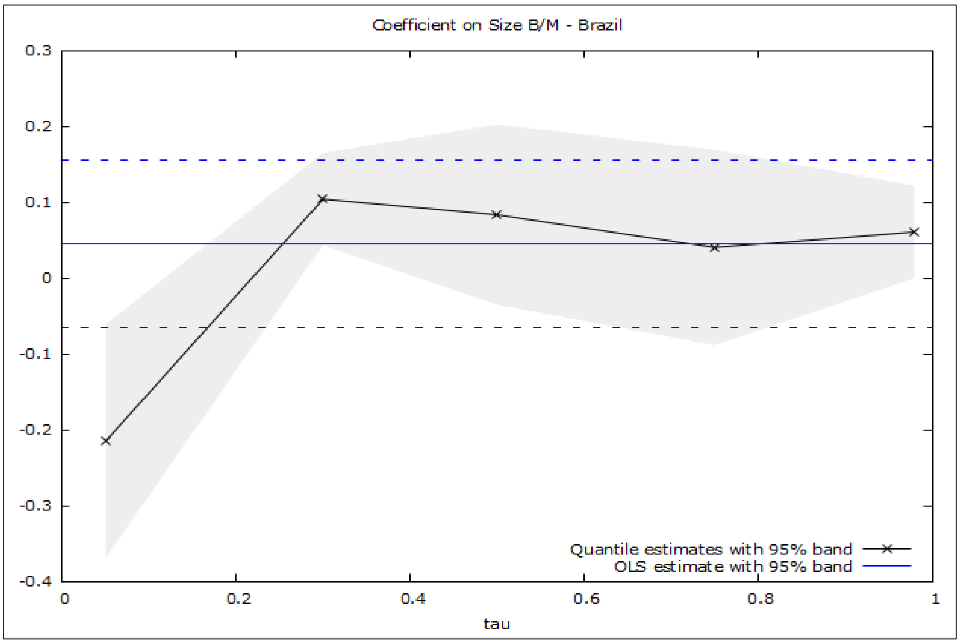

Figure A2. Performance of the $SMB_{B/M}$ risk factor on the percentiles and conditional average of Brazil's GDP growth rate.

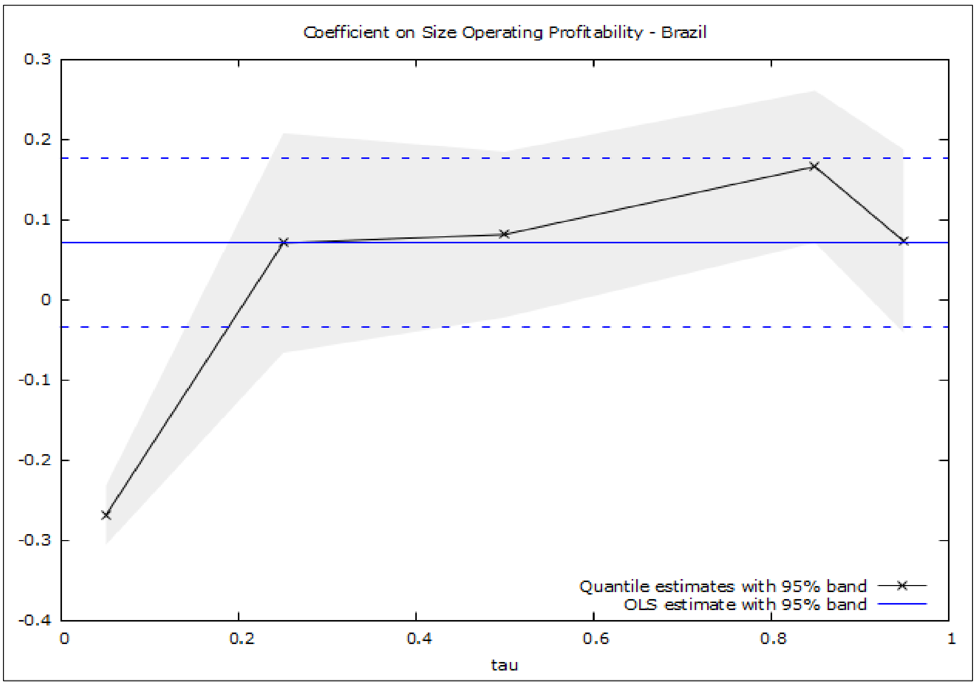

Figure A3. Performance of the SMB_{OP} risk factor on the percentiles and conditional average of Brazil's GDP growth rate.

Appendix B

In order to compare the parameters, Table A2 presents the results of the estimations of seven longitudinal long-panel multiple regression models, with five explanatory variables, the risk factors considered in the Fama and French (2015) model, estimated by: (1) fixed effects, (2) random effects, (3) Pooled by OLS, and considering the existence of first order serial correlation in terms of AR(1) error (4) fixed effect, (5) random effects models, (6) Pooled by OLS and (7) GLS (General Least Squares) method.

Table A2. Long-Panel Data Model Multiple Regression Estimates.

Model			Fixed Effect	Random Effect	Pooled OLS	Fixed Effect AR(1)	Random Effect AR(1)	Pooled OLS AR(1)	GLS AR(1)
$GDP_{it} = \alpha_i + \beta_1 MKT_{i,t-1} + \beta_2 SMB_{i,t-1} + \beta_3 HML_{i,t-1} + \beta_4 RMW_{i,t-1} + \beta_5 CMA_{i,t-1} + \varepsilon_{i,t}$									
6	MKT	Coef	0.04051 ***	0.04048 ***	0.03990 ***	0.03094 ***	0.03137 ***	0.03029 ***	0.02637 ***
		SE	0.00597	0.00602	0.00849	0.00413	0.00451	0.00676	0.00393
	SMB	Coef	−0.00548	−0.00605	−0.01693	0.01129	0.00311	0.00427	0.00358
		SE	0.01642	0.01654	0.02328	0.01546	0.01637	0.02512	0.01441
	HML	Coef	0.02024 *	0.02115 *	0.03849 **	0.00587	0.00429	0.00501	−0.00154
		SE	0.01183	0.01191	0.01662	0.00925	0.01003	0.01518	0.00927
	RMW	Coef	0.05789 ***	0.05683 ***	0.03655	0.02665	0.04323 **	0.03629	0.02464 *
		SE	0.01946	0.01960	0.02750	0.01726	0.01697	0.02547	0.01484
	CMA	Coef	−0.01730	−0.01728	−0.01691	−0.02660 ***	−0.02651 **	−0.02744	−0.02289 **
		SE	0.01586	0.01597	0.02254	0.01164	0.01274	0.01878	0.01036
	Cons	Coef	0.02260 ***	0.02259 ***	0.02232 ***	0.02558 ***	0.02479 ***	0.02488 ***	0.02597 ***
		SE	0.00178	0.00588	0.00253	0.00119	0.00666	0.00434	0.00295

Obs.: Cons: constant; Coef: coefficient; SE, standard error; ***, **, *, $p < 1\%$, 5% and 10%.

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