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A Credibility Theory-Based Robust Optimization Model to Hedge Price Uncertainty of DSO with Multiple Transactions

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Abstract: This paper addresses the deregulated electricity market arising in a distribution system with an electricity transaction. Under such an environment, the distribution system operator (DSO) with a distributed generator faces the challenge of electricity price uncertainty in a spot market. In this context, a credibility theory-based robust optimization model with multiple transactions is established to hedge the uncertain spot price of the DSO. Firstly, on the basis of credibility theory, the spot price is taken as a fuzzy variable and a risk aversion-based fuzzy opportunity constraint is proposed. Then, to exploit the resiliency of multiple transactions on hedging against uncertain spot price, the spot market, option contract and bilateral contract integrating power flow constraints are studied, because it is imperative for DSO to consider the operational constraints of the local network in the electricity market. Finally, the clear equivalence class is adopted to transform the risk aversion constraint into a deterministic robust optimization one. Under the premise of considering the expected cost of the DSO, the optimal electricity transaction strategy that maximizes resistance to uncertain spot price is pursued. The rationality and effectiveness of the model are verified with a modified 15-node network. The results show that the introduction of option contracts and bilateral contracts reduces the electricity transaction cost of DSO by USD 28.5. In addition, under the same risk aversion factor, the cost of the proposed model is reduced by USD 195.18 compared with robust optimization, which avoids the over-conservatism of traditional robust optimization.

Keywords: price uncertainty; DSO; credibility theory; fuzzy chance constraint; robust optimization

MSC: 90-10

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1. Introduction

The distribution system operator (DSO) is responsible for maintaining the security of supply and power quality through investment, construction and reconfiguration of the existing distribution system [1–3]. With the deepening reform of the electricity market environment, as a stakeholder, DSO with distributed generator (DG) plays an important role in the electricity transaction of the distribution system. That is, DSO purchases electricity in the upper wholesale market to meet customer demand as well as maximization of its utility [4–6]. However, the price in the spot market is characterized by uncertainty due to fluctuations in electricity demand, fuel price and renewable power generation [7,8]. Moreover, the forecast error of the spot price is inevitable [9]. Thus, in order to obtain the optimal electricity transaction and expected utility, DSO has to capture the uncertain spot price from the perspective of risk aversion [10]. In view of this, two main questions need to be answered: how to use a portfolio of electricity purchase transactions to hedge against risk brought by uncertain spot price and how to assess the risk when formulating an optimal electricity transaction strategy under the premise of the expected cost?

1.1. Literature Review

For risk decision problems with uncertain electricity price, researchers mainly use three kinds of optimization, including robust optimization [11–15], stochastic optimization [16–20] and fuzzy optimization [21–26]. For instance, in [11], the uncertainty of selling/purchasing price in an electricity market is handled by robust optimization with a polyhedral uncertain set. Accordingly, an economical optimal solution is obtained in consideration of the undesired deviation of the market electricity price from the forecasted one. In [12], an adaptive robust optimization is developed to study the uncertain price in a real-time market. In [13], a maximum–minimum–maximum robust optimization model considering the price deviation in the electricity market is proposed, which improves the robustness of system operation against forecast uncertainty. In [14], the uncertainty of market price is dealt with by the upper deviation of forecast price and the robust electricity trading strategies of risk neutrality and risk aversion are compared. In [15], the uncertainty of the electricity market price is modeled based on robust optimization. In this model, instead of the predicted electricity price, the maximum and minimum amounts of the electricity price are considered. However, robust optimization mainly focuses on the worst-case of an uncertain problem and does not fully investigate the risk preference characteristic of a decision maker [27]. Stochastic optimization is one of the most commonly used methods for managing uncertain price. In [16], by assuming that the uncertain price follows the normal distribution, a scenario generation-based stochastic framework is developed. In this framework, the risk associated with uncertain electricity price is considered through downside risk constraints. In [17], uncertain electricity price is regarded as a random variable and a stochastic optimization model based on the Monte Carlo sampling method is established in microgrid (MG) optimal operation. In [18], a multi-stage stochastic programming method is developed. The bidding strategy in the spot market is described as a Markov decision problem and solved by approximate dual dynamic programming. In [19], the decision-making problem of a retailer under uncertainty is discussed based on stochastic optimization. In [20], the random scenario method is derived to simulate the uncertain spot price and the conditional value at risk is proposed to evaluate the risk of the electricity trading strategy. However, stochastic optimization requires repeated sampling and the solution efficiency is reduced [28].

In addition, the work in [29] states that besides the random feature, the uncertainty also includes the fuzzy feature. Thus, fuzzy optimization, including fuzzy rough set [21], image fuzzy set [22], neutral particle set [23] and so on, has been studied in the operation of a power system over the past few years. In [24], the authors consider the fuzzy feature of uncertain electricity price, in which the fuzzy feature is approximated with a fuzzy number. In [25], a fuzzy set theory-based MG energy management model is established for price uncertainty. In this model, the uncertain electricity price is characterized by triangular fuzzy numbers. In [26], a risk measurement method based on credibility theory is proposed to evaluate the fuzziness of uncertain wind power. However, few studies have addressed the robust power trading model based on credibility theory in view of the uncertainty of spot price in the electricity market.

On the other hand, the existing research on electricity transaction mainly focuses on the discussion of a return model and the formulation of transaction strategy. For example, In [30], the transaction strategy of the power retailer in the spot market is analyzed and the profit model of the retailer under the background of new power reform is discussed. In [31], a deterministic multi-objective optimization model with the goal of profit maximization and peak demand minimization is established to study the short-term decision-making problem of the retailer. In [32], the optimal bidding strategy of an energy hub in the power market is studied under the protection of energy network information privacy. These studies do not take into account the risk assessment of retailers in electricity trading. In [33], a risk decision-making model of electricity transaction with the goal of profit maximization is established. The model analyzes the impact of different electricity transaction combinations on profits from the retailer's point of view. In [34], a risk management model for DSO portfolio with multiple electricity purchase markets is constructed. The impact of different risk appetites on transaction strategy is studied. In [35], the retailer tries to hedge against uncertainty through three trading platforms. The electricity price uncertainty is modeled with the auto regressive integrated moving average method and the retailer's electricity transaction strategy is determined. In [36], based on the portfolio optimization theory, the optimization model of electricity purchase and sale portfolio is constructed to explore the influence of different factors on the purchase and sale risk in the multi-level electricity market. However, in most cases, DSO is not only responsible for trading electricity but also should consider the operational constraints of the distribution system. Therefore, it is necessary to consider network topology as well as power flow constraints of the distribution system when selecting electricity purchase transactions.

Table 1 reports the majority of the studies presented within the last decade; however, most of the existing electricity purchase strategies do not take into account the topological constraints of the network. In addition, to the best of the authors' knowledge of this paper, there are no studies in the literature addressing the robust electricity trading model based on credibility theory in view of the uncertainty of spot price in the electricity market.

Table 1. Taxonomy of recent research works.

Literature	Network Topology Constraint	Deterministic Optimization Model	Risk Assessment Model	Robust Optimization	Stochastic Optimization	Fuzzy Optimization
11–15	✗	✗	✓	✓	✗	✗
16–20	✗	✗	✓	✗	✓	✗
21–26	✗	✗	✓	✗	✗	✓
30–32	✓	✓	✗	✗	✗	✗
33–36	✗	✗	✓	✗	✓	✗
The proposed method	✓	✓	✓	✓	✗	✓

1.2. Our Contributions

To address the above issues, this paper develops a risk aversion DSO electricity transaction model based on the credibility theory. The proposed model can help decision makers determine the optimal combination of electricity purchase transactions under an acceptable risk level, considering the uncertain electricity price and power flow constraints when formulating electricity transaction strategy. The main contributions of this paper are as follows:

- Based on credibility theory, a risk aversion-based fuzzy chance constraint model is proposed. In the model, the uncertain spot price is designed as a fuzzy variable and its credibility distribution is derived to assess the uncertain risk. The proposed model optimizes the credibility that the expected objective is met, from which decision makers can assess the risk of transaction strategy.
- Multiple transactions, including the spot market, option contract and bilateral contract, are considered to hedge the risk caused by uncertain price, and the impact of different electricity transaction combinations on DSO cost is analyzed while considering power flow constraints.
- A clear equivalence class method with fuzzy chance constraint is used to transform the proposed model into a deterministic robust optimization model. The effectiveness of the model is verified with a modified 15-node network.

1.3. Organization of the Research

The rest of this paper is organized as follows. The credibility function associated with forecast error percentage of spot price is derived in Section 2. The multiple electricity transactions model is established in Section 3. A credibility theory-based robust optimization model to hedge uncertain spot price of DSO with multiple transactions is proposed in Section 4. Case studies and related analysis are introduced in Section 5. Finally, this paper concludes in Section 6.

2. Problem Formulation

Fuzzy decision-making is a kind of method to solve problems with fuzzy nature, but the traditional fuzzy decision-making has not established a complete axiomatic system. This leads to unconvincing decision-making conclusions until the credibility theory-based uncertainty measurement is established. It makes up for the disadvantage that possibility measure does not have self-duality [37] and provides a new tool for scholars to study fuzzy decision-making problems. In credibility theory, the credibility measure is developed to describe the credibility of fuzzy events [38]. It holds that events with credibility 1 must occur and events with credibility 0 do not occur, which avoids the decision-making confusion that may be caused by the traditional calculation of membership degree.

The credibility measure can be expressed by the minimum supremum of variable in a fuzzy event set. For any set $A \in \Re$, the credibility measure of fuzzy variable $\xi \in A$ is defined as [39]:

$$\mathrm{Cr}\{\xi \in A\} = \frac{1}{2}\left(\sup_{x \in A} \mu(x) + 1 - \sup_{x \in A^c} \mu(x)\right) \tag{1}$$

where $\sup\limits_{x \in A} \mu(x)$ and $1 - \sup\limits_{x \in A^c} \mu(x)$ denote the possibility measure and necessity measure of A, respectively. A^c represents the complement of the set A, and μ is the membership function of the fuzzy variable. The average value of the possibility measure and the necessity measure in Equation (1) is used to ensure the establishment of duality. In addition, the credibility measure satisfies the following four axioms:

Axiom 1. for a non-empty set $\Theta \in \Re$, $\mathrm{Cr}\{\Theta\} = 1$.
Axiom 2. $\mathrm{Cr}\{A\} \leq \mathrm{Cr}\{B\}$ whenever $A \subseteq B \subseteq \Theta$.
Axiom 3. $\mathrm{Cr}\{A\} + \mathrm{Cr}\{A^c\} = 1$ for any event $A \subseteq \Theta$.
Axiom 4. $\mathrm{Cr}\{\cup_i A_i\} = \sup_i \mathrm{Cr}\{A_i\}$ for any collection of events $\{A_i\}$with $\sup_i \mathrm{Cr}\{A_i\} < 0.5$.

In this paper, the uncertain spot price is designed as a fuzzy variable and the credibility distribution function including possibility measure and necessity measure is derived to evaluate the uncertain risk. The uncertain risk measurement model based on credibility theory also satisfies these four axioms.

Credibility Distribution Function Associated with Forecast Error Percentage of Spot Price

In electricity transactions, there are inevitable errors in the forecast of spot price [40]. Assume that the forecast error percentage of spot price is ε and the mathematical expression is as follows:

$$\varepsilon = \left(\lambda_t^{\mathrm{sm}} - \lambda_t^{\mathrm{sm}\prime}\right) / \lambda_t^{\mathrm{sm}\prime} \tag{2}$$

where λ_t^{sm} and $\lambda_t^{\mathrm{sm}\prime}$ are the actual spot price and the forecast spot price, respectively.

The membership function μ associated with forecast error percentage of spot price can be expressed as the Cauchy distribution [24]. The uncertain spot price is taken as the fuzzy variable and its mathematical expression can be described as:

$$\mu = \begin{cases} \dfrac{1}{1 + \omega(\varepsilon / E_+)^2}, & \varepsilon > 0 \\ \dfrac{1}{1 + \omega(\varepsilon / E_-)^2}, & \varepsilon \leq 0 \end{cases} \tag{3}$$

where E_+ and E_-, respectively, represent the statistical average of positive and negative error percentages and ω is the weighting factor.

After derivation, we can obtain the credibility function of ε:

$$\mathrm{Cr}(\xi \leqslant \varepsilon) = \begin{cases} 1 - \dfrac{1}{2\left[1+\omega(\varepsilon/E_+)^2\right]}, & \varepsilon > 0 \\ \dfrac{1}{2\left[1+\omega(\varepsilon/E_-)^2\right]}, & \varepsilon \leqslant 0 \end{cases} \tag{4}$$

Proof. According to Equation (1), for $\varepsilon \in \mathfrak{R}$, the mathematical expression of the credibility measure is

$$\mathrm{Cr}\{\varepsilon\} = \frac{1}{2}\left(\sup_{y\leq\varepsilon}\mu(x) + 1 - \sup_{y>\varepsilon}\mu(x)\right) \tag{5}$$

If $\varepsilon > 0$, we have

$$\begin{aligned} \sup_{y\leq\varepsilon}\mu(y) &= \max\left\{\sup_{0<y\leq\varepsilon}\mu(y), \sup_{y\leq 0}\mu(y)\right\} \\ &= \max\{\mu(0), \mu(0)\} = 1 \end{aligned} \tag{6}$$

and

$$\sup_{y>\varepsilon}\mu(y) = \sup_{y>\varepsilon>0}\frac{1}{1+\omega(\mathrm{y}/E_+)^2} = \frac{1}{1+\omega(\varepsilon/E_+)^2} \tag{7}$$

Combining Equations (6) and (7), if $\varepsilon > 0$, we have

$$\mathrm{Cr}(\varepsilon) = 1 - \frac{1}{2\left[1+\omega(\varepsilon/E_+)^2\right]}. \tag{8}$$

If $\varepsilon \leq 0$, we have

$$\sup_{y\leq\varepsilon}\mu(y) = \sup_{y\leq\varepsilon\leq 0}\frac{1}{1+\omega\left(y/(E_-)^2\right)} = \frac{1}{1+\omega\left(\varepsilon/(E_-)^2\right)} \tag{9}$$

and

$$\begin{aligned} \sup_{y>\varepsilon}\mu(y) &= \max\left\{\sup_{y\leqslant\varepsilon\leqslant 0}\mu(y), \sup_{y>0}\mu(y)\right\} \\ &= \max\left\{\sup_{y\leqslant\varepsilon\leqslant 0}\frac{1}{1+\omega\left(y/(E_-)^2\right)}, \sup_{y>0}\frac{1}{1+\omega(y/E_-)^2}\right\} \\ &= \mu(0) = 1. \end{aligned} \tag{10}$$

Combining Equations (9) and (10), if $\varepsilon \leq 0$, we have

$$\mathrm{Cr}(\varepsilon) = \frac{1}{2\left[1+\omega\left(\varepsilon/(E_-)^2\right)\right]}. \tag{11}$$

This completes the proof. □

The credibility and membership functions associated with forecast error percentage of spot price are shown in Figure 1, where $E_+ = 10\%$, $E_- = -10\%$, $\omega = 0.33$ and $\varepsilon \in [-0.5, 0.5]$. From the figure, we can see that the credibility function $\mathrm{Cr}(\xi \leqslant \varepsilon)$ is a monotone increasing function. The value of the credibility distribution function refers to the credibility of the fuzzy variable ξ whose value is less than or equal to ε, which can be compared to the probability distribution function of probability theory.

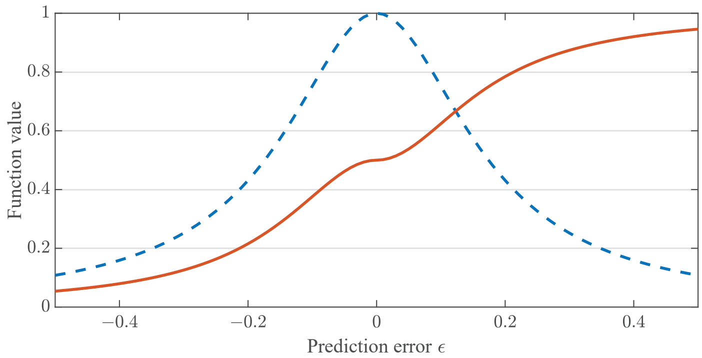

Figure 1. Credibility and membership functions associated with forecast error percentage of spot price.

3. Multiple Electricity Transaction Model under the Deterministic Spot Price

3.1. Objective Function

DSO conducts electricity transaction through the spot market, bilateral contract and option contract. Consider T hour periods and T_1 and T_2 to be, respectively, the peak and non-peak period sets of power demand, satisfying $T_1 + T_2 = T$. Call option contract is only for T_1 and trading volume does not change over time. In addition, suppose there are N bilateral contracts for DSO to choose and the details of the electricity transaction cost are as follows.

(1). The mathematical expression of DSO's cost function C_{sm} in spot market is as follows:

$$C_{\mathrm{sm}} = \sum_{t \in T} \sum_{b \in B} \left[\lambda_t^{\mathrm{sm}} p_{t,b}^{\mathrm{sm}} \right] \tag{12}$$

where $b \in B$ is the range of network node. λ_t^{sm} and $p_{t,b}^{\mathrm{sm}}$ are the electricity price and trading volume of DSO in spot market during time t, respectively.

(2). The cost function C_{bc} of DSO in the bilateral contract electricity transaction can be expressed as:

$$C_{\mathrm{bc}} = \sum_{t \in T_1} \sum_{b \in B} \sum_{n \in N} \left[\lambda_n^{\mathrm{bc}} p_{t,b,n}^{\mathrm{bc}} \right] \tag{13}$$

where λ_n^{bc} is the electricity price with respect to bilateral contract n and $p_{t,b,n}^{\mathrm{bc}}$ is the trading volume of nth bilateral contract selected by node b during time t.

(3). The cost function C_{oc} of DSO from the option contract electricity transaction can be calculated as:

$$C_{\mathrm{oc}} = \sum_{t \in T_2} \sum_{b \in B} \left[\min\{\lambda_{\mathrm{ck}}, \lambda_t^{\mathrm{sm}}\} p_{t,b}^{\mathrm{oc}} + \lambda_0 p_{t,b}^{\mathrm{oc}} \right] \tag{14}$$

where $p_{t,b}^{\mathrm{oc}}$ is the call option contract trading volume of node b during time t, λ_{ck} and λ_0 are the strike price and premium of the call option, respectively. If $\lambda_t^{\mathrm{sm}} > \lambda_{\mathrm{ck}}$, DSO executes the option contract and its option contract purchases electricity at the fixed price; if $\lambda_t^{\mathrm{sm}} < \lambda_{\mathrm{ck}}$, DSO abandons the exercise option and its option contract purchases electricity at the spot market price.

(4). The power generation cost function C_{dg} of DSO can be expressed as:

$$C_{\mathrm{dg}} = \sum_{t \in T} \sum_{b \in B} \left[\lambda^{\mathrm{dg}} p_{t,b} \right] \tag{15}$$

where $p_{t,b}$ is the active power output of DG at node b during time t and λ^{dg} is the power generation cost price of DG.

The mathematical expression of the cost C of the DSO with the spot market, option contract and bilateral contract in an electricity transaction is as follows:

$$C = C_{\mathrm{sm}} + C_{\mathrm{bc}} + C_{\mathrm{oc}} + C_{\mathrm{dg}} \tag{16}$$

3.2. Constraints

In order to ensure that the system operates in a safe and reliable environment, the electricity transaction must meet the following constraints.

(1). Active power output constraint of DG:

$$0 \leq p_{t,b} \leq P_{t,b}^{\max} \tag{17}$$

where $P_{t,b}^{\max}$ is the maximum active power output of DG at node b during time t.

(2). Node voltage constraint:

$$v_{t,b}^{\min} \leq v_{t,b} \leq v_{t,b}^{\max} \tag{18}$$

where $v_{t,b}$ is the square of the voltage at node b during time t. $v_{t,b}^{\min}$ and $v_{t,b}^{\max}$ are the maximum and minimum values of node voltage at node b during time t, respectively.

(3). Contract volume constraint for bilateral contract:

$$p_n^{\mathrm{bc,min}} s_{b,n} \leq p_{t,b,n}^{\mathrm{bc}} \leq p_n^{\mathrm{bc,max}} s_{b,n} \tag{19}$$

where $s_{b,n}$ is a binary variable. If node b selects contract n, then $s_{b,n} = 1$; otherwise, $s_{b,n} = 0$. $p_n^{\mathrm{bc,min}}$, $p_n^{\mathrm{bc,max}}$ are the minimum and maximum contract volumes of bilateral contract n, respectively.

(4). During time t, the total amount of electricity purchased in spot market, bilateral contract and option contract of DSO equals the amount of active power injected from the power grid. Its mathematical expression can be described as:

$$p_{t,b}^{\mathrm{sm}} + \sum_{n \in N} p_{t,b,n}^{\mathrm{bc}} = P_{t,b}^{\mathrm{grid}}, \forall t \in T_1 \tag{20}$$

$$p_{t,b}^{\mathrm{sm}} + \sum_{n \in N} p_{t,b,n}^{\mathrm{bc}} + p_{t,b}^{\mathrm{oc}} = P_{t,b}^{\mathrm{grid}}, \forall t \in T_2 \tag{21}$$

where $P_{t,b}^{\mathrm{grid}}$ is the amount of active power injected from the power grid at node b during time t.

(5). The power flow constraints of the distribution network are as follows:

$$f_{t,l|s(l)=b}^{\mathrm{p}} - P_{t,b}^{\mathrm{grid}} - \sum_{l|r(l)=b} \left(f_{t,l}^{\mathrm{p}} - a_{t,l} R_l \right) - p_{t,b} + D_{t,b}^{\mathrm{p}} + G_b v_{t,b} = 0 \tag{22}$$

$$f_{t,l|s(l)=b}^{\mathrm{q}} - Q_{t,b}^{\mathrm{grid}} - \sum_{l|r(l)=b} \left(f_{t,l}^{\mathrm{q}} - a_{t,l} X_l \right) - q_{t,b} + D_{t,b}^{\mathrm{q}} - B_b v_{t,b} = 0 \tag{23}$$

$$v_{t,b} - 2\left(R_l f_{t,l}^{\mathrm{p}} + X_l f_{t,l}^{\mathrm{q}} \right) + a_{t,l}\left(R_l^2 + X_l^2 \right) = v_{t,b} \tag{24}$$

$$\left(f_{t,l}^{\mathrm{p}} - a_{t,l} R_{t,l} \right)^2 + \left(f_{t,l}^{\mathrm{q}} - a_{t,l} X_{t,l} \right)^2 \leq S_{t,l}^2 \tag{25}$$

$$\left(\left(f_{t,l}^{\mathrm{p}} \right)^2 + \left(f_{t,l}^{\mathrm{q}} \right)^2 \right) / a_{t,l} \leq v_{t,b} \tag{26}$$

$$\left(f_{t,l}^{\mathrm{p}} \right)^2 + \left(f_{t,l}^{\mathrm{q}} \right)^2 \leq S_{t,l}^2 \tag{27}$$

where $l \in L$ is the range of network line. $f^{\mathrm{p}}_{t,l}$, $f^{\mathrm{q}}_{t,l}$ are the active and reactive power flow of line l during time t, respectively. $D^{p}_{t,b}$ and $D^{q}_{t,b}$ are the active load and reactive load of node b during time t, respectively. $a_{t,l}$ is the square of the current of line l during time t. $Q^{\mathrm{grid}}_{t,b}$ is the reactive power injected from the power grid at node b during time t. $S^{2}_{t,l}$ is the upper limit of the apparent power of line l during time t, and R_l, X_l, G_b and B_b are the parameters of resistance, reactance, admittance and conductance of distribution network, respectively. $s(l)$ is the power outflow end of line l and $r(l)$ is the power inflow end of line l. The balance constraints of active and reactive power are shown in Equations (22) and (23). Equation (24) relates the line flow to the node voltage. Equation (25) represents the apparent power flow limitation of each line transmitting node and Equation (26) is a quadratic curve constraint, which convexes the original non-convex AC OPF problem [41]. Under quite unrestricted assumptions, the rationality of this convexity is proved in [42]. Equation (27) represents the apparent power flow limitation of each line receiving node.

4. Robust Optimization Model for DSO Based on Credibility Theory

The forecast error of spot price is inevitable [43]. DSO with different risk preferences needs to hedge the risk caused by forecast error while considering operational cost as well as power flow constraints. Given a certain electricity purchase cost, DSO pursues an electricity transaction strategy that maximizes resistance to the uncertain spot price. In view of this, this paper establishes a credibility theory-based robust optimization model to hedge price uncertainty of DSO with multiple transactions.

$$\max |\varepsilon| \tag{28a}$$

$$\text{s.t.}\begin{cases} \mathrm{Cr}(\max C(\lambda^{\mathrm{sm}}_t, q) \le C_{\mathrm{e}}) \ge \alpha & \text{(28b)} \\ C_{\mathrm{e}} = (1+\sigma)C_0 & \text{(28c)} \\ \lambda^{\mathrm{sm}}_t = (1+\varepsilon)\lambda^{\mathrm{sm}\prime}_t & \text{(28d)} \\ 0 \le \sigma \le 1 & \text{(28e)} \\ 0 \le \alpha \le 1 & \text{(28f)} \\ (17) - (27) & \text{(28g)} \end{cases}$$

where C_0 is the minimum cost of DSO when the spot price equals the forecasted spot price. σ is the risk aversion factor, which indicates the DSO's aversion to the risk due to the uncertain spot price. α is the credibility index and the physical meaning is equivalent to the probability confidence. Equation (28b) is expressed as the credibility that the actual cost of DSO less than the expected cost is not less than α. Equation (28c) represents the expected cost of DSO. When σ is larger, expected cost C_{e} is higher, indicating that DSO has a greater degree of risk aversion. q is the decision variable, which represents the amount of electricity traded by DSO in each market.

Generally, when the actual spot price takes the maximum, the DSO's cost is the highest, so Equation (28b) can be expressed as

$$\mathrm{Cr}\Big(C(\lambda^{\mathrm{sm}}_t, q)|_{\lambda^{\mathrm{sm}}_t=(1+\varepsilon)\lambda^{\mathrm{sm}\prime}_t} \le C_{\mathrm{e}} \Big) \ge \alpha, \quad \varepsilon \ge 0 \tag{29}$$

In view of the fact that the above formula belongs to the fuzzy chance constraint and it is difficult to solve directly, one way to solve the fuzzy chance constraint is to convert it into a clear equivalence class and then use the traditional solving process to calculate the clear equivalence model. According to [44], we can obtain the following theorem:

Theorem 1. *Suppose ξ is degenerated into a one-dimensional fuzzy variable and its membership function is μ. If the function $g(\mathbf{x},\xi)$ has the form $g(\mathbf{x},\xi) = h(\mathbf{x}) - \xi$, then $\mathrm{Cr}\{g(\mathbf{x},\xi) \le 0\} \ge \alpha$, if and only if $h(\mathbf{x}) \le K_\alpha$, where $\mathbf{x}$ and g are the decision vector and constraint, respectively. Moreover,*

$$K_\alpha = \begin{cases} \sup\left\{K \mid K = \mu^{-1}(2\alpha)\right\}, & \alpha < 1/2 \\ \inf\left\{K \mid K = \mu^{-1}(2(1-\alpha))\right\}, & \alpha \ge 1/2 \end{cases} \tag{30}$$

When $\varepsilon \ge 0$, $\alpha \ge 1/2$, according to the credibility measure function and the above theorem, the robust optimization model shown in Equation (28) can be expressed as

$$\max K_\alpha \tag{31a}$$

$$\text{s.t.}\begin{cases} C(\lambda_t^{\mathrm{sm}}, q)|_{\lambda_t^{\mathrm{sm}}=(1+K_\alpha)\lambda_t^{\mathrm{sm}\prime}} \le C_{\mathrm{e}} & \text{(31b)} \\ C_{\mathrm{e}} = (1+\sigma)C_0 & \text{(31c)} \\ 0 \le \sigma \le 1 & \text{(31d)} \\ K_\alpha = \mu^{-1}(2(1-\alpha)) \ge 0 & \text{(31e)} \\ 1/2 \le \alpha \le 1 & \text{(31f)} \\ (17)-(27) & \text{(31g)} \end{cases}$$

The model considers that the actual spot price fluctuates within a certain range of the predicted spot price. The obtained electricity transaction strategy can ensure that the cost of DSO is less than the expected cost and the credibility is not less than α. The solution process of the proposed method is given by Algorithm 1.

Algorithm 1 Solution process

1: Given system data and forecasted spot price;
2: Considering constraints (17)–(27), calculate (16) to obtain the minimum cost C_0 of DSO with predicted spot price;
3: Give DSO risk aversion factor σ or expected cost C_{e};
4: Obtain the membership function associated with forecast error percentage of spot price according to the credibility theory and derive its credibility distribution;
5: By maximizing (28a) and considering constraints (28b)–(28g), a risk measurement model under fuzzy chance constraints is established;
6: Use the clear equivalence class method to transform the above model into a deterministic robust optimization model;
7: Solve the robust optimization model through the SCIP solver and obtain $\varepsilon, \alpha, p_{t,b}^{\mathrm{sm}}, p_{t,b}^{\mathrm{oc}}, p_{t,b,n}^{\mathrm{bc}}$.

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5. Case Analysis

To prove the validity of the proposed model, a modified 15-node distribution network system is selected for numerical study in this paper and the structure of the distribution network system is shown in Figure 2 (the specific parameters are in [45]). Two DGs are set at nodes 1 and 12, respectively. The capacity of each DG is set to 0.15 MW and their power generation cost is USD 30/MWh. The forecast spot price and distribution system load are shown in Figure 3. Suppose that the peak period of electricity consumption is $8:00 \sim 24:00$ and the rest of the period is non-peak period. The call option strike price $\lambda_{\mathrm{ck}} =$ USD 64.3/MWh and the option premium $\lambda_0 =$ USD 2.3/MWh. In addition, set the weighting factor $\omega = 0.33$ in the credibility function.

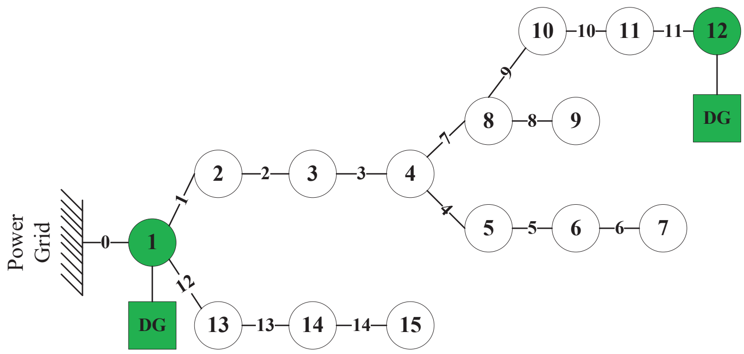

Figure 2. The 15-node network structure diagram.

In the competitive power market, the cost of DSO depends on its own power generation and power purchase plan. DSO can purchase electricity through different combinations of spot market, option contract and bilateral contract. Assume that DSO has five bilateral contracts to choose from non-peak and peak periods, respectively. The detailed parameters of the bilateral contract are shown in Table 2. In order to verify the effectiveness of the credibility theory-based robust optimization model to hedge price uncertainty, this paper selects different electricity transaction scenarios. Scenario 1, DSO only purchases electricity through the spot market; scenario 2, DSO purchases electricity through the spot market and option contract; scenario 3, DSO purchases electricity through the spot market, bilateral contract and option contract.

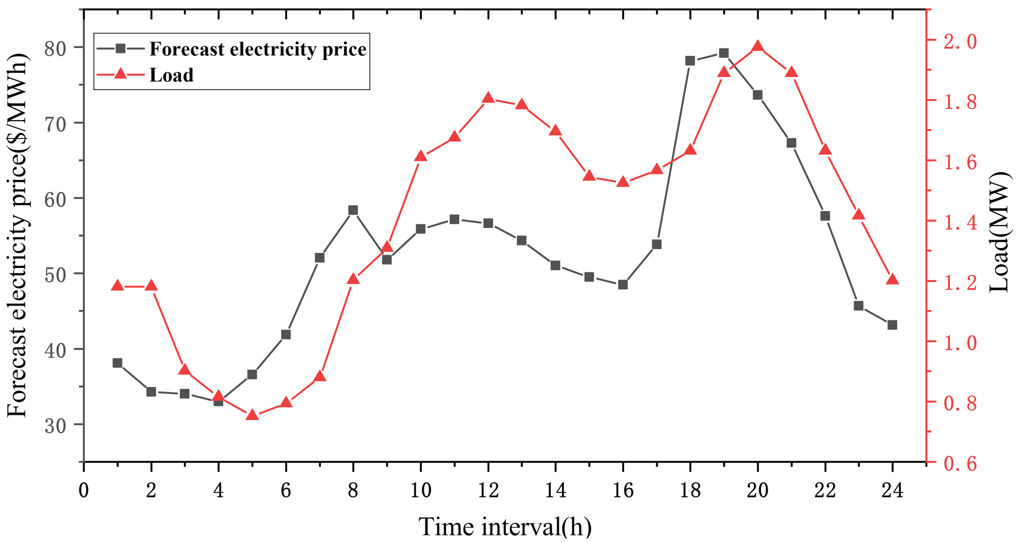

Figure 3. Forecast spot price and distribution system load.

Table 2. Bilateral contract parameters.

Contract Number	Period (h)	Min (MW)	Max (MW)	Contract Price (USD/MWh)
1	Non-peak period	0.006	0.015	43.0
2	Non-peak period	0.008	0.020	42.0
3	Non-peak period	0.010	0.025	38.0
4	Non-peak period	0.010	0.030	35.5
5	Non-peak period	0.012	0.040	33.0
6	Peak period	0.006	0.015	63.5
7	Peak period	0.008	0.020	62.0
8	Peak period	0.010	0.025	59.5
9	Peak period	0.010	0.030	58.5
10	Peak period	0.012	0.040	56.0

5.1. Comparison of Transaction Cost under Deterministic Spot Electricity Price

First of all, it is assumed that the actual spot price equals the predicted value. The minimum cost of DSO is obtained by solving the deterministic electricity transaction model. The optimal DSO electricity transactions in different scenarios are shown in Figure 4. In scenario 1, DG output is 7.20 WMh, the spot market purchase is 26.48 MWh and the cost of DSO is USD 1684.0. In scenario 2, DG output is 7.20 WMh, the spot market and option contract purchase are 13.70 MWh and 12.78 MWh, respectively, and the cost of DSO is USD 1682.5. In scenario 3, the option contract purchase is 8.09 MWh, the bilateral contract purchase is 6.42 MWh, the DG output is 7.20 MWh, the spot market purchase is 11.96 MWh and the cost of DSO is USD 1655.6. It can be seen that with the increase of transaction form, the electricity purchase cost of DSO gradually decreases.

For scenario 3, DSO's DG output and electricity transaction in spot market, bilateral contract and option contract are shown in Figure 5. As can be seen from the figure, the non-peak period electricity transaction market is mainly in the spot market and bilateral contract and peak period electricity transaction market is mainly in the spot market and option contract. Overall, DG output, bilateral contract, spot market and option contract accounted for 21.4%, 19.1%, 35.5% and 24.0% of the total electricity consumption, respectively. The bilateral contract trading volume of each node is shown in Table 3.We can see that bilateral contract transaction is mainly in nodes 2 and 13. Nodes 2 and 13 choose contracts 3, 4, 5, 7, 8, 9, 10 to trade electricity. This is because these two nodes have a high load demand and multiple bilateral contracts can be selected to meet their own demand. The other nodes with low load demand only choose a bilateral contract to trade electricity during peak and non-peak periods. In addition, since the load demand of node 14 is too small, there is no suitable bilateral contract for it to choose, so it meets its own demand through the spot market and option contract.

Table 3. Bilateral contract trading volume of each node.

Nodes	Contract Number									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	-	-	-	-	-	-	-	-	-	-
2	0	0	0.115	0.150	0.252	0	0.184	0.230	0.250	0.428
3	0	0	0	0	0	0	0	0	0	0.273
4	0	0	0	0	0	0	0	0	0	0.273
5	0	0	0	0	0	0	0	0	0.207	0
6	0	0	0	0	0.112	0	0	0	0	0.347
7	0	0	0	0	0	0	0	0	0	0.288
8	0	0	0	0	0	0	0	0	0	0.288
9	0	0	0	0.081	0	0	0	0	0	0.301
10	0	0	0	0.080	0	0	0	0	0	0.296
11	0	0	0	0	0	0	0	0	0	0.286
12	0	0	0	0	0	0	0	0	0	0
13	0	0	0.115	0.150	0.252	0	0.184	0.230	0.250	0.428
14	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0.079	0	0	0	0	0	0.292

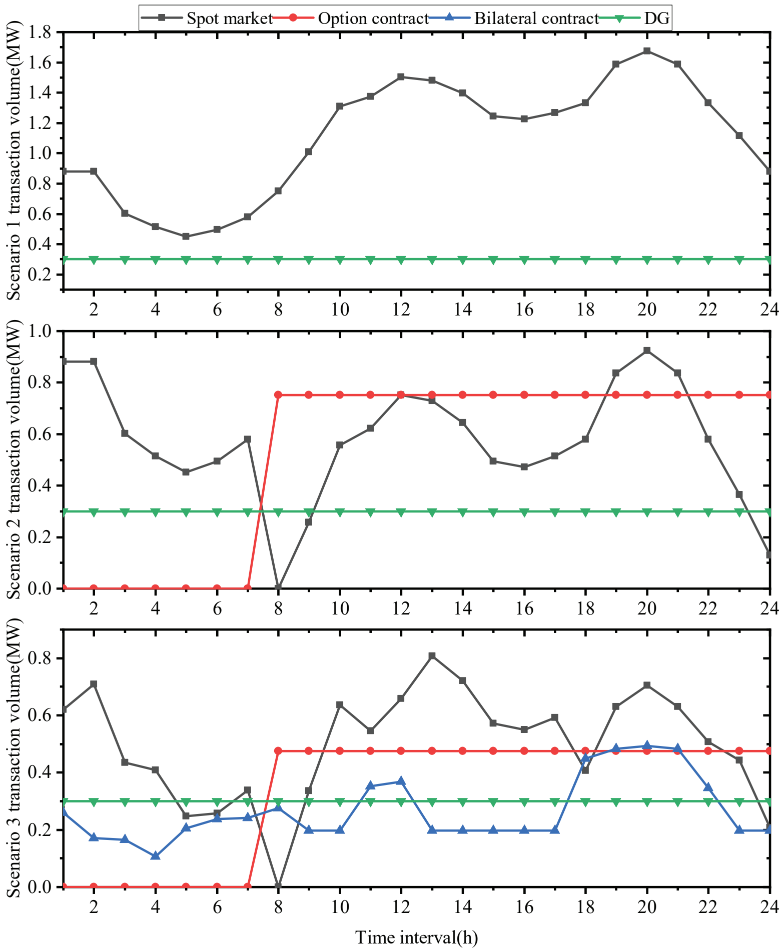

Figure 4. DSO electricity transaction strategy in different scenarios.

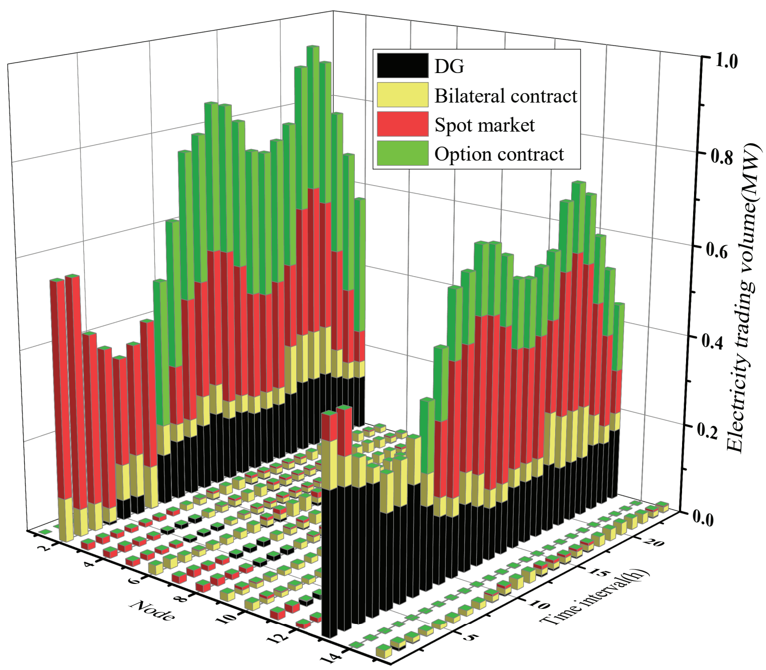

Figure 5. DSO electricity trading volume in each market.

5.2. Robust Optimization Model to Hedge Price Uncertainty of DSO with Multiple Transactions

Assuming that the risk aversion factor σ is 0.1 in scenario 3, the calculated resistible percentage of prediction error is 24.6% and the credibility is 0.83. In this case, if the forecast error percentage of spot price is within the range [0, 24.6%], the cost of DSO is less than or equal to USD 1821.16. If the forecast error percentage of spot price exceeds this range, the actual cost cannot be guaranteed. The electricity transaction strategy of DSO is shown in Figure 6. The credibility associated with the actual cost lower than the expected cost is 0.83, from which the decision maker can assess the risk of the trading strategy.

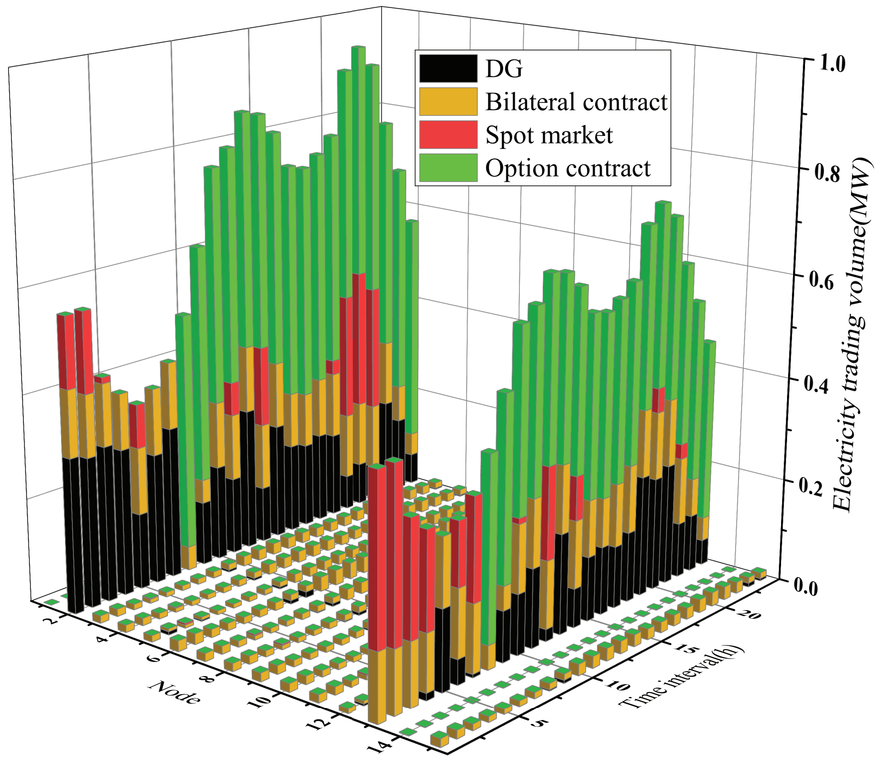

Figure 6. Electricity transaction strategy when the risk aversion factor is 0.1.

As can be seen in Figure 6, the load demand of DSO at non-peak period is mainly satisfied through DG output and electricity purchase in spot market, while at peak period it is mainly satisfied through option contract and power purchase in spot market. In 24 h, the option contract purchase is 13.77 MWh, accounting for 40.90%; the bilateral contract purchase is 9.99 MWh, accounting for 29.67%; the DG output is 6.76 MWh, accounting for 20.08%; and the spot market purchase is 3.15 MWh, accounting for 9.35%. Comparing with Figure 5, it can be seen that considering the uncertainty of spot price, the trading volume of the bilateral contract and the option contract increase, the spot market trading volume decreases and the output of DG decreases slightly.

The bilateral contract trading volume of each node under the uncertainty of spot price is shown in Table 4. We can see that the bilateral contract transaction is still mainly in nodes 2 and 13, but they choose contracts 1–10 to trade electricity and other nodes have also increased the trading volume of bilateral contract. Comparing with the electricity purchase strategy in the deterministic environment, it can be found that in order to reduce the risk caused by the uncertainty of the spot price, DSO increases the trading volume of bilateral contract and option contract.

Table 4. The bilateral contract transaction value of each node under the uncertainty of spot price.

Nodes	Contract Number									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	-	-	-	-	-	-	-	-	-	-
2	0.078	0.128	0.175	0.210	0.280	0.192	0.265	0.363	0.430	0.596
3	0	0.077	0	0	0	0	0	0	0	0.324
4	0	0.077	0	0	0	0	0	0	0	0.324
5	0.056	0	0	0	0	0	0	0	0.273	0
6	0	0	0	0	0.115	0	0	0	0	0.440
7	0	0.083	0	0	0	0	0	0	0	0.349
8	0	0.083	0	0	0	0	0	0	0	0.343
9	0	0	0	0.093	0	0	0	0	0	0.372
10	0	0	0	0.090	0	0	0	0	0	0.356
11	0	0.083	0	0	0	0	0	0	0	0.347
12	0.047	0	0	0	0	0	0.197	0	0	0
13	0.078	0.128	0.175	0.210	0.280	0.192	0.268	0.350	0.430	0.596
14	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0.088	0	0	0	0	0	0.349

5.3. The Influence of Different Risk Aversion Coefficients of DSO on Electricity Transaction

The curve of electricity transaction strategy with risk aversion factor σ is shown in Figure 7. As the value of σ increases, the purchase volumes of bilateral contract and option contract increase, the purchase volume in the spot market decreases and DG's output remains basically unchanged. The results show that as the expected cost increases, DSO increases the trading volumes of option contract and fixed-price bilateral contract, while reducing volume in spot market with uncertain price. In this way, the robustness of the electricity transaction strategy is increased.

The changes of robustness and credibility with risk aversion factor in different scenarios are shown in Table 5. It can be seen that the credibility increases as the risk aversion factor increases. This shows that the stronger the risk aversion awareness of DSO, the higher the credibility of the expected goal realization. This is because the greater the risk aversion factor, the higher the expected cost. The robustness factor increases with the increasing of expected cost. This shows that the greater the expected cost of the DSO, the lower the acceptance of risk. The more conservative the electricity purchase strategy, the stronger the ability of the resulting electricity transaction strategy to resist risk.

Table 5. The changes of robustness coefficient and credibility with risk aversion factors in different scenarios.

σ	Scenario 1		Scenario 2		Scenario 3	
	ε	**Credibility**	ε	**Credibility**	ε	**Credibility**
0	0	0.50	0	0.50	0	0.50
0.05	6%	0.55	7%	0.57	10%	0.61
0.1	11%	0.65	14%	0.70	25%	0.83

In scenario 1, DSO only trades electricity from the spot market. In the event of a bad price that is not conducive to the transaction, there is no electricity purchase plan that can replace or avoid market transactions and it has to accept the market risk caused by price uncertainty. Therefore, the system robustness of scenario 1 is lower than those of other scenarios.

In scenario 2, DSO purchases electricity through bilateral contract and the spot market. The use of fixed-price bilateral contract to purchase electricity avoids to a certain extent the market risk caused by the uncertainty of spot price.

In scenario 3, DSO conducts electricity transaction through spot market, option contract and bilateral contract. It has more means to actively control electricity purchase cost and possible risk losses through reasonable selection of transaction combination and allocation of electricity purchase ratio.

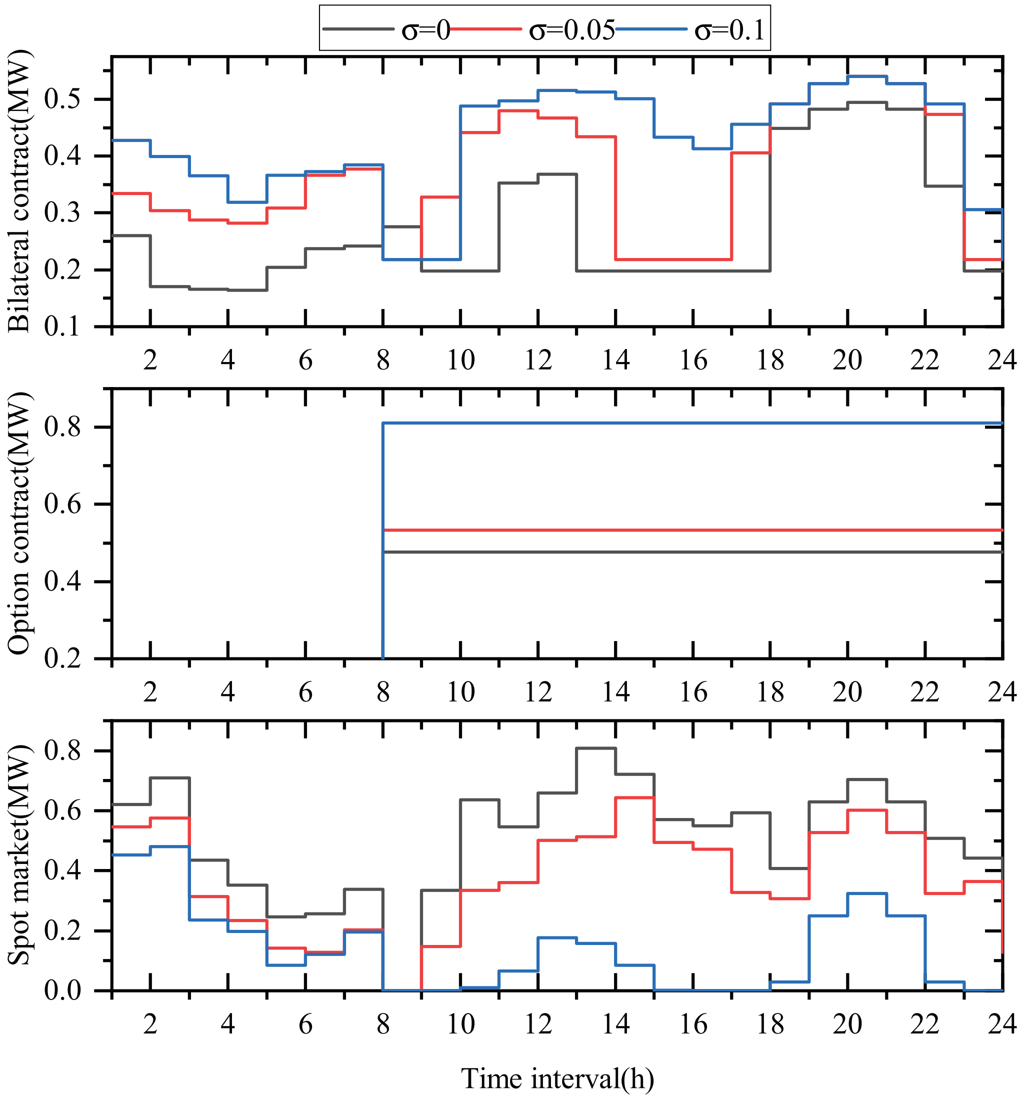

Figure 7. The change curve of electric energy trading volume with σ.

In addition, in order to fully demonstrate the effectiveness of the proposed model in dealing with an uncertain problem, this paper compares the proposed robust optimization model with the robust optimization model (RO) [46] and the stochastic optimization model (SO) [47]. The optimization results obtained by different optimization methods are shown in Table 6.

Table 6. Optimization results under different methods.

	RO	SO	The Proposed Model		
	-	-	σ = 0.04	σ = 0.05,	σ = 0.06
Operation cost	USD 1933.58	USD 1735.7	USD 1721.8	USD 1738.4	USD 1754.9
Optimization time	303.3 s	2310.8 s	527.5 s	461.1 s	694.3 s

It can be seen from the table that when the risk aversion factor is 0.05, the cost of the proposed model is reduced by 11.2% compared with robust optimization. This is due to the fact that RO is the worst-case cost of uncertain variables and the resulting electricity trading strategy is too conservative. Compared with random optimization, the solution speed is increased by 80.0%. In addition, compared with RO and SO, the proposed model considers the degree of risk aversion of the decision maker and the decision maker can choose the appropriate risk aversion factor according to their ability to bear the risk. In addition, in order for readers to better understand the proposed model, the credibility theory-based robust optimization model for a user is provided in Appendix A.

6. Conclusions

Based on the credibility theory, this paper establishes a robust optimization model to hedge price uncertainty of DSO with multiple transactions. This proposed model provides the electricity transaction strategy under different expected cost and the risk-averse DSO achieves the expected goal by rationally allocating the proportion of electricity purchases in different transaction markets. The results of calculation examples show that: (1) Increasing option contract and bilateral contract trading volumes can reduce the electricity transaction cost of DSO by USD 28.5. (2) As the expected cost increases (the degree of risk aversion of DSO increases), DSO will increase the purchase of electricity in option contract and bilateral contract, reduce the trading volume in spot market with uncertain price and increase the robustness of electricity transaction strategy. (3) The proposed robust model takes into account the risk aversion of decision maker and obtains the credibility of the expected goal realization. Compared with random optimization, the solution speed is increased by 80.0%. In addition, under the same risk aversion factor, the cost of the proposed model is reduced by USD 195.18 compared with robust optimization and avoids the over-conservatism of traditional robust optimization. This method provides new tools and ideas for electricity transaction decision maker and risk assessment.

This research work only considers the uncertainty of spot electricity price in electricity transaction. In fact, DSO also faces multiple uncertainties brought by renewable energy and demand. In future research work, we will study how to extend the proposed model to measure the multivariate uncertainty and uncertainty coupling. In order to achieve the goal of energy conservation and emission reduction, the impact of green certificates and carbon emissions trading on electricity trading strategy will be studied in the future.

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Nomenclature

Variables :

ε	forecast error percentage of spot price
α	credibility index
λ_t^{sm}	actual spot price, USD/MWh
C_{sm}	spot market power purchase cost, USD
C_{bc}	bilateral contract power purchase cost, USD
C_{oc}	option contract power purchase cost, USD
C_{dg}	power generation cost, USD
$p_{t,b}^{\mathrm{sm}}$	trading volume in the spot market, MW

$p^{\mathrm{bc}}_{t,b,n}$	trading volume of nth bilateral contract, MW
$p^{\mathrm{oc}}_{t,b}$	call option contract trading volume, MW
$p_{t,b}$	active power output of DG, MW
$s_{b,n}$	binary variable
Indexes :	
$b \in B$	range of network node
$l \in L$	range of network line
$t \in T$	range of time
$\lambda^{\mathrm{sm}\prime}_t$	forecast spot price, USD/MWh
ω	weighting factor
E_+, E_-	statistical average of positive and negative error percentages
λ^{bc}_n	electricity price with respect to bilateral contract, USD/MWh
$\lambda_{\mathrm{ck}}, \lambda_0$	strike price and premium of the call option, USD/MWh
λ^{dg}	power generation cost price of DG, USD/MWh
$P^{\max}_{t,b}$	maximum active power output of DG, MW
$P^{\mathrm{grid}}_{t,b}$	active power injected from the power grid, MW
$Q^{\mathrm{grid}}_{t,b}$	reactive power injected from the power grid, MW
$a_{t,l}$	square of the current of line l
$S_{t,l}$	upper limit of the apparent power of line l
$s(l)$	power outflow end of line l
$r(l)$	power inflow end of line l
σ	risk aversion factor
C_{e}	expected cost of DSO, USD
R_l, X_l	resistance and reactance of distribution network
G_b, B_b	admittance and conductance of distribution network
$p^{\mathrm{bc,min}}_n$	minimum contract volume of bilateral contract, MW
$p^{\mathrm{bc,max}}_n$	maximum contract volume of bilateral contract, MW
$f^{\mathrm{p}}_{t,l}, f^{\mathrm{q}}_{t,l}$	active and reactive power flow
$D^p_{t,b}, D^q_{t,b}$	active load and reactive load, MW
$v^{\min}_{t,b}, v^{\max}_{t,b}$	maximum and minimum values of node voltage

Appendix A

In this example, we assume that a user needs to buy electricity from the spot market, bilateral contracts, options contracts. The model is as follows:

Determine the optimization model

$$C = \sum_{t \in T} \left[\lambda^{\mathrm{sm}}_t p^{\mathrm{sm}}_{t,b} \right] + \sum_{t \in T_1} \sum_{n \in N} \left[\lambda^{\mathrm{bc}}_n p^{\mathrm{bc}}_{t,b,n} \right] + \sum_{t \in T_2} \left[\min\{\lambda_{\mathrm{ck}}, \lambda^{\mathrm{sm}}_t\} p^{\mathrm{oc}}_{t,b} + \lambda_0 p^{\mathrm{oc}}_{t,b} \right] \tag{A1}$$

$$p^{\mathrm{bc,min}}_n s_n \le p^{\mathrm{bc}}_{t,n} \le p^{\mathrm{bc,max}}_n s_n \tag{A2}$$

$$p^{\mathrm{sm}}_{t,} + \sum_{n \in N} p^{\mathrm{bc}}_{t,n} = P^{\mathrm{grid}}_t, \forall t \in T_1 \tag{A3}$$

$$p^{\mathrm{sm}}_t + \sum_{n \in N} p^{\mathrm{bc}}_{t,n} + p^{\mathrm{oc}}_t = P^{\mathrm{grid}}_t, \forall t \in T_2 \tag{A4}$$

where λ^{sm}_t and p^{sm}_t are the electricity price and trading volume of user in spot market during time t, respectively. λ^{bc}_n is the electricity price with respect to bilateral contract n, $p^{\mathrm{bc}}_{t,,n}$ is the trading volume of nth bilateral contract selected during time t. p^{oc}_t is the call option contract trading volume during time t, λ_{ck} and λ_0 are the strike price and premium of the call option, respectively. s_n is a binary variable. If user selects contract n, then $s_n = 1$, otherwise, $s_n = 0$. Equation (A1) represents the power purchase cost of the user, Equations (A2)–(A4) is the constraint on the user.

The proposed robust optimization model

$$\max|\varepsilon| \tag{A5a}$$

$$\text{s.t.}\begin{cases} \mathrm{Cr}(\max C(\lambda_t^{\mathrm{sm}},q) \leq C_{\mathrm{e}}) \geq \alpha & \text{(A5b)} \\ C_{\mathrm{e}} = (1+\sigma)C_0 & \text{(A5c)} \\ \lambda_t^{\mathrm{sm}} = (1+\varepsilon)\lambda_t^{\mathrm{sm}\prime} & \text{(A5d)} \\ 0 \leq \sigma \leq 1 & \text{(A5e)} \\ 0 \leq \alpha \leq 1 & \text{(A5f)} \\ (A2)-(A4) & \text{(A5g)} \end{cases}$$

where C_0 is the minimum cost of user when the spot price equals the forecasted spot price, σ is the risk aversion factor, which indicates the user's aversion to the risk due to the uncertain spot price, α is the credibility index and the physical meaning is equivalent to the probability confidence. Equation (A5b) is expressed as the credibility that the actual cost of user less than the expected cost is not less than α. Equation (A5c) represents the expected cost of user. When σ is larger, expected cost C_{e} is higher, indicating that user has a greater degree of risk aversion. q is the decision variable, which represents the amount of electricity traded by user in each market. When $\varepsilon \geq 0$, $\alpha \geq 1/2$, according to the credibility measure function and above theorem, the robust optimization model shown in Equation (A5) can be expressed as

$$\max K_\alpha \tag{A6a}$$

$$\text{s.t.}\begin{cases} C(\lambda_t^{\mathrm{sm}},q)|_{\lambda_t^{\mathrm{sm}}=(1+K_\alpha)\lambda_t^{\mathrm{sm}\prime}} \leq C_{\mathrm{e}} & \text{(A6b)} \\ C_{\mathrm{e}} = (1+\sigma)C_0 & \text{(A6c)} \\ 0 \leq \sigma \leq 1 & \text{(A6d)} \\ K_\alpha = \mu^{-1}(2(1-\alpha)) \geq 0 & \text{(A6e)} \\ 1/2 \leq \alpha \leq 1 & \text{(A6f)} \\ (A2)-(A4) & \text{(A6g)} \end{cases}$$

References

1. Johansson, P.; Vendel, M.; Nuur, C. Integrating distributed energy resources in electricity distribution systems: An explorative study of challenges facing DSOs in Sweden. *Util. Policy* **2020**, *67*, 101117. [CrossRef]
2. Tsaousoglou, G.; Giraldo, J.S.; Pinson, P.; Paterakis, N.G. Mechanism design for fair and efficient dso flexibility markets. *IEEE Trans. Smart Grid* **2021**, *12*, 2249–2260. [CrossRef]
3. Villanueva-Rosario, J.A.; Santos-García, F.; Aybar-Mejía, M.E.; Mendoza-Araya, P.; Molina-García, A. Coordinated ancillary services, market participation and communication of multi-microgrids: A review. *Appl. Energy* **2022**, *308*, 118332. [CrossRef]
4. Su, H.; Peng, X.; Liu, H.; Quan, H.; Wu, K.; Chen, Z. Multi-Step-Ahead Electricity Price Forecasting Based on Temporal Graph Convolutional Network. *Mathematics* **2022**, *10*, 2366. [CrossRef]
5. Xiang, D.; Wei, E. A general sensitivity analysis approach for demand response optimizations. *IEEE Trans. Netw. Sci. Eng.* **2020**, *8*, 40–52. [CrossRef]
6. Yang, S.; Tan, Z.; Liu, Z.; Lin, H.; Ju, L.; Zhou, F.; Li, J. A multi-objective stochastic optimization model for electricity retailers with energy storage system considering uncertainty and demand response. *J. Clean. Prod.* **2020**, *277*, 124017. [CrossRef]
7. Zhao, H.; Guo, S. Uncertain interval forecasting for combined electricity-heat-cooling-gas loads in the integrated energy system based on multi-task learning and multi-kernel extreme learning machine. *Mathematics* **2021**, *9*, 1645. [CrossRef]
8. Li, Y.; Ding, Y.; Liu, Y.; Yang, T.; Wang, P.; Wang, J.; Yao, W. Dense Skip Attention based Deep Learning for Day-Ahead Electricity Price Forecasting. *IEEE Trans. Power Syst.* **2022**. [CrossRef]
9. Shafiee, S.; Zareipour, H.; Knight, A.M. Developing bidding and offering curves of a price-maker energy storage facility based on robust optimization. *IEEE Trans. Smart Grid* **2017**, *10*, 650–660. [CrossRef]
10. Rodriguez, D.E.; Trespalacios, A.; Galeano, D. Risk transfer in an electricity market. *Mathematics* **2021**, *9*, 2661. [CrossRef]
11. Nazari-Heris, M.; Mohammadi-Ivatloo, B.; Gharehpetian, G.B.; Shahidehpour, M. Robust short-term scheduling of integrated heat and power microgrids. *IEEE Syst. J.* **2018**, *13*, 3295–3303. [CrossRef]
12. Gholami, A.; Shekari, T.; Grijalva, S. Proactive management of microgrids for resiliency enhancement: An adaptive robust approach. *IEEE Trans. Sustain. Energy* **2017**, *10*, 470–480. [CrossRef]

13. Najafi, A.; Pourakbari-Kasmaei, M.; Jasinski, M.; Lehtonen, M.; Leonowicz, Z. A max–min–max robust optimization model for multi-carrier energy systems integrated with power to gas storage system. *J. Energy Storage* **2022**, *48*, 103933. [CrossRef]
14. Nojavan, S.; Nourollahi, R.; Pashaei-Didani, H.; Zare, K. Uncertainty-based electricity procurement by retailer using robust optimization approach in the presence of demand response exchange. *Int. J. Electr. Power Energy Syst.* **2019**, *105*, 237–248. [CrossRef]
15. Nojavan, S.; Najafi-Ghalelou, A.; Majidi, M.; Zare, K. Optimal bidding and offering strategies of merchant compressed air energy storage in deregulated electricity market using robust optimization approach. *Energy* **2018**, *142*, 250–257. [CrossRef]
16. Yu, D.; Ebadi, A.G.; Jermsittiparsert, K.; Jabarullah, N.H.; Vasiljeva, M.V.; Nojavan, S. Risk-constrained stochastic optimization of a concentrating solar power plant. *IEEE Trans. Sustain. Energy* **2019**, *11*, 1464–1472. [CrossRef]
17. Shuai, H.; Fang, J.; Ai, X.; Tang, Y.; Wen, J.; He, H. Stochastic optimization of economic dispatch for microgrid based on approximate dynamic programming. *IEEE Trans. Smart Grid* **2018**, *10*, 2440–2452. [CrossRef]
18. Wozabal, D.; Rameseder, G. Optimal bidding of a virtual power plant on the Spanish day-ahead and intraday market for electricity. *Eur. J. Oper. Res.* **2020**, *280*, 639–655. [CrossRef]
19. Charwand, M.; Gitizadeh, M.; Siano, P. A new active portfolio risk management for an electricity retailer based on a drawdown risk preference. *Energy* **2017**, *118*, 387–398. [CrossRef]
20. Wang, K.; Zhu, Z.; Guo, Z. Optimal Day-Ahead Decision-Making Scheduling of Multiple Interruptible Load Schemes for Retailer With Price Uncertainties. *IEEE Access* **2021**, *9*, 102251–102263. [CrossRef]
21. Yu, B.; Guo, L.; Li, Q. A characterization of novel rough fuzzy sets of information systems and their application in decision making. *Expert Syst. Appl.* **2019**, *122*, 253–261. [CrossRef]
22. Si, A.; Das, S.; Kar, S. An approach to rank picture fuzzy numbers for decision making problems. *Decis. Making Appl. Manag. Eng.* **2019**, *2*, 54–64. [CrossRef]
23. Solis, A.R.; Panoutsos, G. Granular computing neural-fuzzy modelling: A neutrosophic approach. *Appl. Soft Comput.* **2013**, *13*, 4010–4021. [CrossRef]
24. Yan, H.; Luh, P.B. A fuzzy optimization-based method for integrated power system scheduling and inter-utility power transaction with uncertainties. *IEEE Trans. Power Syst.* **1997**, *12*, 756–763.
25. Moradi, M.H.; Eskandari, M. A hybrid method for simultaneous optimization of DG capacity and operational strategy in microgrids considering uncertainty in electricity price forecasting. *Renew. Energy* **2014**, *68*, 697–714. [CrossRef]
26. Chen, J.; Qi, B.; Rong, Z.; Peng, K.; Zhao, Y.; Zhang, X. Multi-energy coordinated microgrid scheduling with integrated demand response for flexibility improvement. *Energy* **2021**, *217*, 119387. [CrossRef]
27. Zhao, P.; Gu, C.; Huo, D.; Shen, Y.; Hernando-Gil, I. Two-stage distributionally robust optimization for energy hub systems. *IEEE Trans. Ind. Inform.* **2019**, *16*, 3460–3469. [CrossRef]
28. Zare, A.; Chung, C.; Zhan, J.; Faried, S.O. A distributionally robust chance-constrained MILP model for multistage distribution system planning with uncertain renewables and loads. *IEEE Trans. Power Syst.* **2018**, *33*, 5248–5262. [CrossRef]
29. Lacagnina, V.; Pecorella, A. A stochastic soft constraints fuzzy model for a portfolio selection problem. *Fuzzy Sets Syst.* **2006**, *157*, 1317–1327. [CrossRef]
30. Peng, X.; Tao, X. Cooperative game of electricity retailers in China's spot electricity market. *Energy* **2018**, *145*, 152–170. [CrossRef]
31. Ghazvini, M.A.F.; Soares, J.; Horta, N.; Neves, R.; Castro, R.; Vale, Z. A multi-objective model for scheduling of short-term incentive-based demand response programs offered by electricity retailers. *Appl. Energy* **2015**, *151*, 102–118. [CrossRef]
32. Li, R.; Wei, W.; Mei, S.; Hu, Q.; Wu, Q. Participation of an energy hub in electricity and heat distribution markets: An MPEC approach. *IEEE Trans. Smart Grid* **2018**, *10*, 3641–3653. [CrossRef]
33. Sun, B.; Wang, F.; Xie, J.; Sun, X. Electricity Retailer trading portfolio optimization considering risk assessment in Chinese electricity market. *Electr. Power Syst. Res.* **2021**, *190*, 106833. [CrossRef]
34. Soroudi, A.; Ehsan, M. IGDT based robust decision making tool for DNOs in load procurement under severe uncertainty. *IEEE Trans. Smart Grid* **2012**, *4*, 886–895. [CrossRef]
35. Golmohamadi, H.; Keypour, R. Stochastic optimization for retailers with distributed wind generation considering demand response. *J. Mod. Power Syst. Clean Energy* **2018**, *6*, 733–748. [CrossRef]
36. Yu, X.; Sun, Y. Trading risk control model of electricity retailers in multi-level power market of China. *Energy Sci. Eng.* **2019**, *7*, 2756–2767. [CrossRef]
37. Zhou, J.; Li, X.; Pedrycz, W. Mean-semi-entropy models of fuzzy portfolio selection. *IEEE Trans. Fuzzy Syst.* **2016**, *24*, 1627–1636. [CrossRef]
38. Gao, J.; Chen, J.; Qi, B.; Zhao, Y.; Peng, K.; Zhang, X. A cost-effective two-stage optimization model for microgrid planning and scheduling with compressed air energy storage and preventive maintenance. *Int. J. Electr. Power Energy Syst.* **2021**, *125*, 106547. [CrossRef]
39. Liu, B. A survey of credibility theory. *Fuzzy Optim. Decis. Mak.* **2006**, *5*, 387–408. [CrossRef]
40. Pinhão, M.; Fonseca, M.; Covas, R. Electricity Spot Price Forecast by Modelling Supply and Demand Curve. *Mathematics* **2022**, *10*, 2012. [CrossRef]
41. Farivar, M.; Low, S.H. Branch flow model: Relaxations and convexification—Part I. *IEEE Trans. Power Syst.* **2013**, *28*, 2554–2564. [CrossRef]

42. Low, S.H. Convex relaxation of optimal power flow—Part II: Exactness. *IEEE Trans. Control. Netw. Syst.* **2014**, *1*, 177–189. [CrossRef]
43. Jahangir, H.; Tayarani, H.; Baghali, S.; Ahmadian, A.; Elkamel, A.; Golkar, M.A.; Castilla, M. A novel electricity price forecasting approach based on dimension reduction strategy and rough artificial neural networks. *IEEE Trans. Ind. Inform.* **2019**, *16*, 2369–2381. [CrossRef]
44. Lu, M. On crisp equivalents and solutions of fuzzy programming with different chance measures. *Information-Yamaguchi* **2003**, *6*, 125–134.
45. Kim, J.; Dvorkin, Y. A P2P-dominant distribution system architecture. *IEEE Trans. Power Syst.* **2019**, *35*, 2716–2725. [CrossRef]
46. Baringo, L.; Conejo, A.J. Offering strategy via robust optimization. *IEEE Trans. Power Syst.* **2010**, *26*, 1418–1425. [CrossRef]
47. Mei, F.; Zhang, J.; Lu, J.; Lu, J.; Jiang, Y.; Gu, J.; Yu, K.; Gan, L. Stochastic optimal operation model for a distributed integrated energy system based on multiple-scenario simulations. *Energy* **2021**, *219*, 119629. [CrossRef]