


Article

The Generalization of the Brusov–Filatova–Orekhova Theory for the Case of Payments of Tax on Profit with Arbitrary Frequency

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Abstract: Both main theories of capital cost and capital structure—the Brusov–Filatova–Orekhova (BFO) theory and its perpetuity limit, the Modigliani–Miller theory—consider the payments of tax on profit once per year, while in real economy these payments are made more frequently (semi-annual, quarterly, monthly etc.). Recently the Modigliani–Miller theory has been generalized by us for the case of tax on profit payments with an arbitrary frequency. Here for the first time, we generalized the Brusov–Filatova–Orekhova (BFO) theory for this case. The main purpose of the paper is bringing the BFO theory closer to economic practice, taking into account one of the features of the real functioning of companies, the frequent payments of tax on profit. We derive modified BFO formulas and show that: (1) All BFO formulas change; (2) all main financial parameters of the company, such as company value, V , equity cost, k_e , and the weighted average cost of capital, $WACC$, depend on the tax on profit payments frequency. The increase of the frequency of payments of income tax leads to a decrease in the cost of attracting capital, $WACC$, and increase in the capitalization of the company, V . At a certain age n of the company and at certain frequency of tax on profit payments p , a **qualitatively new anomalous effect** takes place: the equity cost, $k_e(L)$, decreases with an increase in the level of leverage L . This radically changes the company's dividend policy, since the economically justified amount of the dividends is equal to the cost of equity. For both parties—for the company and for the tax regulator more frequent payments of tax on profit are beneficial: for the company, because this increases the company capitalization, and for the tax regulator, because earlier payments are beneficial for it due to the time value of money.

Keywords: generalized Brusov–Filatova–Orekhova (BFO) theory; frequency of payment of income tax; the Modigliani–Miller theory; the weighted average cost of capital; the equity cost; the company capitalization; a qualitatively new anomalous effect

MSC: 91B24



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1. Introduction

The paper has the following structure:

In Introduction, we describe some approaches of the capital structure theory: traditional approach, Modigliani–Miller theory, and some of its modifications.

We generalize the Brusov–Filatova–Orekhova (BFO) theory for the case of arbitrary frequency of payments of income tax and get generalized Brusov–Filatova–Orekhova (BFO)

formulas for all main company financial indicators: for the weighted average cost of capital, WACC, for the company value, V , for the equity cost, k_e .

Within the generalized Brusov–Filatova–Orekhova (BFO) theory, the dependence of the main financial parameters of the company (the weighted average cost of capital, WACC, equity cost, k_e , company capitalization, V) on leverage level L at different frequency of payments of income tax p is studied.

Obtained results allow coming to some very important conclusions and present recommendations on how frequently a company should pay income tax in order to decrease the cost of attracting capital and to increase its capitalization.

1.1. Capital Structure of the Company

This article is the first article of Special issue “Recent Development of Mathematical Methods in Financial Management”, written by the editors, which could be considered as an Editorial. It is devoted to one of the most important problems of financial management—problem of capital cost and capital structure of the company. Management by capital structure of the company (the relationship between equity and debt capital of the company) allows the company’s management to solve the main task—increasing the company value. This also relates with the problem of optimal capital structure, which is one of the most important problems that needs to be solved in financial management: i.e., with the capital structure, minimizing the weighted average cost of capital, WACC, and maximizing the company capitalization, V . The first quantitative study of influence of capital structure of the company on its (company) financial indicators was the work by Nobel Prize winners Modigliani and Miller (1958) [1]. Before their work, the traditional approach existed, based on empirical data analysis. In 2008 the modern capital cost and capital structure theory, Brusov–Filatova–Orekhova (BFO) theory, was developed [2], which made Modigliani–Miller theory its particular case. Within BFO theory many qualitatively new effects have been discovered, which are absent within the Modigliani–Miller theory. BFO theory has destroyed some main existing principles of financial management, among them the world famous trade off theory [3–5], which during many decades was considered as keystone of formation of optimal capital structure of the company. Brusov–Filatova–Orekhova have proven the bankruptcy of trade off theory and have found the cause of this (Section 4 in monograph [6]). Below we briefly describe the historical development of the theory of the capital cost and the capital structure.

Figure 1 shows the historical development of the theory of capital cost and capital structure (from the empirical traditional approach, through perpetuity Modigliani–Miller theory to general theory of capital cost and capital structure—Brusov–Filatova–Orekhova (BFO) theory [6]).

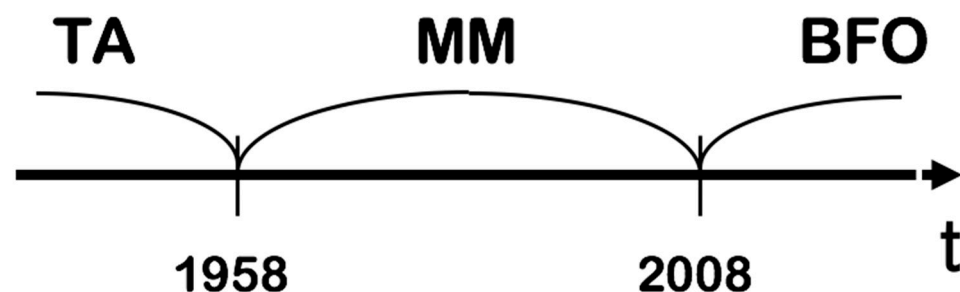


Figure 1. Historical aspects of capital cost and capital structure theory: here empirical traditional approach—TA, Modigliani–Miller theory—MM, Brusov–Filatova–Orekhova theory—BFO.

Myers [7] in 2001 considered the case of one-year company and showed that WACC in this case is bigger than in the case of Modigliani–Miller, and the company value, V , is less than in the case of Modigliani–Miller.

Before 2008 there were only two results for capital structure of the company: Modigliani–Miller one for perpetuity company [1,8,9] and Myers [7] for one-year company (see

Figure 2). Wherein the Brusov–Filatova–Orekhova theory filled out the whole interval between $t = 1$ and $t = \infty$. Brusov–Filatova–Orekhova theory allows calculating the company value, V , the weighted average cost of capital, WACC, the equity cost k_e , and other financial parameters for arbitrary age companies and for arbitrary life-time companies. A lot of new meaningful effects, which are absent in Modigliani–Miller theory [6] have been discovered in the BFO theory.

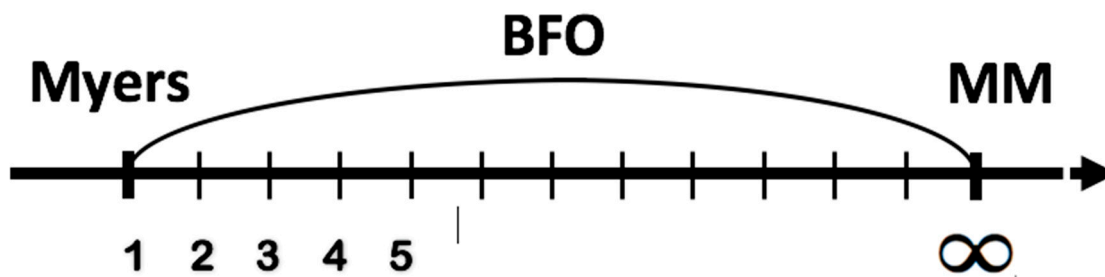


Figure 2. Myers result describes one-year company, Modigliani–Miller theory (MM) corresponds to perpetuity limit, while Brusov–Filatova–Orekhova (BFO) theory fills the whole numeric axis (from $n = 1$ up to perpetuity limit $n = \infty$).

The main purpose of this paper is to generalize and further develop the Brusov–Filatova–Orekhova (BFO) theory, taking into account the payment of tax on profit with an arbitrary frequency (monthly, quarterly, semi-annually, or annually), which takes place in real economic practice. Note that both major theories on the cost of capital and capital structure of the Brusov–Filatova–Orekhova (BFO) theory and its perpetuity limit, the Modigliani–Miller theory, take into account the annual income tax payments. The latter was recently generalized by us to the case of an arbitrary periodicity in the payment of income tax [10], which showed a significant change in the results of the classical Modigliani–Miller theory. Here we make this generalization for the Brusov–Filatova–Orekhova (BFO) theory.

Below we discuss the traditional approach, the Modigliani–Miller theory and its different modifications, the Brusov–Filatova–Orekhova (BFO) theory.

1.2. The Modigliani–Miller Theory

Before the Modigliani–Miller work, the traditional approach existed, based on empirical data analysis. In the traditional approach, weighted average cost of capital, WACC, and the company value, V , depend on the capital structure, on the leverage level, L . The cost of debt is lower than cost of equity via the fact that first one has lower risk, because creditor claims are met prior to shareholders claims in the event of bankruptcy. Thus, the increase of the share of lower-cost debt capital up to the values that do not violate the financial sustainability and do not lead to growth of risk of bankruptcy leads to lower weighted average cost of capital, WACC. The profitability, required by investors, which is equal to the equity cost, is growing; but its growth does not compensate for the benefits from use of lower-cost debt capital. Therefore, in the traditional approach, the increase of leverage level (at low leverage level) leads to decrease of WACC and to the associated increase of the value of company V . At high leverage level, financial difficulties and the risk of bankruptcy appear. The debt capital cost increases as well as the equity cost (which always increases with leverage level), and these lead to increase of WACC and decrease of company value V . Thus, competition between the advantages of debt financing at a low level of leverage and its disadvantages at a high level of leverage forms the optimal capital structure, when WACC reaches minimum and capital value V reaches maximum. This is the main point of the traditional approach and as well as trade off theory. This empirical approach existed up to creation of the first quantitative theory by Nobel Prize Winners Modigliani and Miller (1958) [1].

In their first paper [1] Modigliani and Miller (MM) have come to the conclusion, that choosing the ratio between the debt and equity capital does not affect the capitalization

of the company, V , as well as weighted average cost of capital, WACC. This conclusion was absolutely different from the results of traditional approach. The main assumptions of MM were that there are no taxes, no bankruptcy costs, no transaction costs, perfect financial markets with symmetry information exist, the borrowing costs are equal for both companies and investors, etc.

Modigliani and Miller analyzing the influence of financial leverage on V and V_0 , weighted average cost of capital, WACC and k_e , get the following results: (here, EBIT—earnings before interest and taxes, k_0 is discount rate, V and V_0 are the values of levered and unlevered companies, k_e is the equity cost)

$$V = V_0 = \frac{EBIT}{k_0} \tag{1}$$

This leads to the following expressions for weighted average cost of capital, WACC:

$$WACC = k_0 \tag{2}$$

Here k_0 is the equity cost for financially independent company. For financially dependent company, k_0 is the equity cost at zero leverage level ($L = 0$).

From (1), and formula for WACC definition

$$WACC = k_0 = k_e w_e + k_d w_d \tag{3}$$

One can derive the formula for the cost of equity capital, k_e .

Finding k_e , one gets

$$k_e = \frac{k_0}{w_e} - k_d \frac{w_d}{w_e} = \frac{k_0(S + D)}{S} - k_d \frac{D}{S} = k_0 + (k_0 - k_d) \frac{D}{S} = k_0 + (k_0 - k_d)L \tag{4}$$

Here,

D	debt capital value;
S	equity capital value;
$k_d, w_d = \frac{D}{D+S}$	debt capital cost and its share;
$k_e, w_e = \frac{S}{D+S}$	equity capital cost and its share;
$L = D/S$	leverage level
WACC	weighted average cost of capital.

Thus, we have the following formula for equity cost

$$k_e = k_0 + L(k_0 - k_d) \tag{5}$$

From (5) it follows that equity cost, k_e , linearly increases with leverage level L .

In 1963, Modigliani and Miller [8] accounted the tax on profit and got the following formula for the cost of financially dependent company, V ,

$$V = V_0 + DT, \tag{6}$$

where V_0 is the value of financially independent company, D is debt value, and T is the tax on profit.

Below we get the expression for WACC and the equity cost k_e under the existence of corporate taxes.

1.3. Weighted Average Cost of Capital, WACC

Substituting into (6) $D = w_d V$, one gets

$$V(1 - w_d T) = V_0 \tag{7}$$

Putting $V = \frac{CF}{WACC}$; $V_0 = \frac{CF}{k_0}$ into Formula (7), we get

$$\frac{CF}{WACC}(1 - w_d T) = \frac{CF}{k_0} \tag{8}$$

Here CF is the income of the company for one period.

From (8), we arrive to the expression for weighted average cost of capital, WACC

$$WACC = k_0 \cdot (1 - w_d t) \tag{9}$$

This formula for WACC is one of the main results of Modigliani–Miller theory with taxes. The equity cost, k_e

Let us derive formula for equity cost.

The weighted average cost of capital WACC in the presence of corporate taxes has the following form

$$WACC = k_0 w_e + k_d w_d (1 - T). \tag{10}$$

Equating Equations (9) and (10), one gets

$$k_0(1 - w_d T) = k_0 w_e + k_d w_d (1 - T) \tag{11}$$

From (11), we get the following expression for equity cost, k_e [8]:

$$\begin{aligned} k_e &= k_0 \frac{(1 - w_d T)}{w_e} - k_d \frac{w_d}{w_e} (1 - T) = k_0 \frac{1}{w_e} - k_0 \frac{w_d}{w_e} T - k_d \frac{D}{S} (1 - T) \\ &= k_0 \frac{D + S}{S} - k_0 \frac{D}{S} T - k_d \frac{D}{S} (1 - T) = k_0 + L(1 - T)(k_0 - k_d). \end{aligned} \tag{12}$$

The Formula (12) is different from Formula (5) (case without tax) only by the multiplier $(1 - T)$ in term, meaning a premium for risk. The multiplier $(1 - T)$ is less than unit, thus the corporate income tax leads to decrease of the slope of the curve $k_e(L)$ with respect to L -axis.

From Formulas (5), (9), and (12) we get the following conclusions. When leverage level grows, (1) value of company, V , increases; (2) WACC decreases from k_0 up to $k_0(1 - T)$ (at $L = \infty$); (3) equity cost increases linearly from k_0 up to infinity (Figure 3).

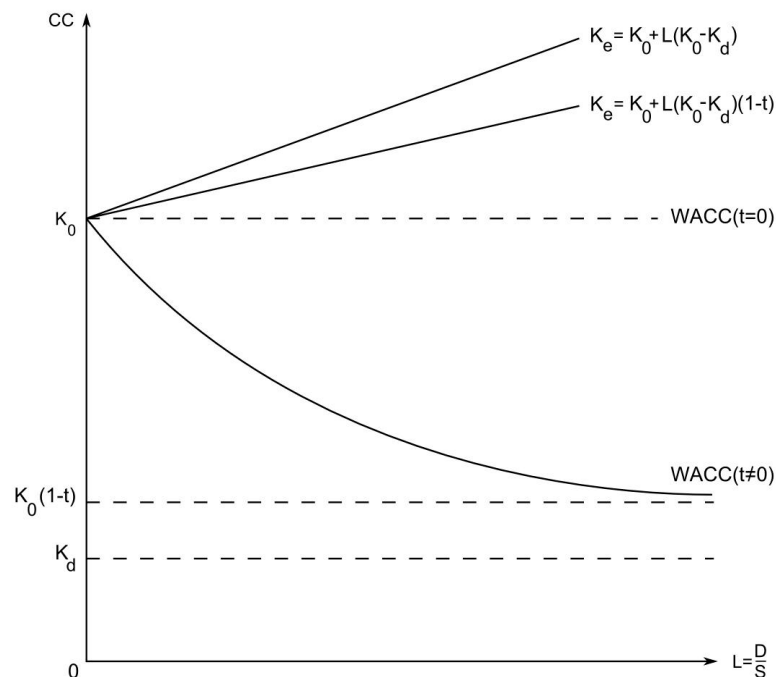


Figure 3. Impact of leverage level on equity capital cost k_e , and WACC within the Modigliani–Miller theory without taxes ($t = 0$) and with taxes ($t \neq 0$).

2. Some Modifications of Modigliani–Miller Theory

2.1. Hamada Model: Accounting Market Risk

The Modigliani–Miller theory were united with Capital Asset Pricing Model (CAPM) in 1969 [11] by R. Hamada, who has derived the following formula for the leveraged company equity cost, accounting both financial and business risk of company:

$$k_e = k_F + (k_M - k_F)b_U + (k_M - k_F)b_U \frac{D}{S}(1 - T), \quad (13)$$

where b_U is the β -coefficient of the same group of business risk company, that the considering company, but with $L = 0$. The Formula (13) describes the profitability of equity capital k_e and consists of a sum of three components: risk-free profitability k_F , which compensates a temporary value of their money to shareholders, premium for business risk $(k_M - k_F)b_U$, and premium for financial risk $(k_M - k_F)b_U \frac{D}{S}(1 - T)$. Without debt financing, the financial risk is equal to zero, and only the business risk premium will be received by shareholders.

2.2. The Account of Corporate and Individual Taxes (Miller Model)

In 1963, Modigliani and Miller [8] have accounted the taxation of corporate income, but the individual taxes of investors were not taken into account.

Both corporate and individual taxes has been taken into account by Merton Miller in 1977 [12], who studied the influence of debt financing on the company capitalization. In Miller model, the following definitions have been used: T_C —the rate of tax on corporate income, T_S —the tax rate on income of an individual investor from his ownership by corporation stock, T_D —tax on interest income from the provision of investor-individuals of credits to other investors and companies rate. The income from shares comes in the form of a dividend and as capital profits, so T_S is a weighted average value of the tax on capital profits on shares and on dividends rates. The profit from the provision of loans comes in the form of the interests. The former are usually taxed at a higher rate. For the capitalization of the financially independent company one has:

$$V_U = \frac{EBIT(1 - T_C)(1 - T_S)}{k_0}. \quad (14)$$

Term $(1 - T_S)$ accounts for the individual taxes. Numerator shows the part of the operating company's profit which remains in the possession of the investors, after the company pays taxes on income, and then shareholders pay individual taxes on profit from stock ownership. Since individual taxes reduce profits, remaining at investors, the last reduce as well an overall assessment of the capitalization of financially independent company.

2.3. More General Case for WACC Formula

In [13–16] more general than Modigliani–Miller (MM) formula for the WACC has been derived: it is described by the following formula [13]

$$WACC = k_0(1 - w_d T) - k_d t w_d + k_{TS} t w_d \quad (15)$$

Here k_0 , k_d , and k_{TS} are the expected returns respectively on the financially independent company, the debt and the tax shield, V is the capitalization of the financially dependent company, V_{TS} is the tax shield value, D is the debt value, and T_C is the rate of corporate tax.

Formula (15) is derived from the WACC definition and the balance identity (see as well Berk and De Marzo, 2007 [15]). Although Equation (15) is general enough, some additional conditions are required for its practical applicability. If the WACC is constant over time, as it is stated in [13], the levered company value can be found by discounting with the WACC for the unlevered free cash-flows. The resulting formulas, which describe the special cases, where the WACC is constant, can be found in textbooks [15,16].

First, Modigliani and Miller in 1963 [8] assume that the level of debt D is constant. Then, as the expected after-tax cash-flow of the financially independent company is fixed,

V_0 is also constant. By assumption, $k_{TS} = k_d$ and the tax shield value is $TS = tD$. Thus, for the financially dependent company capitalization, V , we arrive to the following formula for WACC instead of Formula (15):

$$WACC = k_0(1 - w_d T)$$

We think that “classical” Modigliani–Miller (MM) theory, suggesting the equality of the expected returns on the debt k_d and of the tax shield k_{TS} (via the fact that both of them have debt nature), is much more reasonable and namely the “classical” Modigliani–Miller (MM) theory, which is still widely used in practice, has been modified by us in [10].

2.4. Fiscal Pressure, Financial Liquidity, Financial Solvency and Financial Leverage

The problem of fiscal pressure is more current than ever in most countries. The article by Batrancea [17] was the first empirical study of the impact of fiscal pressure on the financial equilibrium of energy companies. Empirical results obtained by econometric models showed that fiscal pressure has stronger impacts on the short- and long-term equilibrium of oil and electricity companies than of gas companies. The research could be useful to managers of energy companies when estimating the evolution of equilibrium state of the company considering different possible financial crises.

How financial liquidity and financial solvency impact the performance of healthcare companies has been studied by Batrancea [18], who used econometric models with two-stage least squares (2SLS) panel and panel generalized method of moments (GMM). From an empirical evidence it follows that the financial parameters, such as current liquidity ratio, quick liquidity ratio, and financial leverage significantly influenced company performance measured by gross margin ratio, return on assets, operating margin ratio, earnings before interest, tax, depreciation, and amortization. Strategies were also addressed with the intention to improve business performance based on liquidity and solvency insights.

2.5. Brusov–Filatova–Orekhova (BFO) Theory

The suggestion about perpetuity of the companies and of all financial flows is the most serious limitations of the Modigliani–Miller theory. This limitation has been lifted up in 2008 by Brusov, Filatova, and Orekhova [4], who have created the modern theory of capital cost and capital structure, generalizing the Modigliani–Miller theory for the companies of arbitrary age (arbitrary life–time). They have shown that in this case, all Modigliani–Miller results [1–3] show significant changes: in the presence of corporative taxes the company value, V , is changed, as well as the equity cost, k_e , and the weighted average cost of capital, WACC. A number of qualitatively new effects in corporate finance, obtained in Brusov–Filatova–Orekhova theory [4], are absent in the Modigliani–Miller theory [1,8,9].

The Brusov–Filatova–Orekhova formula for weighted average cost of capital, WACC for the company of arbitrary age n , takes the following form [4]

$$\frac{1 - (1 + WACC)^{-n}}{WACC} = \frac{1 - (1 + k_0)^{-n}}{k_0 [1 - w_d T (1 - (1 + k_d)^{-n})]} \tag{16}$$

Here, S —the equity capital value, $w_d = \frac{D}{D+S}$ —the debt capital share; $k_e, w_e = \frac{S}{D+S}$ —the equity capital cost and its share, $L = D/S$ —leverage level.

Substituting in (14) $n \rightarrow \infty$, we easily arrive at the perpetuity (Modigliani–Miller) limit formula for weighted average cost of capital, WACC.

$$WACC = k_0(1 - w_d T) \tag{17}$$

Results of BFO theory are well-known in the world literature (for example, see references [19–25]). Some papers [25] use the BFO theory in practical calculations.

The similarities and differences between BFO and MM results are shown in Table 1:

Table 1. The similarities and differences between BFO and MM results.

	BFO	MM
Time parameter, n	arbitrary age of companies, arbitrary life–time of companies	perpetuity companies, perpetuity financial flows; $n = \infty$
Company age and life–time	Are different	Are the same and equal infinity
Dependence of all main financial indicators: WACC, V , k_e on time parameter, n	Allow study this dependences	Time parameter, n is absent
all main financial indicators: WACC, V , k_e depend on debt cost k_d	yes	none
WACC	Correct estimation; decreases with leverage level L	Underestimation; decreases with leverage level L
V	Correct estimation; increases with leverage level L	Overestimation; increases with leverage level L
Equity cost k_e	increases linearly with leverage level L	increases linearly with leverage level L
Equity cost k_e : the slope of the curve $k_e(L)$	Less than in MM theory	Bigger than in BFO theory
k_e : the slope of the curve $k_e(L)$	could be negative at $T > T^*$ (qualitatively new effect)	always is positive new effect is absent
Golden and silver age effects [6]	exist	absent

2.6. Trade–Off Theory

As we mentioned above, the main theory of optimal capital structure of the company during many decades was the world famous trade-off theory [3–5], which is still used now for decision on capital structure-making. However, in 2013, Brusov et al. [6] have proven the trade-off theory’s bankruptcy. In opposite to waiting result, it was shown that suggestion of risky debt financing which causes growth of credit rate near bankruptcy, does not lead to the weighted average cost of capital, WACC, growing, which still decreases with leverage level. Thus, the minimum in the WACC dependence on leverage level is absent as well. This means, that the world famous trade-off theory lacks an optimal capital structure. Brusov et al. in 2013 [6] provided the explanation of this fact by analyzing the equity capital cost dependence on the leverage level on the risky debt capital assumption.

3. Modification of the Brusov–Filatova–Orekhova (BFO) Theory for Companies with Frequent Payments of Tax on Income

Below we use the following definitions:

- D debt capital value
- S equity capital value
- $k_d, w_d = \frac{D}{D+S}$ debt capital cost and its share
- $k_e, w_e = \frac{S}{D+S}$ debt capital cost and its share
- $L = D/S$ leverage level
- WACC weighted average cost of capital
- k_0 is the equity cost for financially independent company;
- p is the number of payments of tax on profit per year;
- T is tax on profit.

3.1. Calculation of the Tax Shield

We start from the calculation of the tax shield within Brusov–Filatova–Orekhova theory for the case of p payments of tax on profit per year (payments are made at the end of

periods). The tax shield, TS , for period of n -years is equal to the sum of discounted values of benefits from the use of tax incentives

$$(TS)_n = \frac{k_d Dt}{p(1+k_d)^{1/p}} + \frac{k_d Dt}{p(1+k_d)^{2/p}} + \dots + \frac{k_d Dt}{p(1+k_d)^{np/p}} \tag{18}$$

We have a geometric progression with denominator $q = \frac{1}{(1+k_d)^{1/p}}$.
After summing the progression, one obtains:

$$(TS)_n = \frac{k_d Dt (1 - (1+k_d)^{-n})}{p(1+k_d)^{1/p} (1 - (1+k_d)^{-1/p})} = \frac{k_d Dt (1 - (1+k_d)^{-n})}{p((1+k_d)^{1/p} - 1)} \tag{19}$$

In the classical Brusov–Filatova–Orekhova theory (at $p = 1$):

$$(TS)_n = Dt (1 - (1+k_d)^{-n}) \tag{20}$$

It is easy to obtain this result from (19), putting the frequency of payments of tax on profit $p = 1$.

3.2. Derivation of the Modified BFO Formula for Weighted Average Cost of Capital (WACC)

We derive now the modified formula BFO for weighted average cost of capital (WACC) for the case of p payments of tax on profit per year (payments at the end of periods).

One has for the financially dependent company value V :

$$V = V_0 + (TS)_n \tag{21}$$

Here V_0 is the value of a financially independent company. Putting expression (19) for TS , one has:

$$V = V_0 + \frac{k_d Dt (1 - (1+k_d)^{-n})}{p((1+k_d)^{1/p} - 1)} \tag{22}$$

After substituting $D = w_d V$, we get:

$$V \left(1 - \frac{k_d w_d t (1 - (1+k_d)^{-n})}{p((1+k_d)^{1/p} - 1)} \right) = V_0 \tag{23}$$

Accounting that the values of financially dependent company, V , and financially independent company, V_0 , are respectively equal to

$$V = \frac{CF(1 - (1+WACC)^{-n})}{WACC}; V_0 = \frac{CF(1 - (1+k_0)^{-n})}{k_0} \tag{24}$$

one gets:

$$\frac{CF(1 - (1+WACC)^{-n})}{WACC} \cdot \left(1 - \frac{k_d w_d t (1 - (1+k_d)^{-n})}{p((1+k_d)^{1/p} - 1)} \right) = \frac{CF}{k_0} \cdot (1 - (1+k_0)^{-n}) \tag{25}$$

From here we derive now the modified formula BFO for weighted average cost of capital (WACC) for company of age n years for the case of p payments of tax on profit per year (payments at the end of periods).

$$\frac{1 - (1 + WACC)^{-n}}{WACC} = \frac{CF(1 - (1 + k_0)^{-n})}{k_0 \left(1 - \frac{k_d w_d t (1 - (1 + k_d)^{-n})}{p((1 + k_d)^{1/p} - 1)} \right)} \tag{26}$$

At $p = 1$ we get the **classical** BFO formula

$$\frac{1 - (1 + WACC)^{-n}}{WACC} = \frac{1 - (1 + k_0)^{-n}}{k_0 \cdot \left(1 - w_d t \left[1 - (1 + k_d)^{-n} \right] \right)} \tag{27}$$

3.3. Formulas for Capital Value, V , and Equity Cost, k_e

Below, in Section 3, we investigate the dependence of the weighted average cost of capital, WACC, capital value, V , equity cost, k_e , on leverage level L at different frequencies of payment of tax on profit p for three-year and six-year companies, using Microsoft Excel. For WACC, we use Formula (10) and for capital value, V , and equity cost, k_e , we use Formulas (28) and (29) respectively (see below).

Company of age n capitalization could be calculated by the following formula

$$V = \frac{CF \cdot (1 - (1 + WACC)^{-n})}{WACC} \tag{28}$$

k_e should be found from the equation

$$WACC = k_e w_e + k_d w_d (1 - t) \tag{29}$$

$$k_e = \frac{WACC}{w_e} - \frac{k_d (1 - t) w_d}{w_e} = WACC(1 + L) - L k_d (1 - t)$$

where one should substitute WACC in the Formula (26).

4. Results

In this section we study the dependence of the weighted average cost of capital, WACC, capital value, V , equity cost, k_e , on leverage level L at different frequencies of payment of tax on profit p for three-year and six-year companies, using Microsoft Excel. As we mentioned above, for WACC we use Formula (10) and for capital value, V , and equity cost, k_e , we use Formulas (11) and (13) respectively.

We use the following parameters: $k_0 = 0.2$; $k_d = 0.18$; $t = 0.2$; $n = 3$; 6 ; $CF = 100$.

4.1. Dependence of the Weighted Average Cost of Capital, WACC, Capital Value, V , Equity Cost, k_e , on Leverage Level L at Different Frequency of Payment of Tax on Profit p for Three-Year Company

4.1.1. Dependence of the Weighted Average Cost of Capital, WACC, on Leverage Level L at Different Frequency of Payment of Tax on Profit p for Three-Year Company

From Table 2 and Figure 4 it following that the weighted average cost of capital, WACC, decreases with leverage level L at any frequency of payments of tax on profit p . The difference between the WACC (L) curves is maximum when moving from annual ($p = 1$) to semi-annual ($p = 2$) income tax payments and decreases when moving from semi-annual ($p = 2$) to quarterly ($p = 4$) payments and from quarterly ($p = 4$) to monthly ($p = 12$) payments.

Table 2. Dependence of the weighted average cost of capital, WACC, on leverage level L at different frequency of payments of tax on profit p for three-year company.

L	w_d	WACC				
		$p = 1$	$p = 2$	$p = 4$	$p = 6$	$p = 12$
1	0.5	0.1749	0.1738	0.1732	0.1730	0.1728
2	0.666667	0.1664	0.1650	0.1642	0.1640	0.1637
3	0.75	0.1622	0.1605	0.1597	0.1594	0.1591
4	0.8	0.1596	0.1579	0.1570	0.1567	0.1564
5	0.833333	0.1579	0.1561	0.1552	0.1549	0.1545
6	0.857143	0.1567	0.1548	0.1539	0.1536	0.1532
7	0.875	0.1558	0.1539	0.1529	0.1526	0.1522
8	0.888889	0.1551	0.1532	0.1522	0.1518	0.1515
9	0.9	0.1545	0.1526	0.1515	0.1512	0.1509
10	0.909091	0.1541	0.1521	0.1511	0.1507	0.1504

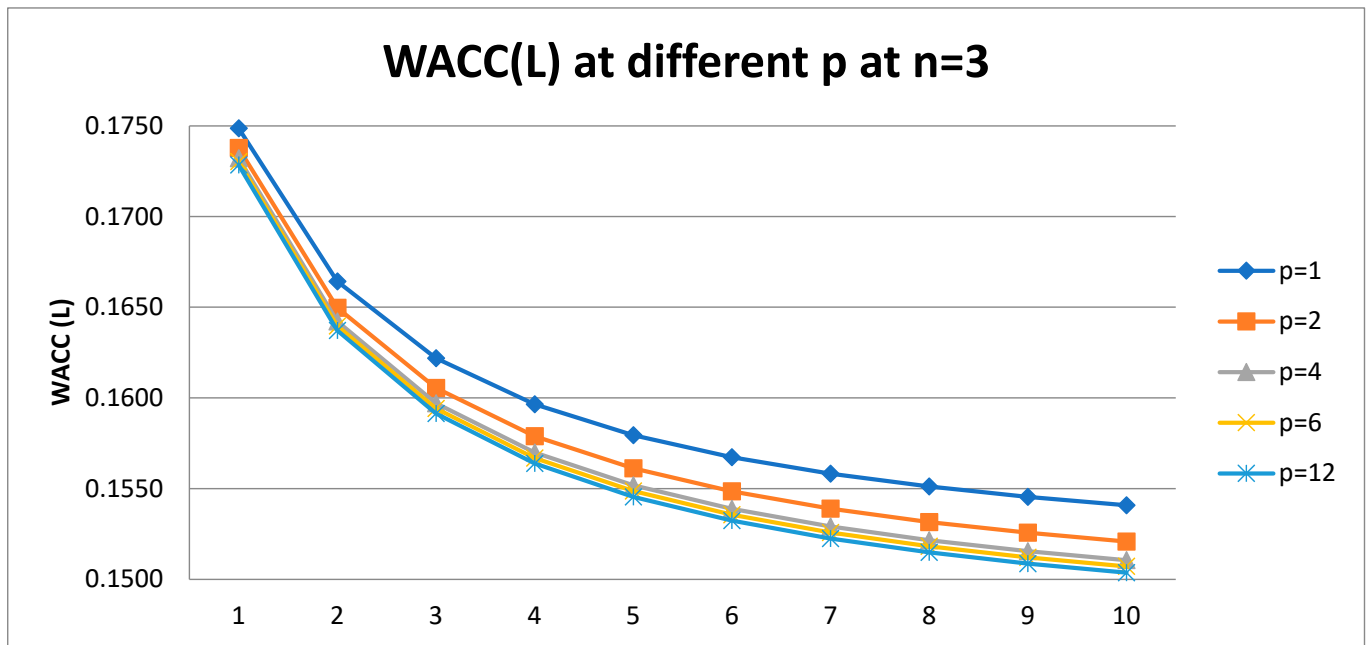


Figure 4. Dependence of the weighted average cost of capital, WACC, on leverage level L at different frequency of payments of tax on profit p for three-year company.

4.1.2. Dependence of the Company Value, V , on Leverage Level L at Different Frequency of Payment of Tax on Profit p for Three-Year Company

From Table 3 and Figure 5 it follows that the value of company V increases with the frequency p . The largest increase occurs when moving from annual ($p = 1$) to semi-annual ($p = 2$) income tax payments and decreases when moving from semi-annual ($p = 2$) to quarterly ($p = 4$) payments and from quarterly ($p = 4$) to monthly ($p = 12$) payments.

Table 3. Dependence of the company value, V , on leverage level L at different frequency of payments of tax on profit p for three-year company.

L	w_d	V				
		$p = 1$	$p = 2$	$p = 4$	$p = 6$	$p = 12$
1	0.5	219.2281	219.6139	219.8116	219.8781	219.9451
2	0.666667	222.2455	222.7746	223.0459	223.1373	223.2292
3	0.75	223.7856	224.3893	224.6989	224.8033	224.9082
4	0.8	224.7199	225.3694	225.7025	225.8149	225.9278
5	0.833333	225.3471	226.0275	226.3766	226.4944	226.6127
6	0.857143	225.7973	226.5000	226.8606	226.9822	227.1045
7	0.875	226.1361	226.8557	227.2250	227.3495	227.4747
8	0.888889	226.4003	227.1331	227.5091	227.6360	227.7635
9	0.9	226.6122	227.3555	227.7370	227.8657	227.9951
10	0.909091	226.7858	227.5378	227.9238	228.0540	228.1849

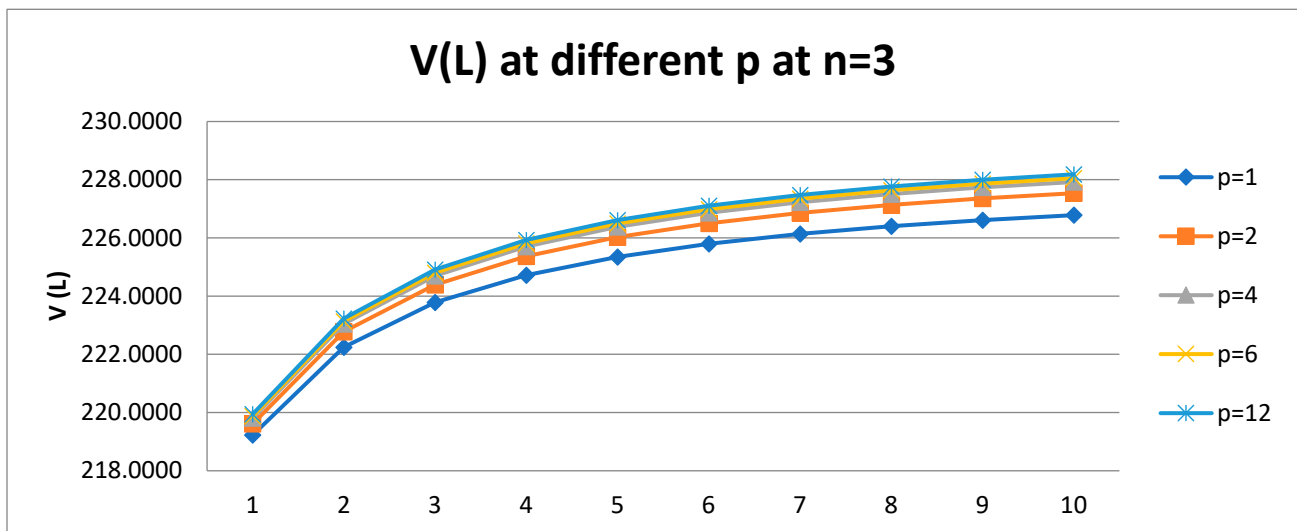


Figure 5. Dependence of the company value, V , on leverage level L at different frequency of payments of tax on profit p for three-year company.

4.1.3. Dependence of the Equity Cost, k_e , on Leverage Level L at Different Frequency of Payment of Tax on Profit p for Three-Year Company

From Table 4 and Figure 6 it following that the cost of equity k_e increases linearly with the level of leverage L . The slope of the curve $k_e(L)$ depends on the frequency of paying income tax: it decreases with increasing p , most rapidly when moving from annual ($p = 1$) to semi-annual ($p = 2$) payments of income tax and slower in the transition from semi-annual ($p = 2$) to quarterly ($p = 4$) payments and from quarterly ($p = 4$) to monthly ($p = 12$) payments.

Table 4. Dependence of the equity cost, k_e , on leverage level L at different frequency of payments of tax on profit p for three-year company.

L	w_d	k_e				
		$p = 1$	$p = 2$	$p = 4$	$p = 6$	$p = 12$
1	0.5	0.2057	0.2036	0.2024	0.2021	0.2017
2	0.666667	0.2113	0.2069	0.2046	0.2039	0.2031

Table 4. Cont.

L	w_d	k_e				
		$p = 1$	$p = 2$	$p = 4$	$p = 6$	$p = 12$
3	0.75	0.2168	0.2102	0.2068	0.2057	0.2045
4	0.8	0.2222	0.2134	0.2089	0.2074	0.2059
5	0.833333	0.2277	0.2167	0.2111	0.2092	0.2073
6	0.857143	0.2331	0.2199	0.2132	0.2109	0.2086
7	0.875	0.2386	0.2232	0.2153	0.2126	0.2100
8	0.888889	0.2440	0.2264	0.2174	0.2144	0.2113
9	0.9	0.2495	0.2296	0.2195	0.2161	0.2127
10	0.909091	0.2549	0.2329	0.2216	0.2178	0.2140

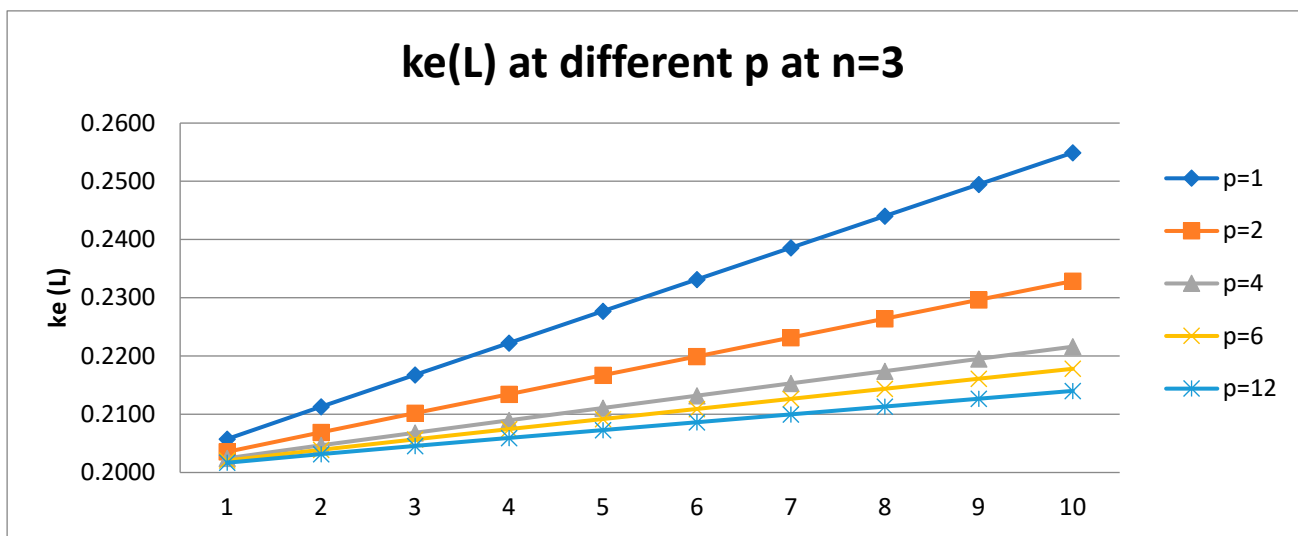


Figure 6. Dependence of the equity cost, k_e , on leverage level L at different frequency of payments of tax on profit p for three-year company.

4.2. Dependence of the Weighted Average Cost of Capital, WACC, Capital Value, V , Equity Cost, k_e , on Leverage Level L at Different Frequency of Payment of Tax on Profit p for Six-Year Company

4.2.1. Dependence of the Weighted Average Cost of Capital, WACC, on Leverage Level L at Different Frequency of Payment of Tax on Profit p for Six-Year Company

We use the following parameters: $k_0 = 0.2$; $k_d = 0.18$; $t = 0.2$; $n = 6$;

From Table 5 and Figure 7 it following that the weighted average cost of capital, WACC, decreases with leverage level L at any frequency of payments of tax on profit p . The difference between the WACC(L) curves is maximum when moving from annual ($p = 1$) to semi-annual ($p = 2$) income tax payments and decreases when moving from semi-annual ($p = 2$) to quarterly ($p = 4$) payments and from quarterly ($p = 4$) to monthly ($p = 12$) payments.

Table 5. Dependence of the weighted average cost of capital, WACC, on leverage level L at different frequency of payments of tax on profit p for six-year company.

L	w_d	WACC				
		$p = 1$	$p = 2$	$p = 4$	$p = 6$	$p = 12$
1	0.5	0.1743	0.1732	0.1726	0.1725	0.1723
2	0.666667	0.1657	0.1641	0.1634	0.1631	0.1629

Table 5. Cont.

<i>L</i>	<i>w_d</i>	WACC				
		<i>p</i> = 1	<i>p</i> = 2	<i>p</i> = 4	<i>p</i> = 6	<i>p</i> = 12
3	0.75	0.1613	0.1596	0.1587	0.1584	0.1581
4	0.8	0.1586	0.1568	0.1559	0.1556	0.1553
5	0.833333	0.1569	0.1550	0.1540	0.1537	0.1534
6	0.857143	0.1556	0.1537	0.1527	0.1523	0.1520
7	0.875	0.1547	0.1527	0.1517	0.1513	0.1510
8	0.888889	0.1539	0.1519	0.1509	0.1505	0.1502
9	0.9	0.1534	0.1513	0.1502	0.1499	0.1495
10	0.909091	0.1529	0.1508	0.1497	0.1494	0.1490

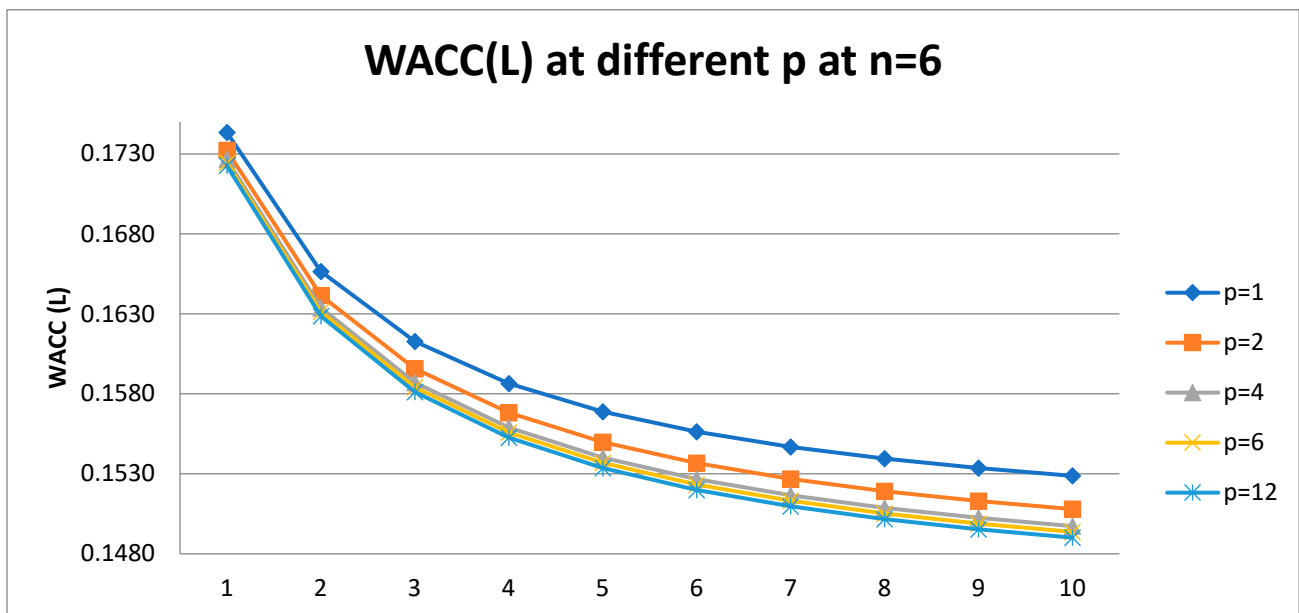


Figure 7. Dependence of the weighted average cost of capital, WACC, on leverage level *L* at different frequency of payments of tax on profit *p* for six-year company.

4.2.2. Dependence of the Company Value, *V*, on Leverage Level *L* at Different Frequency of Payment of Tax on Profit *p* for Six-Year Company

From Table 6 and Figure 8 it following that the value of company *V* increases with the frequency *p*. The largest increase occurs when moving from annual (*p* = 1) to semi-annual (*p* = 2) income tax payments and decreases when moving from semi-annual (*p* = 2) to quarterly (*p* = 4) payments and from quarterly (*p* = 4) to monthly (*p* = 12) payments.

Table 6. Dependence of the company value, *V*, on leverage level *L* at different frequency of payments of tax on profit *p* for six-year company.

<i>L</i>	<i>w_d</i>	<i>V</i>				
		<i>p</i> = 1	<i>p</i> = 2	<i>p</i> = 4	<i>p</i> = 6	<i>p</i> = 12
1	0.5	354.8940	355.9256	356.4549	356.6333	356.8127
2	0.666667	363.0242	364.4649	365.2052	365.4550	365.7063

Table 6. Cont.

L	w _d	V				
		p = 1	p = 2	p = 4	p = 6	p = 12
3	0.75	367.2305	368.8900	369.7435	370.0315	370.3214
4	0.8	369.8015	371.5971	372.5210	372.8329	373.1468
5	0.833333	371.5355	373.4240	374.3960	374.7242	375.0545
6	0.857143	372.7841	374.7399	375.7468	376.0869	376.4291
7	0.875	373.7261	375.7330	376.7664	377.1154	377.4667
8	0.888889	374.4620	376.5090	377.5632	377.9193	378.2776
9	0.9	375.0529	377.1322	378.2031	378.5648	378.9289
10	0.909091	375.5377	377.6435	378.7283	379.0947	379.4635

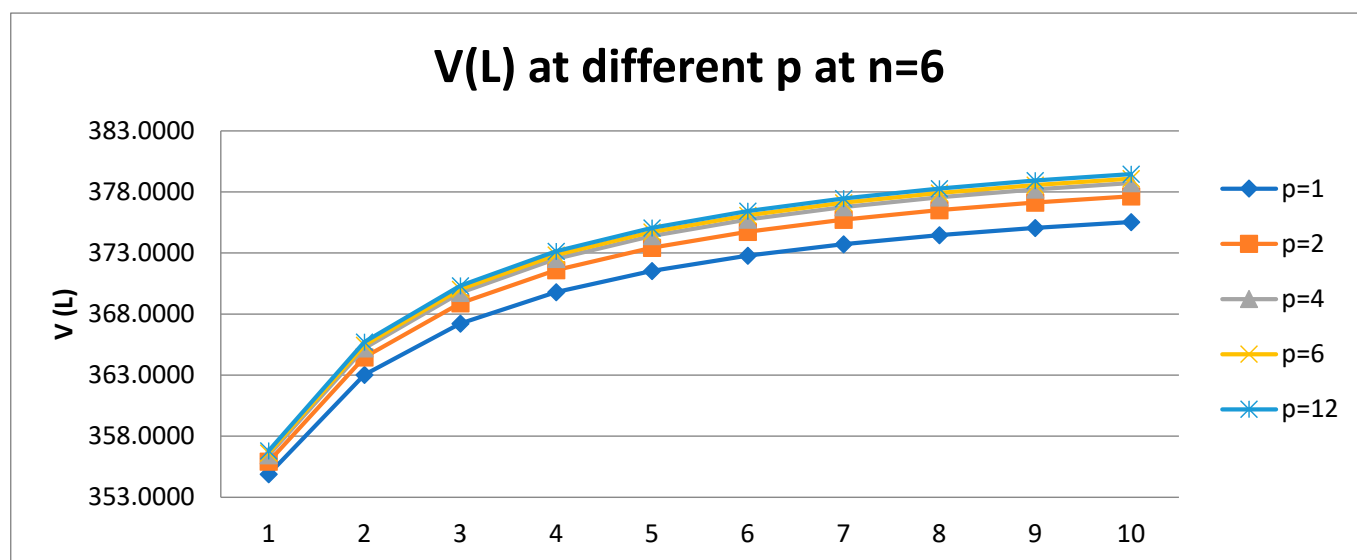


Figure 8. Dependence of the company value, V, on leverage level L at different frequency of payments of tax on profit p for six-year company.

4.2.3. Dependence of the Equity Cost, k_e , on Leverage Level L at Different Frequency of Payment of Tax on Profit p for Six-Year Company

From Table 7 and Figure 9 it following that the cost of equity k_e increases linearly with the level of leverage L. The slope of the curve $k_e(L)$ depends on the frequency of paying income tax: it decreases with increasing p, most rapidly when moving from annual ($p = 1$) to semi-annual ($p = 2$) payments of income tax and slower in the transition from semi-annual ($p = 2$) to quarterly ($p = 4$) payments and from quarterly ($p = 4$) to monthly ($p = 12$) payments.

One can see that for a six-year-old company with monthly income tax payments, **a qualitatively new anomalous effect takes place**: $k_e(L)$ decreases with an increase in the level of leverage L. This radically changes the company's dividend policy, since the economically justified amount of the dividends is equal to the cost of equity.

Table 7. Dependence of the equity cost, k_e , on leverage level L at different frequency of payments of tax on profit p for six-year company.

L	w_d	k_e				
		$p = 1$	$p = 2$	$p = 4$	$p = 6$	$p = 12$
1	0.5	0.2047	0.2024	0.2013	0.2009	0.2005
2	0.666667	0.2090	0.2044	0.2021	0.2013	0.2006
3	0.75	0.2131	0.2063	0.2028	0.2016	0.2005
4	0.8	0.2172	0.2081	0.2035	0.2019	0.2003
5	0.833333	0.2213	0.2099	0.2041	0.2021	0.2001
6	0.857143	0.2254	0.2117	0.2047	0.2023	0.1999
7	0.875	0.2294	0.2134	0.2052	0.2025	0.1997
8	0.888889	0.2335	0.2152	0.2058	0.2027	0.1995
9	0.9	0.2376	0.2169	0.2064	0.2029	0.1993
10	0.909091	0.2416	0.2187	0.2070	0.2030	0.1991

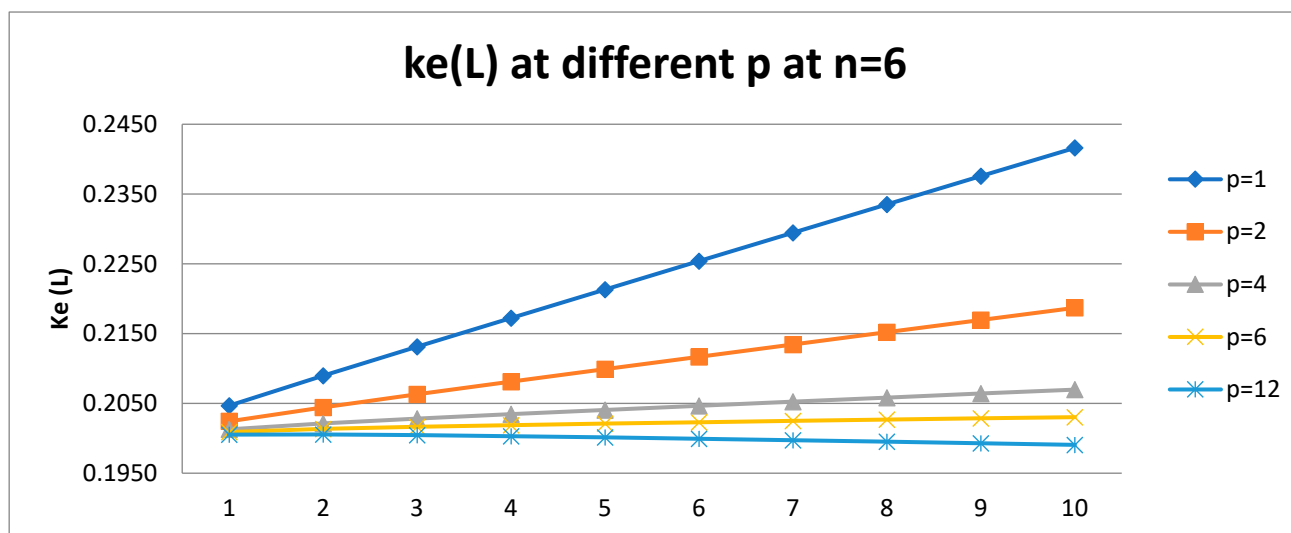


Figure 9. Dependence of the equity cost, k_e , on leverage level L at different frequency of payments of tax on profit p for six-year company.

5. The Discussion and Conclusions

The main purpose of the paper is to bring the Brusov–Filatova–Orekhova (BFO) theory closer to economic practice, taking into account one of the features of the real functioning of companies—the frequent payments of tax on profit. We generalize for the first time the Brusov–Filatova–Orekhova (BFO) theory for the case of tax on profit payments with an arbitrary frequency. We derive modified BFO formulas and show that: (1) All BFO formulas change; (2) all main financial parameters of the company, such as company value, V , the weighted average cost of capital, WACC, and equity cost, k_e , depend on the tax on profit payments frequency. The increase of the payments of tax on income frequency leads to a decrease in attracting capital cost and to an increase in the capitalization of the company. More frequent payments of tax on profit are beneficial for both parties—for the company and for the tax regulator; for the company, this leads to an increase in the company capitalization, and for the tax regulator, earlier payments are beneficial due to the time value of money.

We numerically studied the dependence of the weighted average cost of capital WACC, value of company, V , cost of equity k_e on the level of leverage L at different frequencies

of income tax payments p for three- and six-year companies and obtained the following results. The calculation of financial indicators are made for some typical parameters of enterprises (equity and debt costs etc.).

The weighted average cost of capital, WACC, decreases with leverage level L at any frequency of payments of tax on profit p . The difference between the WACC(L) curves is maximum when moving from annual ($p = 1$) to semi-annual ($p = 2$) income tax payments and decreases when moving from semi-annual ($p = 2$) to quarterly ($p = 4$) payments and from quarterly ($p = 4$) to monthly ($p = 12$) payments.

The value of company V increases with the frequency p . The largest increase occurs when moving from annual ($p = 1$) to semi-annual ($p = 2$) income tax payments and decreases when moving from semi-annual ($p = 2$) to quarterly ($p = 4$) payments and from quarterly ($p = 4$) to monthly ($p = 12$) payments.

The cost of equity k_e increases linearly with the level of leverage L . The slope of the curve $k_e(L)$ depends on the frequency of paying income tax: it decreases with increasing p , most rapidly when moving from annual ($p = 1$) to semi-annual ($p = 2$) payments of income tax and slower in the transition from semi-annual ($p = 2$) to quarterly ($p = 4$) payments and from quarterly ($p = 4$) to monthly ($p = 12$) payments.

At a certain age of the company, a **qualitatively new anomalous effect** takes place: $k_e(L)$ decreases with an increase in the level of leverage L . We obtain this result for a six-year-old company with monthly income tax payments, but it is clear that this effect should take place for the companies of different age and at different frequencies of income tax payment. This radically changes the company's dividend policy, since the economically justified amount of the dividends is equal to the cost of equity.

Note that this result reminds us of the discovery made by us earlier (Section 8 in monograph [6]): the abnormal dependence of equity cost, k_e , on leverage level L : above some value of tax on income, T^* , equity cost k_e decreases with the leverage level L .

Thus, it is interesting to note that an increase in frequency of income tax payments p and increase of tax on profit T lead to the similar abnormal effect: decrease of equity cost k_e with leverage level L . If in the first case there are threshold values for the frequency of paying income tax, p^* , and the age of the company, n^* , then in the second case there is a threshold value for income tax T^* .

In Table 8 we compare the impact of the frequency of income tax payments in BFO theory and in MM theory

Table 8. Comparison of the impact of the frequency of income tax payments in BFO theory and in MM theory.

	BFO	MM
theorems, statements, and formulas	change	change
all main financial indicators: WACC, V , k_e	depend on the frequency of tax on profit payments p	depend on the frequency of tax on profit payments p
all main financial indicators: WACC, V , k_e	value of the effect of p on all indicators is higher, than in MM	value of the effect of p on all indicators is smaller, than in BFO
all main financial indicators: WACC, V , k_e depend on debt cost k_d	at all p	at $p \neq 1$ only
WACC	decreases with p	decreases with p
V	increases with p	increases with p
k_e : the slope of the curve $k_e(L)$	decreases with increasing p	decreases with increasing p
k_e : the slope of the curve $k_e(L)$	could be negative (qualitatively new effect)	always is positive

6. Recommendations

From the analysis of the results of the current paper the following recommendations are made:

For company: company should pay tax on profit as frequently as it is possible, because in this case the company value increases.

For regulator: should encourage more frequent payments of tax on profit, because earlier payments are beneficial for budget due to time value of money.

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