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Spotting Error Patterns in Input–Output Projections Using Location Quotients

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Abstract: The Sustainable Development Goals (SDGs) stated by the United Nations (UN) constitute a universal agenda committed to human rights. In this context, mathematics can perform a fundamental role. Exploring possible contributions to these goals could be considered an interesting approach. Input–output (IO) tables provide detailed information for socio-economic quantifications. Thus, they allow for more precise policy decision-making and application in the SDG strategy. However, the smaller the subnational unit to be considered, the less statistical information that is available. Survey-based IO tables with large product/industry disaggregation are seldom published. Therefore, non-survey methods to estimate subnational IO tables based on the national are needed. These methodologies should yield optimal results. In the present investigation, different formulations for these non-survey regionalization methods are analyzed. The work focuses on the methodologies based on location quotients (*LQ*). As a result, some error patterns associated with current formulations present in literature are described. A slight refinement of these methodologies is proposed in order to improve the estimation's accuracy.

Keywords: regional input–output tables; location quotients; non-survey; 2D-*LQ*; *AFLQ*

MSC: 91B72

1. Introduction

The Sustainable Development Goals (SDGs) stated by the United Nations (UN) constitute a universal agenda committed to human rights. In this context, mathematics can perform a fundamental role in providing basic information for the evaluation of sustainable development strategies that can be established at different levels of territorial disaggregation. Exploring possible contributions to these goals could be considered an interesting approach. People collectively organize on the basis of different territories (nations). Moreover, nations are structured in subnational units (regions, counties, small areas, cities, etc.). In order to efficiently defend human rights, it is necessary to measure different items at these lower levels. For example, to achieve SDG 7 (Ensure access to affordable, reliable, sustainable, and modern energy for all), SDG 12 (Ensure sustainable consumption and production patterns), or especially SDG 8 (Promote inclusive and sustainable economic growth, employment, and decent work for all), it is key to know how a certain territory produces and/or consumes and to observe to what extent dependencies on non-sustainable energies appear. Within a nation, it is likely that different subnational units present heterogeneous behavior with considerable divergences. In this context, applying a homogeneous sustainable development policy will be ineffective in some sub-territories, while it will certainly favor the development of others. Thus, achieving the SDGs at both the national and the regional levels requires a rigorous impact assessment.

Input–output (IO) tables provide detailed information for socio-economic quantifications. Thus, they allow for more precise policy decision-making and application in the SDG strategy. However, the smaller the subnational unit to be considered, the less statistical information that is available. Survey-based IO tables with large product/industry disaggregation are seldom published at subnational levels. Implementing the IO methodologies at the regional or the local levels generally proves cumbersome due to the scarcity of accounting frameworks. Non-survey techniques are therefore commonly used to project nonexistent, incomplete, or outdated IO frameworks. A number of alternatives are used for this purpose [1–3], the most salient of which are *LQ*s. Numerous studies have mentioned the usefulness of these quotients [4–6]. There are, however, different *LQ* variants, and it is therefore essential to know which formulation is the most efficient. This debate is still open, with [3,7] favoring the prevalence of the Flegg formula (*FLQ*) or its modified version (*AFLQ*), while [3,8] advocate for other quotients. Pereira-López et al. [9,10] propose a two-dimensional reformulation of *LQ*s. Subsequently, they develop a performance of this new technique and the results support it as more efficient than previous formulations. Location quotients are at times also supplemented by adjustment techniques (variants of RAS or cross-entropy) that draw on aggregate information as a constraint [11–13].

The search for the most suitable *LQ* is justified through certain statistics that measure the degree of similarity between estimated matrices of technical coefficients and the true matrices. These statistics nevertheless measure the deviations between the projected and the true matrices in absolute value distances (or through quadratic expressions), and therefore mask relevant information that should be useful for testing non-survey methods, possibly even helping to bring about methodological improvements. Additionally, in certain cases the contrasts are not performed element by element, and they focus only on the column sums of the matrices. Therefore, they may incorporate an interpretive bias.

It is therefore considered appropriate to deal in a complementary way with possible estimation deviations according to their sign: overestimations and underestimations. Different deviation scenarios may occur for the same value of a statistic, and they may also be located in different parts of the matrix, certain rows or columns, or especially in the main diagonal. Focusing only on coefficients can also be misleading, and therefore the multipliers associated with these estimations should be tested.

Ideally, the rectification matrices should achieve a perfect estimation. For applied purposes, the available data render detecting such matrices possible and they can therefore be contrasted with the rectification matrices via *LQ* to ascertain the extent to which the construction of the different formulations fit the target to be projected. The application shall be carried out in relation to the technical (total) quotients with a view to ascertaining possible *LQ* variants rectifying coefficients upward. These modifications will not be compatible with the proposal of Jensen et al. [14].

Underestimates of IO coefficients are present in the *LQ* projections. Indeed, this is a problem since the default estimation of intermediate inputs implies erroneous impact quantifications. Thus, the tasks of designing and of implementing economic and social policies can be claimed to be optimal. Within the possible methodological extensions of the *LQ*s, the main diagonal elements can be estimated differently from the other elements. This could be a suitable alternative given that self-consumption generally accumulates a considerable portion of flows, although this would involve incorporating another parameter into the optimization process. In any case, this higher degree of exploitation of the given constraints is computationally salvageable. The 2D-*LQ* is taken as a reference, although possible extensions will be extrapolated to the other *LQ*s.

2. Literature Review

The main advantage of *LQ*s lies in their simplicity and the near real-time availability of information to quantify the share of regional requirements for a given sector in a specific region. However, projections can affect intermediate flows, coefficients, and even multipliers. In this case, the focus will be on the technical coefficients. Here, we mention the most common *LQ*s that stand out for their simplicity, even though we know that there are other—also basic—variants or more complicated generalizations.

The basic idea of the *LQ*s [14] is to estimate the regional technical coefficients (a_{ij}^{R}) by rectifications of the national coefficients (a_{ij}^{N}) through a multiplicative effect:

$$a_{ij}^{R} = a_{ij}^{N} LQ_{ij}, \quad i, j = 1, 2, \ldots, n. \tag{1}$$

The subscripts i and j refer to the purchasing and the supplying sectors, respectively. For the rest, a_{ij}^{R} is defined as the regional quantity of input i needed to produce one unit of product j, and the generic factor LQ_{ij} is associated with the share of the corresponding industry in regional trade.

The regional technical coefficients are constrained by the following criterion:

$$\begin{array}{ll} a_{ij}^{R} = a_{ij}^{N} LQ_{ij}, & if\ LQ_{ij} < 1 \\ a_{ij}^{R} = a_{ij}^{N}, & if\ LQ_{ij} = 1 \end{array} \tag{2}$$

Therefore, if the region is self-sufficient, the regional coefficient is exactly the same as the coefficient relative to the national intermediate consumption matrix. In contrast, if the region is a net importer, the regional coefficient will be lower than the national coefficient [15].

The *SLQ* is the most common approach, which compares the relative weight of a certain sectoral magnitude of a sub-territory with its relative weight in the territory. Analytically,

$$SLQ_i = \frac{wx_i^R}{wx^R} \tag{3}$$

where wx_i^R represents the weight of the production of sector i of region R in the production of sector i of the total economy and wx^R corresponds to the participation of the production of region R in the total production of the country. At the local level, it is very likely that disaggregated information by production is not available, therefore employment or even value-added data can be used as an alternative.

The above formulation only works on a row-by-row basis if the degree of specialization is less than 1. It is therefore feasible to enter projection information on a column-by-column basis as well. The Cross-Industry Location Quotient (*CILQ*) considers the relative importance of the selling industry with respect to the purchasing industry, as shown below:

$$CILQ_{ij} = \frac{SLQ_i}{SLQ_j} = \frac{wx_i^R}{wx_j^R} \tag{4}$$

Given that the formulation above excludes, for the sake of simplification, the size of the region in the process, Flegg and Webber [4], proposed the *FLQ* method, which is defined as follows:

$$FLQ_{ij} = CILQ_{ij} \cdot \left[\log_2\left(1 + wx^R\right)\right]^{\delta},\ 0 \leq \delta < 1 \tag{5}$$

The effect of region size is usually abbreviated as:

$$\lambda = \left[\log_2\left(1 + wx^R\right)\right]^{\delta}. \tag{6}$$

In this expression, the parameter δ is a coefficient associated with interregional imports, after which λ works as a corrective element of the *CILQ*. Following the standard procedure, the regional a_{ij}^R technical coefficients are the result of corrections to the national coefficients a_{ij}^N, namely:

$$\begin{array}{ll} a_{ij}^R = a_{ij}^N FLQ_{ij} & if\ FLQ_{ij} \leq 1 \\ a_{ij}^R = a_{ij}^N & if\ FLQ_{ij} > 1 \end{array} \tag{7}$$

Subsequently, Flegg and Webber [5] then rectified columns (semi-logarithmic smoothing) for specialized purchasing sectors. This resulted in the Augmented *FLQ* (*AFLQ*):

$$AFLQ_{ij} = \begin{cases} FLQ_{ij} \cdot \log_2(1 + SLQ_j) & if \quad SLQ_j > 1 \\ FLQ_{ij} & if \quad SLQ_j \leq 1 \end{cases} \tag{8}$$

Greater sectoral specialization thus leads to an increase in the coefficient, and a consequent reduction in imports.

Numerous studies address this in greater detail, including but not limited to [1,2,4,5,15,16].

In accordance with the logic used in the matrix updating processes, Pereira-López et al. [9] propose a two-dimensional approach (2D-*LQ*) to estimate domestic coefficients at the sub-territorial level. This technique can be extrapolated to other contexts, for example, to generate flow matrices, total coefficients, or multipliers.

This two-dimensional approach is represented by the following matrix expression:

$$\widetilde{A}^R = R(\alpha) A^N S(\beta) \tag{9}$$

where A is a matrix of intermediate domestic coefficients, and $R(\alpha)$ and $S(\beta)$ are diagonal matrices, whose elements appear on the main diagonal function as weighting factors.

The generic element of the projected matrix, $\widetilde{A}^R$, through the proposed alternative is as follows:

$$\widetilde{a}_{ij}^R = \begin{cases} (SLQ_i)^\alpha a_{ij}^N \left(wx_j^R \right)^\beta & if \quad SLQ_i \leq 1 \\ \left[\frac{1}{2} \tan\ \mathrm{h}(SLQ_i - 1) + 1 \right]^\alpha a_{ij}^N \left(wx_j^R \right)^\beta & if \quad SLQ_i > 1 \end{cases} \tag{10}$$

The scalars α and β are the influential parameters in the row and column corrections, respectively. This new approach breaks drastically with the *CILQ* and, consequently, with the *FLQ* and the *AFLQ*. The direct (α = 1) and the inverse (β = -1) relationships of the degrees of specialization are then leveled by smoothing, acting on the degrees of specialization and branch weights at the sub-territorial level. Another novelty occurs through activating the function, $\left[\frac{1}{2} \tan\ \mathrm{h}(SLQ_i - 1) + 1 \right]^\alpha$, which allows the coefficients to be slightly higher (when $SLQ_i > 1$) than those of the reference table. That is, it also breaks with the conditionals of the expression (2). However, the authors note that *LQ* techniques tend to underestimate inter-sectoral flows, especially *CILQ*, *FLQ*, and *AFLQ*. It is therefore appropriate to investigate methodological improvements in this area of work.

3. Improvement Opportunities in the State of Art

The above formulas provide corrective effects on a frame of reference. Unless specialists in this field of work are able to make an intuitive guess, it is difficult to ascertain which estimation errors are made when applying one *LQ* formula or another. In this regard, it is considered appropriate to illustrate the projections that would incur a null error, cell by cell, for four EA-19 countries, namely Austria, Belgium, Germany, and France. As an example, the calculations for Austria are available as a Supplementary Materials. In this regard, we downloaded a Eurostat database (see Supplementary Materials: Projection via *LQ*s for Austria 2015) table (https://ec.europa.eu/eurostat/web/esa-supply-use-input-tables/data/database, accessed on 25 March 2020) containing the symmetric matrices of total flows for 2015 at basic prices, 64 × 64 product per product [naio_10_cp1700]. While these four countries have different productive dimensions, they may be conducive to detecting possible common patterns. To include more countries would imply a perhaps too extensive development, although some more could be selected given that almost all accounting frameworks are available.

Figure 1 presents the values of the technical coefficients for both the reference territory, EA-19, and the selected countries (year of reference: 2015). Indirectly, one can see how self-consumption represents a substantial part of the intersectoral flows for the total and the chosen portions. We can also observe that some of these values for the four selected countries, especially for Austria and Belgium, exceed the EA-19 values. This discrepancy indicates a problem in the estimations, because the usual formulations do not provide for such upward projections, i.e., a priori, insurmountable errors accumulate.

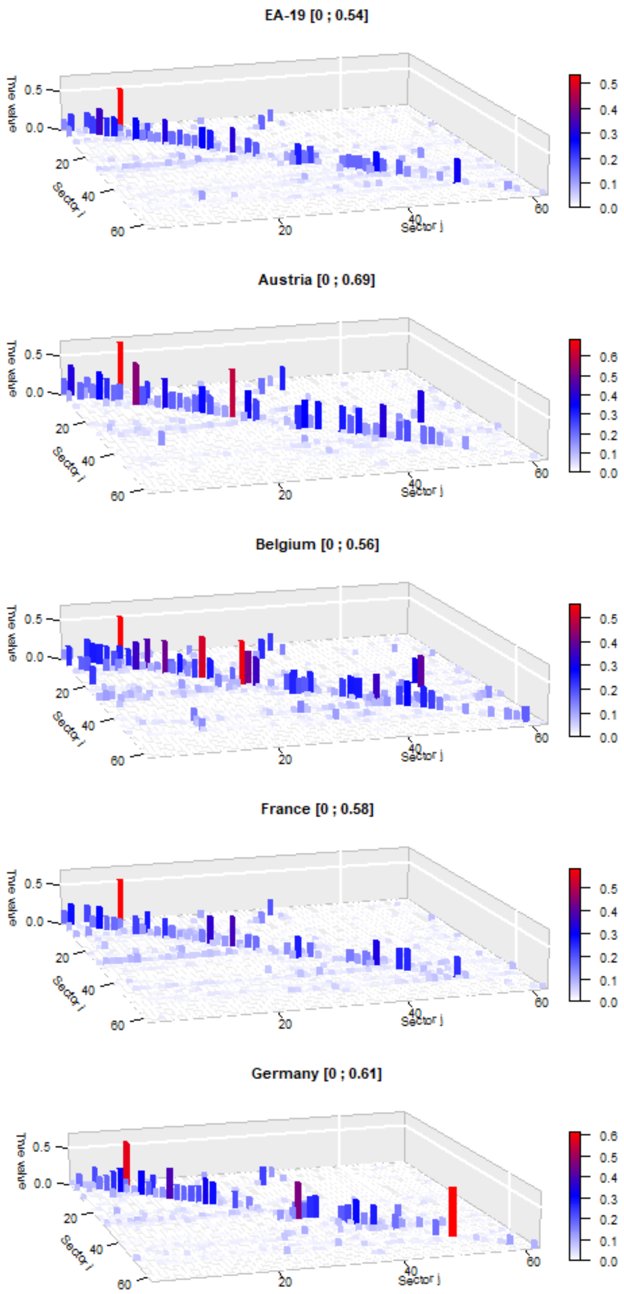

Figure 1. Viewing the true EA-19 technical coefficients (2015). Source: internal based on Eurostat data [naio_10_cp1700].

Matrix contrasts are performed using the Standardized Total Percentage Error (STPE), an overall measure obtained by means of the distances—through the absolute value—between estimated and true coefficients, relativized by the sum of the true coefficients [17–21].

$$\text{STPE} = 100 \sum_{i=1}^{n}\sum_{j=1}^{n}\left|\tilde{a}_{ij}^{R} - a_{ij}^{R}\right| \Big/ \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{R} \tag{11}$$

Other measures are also used in this area of work. Specifically, the mean absolute difference (MAD) [1,3,12,22,23], the mean absolute percentage error (MAPE) [7,21,24–26], the standard deviation of the mean absolute difference (SD-MAD) [22,27] and the Theil index (U) [7,17,19–22,28]. However, by using quadratic expressions, the optimization processes obtain identical or similar results.

For the sake of clarity, the information on the target LQ_{ij} and data derived by the *CILQ*, *AFLQ*, and 2D-*LQ* techniques for each cell are presented for the four indicated countries. The other formulas are excluded here because they are either slight modifications or previous proposals of some of those mentioned. Nevertheless, the cells were processed to prevent visual saturation of the rectifying effects. Thus, the technical coefficients with values below 0.02 were refined (assigning the color white beforehand). Then, the resulting LQ_{ij} were relativized to the optimal projections given by the *CILQ*, *AFLQ*, and 2D-*LQ*; namely by $\tanh\left(LQ_{ij} - 1\right) + 1$. The degrees of rectification, whether upward or downward, are thus easier to interpret, since they lie within the interval (0, 2), and the middle means that there is no rectification.

Accordingly, Figure 2 is incorporated for Austria. Initially, the necessary rectifications are represented by increments (red) and decrements (blue) to achieve a perfect projection via LQ_{ij} *target*. This will obviously not occur in practice, but this approach will be enormously beneficial. Thus, when entering the estimations via the *CILQ*, we observe that in the optimum (STPE = 59.97) all the rectifications are downward, and their number is not very high. No coefficient is corrected upward, in line with formula (9). In addition, the *CILQ* does not rectify the elements of the main diagonal, which already has a large proportion of the total. The *AFLQ* is a slight improvement with a STPE of 59.12. The 2D-*LQ* fails to reduce the value of the statistic, but the corresponding mapping is much closer to the desired result. In effect, several cells emerge in which increments are applied. The last method was designed to estimate domestic flows and here it was extrapolated to this context without altering the formulation.

Figure 3 shows the same analytical arrangement for Belgium, which bears a close resemblance to the previous, though other aspects arise. Always incurring underestimations, *CILQ* again fails to correct many cells. Here, we can see how the *AFLQ* applies some upward correction, which is in fact permitted by its formulation. The *FLQ* would not allow it, but the enhanced version may contemplate it in some cases. The error using the 3 *LQ* techniques decreases from *CILQ* to 2D-*LQ*, going from 62.73 to 60.65. This is a frequent peculiarity, though there are some exceptions, such as in the case of Austria.

Figure 4 depicts the projections for France, which are very similar to the previous country graphs. At the outset, there is a problem since the required rectifications would be upward, but none of the *LQ* methods perform these rectifications, except for the *AFLQ* (and only very slightly). Once again, the higher match in modified cells comes from the 2D-*LQ*, and it helps to reduce the statistic.

Turning to Germany, Figure 5 shows that the three *LQ*s used here succeed in rectifying certain cells, although the "ideal" objective is not achieved. The STPE gradually decreases from 51.43 for *CILQ* to 49.94 for 2D-*LQ*. As a common pattern in the projected accounting frameworks, *CILQ* acts on a much more limited number of cells than the other techniques, apart from not affecting the main diagonal. The 2D-*LQ* is surprisingly positive since it intervenes in the necessary cells, yet it does not perform the rectifications in the desired direction. This circumstance suggests the possibility of introducing additional restrictions, or conditioning factors, in the basic formulation and its subsequent variants.

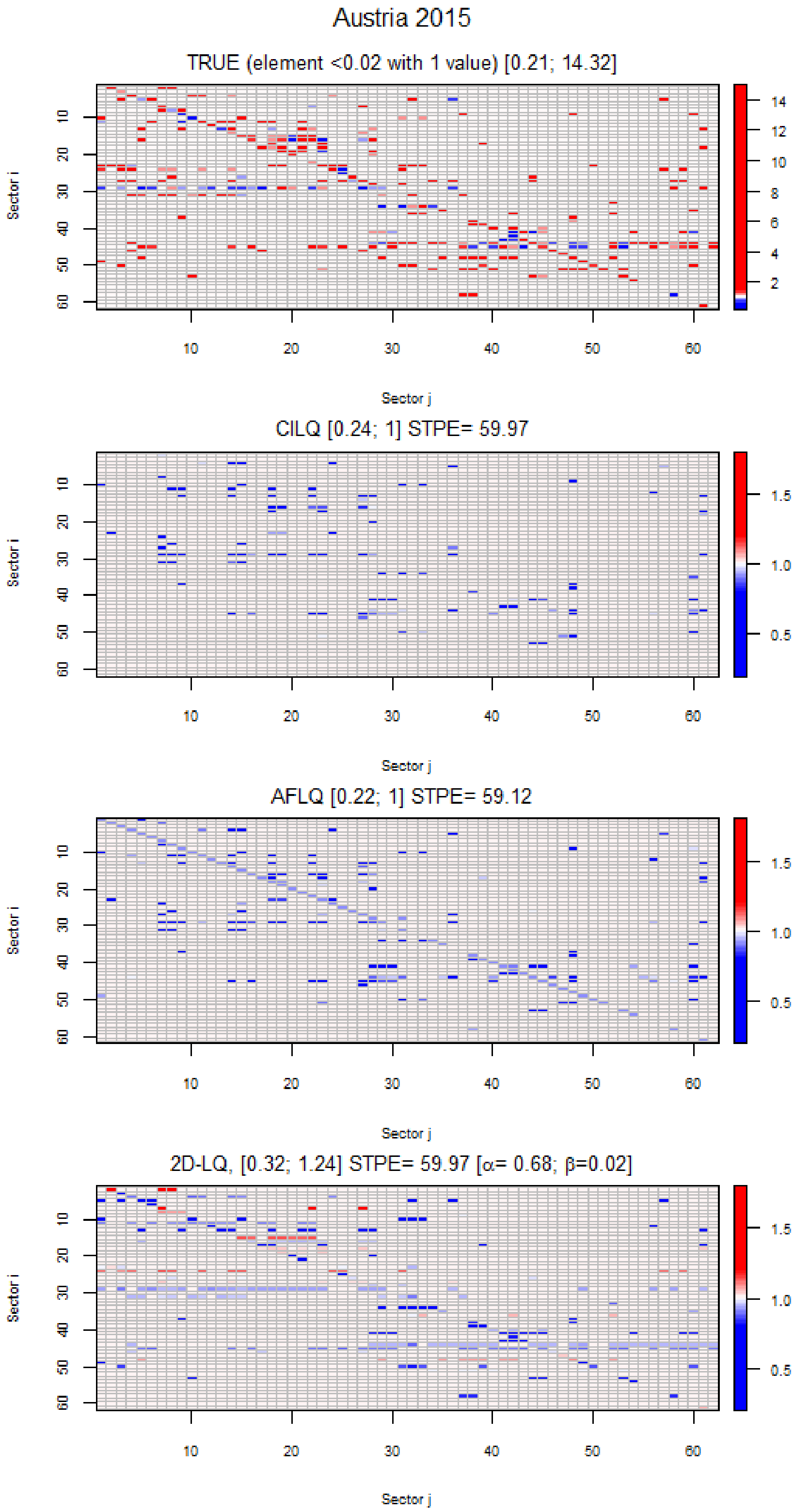

Figure 2. Projections of technical coefficients via *LQ* versus ideal rectifications (Austria). Source: internal based on Eurostat data [naio_10_cp1700].

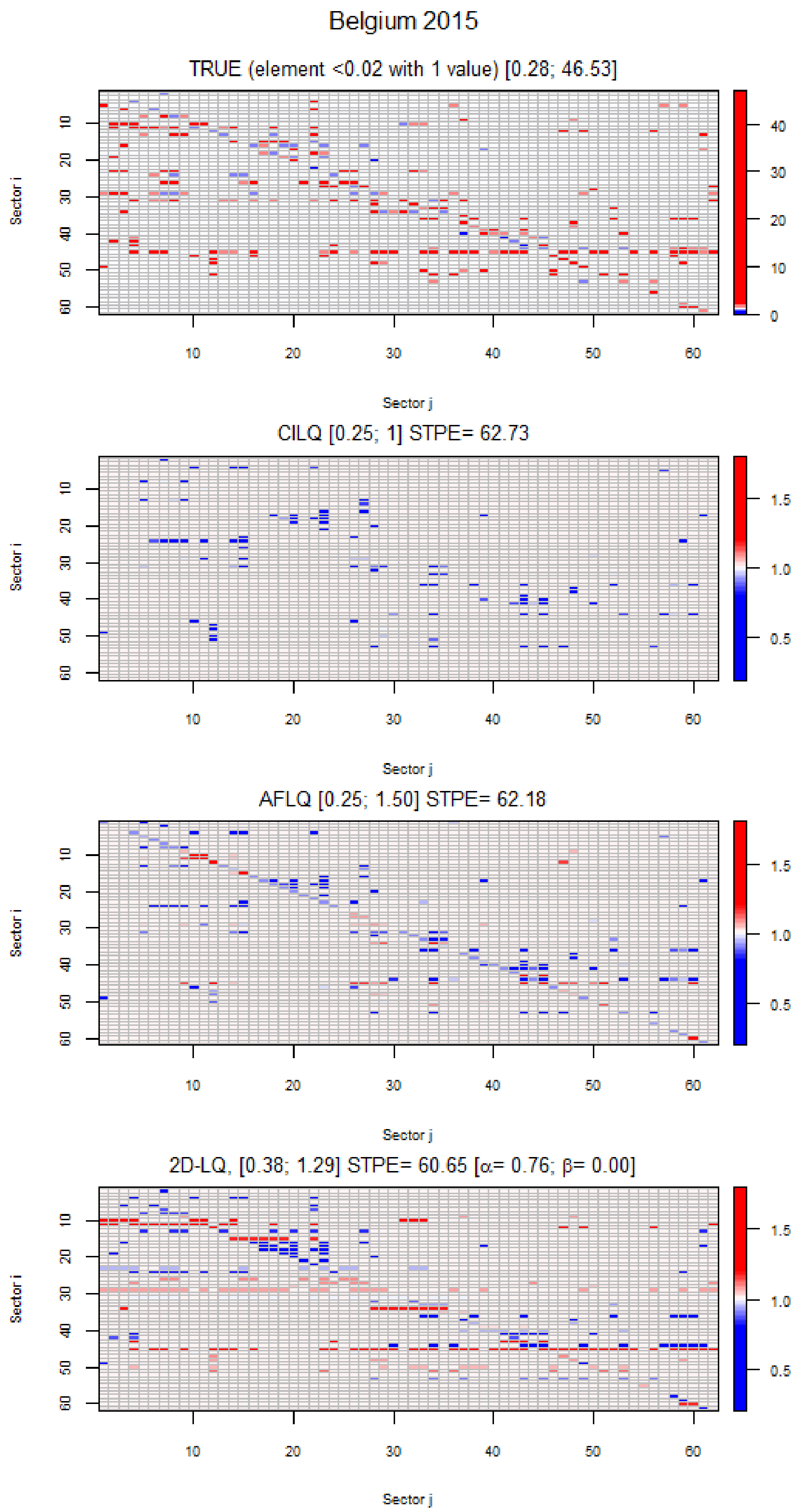

Figure 3. Projections of technical coefficients via *LQ* versus ideal rectifications (Belgium). Source: internal based on Eurostat data [naio_10_cp1700].

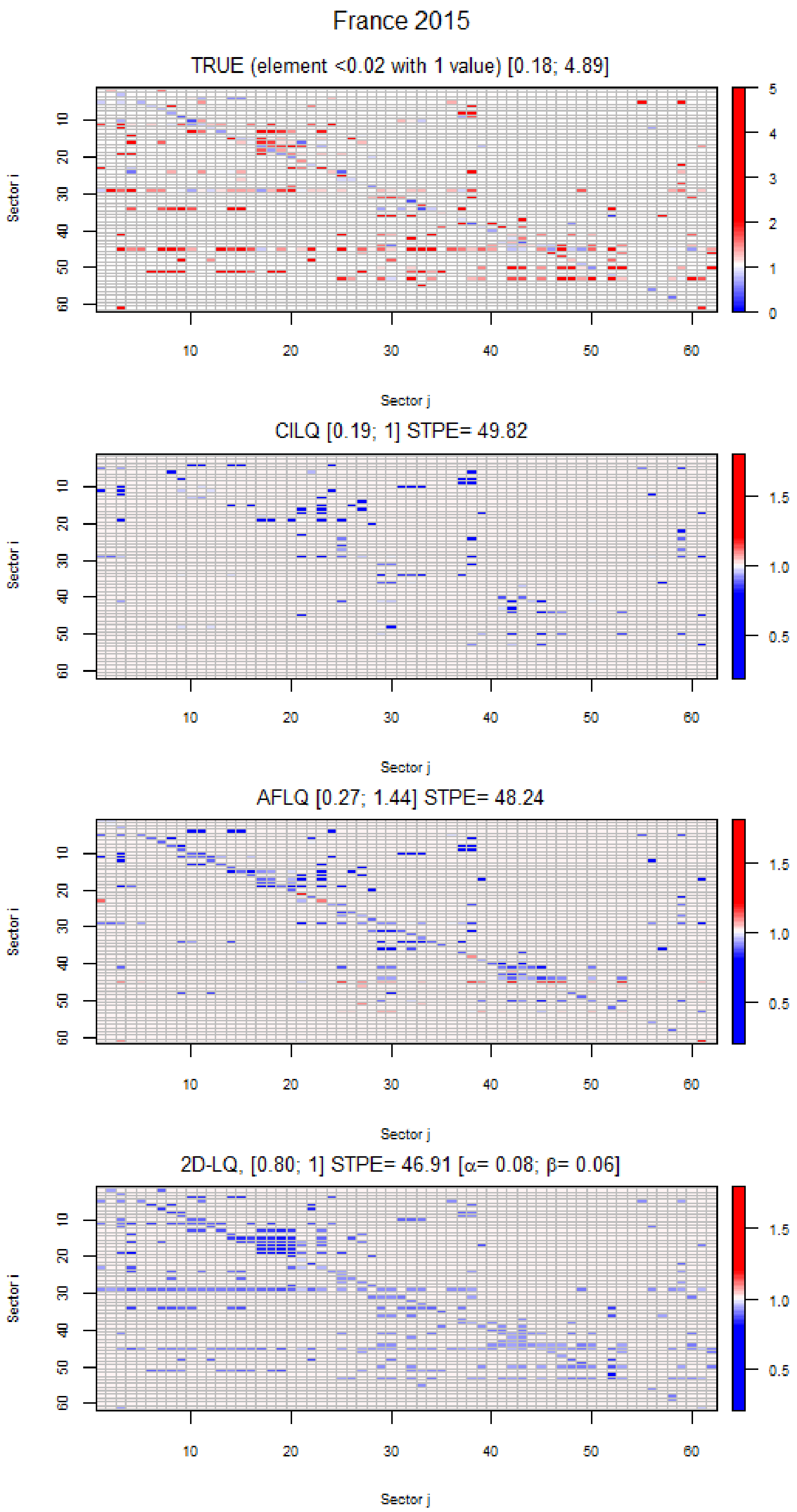

Figure 4. Projections of technical coefficients via *LQ* versus ideal rectifications (France). Source: internal based on Eurostat data [naio_10_cp1700].

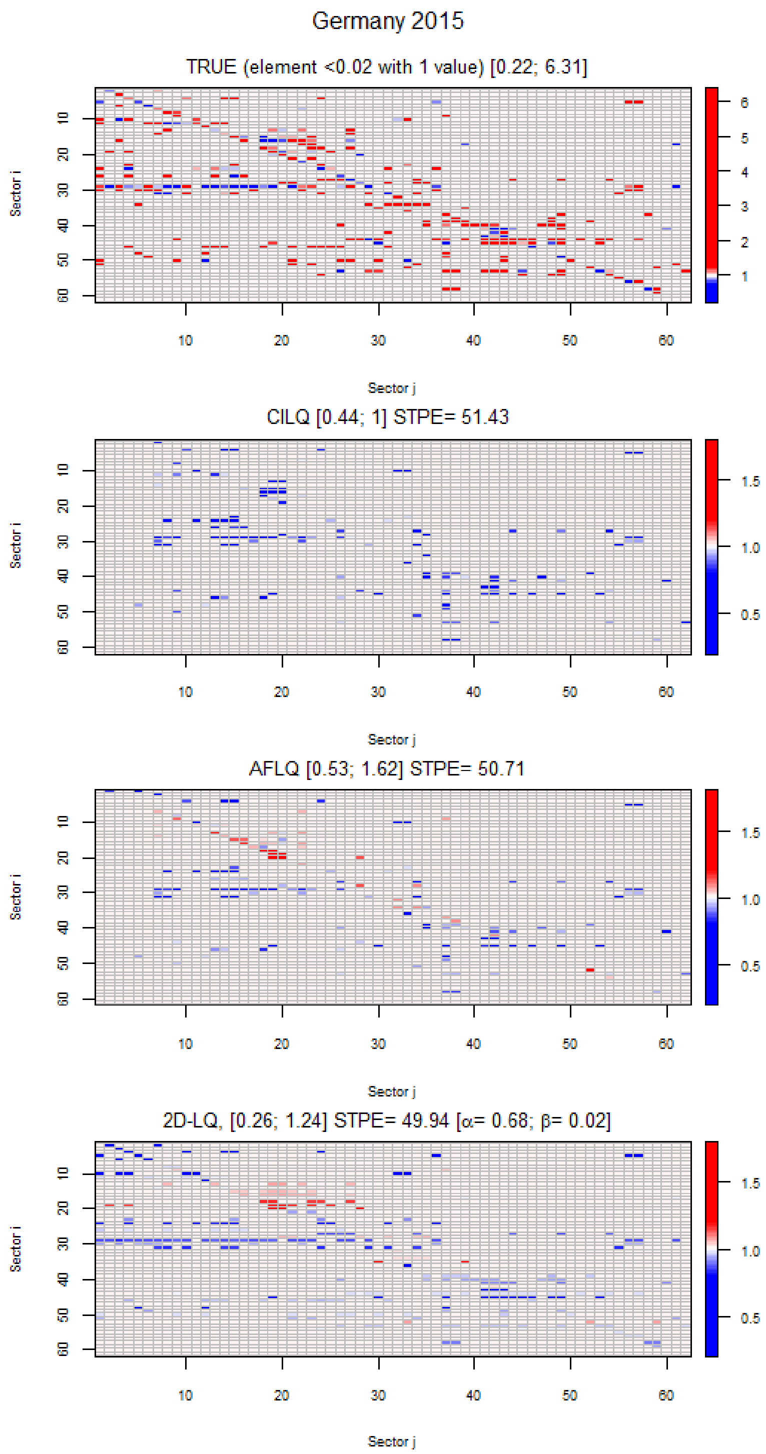

Figure 5. Projections of technical coefficients via *LQ* versus ideal rectifications (Germany). Source: internal based on Eurostat data [naio_10_cp1700].

4. Methodological Proposal

Based on the previous mappings, we can affirm that the erroneous projection of the elements of the main diagonal contributes decisively to an increase in the value of the statistics. Unlike the other techniques, *CILQ* does not consider the main diagonal elements, yet the rectifications almost always go against the desired direction, leaving the 2D-*LQ* better positioned. There is therefore a weakness in the *LQ*s open to review. Another estimation bias is caused by not raising any technical coefficient, or virtually none at all, in the disaggregation process. This gives rise to the characteristic underestimations. The first issue will be dealt with here because the second is easier to solve, as it would suffice to admit only slight corrections to the coefficients by means of "prudent" smoothing of the degrees of sector specialization.

Therefore, we propose the following modification to 2D-*LQ*:

$$\begin{array}{l} i \neq j;\ \widetilde{a}_{ij}^{R} = \begin{cases} (SLQ_i)^{\alpha} a_{ij}^{N} \left(wx_j^{R}\right)^{\beta}, & if\ SLQ_i \leq 1 \\ \left[\dfrac{1}{2}\tan\ \mathrm{h}(SLQ_i - 1) + 1\right]^{\alpha} a_{ij}^{N} \left(wx_j^{R}\right)^{\beta} & if\ SLQ_i > 1 \end{cases} \\ i = j;\ \widetilde{a}_{ij}^{R} = (SLQ_i)^{\gamma} a_{ij}^{N} \left(wx_j^{R}\right)^{\beta}, \qquad \forall SLQ_i \end{array} \tag{12}$$

Parameter γ (in relation to main diagonal elements) is thus incorporated by rows, and it is not linked to α, i.e., the elements of the main diagonal are arranged to correct differently from the others.

Incorporating a new parameter poses no great difficulty to solve the optimization process. There is clearly a need to derive a ternary $(\alpha^*,\ \beta^*, \gamma^*)$ to minimize the corresponding statistic, in this case the STPE.

5. Results

This section deals with the projections calculated for the four countries under study using the proposed 2D-*LQ* extension, though it could be perfectly extrapolated to the other *LQ* techniques. The ultimate goal is to determine the margin for improvement relative to previous *LQ*s.

An iterative procedure is chosen to find the ternary $(\alpha^*,\ \beta^*, \gamma^*)$ that minimizes the STPE. It is described below. Each iteration encompasses two phases. We start from the initial $\left(\alpha^{(0)},\ \beta^{(0)}\right)$ pair that optimizes the basic 2D-*LQ*. From there, the associated $\gamma^{(1)}$ is calculated, which will then be used to simultaneously recalculate $\left(\alpha^{(1)},\ \beta^{(1)}\right)$. The following iterations continue similarly until the optimal ternary $(\alpha^*,\ \beta^*, \gamma^*)$ is achieved by convergence. A graphic depiction of the search process for this ternary should be helpful. Figure 6 shows the iterative processes in the technical coefficient matrix for the selected countries. For the sake of clarity in the interpretation, Appendix A presents the data for these processes (Table A1 for technical coefficients and Table A2 for multipliers). Incorporating another parameter naturally modifies the values of α and β, albeit the latter only slightly. Convergence is immediate for Austria, and it materializes fairly quickly for the other three countries. In any case, the number of iterations is a minor issue because the results achieved are of greater interest. There is indeed a reduction in STPE in all the cases tested. Table A3 in Appendix A presents the optimal parameters for the different *LQ*s, including those related to this methodological extension.

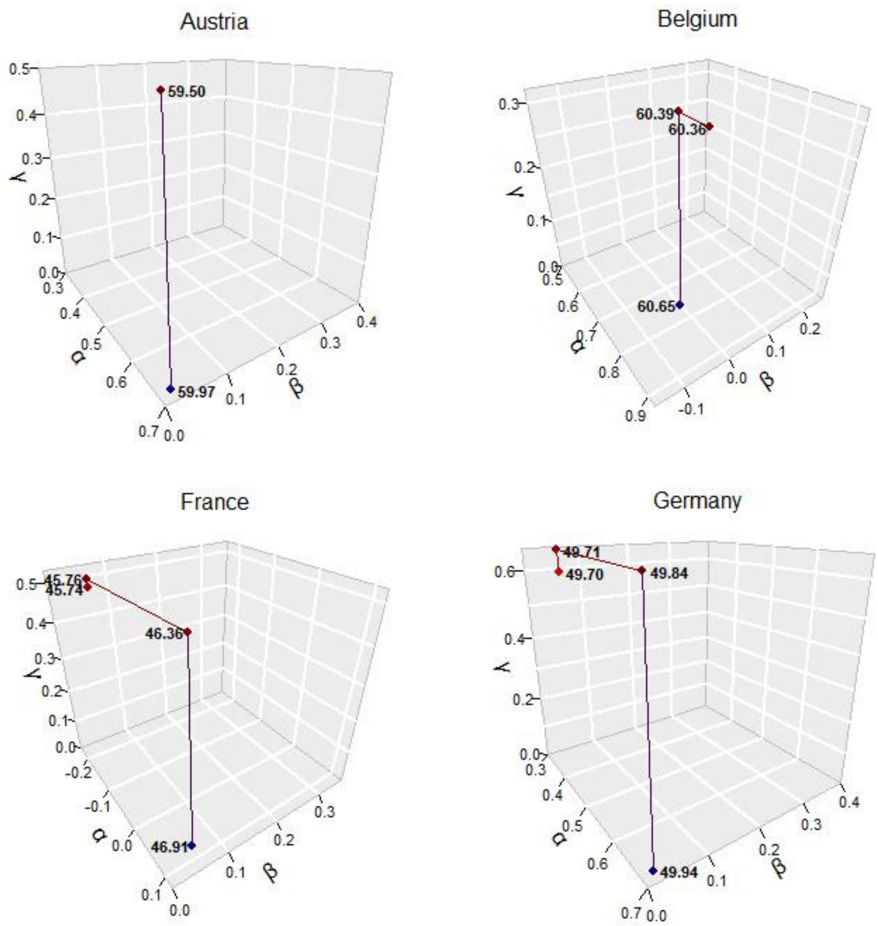

Figure 6. Extended 2D-*LQ* optimums for EA-19 countries (2015). Source: internal based on Eurostat data [naio_10_cp1700].

After formalizing the projections through the 2D-*LQ* extension, the next step entails checking each element according to the previously developed mapping. Figure 7 confirms that there are generally no stark differences in relation to what was previously seen in the 2D-*LQ* plots. Indeed, the color scale is only slightly modified.

The main diagonal elements are also not conditioned by compensation effects, since they are not linked to the rectification of the other elements. Figure 8 shows the corresponding comparisons between the respective *LQ* techniques. The rectifications on the main diagonals appear to be more opportune so that the implementation of a conditional for these elements provides the desired effect. It is also clear how *LQ* techniques are not designed to avoid underestimations. These graphs show how the estimation errors in the overestimations are reduced as the *LQ* techniques become more sophisticated, especially in the first sectors. However, the errors accumulating in the other estimation direction are virtually unchanged. Another weakness therefore emerges in the formulations to be reviewed in the future.

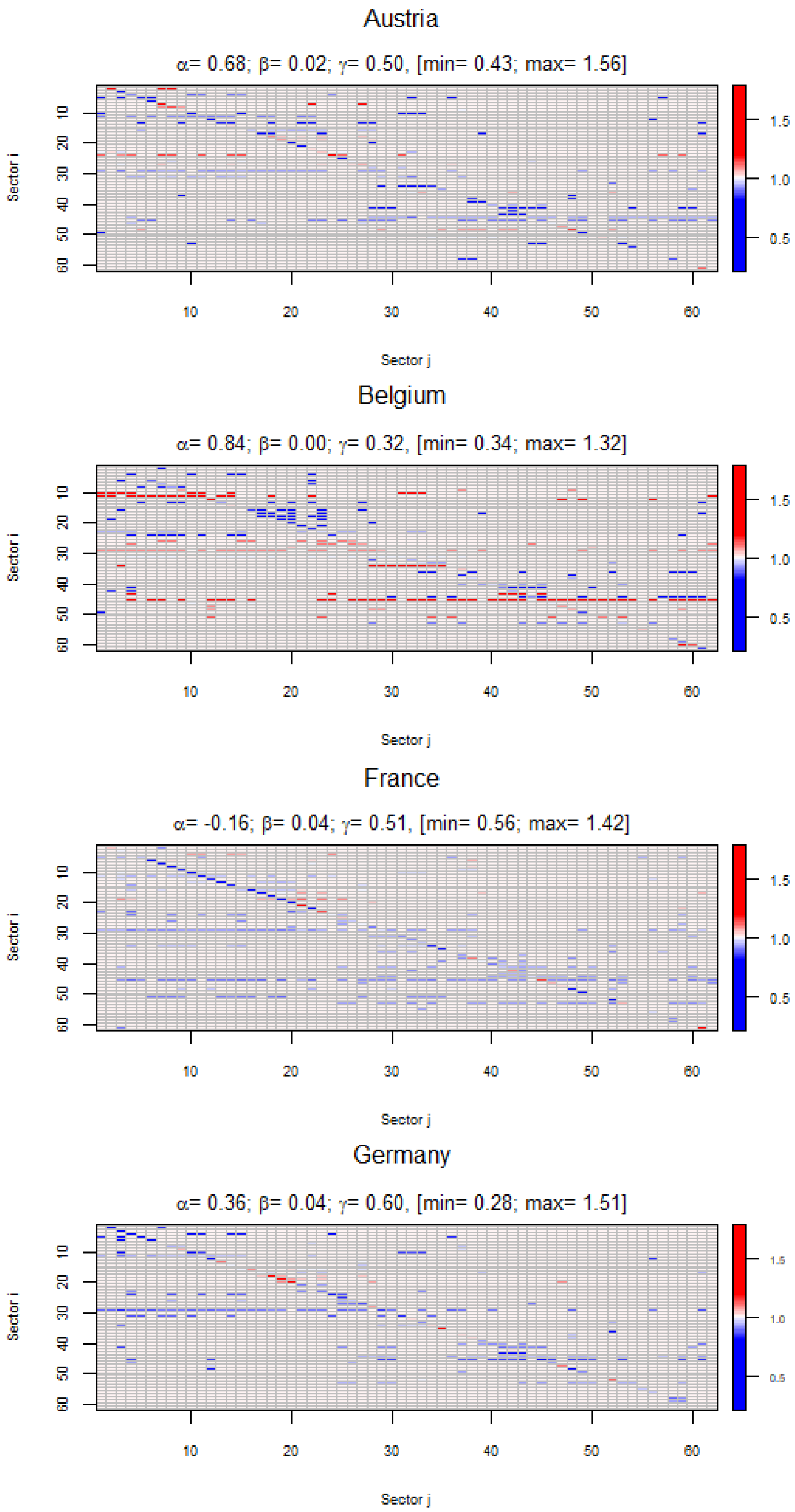

Figure 7. Errors in technical coefficients via extended 2D-*LQ* for EA-19 countries. Source: internal based on Eurostat data [naio_10_cp1700].

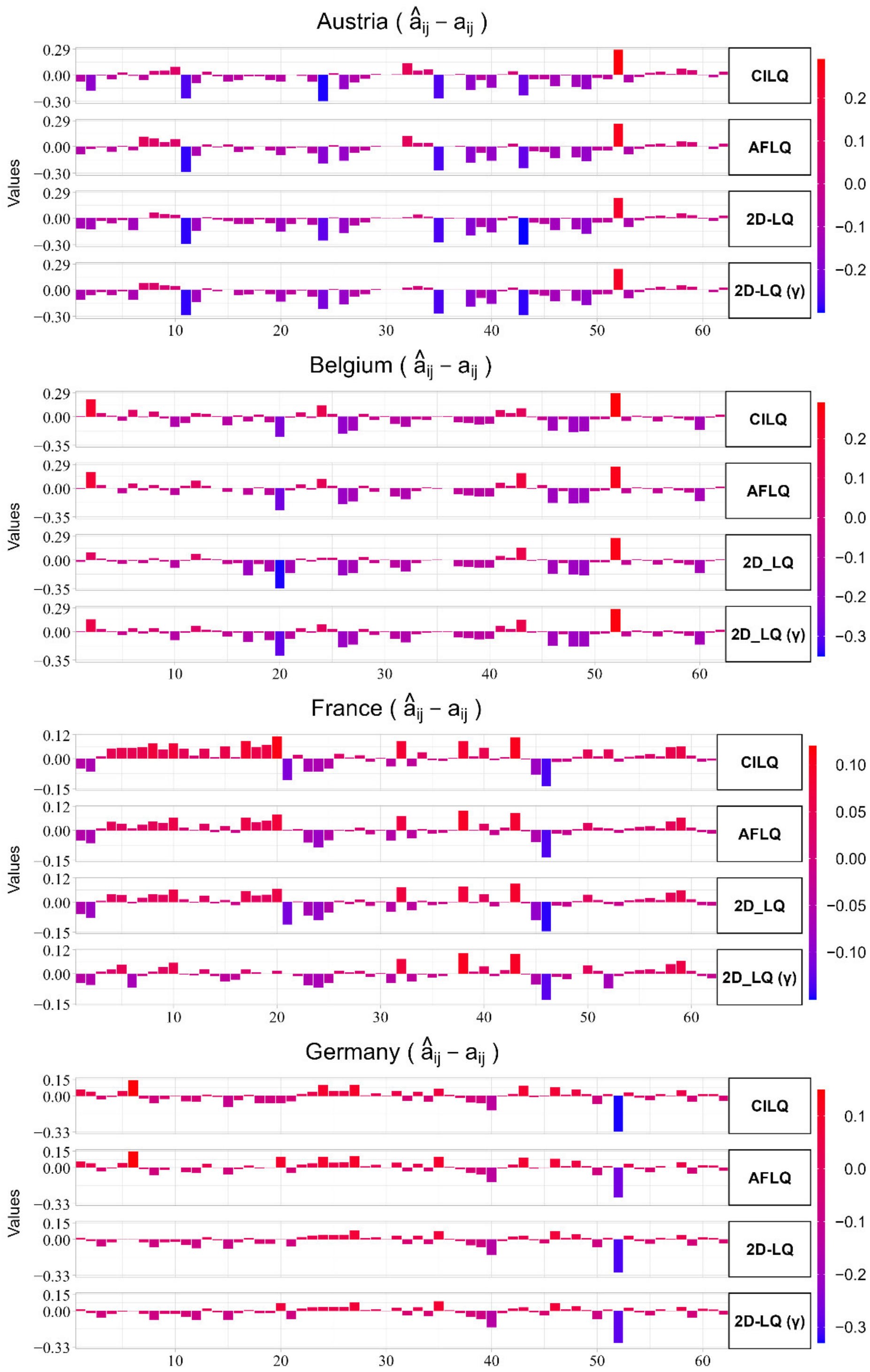

Figure 8. Viewing *LQ* effects on the main diagonal. Source: internal based on Eurostat data [naio_10_cp1700].

In relation to the Leontief inverse multipliers, only the sum by columns is taken into account, just as in other research studies [20,29,30]. Figure 9 shows the effect of *LQ* projections on the multipliers. For Belgium, underestimations predominate for the *CILQ* and the *AFLQ*. Germany, in contrast, presents certain compensation of estimation signs. In general, the *CILQ* incurs more underestimations—although no major improvements are seen for the other three *LQ* methods—as here the STPEs have lower values than those related to the technical coefficients, and this aspect is reproduced in other studies [7]. This circumstance is of vital importance given that the IO frameworks are built for the purpose of exploiting them with multiple IO techniques to redefine planning actions that contribute to a sustainable development of the economy under study.

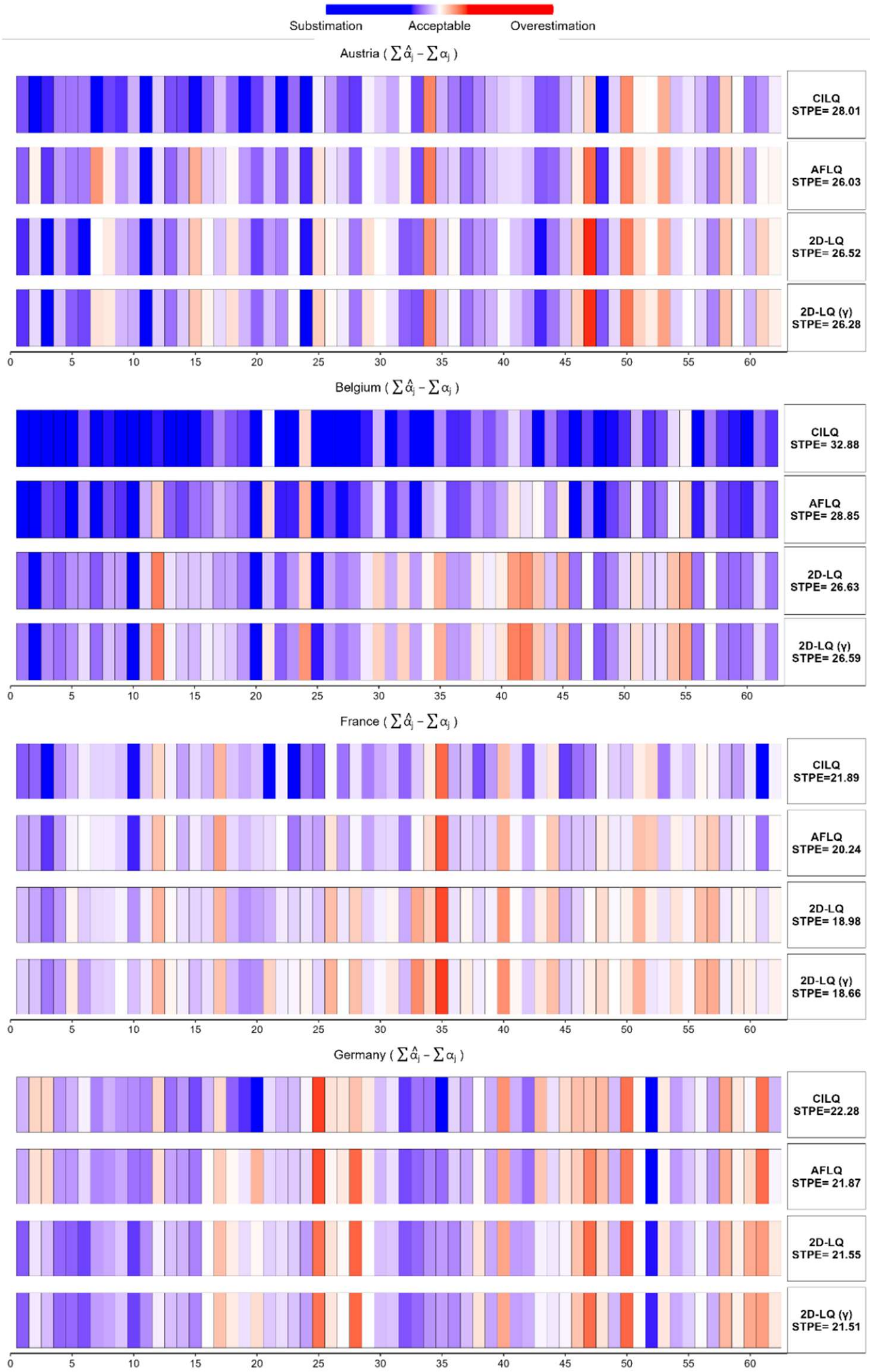

Figure 9. Errors in multipliers using *LQ*s for EA-19 countries. Source: internal based on Eurostat data [naio_10_cp1700].

The projections obtained for these four EA-19 countries prove that there is some room for *LQ* improvement. We should note that we do not intend to fundamentally rethink the formulation of the *LQ*s. In fact, it has been subject to constant refinement, but we suggest allowing duly controlled rectifications that lead to an increase in certain coefficients.

Starting from a reference matrix, all cells are susceptible to rectification with the exception of the null cells.

Self-consumption represents a considerable proportion of total flows, and it therefore deserves particular attention. The result in the *LQ* formulation is associated with an optimum driven by parameters. However, it is understood that forcibly applying corrections to cells, or groups of cells, according to the remaining cells may be excessively limited, resulting in a "null sum" type of compensation effect so that the value of the statistic used hardly decreases.

6. Conclusions

In order to promote sustainable development, there are different actions that can be grouped into two alternatives (not mutually exclusive). On the one hand, there are measures aimed at refining the various behaviors of individuals. On the other hand, there are actions—of a global nature—that seek technological advances in production structures. However, to enhance this second alternative, it is necessary to have appropriate methods and models that are based on accurate information [31]. In other words, it is difficult to achieve sustainable development goals through sophisticated modeling of socio-economic and environmental impacts if the IO accounting frameworks are inaccurate, or they do not adequately reproduce the economy under study; and, alternatively, it is chosen to generalize the observed effects to supra-territorial scales.

This paper emphasized certain inappropriateness of *LQ* formulations. A methodological proposal was made to correct one of the error patterns in the accounting projections, which affects inter-industry self-consumption. The use of matrix mapping has been fruitful for visualizing error patterns. The truth is that it is an elementary mechanism. In this area of work, there is a tendency to resort to overall measurements, leaving aside what happens on a cell-by-cell basis. The results suggest that it is appropriate to decouple the diagonal elements from the rest of the cells. In the future, it would be desirable to contrast the proposed methodology using other accounting bases.

In general, underestimating IO coefficients at the sub-territorial level has consequences. The methodological solution in the future will likely involve the search for more generic *LQ* formulations in which the smoothing of the degrees of specialization does not take from the outset and the rectifications take place row-by-row, even if the degree of specialization is higher than one.

Lastly, our computational capabilities allow for the execution of other contrasts at a disaggregated level. Future research is expected to examine other error patterns present in two-dimensional rectification processes. Thus, formalizing a detailed check to ascertain the rows and the columns with the highest number of errors will help to reflect on the design of the formulations. The productive branches from the two accounting points of view are very disparate, so they easily merit different treatment from each other. The starting assumption would be that treating heterogeneous sectors homogeneously—by rows or by columns—is possibly an inaccurate procedure, and it contributes to some extent to increasing an overall error.

Supplementary Materials: The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/math10091474/s1. Spotting error patterns in input-output projections using location quotients.

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Appendix A. Extended 2D-*LQ* Methodology Assessment

Table A1. Extended 2D-*LQ*: parameter values in each iteration process. Technical coefficient matrix.

Countries	Iteration	α	β	γ	STPE
Austria	0	0.68	0.02		59.97
	1.1	0.68	0.02	0.50	59.50
	1.2	0.68	0.02	0.50	59.50
Belgium	0	0.76	0.00		60.65
	1.1	0.76	0.00	0.32	60.39
	1.2	0.84	0.00	0.32	60.36
	2.1	0.84	0.00	0.32	60.36
France	0	0.08	0.06		46.91
	1.1	0.08	0.06	0.53	46.36
	1.2	−0.16	0.04	0.53	45.76
	2.1	−0.16	0.04	0.51	45.74
	2.2	−0.16	0.04	0.51	45.74
Germany	0	0.68	0.02		49.94
	1.1	0.68	0.02	0.66	49.84
	1.2	0.36	0.04	0.66	49.71
	2.1	0.36	0.04	0.60	49.70
	2.2	0.36	0.04	0.60	49.70

Source: internal based on Eurostat data [naio_10_cp1700].

Table A2. Extended 2D-*LQ*: parameter values in each iteration process. Multiplier matrix.

Countries	Iteration	α	β	γ	STPE
Austria	0	0.72	0.00		26.52
	1.1	0.72	0.00	0.34	26.35
	1.2	0.96	0.00	0.34	26.28
	2.1	0.96	0.00	0.33	26.28
	2.2	0.96	0.00	0.33	26.28
Belgium	0	0.72	−0.03		26.63
	1.1	0.72	−0.03	0.30	26.62
	1.2	0.84	−0.03	0.30	26.59
	2.1	0.84	−0.03	0.30	26.59
France	0	0.20	0.00		18.98
	1.1	0.20	0.00	0.34	18.84
	1.2	0.08	0.00	0.34	18.70
	2.1	0.08	0.00	0.46	18.67
	2.2	0.04	0.00	0.46	18.66
	3.1	0.04	0.00	0.46	18.66
Germany	0	0.44	0.04		21.55
	1.1	0.44	0.04	0.32	21.55
	1.2	0.32	0.05	0.32	21.52
	2.1	0.32	0.05	0.33	21.51
	2.2	0.32	0.05	0.33	21.51

Source: internal based on Eurostat data [naio_10_cp1700].

Table A3. *LQ*s: coefficients and multipliers (2015).

STPE 2015								
	Coefficients				Multipliers j			
Countries	*CILQ*	*AFLQ* (δ)	2D-*LQ* (α; β)	Extd. 2D-*LQ* (α; β; γ)	*CILQ*	*AFLQ* (δ)	2D-*LQ* (α; β)	Extd. 2D-*LQ* (α; β; γ)
Austria	59.97	59.12 (0.03)	59.97 (0.68; 0.02)	59.50 (0.68; 0.02; 0.50)	28.01	26.26 (0.03)	27.15 (0.68; 0.02)	26.78 (0.68; 0.02; 0.50)
Belgium	62.73	62.18 (0.03)	60.65 (0.76; 0.00)	60.36 (0.84; 0.00; 0.32)	32.88	30.30 (0.03)	28.32 (0.76; 0.00)	28.06 (0.84; 0.00; 0.32)
France	49.82	48.24 (0.10)	46.91 (0.08; 0.06)	45.74 (−0.16; 0.04; 0.51)	21.89	22.00 (0.10)	20.68 (0.08; 0.06)	19.32 (−0.16; 0.04; 0.51)
Germany	51.43	50.71 (0.00)	49.94 (0.68; 0.02)	49.70 (0.36; 0.04; 0.60)	22.28	22.24 (0.00)	21.71 (0.68; 0.02)	21.73 (0.36; 0.04; 0.60)

Note: parameter values are indicated in brackets, optimal values in bold. Source: internal based on Eurostat data [naio_10_cp1700].

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