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Editorial

Well-Posedness, Dynamics, and Control of Nonlinear Differential System with Initial-Boundary Value

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1. Introduction

Well-posedness, dynamics, and control of nonlinear differential system with an initial-boundary value involve many mathematical, physical, and engineering problems. Classical boundary value problems cover fluid mechanics, gas combustion theory, neural network systems, economic dynamic systems, and so on. In recent decades, not only reaction diffusion differential systems but also fractional order dynamic systems have been involved in boundary value problems. In particular, the stability of equilibrium points involves the existence, uniqueness, and regularity of typical boundary value problems. On the other hand, the stability and synchronization control of dynamic systems are also among the main contents of cybernetics, and they are part of well-posed problems. Fifteen papers [1–14] in this Special Issue deal with the stabilization and synchronization control of initial boundary value problems of partial differential equations or ordinary differential equations, and jointly complete the original work plan of this Special Issue. The papers in this Special Issue can be divided according to the following scheme considering their main purposes:

(1) Analytical theory;
(2) Numerical verifications;
(3) Applications.

2. Analytical Theory

We may begin with a research paper by Xinggui Li and the Guest Editors Xinsong Yang and Ruofeng Rao [1]: "Impulsive Stabilization on Hyper-Chaotic Financial System under Neumann Boundary". The equilibrium solution of the Hyper-Chaotic financial system is the Sobolev solution of the elliptic partial differential equations with a Neumann boundary value. We construct a positive cone and positive, compact, and linear operators in an infinite dimensional function space, which leads to the unique existence of a positive solution. Furthermore, the contraction mapping principle leads to the global stability of the equilibrium solution.

Shuoting Wang, Kaibo Shi and Jin Yang [2] studied the asymptotic stability of neural networks with time-varying delays. Firstly, a new sufficient and necessary condition for general polynomial inequalities is given. On this basis, a new augmented Lyapunov–Krasovsky function (LKF) is constructed, which effectively introduces some new terms related to the previous information of neuron activation functions. In addition, based on the appropriate LKF and the negative condition of the general polynomial, two conservative criteria in the form of linear matrix inequality are derived. Finally, two numerical examples are used to verify the superiority of the proposed criterion and obtain a larger allowable upper limit for delay.

Regarding the Mittag–Leffler stability of fractional order dynamical systems, Bingrui Xu and Bing Li [3] studied the event-triggered state-estimation problem for a class of fractional order neural networks. An event-triggering strategy is proposed to reduce

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the transmission frequency of output measurement signals and ensure state estimation performance requirements. Based on the Lyapunov method and the property of fractional calculus, a sufficient criterion for the Mittag–Leffler stability of the estimation error system is established. Taking full advantage of the property of the Caputo operator and the Mittag–Leffler function, the evolution dynamics of measurement errors are analyzed to eliminate the unexpected Zeno phenomenon in event-triggering strategies.

3. Numerical Verifications

As pointed out in [1], all the variables of the Hyper-Chaotic financial system are actually bounded as a result of the limited resources in human society. So, in [1], Xinggui Li, Xinsong Yang and Ruofeng Rao proposed the boundedness assumptions on the variables. However, it is necessary to verify whether the boundedness assumptions can be fit for the dynamics of the Hyper-Chaotic financial system. Hence, numerical examples are designed to show the effectiveness of the criterion with the boundedness restrictions. Indeed, the computer simulation vividly showed the boundedness of the dynamic trajectory of the Hyper-Chaotic financial system (see Figures 14–17 in [1]).

In [4] by Shuyan Qi, Jun Zhao, and Li Tang, the problem of adaptive output feedback control for switched uncertain nonlinear systems with input quantization, unmeasured system state and state constraints is studied. Firstly, a fuzzy logic system is introduced to identify the uncertainty of the system, and then a fuzzy-based observer is constructed to estimate the unavailable state. Secondly, combining backstepping technology and barrier Lyapunov function method, an adaptive fuzzy output feedback control law is designed to ensure that all signals in the closed-loop system are bounded, the system outputs tracking reference signals, and the system state meets corresponding constraints. Particularly, the simulation verifications show that the control scheme has good performance.

The objective of [5] is to provide a set of infinite uncertainty matrices J_α that can adapt to control systems under uncertain conditions. The numerical simulation results illuminate that the new method can control the system in a short time and has low computational complexity. The authors of [6] studied the stability and synchronization of complex-valued inertial neural networks using Caputo fractional derivatives, and particularly vividly demonstrated the dynamic synchronization phenomenon of complex-valued dynamic trajectories in both real and imaginary parts through numerical simulations (see, e.g., Figures 7–14 in [6]). Chen et al proposed in [7] an uncertain sandwich control system with impulse time windows, which was well displayed in the computer simulations (Figures 2–5 in [7]). Reference [8] reported a new pulse control mode, which is a three-level pulse. It is a cyclic system with three parts: continuous inputs are applied to the first and third parts of the cycle without the second part of the input cycle. Pulses are applied to each continuous subsystem, and the controller structure is simple. The exponential stability of the hyperchaotic system is achieved, and computer simulation confirms the effectiveness of the algorithm (Figure 9 in [8]). A new Lyapunov functional is proposed in reference [9], which does not require all matrices in the quadratic form of the Lyapunov functional to be positively definite, while the quadratic form remains positively definite, making the estimation more relaxed. Moreover, a numerical example shows the effectiveness of the proposed method. The authors of [10] obtained a sufficient condition criterion for the realization of consistency of a random multi-agent system by using the Lyapunov orientation method, and computer simulation shows the feasibility of the proposed coupling method (Figures 1–8 in [10]). In [11], the Lyapunov analysis theory and an improved event triggered controller were used to achieve intelligent control, ensuring the synchronization of complex dynamic networks, and computer simulation demonstrated the feasibility of exponential synchronization (see, e.g., Figure 16 in [11]). The authors of reference [2] used computer numerical simulations to test the asymptotic stability criteria, which they employed the Lyapunov Krasovskii functional method to derive. In reference [12], numerical simulations were conducted to verify the feasibility of the control scheme and synchronization criteria based on a pure power-law T-S fuzzy control strategy. In reference [13], a polynomial random noise is introduced to suppress the potential explosion behavior of nonlinear impulsive differential systems with time-varying delays. It is proved that there is a unique bounded global solution to the impulsive delay differential systems with stochastic control, and the boundedness of this global solution is illuminated by numerical simulations. In reference [14], a second-order precision tracking controller for multi-integrator systems was constructed using the delayed Zhang neural network method, and numerical experiments were conducted to verify the theoretical results of the Zhang neural network method.

4. Applications

The series of papers in this Special Issue reflect the application of dynamic system theory in finance, neural networks, and other control systems [1–14]. For example, the authors of [1] studied the stability of PSS in hyperchaotic financial systems. In fact, the financial system is stable in terms of positive interest rates and other economic indicators, which is in line with the national conditions of most countries, while many countries previously only involved positive interest rates. The stability criterion of the financial system in [1] has a large pulse allowable range, and does not limit that the pulse must be less than 1, thus reducing conservatism, which is common in the actual financial market, that is, investors and financial management departments have difficulty ensuring whether their impulsive measures are conducive to the stability of the financial system, while the impulsive stability criterion in [1] is more suitable for guiding the impulsive stability plan in the actual financial engineering. Next, the mathematical model in [5] involves nonlinear system modeling in medicine, agriculture and biology. In addition, the authors of [6] used a direct analysis method to study the asymptotic stability and synchronization of a fractional order complex valued inertial neural network. Additionally, [7] reports that a new system was established, which is an uncertain sandwich pulse control system with a pulse time window. The calculation results indicate that the linear entry matrix of the system is uncertain. In order to make this conclusion more general, the authors of [7] inserted a countable number of pulses during the T period and studied its exponential stability. Additionally, the authors of [8] successfully constructed a hyper-chaotic circuit system on the basis of the three-dimensional Chen system, using the smooth continuous nonlinear flux controlled reluctance model as the positive feedback term of the system, and realized the exponential stabilization of the memristor-based hyper-chaotic system by using the Lyapunov function and linear inequality matrix techniques.

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