mathematics

Review

A Mini-Review on Recent Fractional Models for Agri-Food Problems

Stefania Tomasiello [1] and Jorge E. Macías-Díaz [2,3,*]

[1] Institute of Computer Science, Faculty of Science and Technology, University of Tartu, 51009 Tartu, Estonia
[2] Department of Mathematics and Didactics of Mathematics, School of Digital Technologies, Tallinn University, 10120 Tallinn, Estonia
[3] Departamento de Matemáticas y Física, Universidad Autónoma de Aguascalientes, Avenida Universidad 940, Ciudad Universitaria, Aguascalientes 20100, Mexico
* Correspondence: jemacias@correo.uaa.mx; Tel.: +52-449-9108400

Abstract: This work aims at providing a concise review of various agri-food models that employ fractional differential operators. In this context, various mathematical models based on fractional differential equations have been used to describe a wide range of problems in agri-food. As a result of this review, we found out that this new area of research is finding increased acceptance in recent years and that some reports have employed fractional operators successfully in order to model real-world data. Our results also show that the most commonly used differential operators in these problems are the Caputo, the Caputo–Fabrizio, the Atangana–Baleanu, and the Riemann–Liouville derivatives. Most of the authors in this field are predominantly from China and India.

Keywords: agri-food problems; fractional differential equations; mathematical models; advection–diffusion–reaction systems; Caputo derivative; Caputo–Fabrizio derivative; Atangana–Baleanu derivative; Riemann–Liouville derivative

MSC: 34A08; 35R11

Citation: Tomasiello, S.; Macías-Díaz, J.E. A Mini-Review on Recent Fractional Models for Agri-Food Problems. *Mathematics* **2023**, *11*, 2316. https://doi.org/10.3390/math11102316

Academic Editors: Dongfang Li and Hongyu Qin

Received: 12 April 2023
Revised: 9 May 2023
Accepted: 11 May 2023
Published: 16 May 2023

1. Introduction

Agriculture plays a vital role in the economic growth of any country. Currently, modern agriculture and food production systems are facing increasing pressure from climate change, land, and water availability and, more recently, the COVID-19 pandemic. Building resilient and sustainable farming systems is the ultimate goal of the agri-food production system. To that end, information and communication technology (ICT) is pervading the agri-food sector while passing through the Agriculture (or Agri-Food) 4.0 revolution [1]. In such a context, new words have been coined (such as *smart farming* or *precision agriculture*) as a result of the application of the Internet of Things (IoT) and machine learning (ML) algorithms. The main focus of precision farming is to reduce production costs and environmental effects while increasing the farm's profitability. Some application examples are water resources management, farm operation scheduling, and delivery plans. More recently, a new paradigm (called Agriculture 5.0) based on robots has emerged (robots are supposed to solve the workforce shortage in farms) [2].

In addition to the farming objectives mentioned above, there is also the need to limit carbon emissions, in harmony with the EU "Green Deal" policy. This aspect is particularly important to optimize the farming supply chain and takes into account production sustainability. Most techniques used to achieve the above-mentioned goals are model-free, falling into the broad field of data science (for example, see [3]). The aim of this review is to examine recent model-based approaches and, in particular, the use of fractional models. More precisely, we will focus our attention on problems modeled by fractional differential equations (FDEs). Unfortunately, the related literature is still fragmented and incomplete, and the present review represents the first attempt to offer an overview of what has been done so far, discussing the limits and potential of future research. Hence, this review aims to answer the following research questions:

- Are there fractional models to tackle agri-food problems? If so, which kind of fractional operators have been used?
- Has real-world data been employed along with such models?

The article is structured as follows. In the next section, the mathematical models for the agri-food sector are recalled. The third section is devoted to the literature review on fractional models, followed by a discussion on fractional versus non-fractional in the fourth section. The article closes with some conclusions and an appendix recalling the operators mentioned in this paper (see Appendix A).

2. Mathematical Models

Global food, fuel, and fiber demand is estimated to increase to 11.6 billion tons by 2050 [4]. This increase in the demand for food requires the optimization of production on existing cropland and further expansion of arable land. According to the global yield gap analysis, the production could be increased by 30% through improved management of soil constraints and fertilizer application [5]. From here follows the importance of defining proper crop models that can function as decision-support tools. Crop models are typically constructed as differential equations that model dynamical systems. More precisely, one has to distinguish between dynamic crop models (that is, a set of differential equations that are then integrated in time to simulate the crop responses of interest at each time point) and crop-response models (which relate crop responses directly to inputs [6]).

The main state variables of crop models are usually above-ground biomass, leaf area index, harvesting yield, and water and nitrogen balances [6]. All of the crop models involve empirical components and are of varying levels of complexity, depending on the particular goals of the model and on the availability of the input data. There are several studies that adopt a dynamical systems framework, such as those reports that examine grass ecosystems and grazing, forest ecosystems, soil salinity, the evolution of canopy cover, soil moisture, and soil nitrogen (see [7] and references therein). Such models aim mostly to support the decisions on irrigation and fertilization. It is worth pointing out that not all of the works in the specialized literature use dynamical systems theory to understand the mathematical behavior and properties of the models. Using dynamical systems theory to study crop models allows one to capture many critical aspects of crop systems, such as their stability with respect to parameters changes; the feedback between water, carbon, and nutrient cycling; and the optimal conditions for growth and the impact of external inputs (such as fertilization and irrigation).

As an example, thanks to dynamical systems theory, optimal yield, and profitability under different climate scenarios, irrigation strategies and fertilization strategies were examined in [7]. Another class of problems requiring dynamic systems background is plant diseases and pest control. Plant viral diseases have devastating effects on agricultural production. It is estimated that around 42% of the world's food is exhausted because of pests. Chemical pesticides are still employed to control pests. In any case, the intense use of chemical pesticides in farming causes several side effects. Problems such as pest renaissance and secondary pest outbursts have to be carefully considered. The equilibria and stability of the adopted dynamical models help one to understand the disease spread considering the incubation period [8,9]. In particular, to provide suitable interventions for crop pests (that is, to decrease the number of pests in the crop field), optimal control theory is a valuable mathematical tool [8].

3. Fractional Models

This section is devoted to providing a recent literature review. To that end, we firstly describe the review methodology. The problems and the type of fractional operators employed in the mathematical models are summarized in Table 1. As mentioned previously, the present review focuses on models that are based on FDEs. The search process revealed a few different studies in the agri-food context that we cite here for the sake of completeness. In such studies, the adjective "fractional" is related to types of machine learning algorithms [10,11], regression models [12] or empirical models [13,14].

Table 1. Problems and types of fractional operators adopted. For the sake of convenience, we employ the following abbreviations: Caputo (C), Caputo–Fabrizio (CF), Atangana–Baleanu (AB), and Riemann–Liouville (RL).

Problems	Operators	References
Biomasses, biogases, and bio-fertilizers	C, CF, AB	[15–18]
Environmental issues (CO_2, nitrogen estimate)	C, CF, RL	[19–22]
Food science	RL	[23,24]
Livestock and fishery	C, CF, AB, RL	[25–28]
Plant diseases	C, CF, AB	[29–32]
Transportation of contaminants and water issues	C	[33–40]

It is important to point out that, in the retrieved articles, most authors did not use real-world data. A list of models calibrated by using real-world data is reported in Table 2.

Table 2. Real-world data calibrated models: application problems and corresponding adopted fractional operators. For the sake of convenience, we employ the following abbreviations: Caputo (C), Riemann–Liouville (RL).

Problems	Operators	References
CO_2 emissions/dynamics	C, RL	[20,21]
Fishery	RL	[27]
Food science	RL	[23,24]
Soil moisture	C	[39]

It is also worth mentioning that not all of the articles present mathematical properties (e.g., existence and uniqueness, convergence, positivity, and boundedness) and a proper study of the dynamics (e.g., the stability of the equilibrium, bifurcation analysis). Only in 13 articles out of 31 it is possible to find a formal discussion. Figure 1 shows the distribution of the classes of the formal investigation.

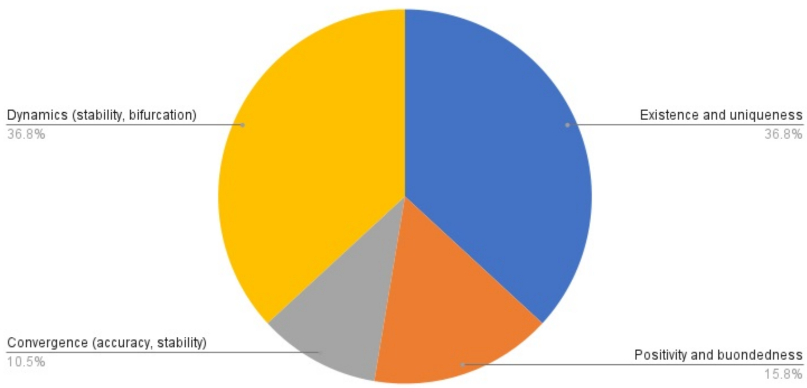

Figure 1. Classes of formal investigation.

3.1. Review Methodology

The review process was carried out by following the main steps of the preferred reporting items for systematic reviews and meta-analyses (PRISMA) methodology [41]. The main objective of systematic reviews is to present a detailed review study for a specific research area and date interval. The methodology consists of steps such as identification, screening, and eligibility. We adapted our review process to the PRISMA methodology.

In the first step, we considered the main academic databases (such as Scopus and WoS) to present a review of fractional models in agri-food. The related papers were checked one by one. The literature analysis was realized based on the following keywords, by using the Boolean operators **OR** and **AND**:

((agri*) **OR** (food) **OR** (plant) **OR** (crop) **OR** (livestock) **OR** (fish)) **AND** (fractional)

Here, "*" means any string after "agri" and the **OR** operators are processed before **AND**. The search was performed on the title, abstract, and keywords. To refine the search, we replaced the words "crop" and "plant" with specific terms such as "vegetable", "potato", "olive", "tomato", "wheat", "rice", etc. We fixed the year range [2003, 2023], but the fact that the related works are mostly from the last five years shows that the interest in this topic has been growing lately (see Table 3).

Table 3. Number of articles per year reporting on recent fractional models in agri-food investigation.

Year	Number of Publications
2003	1
2008	1
2012	1
2013	2
2014	1
2015	1
2018	1
2019	4
2020	1
2021	4
2022	10
2023 (April)	1

The second step was about screening the papers in order to determine irrelevant or duplicated works. Irrelevant papers were those in which the title was misleading, presenting content not strictly related to the topics here considered. In the third step, irrelevant or duplicated papers were removed. The initial screening was of around 100 papers, but only 25 papers were relevant. The number of considered reports with the publication year is shown in Table 3. The countries of the corresponding authors are shown in Figure 2. From those results, it is shown that authors with Chinese and Indian affiliations are predominant. Multiple affiliations have been taken into account.

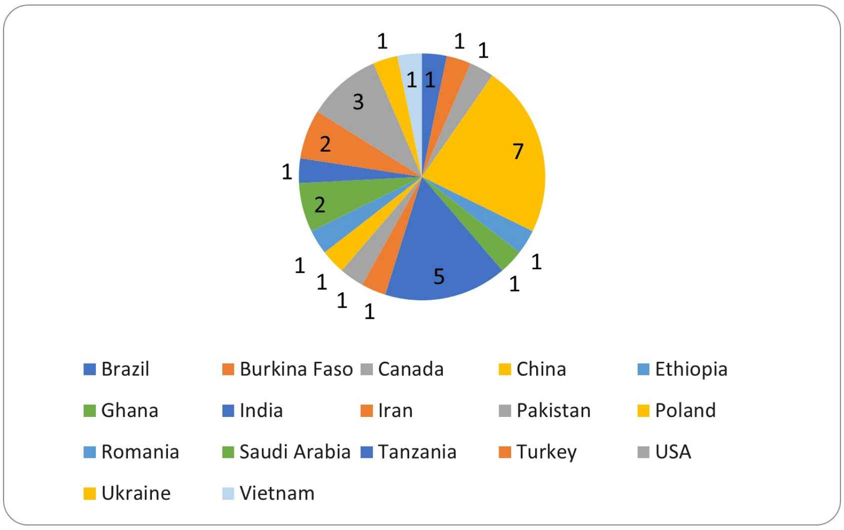

Figure 2. Countries of the corresponding authors.

3.2. Caputo-Type Models

According to our results, the use of Caputo-type fractional differential operators is dominant. This can be easily observed in Figure 3, where the pie chart covers the different fractional operators employed in the relevant literature of this review. We must mention that one of the main problems tackled by FDEs is the transportation of biological or chemical contaminants into groundwater and soils. In [33], the author used a fractional advection-dispersion equation, which was solved by Jacobi's collocation method. The author conducted a formal convergence analysis of the proposed method. The authors of [34] aimed to model the dynamics of colloids through a soil-vegetation system. To this end, they also used a fractional advection-dispersion with a few empirical parameters. The authors did not explicitly mention the Caputo differential operator, but from the context, it is obvious that it is so. The review article [35] also mentions fractional advection-dispersion equations to investigate the groundwater quality of aquifers. Relevant problems in this regard are induced saltwater intrusion, hydraulic fracturing, carbon dioxide (CO_2) sequestration, and deep geologic storage of nuclear waste.

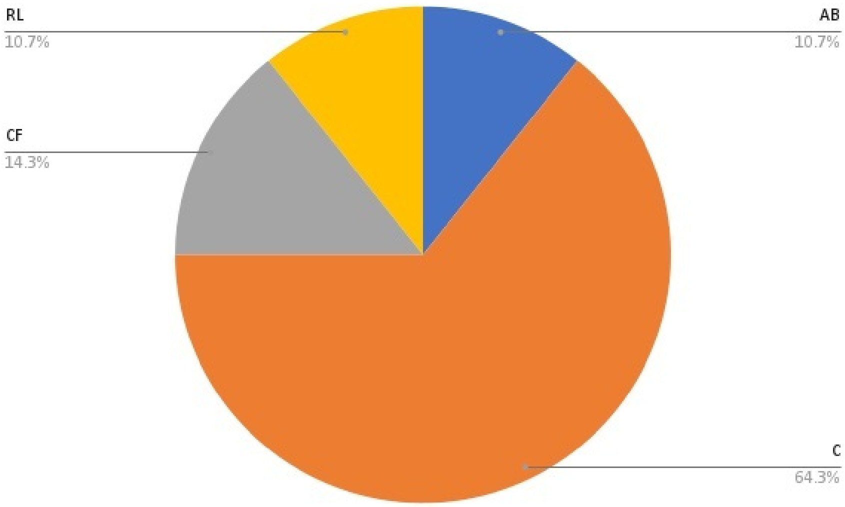

Figure 3. Distribution of the different fractional operators. For the sake of convenience, we employ the following abbreviations: Caputo (C), Caputo–Fabrizio (CF), Atangana–Baleanu (AB), and Riemann–Liouville (RL).

The problem of transport of contaminants in groundwater or porous soil was also tackled in [36]. The authors of that report solved the fractional diffusion equation by employing the finite element methodology. The focus was mainly on the adopted numerical technique. Somewhat related is the problem of a suitable model for the water table profile in agricultural soils. The authors of [37] used a one-dimensional fractional Boussinesq equation. The solution was obtained using a spectral representation of the fractional derivative. The computed values were compared to the observed ones in the field experiments, and they showed good accuracy. Models of water transport in soils represent the basis for decision-support systems for irrigation. Their aim is to predict soil moisture content and to determine irrigation schedules. There are works on the fractional version of Richards' equation to investigate soil moisture mainly from a theoretical perspective [38] and from a practical one, with a real-world application [39]. In [38], the basic Caputo derivative with respect to the time was adopted, while in [39], a generalization of it, that is, the ψ-Caputo derivative, was used. In particular, in [39] the authors used the particle swarm optimization algorithm to identify the parameters of the fractional Richards equation, given some inputs, namely, the suction pressure (from Watermark sensors) and the evapotranspiration. The aim was to investigate how different derivatives' functional parameters may influence the accuracy of soil moisture dynamics simulation.

Other application problems fall into the area of biomass and biogas. In [15], the author intended to model the biochemical reaction via anaerobic digestion. This is the biochemical process for producing biogas as a result of the biological degradation of biomass. A system of fractional differential equations was obtained, and the solution was sought by means of the Adomian decomposition method. In [16], a fractional dynamical system for maize biomass and fall armyworm interaction was proposed. The fall armyworm (*spodoptera frugiperda*) is a harmful pest that may affect fields in North and South America. The model parameters and their baseline values were retrieved from the literature or estimated. In their work, the authors proved the positivity and boundedness of the solutions. Their study was completed by the discussion on the local stability of the equilibrium points of their model. There is also another work about biomass [17], but we will recall it in the next subsection since the authors considered other additional fractional operators.

Another interesting topic was discussed in [18]. The authors introduced a fractal-fractional dynamical model in the sense of Caputo's derivative to study the reuse of dead *algae detritus* as fertilizer for crops. The algae recover nitrogen and phosphorus from water, and they can be reused in agriculture by replacing chemical fertilizers. The values of the parameters' model were taken from the literature. The numerical scheme to solve the model equation was based on Lagrange interpolation polynomials. The authors proved the existence and uniqueness of the solution, employing then the Ulam–Hyers approach for the model stability analysis. In [42], a parametric fractional differential equation was used to process the data obtained from image processing to analyze mixtures of olive and soybean oil, through the RGB colour system. A hybrid scheme based on genetic algorithms and a simplex-based algorithm was adopted for parameter estimation. Evapotranspiration and the effective utilization of agricultural water resources were investigated in [40]. Here, the authors converted a model previously proposed in [43] into a time-fractional model.

An example of fractional model with Caputo-type derivative is offered in [32] to study the spread of pests in tea plants. The authors considered a tritrophic food chain system using the Caputo derivative, proposing an iterative scheme to seek the solution. They formally investigated the stability analysis of this iterative method and proved the existence and uniqueness of the solution. In their work, they also considered a Tuofik–Atangana scheme with the Caputo derivative for comparative purposes.

Finally, since global emissions due to agriculture were recently estimated to be 9.3 billion tonnes of CO_2 equivalent [44], it is worth mentioning fractional models for the prediction of carbon dioxide. In [19], the authors presented a fractional-order model to estimate the association between economic growth, electricity consumption, agriculture, and CO_2 emissions in Turkey in the year range [1971,2014]. The authors used real-world data. Instead, in [20], the authors used a fractional grey Riccati model to predict the CO_2 emissions of the United States, China, and Japan. The data sets used were from the M-competition for forecasting. There is another work [21] dealing with carbon dioxide, but we will recall it in the next subsection.

3.3. Other Fractional Models

In [25], a model for the Q fever (*coxiellosis*) transmission in livestock was studied. The authors investigated the use of conventional derivatives as well as Caputo, Caputo–Fabrizio, and Atangana–Baleanu fractional differential operators. In order to show that their integer-order model was epidemiologically well-stated, the authors formally proved the positivity and boundedness of the solutions, and they conducted a bifurcation analysis. Then, they proved the existence and uniqueness of the solutions of the fractional models. On the other hand, there are several papers dealing with plant diseases. For example, the authors of [29] investigated a plant-disease transmission model for two-stage infection. More precisely, the authors considered a first phase for exposed individuals before becoming infectious, and they then considered the transmission of the disease. The authors used two fractional operators, namely, the Caputo and the Caputo–Fabrizio fractional derivatives. In [30], the authors studied the dynamics of the treatments of plant diseases via the Atangana–Baleanu derivative. They proved the existence and the uniqueness of the solution. Their numerical approach was based on Lagrange interpolation. They found that by increasing the roguing rate for the most infected plant or decreasing the rate of planting in the infected area, disease transmission was reduced.

The authors of [31] investigated the transmission dynamics of the potato leaf roll virus using integer and fractional-order differential equations. The models were built by dividing the potato population into susceptible and infected individuals. The model parameter values were gathered from the literature or assumed by the authors. Here, it is important to recall that the use of the type of fractional operator was not explicitly mentioned. The authors proved the positivity and boundedness of the solution. They also formally discussed the local stability of the disease-free equilibrium. In [17], the authors proposed a mathematical model for ethanol production based on fractal-fractional operators. Bioethanol is produced from biomass and bioenergy crops, and it has gained an increasing amount of attention as an effective alternative to fossil fuels. The authors employed the Caputo, the Caputo–Fabrizio, and the Atangana–Baleanu operators. The solution was sought by adopting the Adams–Bashforth approach, which is a numerical technique essentially based on the Lagrangian interpolation method. The authors proved the existence and uniqueness of the solution. Moreover, they formally discussed the Ulam–Hyres stability by using nonlinear analysis.

On the other hand, there are a few papers dealing with fisheries. In [26], the authors formally investigated random fuzzy differential equations with the Riemann–Liouville fractional derivative. Their application example was fish harvesting. The authors considered the fuzzy fish population growth model, assuming that fish population size, birth, and death rates were all fuzzy-valued. They proved the existence and uniqueness of the solution and proposed a technique to find the analytical solutions of fractional random fuzzy differential equations by using the solutions of random fuzzy differential equations. In [27], the focus was on the parameter estimation problem for stochastic differential equations with both ordinary and fractional Brownian motions. The proposed approach converted the stochastic differential equation into a system of ordinary differential equations. For their numerical experiments, the authors used the data on the North-Atlantic herring population counted by several independent observatories over the years in the range [1940, 2010]. In [28], the authors modeled commercial fishing as a dynamical system via fractional differential equations. In particular, they employed a predator–prey model. They discussed the stability of the equilibrium points of their model, the non-existence of the limit cycle, and the periodic solution for the movement of the considered fish stocks belonging to two species.

The aim of [22] was to study the feasibility of detecting nitrogen in crops by a fractional-order differential algorithm. The authors used spectral data with the help of a Grunwald–Letnikov fractional differential equation. It is worth mentioning that nitrogen is one of the most used fertilizers, and it influences the growth, development, yield, and quality of crops. The rapid and accurate assessment of nitrogen content in crops is critical for nutrition diagnosis and growth monitoring. In [45], agriculture was only mentioned as an application field. Anyhow, considering the theoretical nature of most related works in literature, we deem it relevant. The authors studied the non-linear space-fractional Fisher–Kolmogorov–Petrovskii–Piskunov equation, where the fractional derivatives were taken in the modified Riemann–Liouville sense. Using the travelling-wave transformation, the original equation was converted into an ordinary differential equation and the dynamics were investigated.

Other interesting applications based on the Riemann–Liouville differential operator are related to the broad area of food science. In [23], fractional differential equations were used to predict the nonlinear survival and growth curves of food-borne pathogens. The curves were validated using experimental data in the literature. Thanks to the model, it was possible to predict the tails in survival curves, which was not possible using Weibull or linear models. The problem in [24] was the consecutive intakes of contaminated food when the human immune system was not properly working. The fractional differential equation of order one-half was proposed to model the dynamical process of the accumulation and elimination of the contaminant in the human body. The Adomian decomposition method was used to obtain the approximate solution. The data for the numerical experiments was obtained upon request from the National Center of Food in Burkina Faso.

As in the previous subsection, we close this literature review by citing a work dealing with carbon dioxide. In [21], the authors discussed a system of fractional differential equations modeling the atmospheric dynamics of carbon dioxide. This model was obtained by taking into account the atmospheric level, the human population and the forest biomass. The authors used the Caputo–Fabrizio fractional operator and the Laplace transform with the q-homotopy approach to obtain the approximate solution. The values of the parameters were retrieved from reports available in the literature. Besides, the authors proved the existence and the uniqueness of the solution.

4. Fractional Versus Non-Fractional

Various works tackle the comparison between the non-fractional and the fractional scenarios [25,29,31]. In some cases, the numerical results show that as the value of the fractional order increases, the behavior of the fractional-order model solution approaches the integer-order scenario [31]. This fact is entirely expected, of course. However, it may also happen that the trajectories of the fractional-order systems follow different directions and do not converge to the equilibrium point when they approach the integer order [25]. In general, the fractional derivative provides an excellent instrument for modeling real-life phenomena, such as memory and hereditary properties in the context of plant diseases [29]. In [29], the authors discussed the positiveness and boundedness of the solution and formally investigated the local asymptotically stability of the disease-free equilibrium. This ability of the fractional-based models is also reflected in the study comparing the classical Richards model for water transport with the generalized one based on fractional derivatives [38]. It was shown that the classical Richards' equation was able to predict a decrease in the soil water diffusivity while the infiltration progressed, but the generalized Richards' equation could describe all of the observations properly by means of a single diffusivity function. In fact, there is evidence in the literature that Richards'equation fails to model water transport in horizontal soil columns. The generalized Richards' equation indicates the presence of memory effects in soil–water transport phenomena. Another comparison between the classical Richards model and the fractional one is discussed in [39]. The fractional model turned out to be the most accurate in the two highest layers of soil (15 cm and 35 cm), i.e., the layers containing most of the crop's roots. The classical model in these layers significantly overestimated evapotranspiration. Such forecasts could contribute to soil over-moistening, potential crop-yield losses, and inefficient irrigation water usage. Compared to the classical model, in the first soil layer (up to 15 cm depth), the fractional models gave 5–7 times better accuracy.

On the other hand, studies carried out using integer-order models have the advantage that they can be derived from some well-known conservation laws. This situation prevails in other physical and biological problems, where differential equations can be obtained from some principle of conservation of mass, energy, momentum, etc., [46]. This is the case for many advection–diffusion–reaction equations from mathematical physics and biology. In the fractional-case scenario, however, science has not been able to establish empirical conservation laws. To clarify this statement, it is important to mention that conservation laws have been established analytically for some systems of fractional-order partial differential equations. As an example, one of the authors of this mini-review has devoted some recent works to design conservative schemes for Riesz-fractional partial differential equations of the hyperbolic type [47–49]. These facts have been established thanks to some Riesz-fractional forms of the formula for integration by parts derived by Tarasov [50]. However, there are no reports of fractional-order conservation laws established experimentally. From that perspective, the deduction of fractional-order differential equations from experimental fractional laws is not possible at all. In the best case, these models are the only possible candidates to describe real-world problems. In some cases, those models provided good descriptions of phenomena. Such is the case of the article [20], in which the authors applied a fractional-order Riccati model to predict the carbon dioxide emissions of various countries.

To this day, the relevance of fractional calculus is still under scientific scrutiny. Beyond the consequent development of the mathematical theory that fractional calculus has provoked, there are still many questions on the rigorous applicability of fractional mathematics. Indeed, it is well known that some systems consisting of particles with long-range interactions yield fractional derivatives of the Riesz type in the continuum limit [51]. Moreover, a variational calculus for Riesz fractional operators has been developed in the literature [50], allowing for the possibility of developing a Hamiltonian theory of mechanics for systems with long-range interactions and memory. However, a physical interpretation of these operators is still lacking in the literature. Nevertheless, beyond that limitation, fractional differential equations have been applied successfully in the description of many complex phenomena and, in some situations, yield better results than integer-order systems.

5. Conclusions

In this work, we investigated the state of the art on the use of fractional differential equations to solve agri-food problems. The problems considered in this work span various areas related to agriculture, food, plants, crops, livestock, and fish, and all of them had the same common denominator: they were modeled by fractional differential equations. Following a standard methodology (PRISMA), it was possible to identify recent progress in the area. The investigation was confined to the main academic databases, and various conclusions were drawn from our search. To start with, we identified that most of the contributions have been authored by researchers from China and India. Moreover, a dramatic increase in the number of publications in the area was recorded in the past year. Additionally, it has been established that the most common differential operators employed in these fractional models are Caputo-type operators, the Atangana–Baleanu derivative, and the Riemann–Liouville operators. In all of these reports, the fractional operators have been applied to the temporal variable, which means that these systems model some associated memory effects.

Author Contributions: Conceptualization, S.T.; methodology, S.T.; validation, S.T. and J.E.M.-D.; formal analysis, S.T. and J.E.M.-D.; investigation, S.T. and J.E.M.-D.; resources, S.T. and J.E.M.-D.; data curation, S.T. and J.E.M.-D.; writing—original draft preparation, S.T. and J.E.M.-D.; writing—review and editing, S.T. and J.E.M.-D.; visualization, S.T. and J.E.M.-D.; supervision, S.T. and J.E.M.-D.; project administration, S.T. and J.E.M.-D.; and funding acquisition, S.T. and J.E.M.-D. All authors have read and agreed to the published version of the manuscript.

Funding: S.T. was funded by the European Social Fund via the IT Academy program and by the Estonian Research Council through the funding of SusAn, FACCE ERA-GAS, ICT-AGRI-FOOD, and SusCrop ERA-NET. On the other hand, J.E.M.-D. was funded by the National Council of Science and Technology of Mexico (CONACYT) through grant A1-S-45928.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The authors confirm that the data supporting the findings of this study are available within the article.

Acknowledgments: The authors would like to thank the anonymous reviewers and the associate editor in charge of handling this paper for their comments and criticisms. Their suggestions were literally followed, and they helped in improving the quality of this mini-review.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Fractional Operators

Beforehand, it is worthwhile to point out that dramatic progress has recently been made in the field of fractional calculus [52,53]. Many non-equivalent fractional derivatives have been recently introduced in the literature in order to extend the classical calculus to the fractional order (see also [54] and references therein). Moreover, the literature provides an account of many new analytical results that have been derived in the way. The purpose of this appendix is to recall the definitions of the fractional derivatives mentioned in this work. Throughout, we will suppose that Ω is a nonempty domain of $\mathbb{R}^n$ and that $u : \Omega \times (0, \infty) \to \mathbb{R}$ is a sufficiently regular function on the vector $(x, t) \in \Omega \times (0, \infty)$. If $\alpha \in (0, 1)$, then the Caputo fractional derivative of u with respect to t of order α at the point (x, t) is defined as (see [55])

$$ {}^{C}D_t^{\alpha} u(x,t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} \frac{\partial u(x,\tau)}{\partial t} d\tau, \quad \forall (x,t) \in \Omega \times (0,\infty). \tag{A1} $$

The Caputo–Fabrizio fractional derivative of u with respect to t of order α at (x, t) is given by

$$ {}^{CFC}D_t^{\alpha} u(x,t) = \frac{B(\alpha)}{1-\alpha} \int_0^t \exp\left[-\frac{\alpha(t-\tau)}{1-\alpha}\right] \frac{\partial u(x,\tau)}{\partial t} d\tau, \quad \forall (x,t) \in \Omega \times (0,\infty). \tag{A2} $$

Here, B is a positive normalization function defined on $[0, 1]$, which satisfies $B(0) = B(1) = 1$ (see [56]). In turn, the Atangana–Baleanu fractional derivative of u of order α with respect to t at (x, t) is defined by the expression

$$ {}^{ABC}D_t^{\alpha} u(x,t) = \frac{B(\alpha)}{1-\alpha} \int_0^t E_{\alpha}\left[-\frac{\alpha(t-\tau)^{\alpha}}{1-\alpha}\right] \frac{\partial u(x,\tau)}{\partial t} d\tau, \quad \forall (x,t) \in \Omega \times (0,\infty). \tag{A3} $$

The function B is a function defined on $[0, 1]$, which satisfies the same properties in the definition of the Caputo–Fabrizio derivative [57]. The Riemann–Liouville fractional derivative of u with respect to t of order α at the point (x, t) is defined by (see [58])

$$ {}^{RL}D_t^{\alpha} u(x,t) = \frac{1}{\Gamma(1-\alpha)} \frac{\partial}{\partial t} \int_0^t (t-\tau)^{-\alpha} u(x,\tau) d\tau, \quad \forall (x,t) \in \Omega \times (0,\infty). \tag{A4} $$

Finally, if $f, \psi \in C^{n+1}([a, b])$ and $\alpha > 0$, then the ψ-Caputo derivatives are defined as

$$ \begin{aligned} {}^{C}D_{a+}^{\alpha,\psi} f(x) = {} & \frac{(\psi(x)-\psi(a))^{n-\alpha}}{\Gamma(n+1-\alpha)} f_{\psi}^{[n]}(a) \\ & + \frac{1}{\Gamma(n+1-\alpha)} \int_a^x (\psi(x)-\psi(t))^{n-\alpha} \frac{d}{dt} f_{\psi}^{[n]}(t) dt \end{aligned} \tag{A5} $$

and

$$ \begin{aligned} {}^{C}D_{b-}^{\alpha,\psi} f(x) = {} & (-1)^n \frac{(\psi(b)-\psi(x))^{n-\alpha}}{\Gamma(n+1-\alpha)} f_{\psi}^{[n]}(b) \\ & - \frac{1}{\Gamma(n+1-\alpha)} \int_x^b (\psi(t)-\psi(x))^{n-\alpha} (-1)^n \frac{d}{dt} f_{\psi}^{[n]}(t) dt. \end{aligned} \tag{A6} $$

In general, the definitions are not all equivalent to each other. However, they satisfy various properties that can be found in the literature.

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