*mathematics*

*Editorial*

# Advances in Mathematical Inequalities and Applications

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## 1. Introduction

Why do we study inequalities? The answer to this question was given by Bellman in [1], in a very concrete and elegant fashion: "There are three reasons for the study of inequalities: practical, theoretical and aesthetic. In many practical investigations, it is necessary to bind one quantity to another. The classical inequalities are very useful for this purpose. From the theoretical point of view, very simple questions give rise to entire theories. For example, we may ask when the nonnegativity of one quantity implies that to another. This simple question leads to the theory of positive operators and theory of differential inequalities. Another question which gives rise to much interesting research is that of finding equalities associated with inequalities. We use the principle that every inequality should come from an equality which makes the inequality obvious. Along these lines, we may also look for representation which makes inequalities obvious. Often, these representations are the maxima or minima of certain quantities. Finally, let us turn to aesthetic aspects. As has been pointed out, beauty is in the eyes of the beholder. However, it is generally agreed that certain pieces of music, art or mathematics are beautiful. There is an elegance to inequalities that makes them very attractive".

In this Special Issue, we present new results related to classical inequalities, such as the Jensen inequality, Jensen–Steffensen inequality, Jessen inequality, Grüss inequality, Chebyshev inequality, etc. They have various applications in various branches of mathematics, among which are numerical analysis, probability and statistics, as well as in other sciences, such as information theory.

## 2. Statistics of the Special Issue

A total of 30 papers were submitted for this Special Issue, of which 10 were published (33.33%) and 20 were rejected (66.67%), indicating a rigorous peer review process.

## 3. Overview of the Contributions to the Special Issue

1. Barotov, D.; Barotov, R.; Soloviev, V.; Feklin, V.; Muzafarov, D.; Ergashboev, T.; Egamov, K. The Development of Suitable Inequalities and Their Application to Systems of Logical Equations. Mathematics 2022, 10(11), 1851; https://doi.org/10.3390/math10111851.

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In this paper, two uncomplex inequalities are invented and intricately established, adequately describing the behaviour of discrete logical functions ***or***(x1,x2, . . . ,xn) and ***and***(x1,x2, . . . ,xn). Based on these proven inequalities, infinitely differentiable extensions of the logical functions ***or***(x1, x2, . . . , xn) and ***and***(x1, x2, . . . ,xn) were defined. These suitable extensions were applied to systems of logical equations. Specifically, the system of m logical equations is first transformed in Rn into an equivalent system of m smooth rational equations (**SmSRE**) in a constructive way, without adding any equations (field equations or otherwise), such that the solution of **SmSRE** can be reduced to the problem minimization of the objective function, and any numerical optimization methods can be applied, since the objective function will be infinitely differentiable. Transforming **SmSRE** into an equivalent system of m polynomial equations (**SmPE**) means that any symbolic



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methods for solving polynomial systems can be used to solve and analyse an equivalent **SmPE**. The equivalence of these systems has been justified in detail. Based on this evidence and results, in the next paper, we plan to study the practical applicability of numerical optimization methods for **SmSRE** and symbolic methods for **SmPE**.

2. Barić, J. Hermite-Hadamard-Type Inequalities and Two-Point Quadrature Formula. Mathematics 2022, 10(9), 1432; https://doi.org/10.3390/math10091432.

https://www.mdpi.com/2227-7390/10/9/1432.

Convexity plays an important role in many aspects of mathematical programming (e.g., for obtaining sufficient optimality conditions and in duality theorems), and one of the most important inequalities for convex functions is the Hermite–Hadamard inequality. As such, this paper is critical in providing new improvements for convex functions and guidance in studying new variants of the Hermite–Hadamard inequality. The first part of the article includes familiar concepts regarding convex functions and the related inequalities. In the second part of the study, a derivation of the Hermite–Hadamard inequality for convex functions of higher order is given, emphasizing the purpose and significance of quadrature formulas. In the third section, the applications of the main results are presented by obtaining Hermite–Hadamard-type estimates for various classical quadrature formulas, such as the Gauss–Legendre two-point quadrature formula and the Gauss–Chebyshev two-point quadrature formulas of the first and second kind.

3. Khan, A.; Nabi, H.; Pečarić, J. On Ostrowski Type Inequalities via the Extended Version of Montgomery's Identity. Mathematics 2022, 10(7), 1113; https://doi.org/10.3390/math10071113.

https://www.mdpi.com/2227-7390/10/7/1113.

The authors establish new Ostrowski-type inequalities by using the extended version of Montgomery identity and Green's functions. They also gave estimations of the difference between the two integral means.

4. Rodić, M. On the Converse Jensen-Type Inequality for Generalized f-Divergences and Zipf-Mandelbrot Law. Mathematics 2022, 10(6), 947; https://doi.org/10.3390/math10060947.

https://www.mdpi.com/2227-7390/10/6/947.

Motivated by recent investigations relating the sharpness of the Jensen inequality, this paper concerns with the sharpness of the converse of the Jensen inequality. These results are then used for deriving new inequalities for different types of generalized f-divergences. As divergences measure the differences between probability distributions, these new inequalities are then applied on the Zipf–Mandelbrot law as a special kind of a probability distribution.

5. Alomari, M.; Klaričić Bakula, M. An Application of Hayashi's Inequality for Differentiable Functions. Mathematics 2022, 10(6), 907; https://doi.org/10.3390/math10060907.

https://www.mdpi.com/2227-7390/10/6/907.

In this paper, the authors introduce new applications of the Hayashi's inequality for differentiable functions by determining new error estimates of the Ostrowski and trapezoid-type quadrature rules.

6. Aslam, G.; Ali, A.; Mehrez, K. A Note of Jessen's Inequality and Their Applications to Mean-Operators. Mathematics 2022, 10(6), 879; https://doi.org/10.3390/math10060879.

https://www.mdpi.com/2227-7390/10/6/879.

The authors ascertain a variant of the Jessen-type inequality for a semigroup of positive linear operators, defined on a Banach lattice algebra. The corresponding mean value theorems lead to a new family of mean operators.

7. Pečarić, J.; Perić, J.; Varošanec, S. Refinements of the Converse Hölder and Minkowski Inequalities. Mathematics 2022, 10(2), 202; https://doi.org/10.3390/math10020202.

https://www.mdpi.com/2227-7390/10/2/202.

The authors refine the converse Hölder inequality for functionals using an interpolation result for Jensen's inequality. Additionally, they obtain similar improvements of the converse of the Beckenbach inequality. They consider the converse Minkowski inequality for functionals and of its continuous form, and refine it. Thus, an application to integral mixed means is given.

8. Klaričić Bakula, M.; Pečarić, J. Chebyshev-Steffensen Inequality Involving the Inner Product. Mathematics 2022, 10(1), 122; https://doi.org/10.3390/math10010122.

https://www.mdpi.com/2227-7390/10/1/122.

In this paper, the authors prove the Chebyshev–Steffensen inequality, integrating the inner product on the real m-space. Some upper bounds for the weighted Chebyshev-Steffensen functional, as well as the Jensen-Steffensen functional involving the inner product under various conditions are also given.

9. Andrić, M. Jensen-Type Inequalities for (h, g; m)-Convex Functions. Mathematics 2021, 9(24), 3312; https://doi.org/10.3390/math9243312.

https://www.mdpi.com/2227-7390/9/24/3312.

The author substantiates Jensen-type inequalities for the recently introduced class of (h,g;m)-convex functions, and indicates special results. These results generalize and extend the corresponding inequalities for the classes of convex functions that already exist in the literature.

10. Ullah, H.; Adil Khan, M.; Saeed, T. Determination of Bounds for the Jensen Gap and Its Applications. Mathematics 2021, 9(23), 3132; https://doi.org/10.3390/math9233132.

https://www.mdpi.com/2227-7390/9/23/3132.

The Jensen inequality is considered one of the most consequential inequalities, finding a variety of applications within various science fields. For this reason, the Jensen inequality has become one of the most discussed developmental inequalities in the current literature on mathematical inequalities. The main goal of this paper is to find some new bounds for the Jensen inequality using certain classes of doubly differentiable convex functions. The authors obtain the proposed bounds by using the power mean and the Hölder inequality, the notion of convexity, and the Jensen inequality for concave functions. The authors derive several inequalities for power and quasi-arithmetic means as an outcome of the main results. They also establish several improvements of the Hölder inequality and present some applications of the main results in information theory.

## 4. Acknowledgments to the Authors and Reviewers

As the Guest Editor of the Special Issue "Advances in Mathematical Inequalities and Applications", I am grateful to all authors who contributed. I would also like to thank all the reviewers for their careful work and valuable comments, which have helped improve the quality of the submitted papers.

## 5. Conclusions

The goal of this Special Issue was to present some new results in the field of mathematical inequalities and their applications. It is my hope that these selected research papers will be recognized by the international scientific community as interesting and significant, and that they may form the basis for further research in the field of mathematical inequalities and their applications.

**Conflicts of Interest:** The authors declare no conflict of interest.

## Reference

1. Bellman, R. Why Study Inequalities? In *General Inequalities 2. International Series of Numerical Mathematics/Internationale Schriftenreihe zur Numerischen Mathematik/Série Internationale d'Analyse Numérique*; Beckenbach, E.F., Ed.; Birkhäuser: Basel, Switzerland, 1980; Volume 47. [CrossRef]